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# Quantum security analysis of a lattice-based oblivious transfer protocol

**Key words:** Oblivious transfer; Post-quantum; Lattice-based; Learning with errors; Universally composable

Contact: Mo-meng Liu

E-mail: [liumomeng@gmail.com](mailto:liumomeng@gmail.com)

 ORCID: <http://orcid.org/0000-0002-8545-5551>

# Problems

- OTs built upon traditional number theoretic problems (discrete logarithm & factoring) cannot be secure against quantum attacks.
- OTs built upon post-quantum cryptography lack of quantum security proofs.

# Motivation & Positive Results

## Quantum security analysis of [PVW08]:

- **[PVW08] A Framework for Efficient and Composable Oblivious Transfer**  
*Chris Peikert, Vinod Vaikuntanathan, Brent Waters. CRYPTO'08*

## Positive Results:

- **[Unr10] Universally Composable Quantum Multi-Party Computation**  
*Dominique Unruh. CRYPTO10*
- **[HSS11] Classical Cryptographic Protocols in A Quantum World**  
*Sean Hallren, Adam Smith, Fang Song. CRYPTO11*
- **[Son14] A Note on Quantum Security for Post-Quantum Cryptography**  
*Fang Song. PQC2014*

# Lattice-based OT in [PVW08]

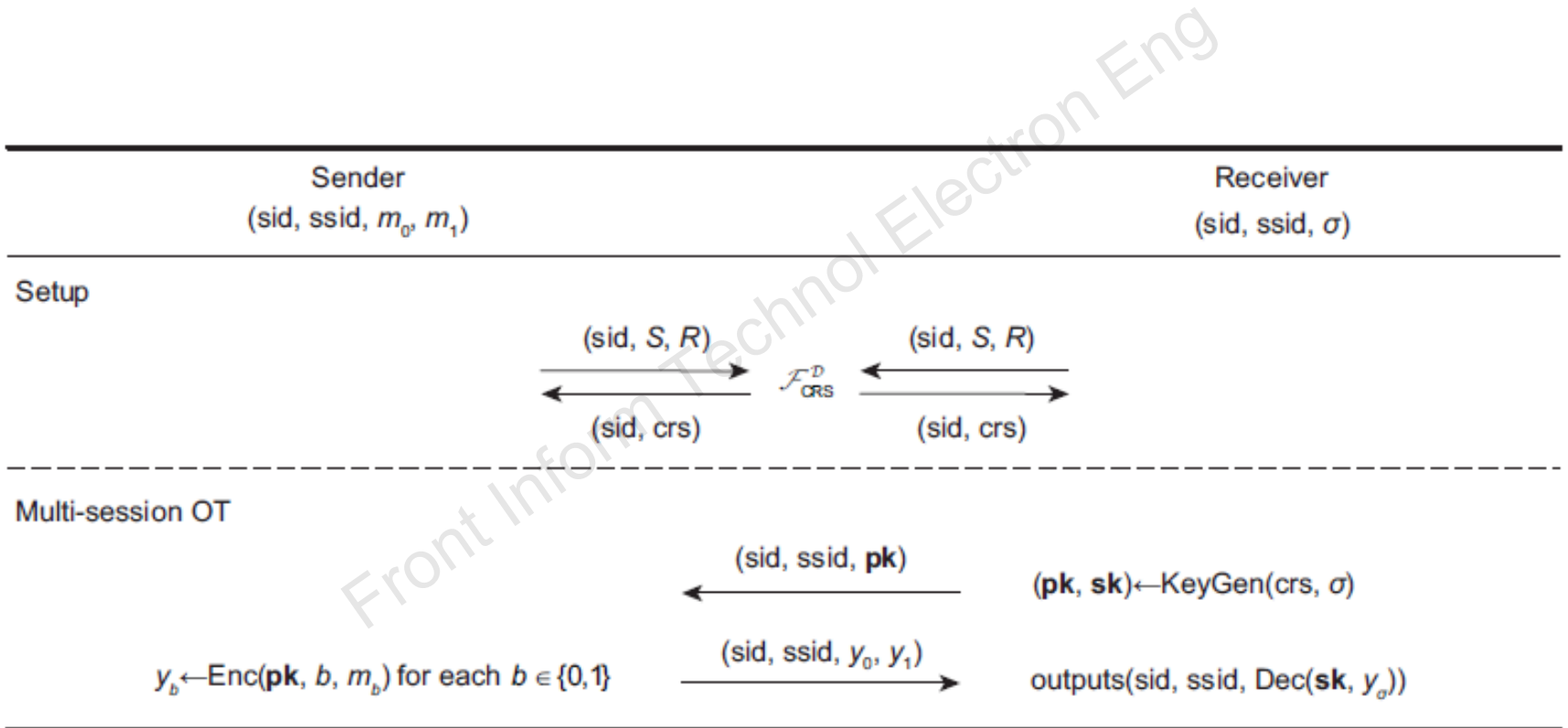


Fig. 1 Protocol  $\text{dm}^{\text{mode}}$  for oblivious transfer

# [Unr10]: Statistically Quantum Lifting Theorem

**Theorem 13 (Quantum lifting theorem).** *Let  $\pi$  and  $\rho$  be classical protocols. Assume that  $\pi$  statistically classical-UC-emulates  $\rho$ . Then  $\pi$  statistically quantum-UC-emulates  $\rho$ .*

$$\pi^c \cup \{\hat{Z}, \hat{Adv}\}$$

 $\approx^p$ 


$$\pi^c \cup \{Z, Adv\}$$

 $\approx^s$ 

$$\rho^c \cup \{Z, Sim\}$$

$$\rho^c \cup \{\hat{Z}, \hat{Sim}\}$$


 $\approx^p$ 
 $\hat{M}$ :QIM

 $M$ :ITM

# [Unr10]: Computationally Quantum Lifting Theorem

A directly computational analogue of Quantum Lifting Theorem 



QPPT machines:  $N \cup \{M\} \approx^p N \cup \{\hat{M}\}$ , then  $M$  is QPPT, denoted as  $M'$



**Computationally** Quantum Lifting Theorem (QPPT)

$$\pi^c \cup \{\hat{Z}, \hat{Adv}\}$$

$$\rho^c \cup \{\hat{Z}, \hat{Sim}\}$$

$\approx^p$



$\approx^p$



$\hat{M}$ :QIM  
 $M$ :ITM  
 $M'$ :QPPT

$$\pi^c \cup \{Z', Adv'\} \approx^c \rho^c \cup \{Z', Sim'\}$$

# Quantum Analysis on [PVW08]

In different corruption case: **Statistical** & **Computational**

**Statistical**: Use Statistically-QLT directly

**Messy mode:**

(1) Adv+R: **statistical security** for S

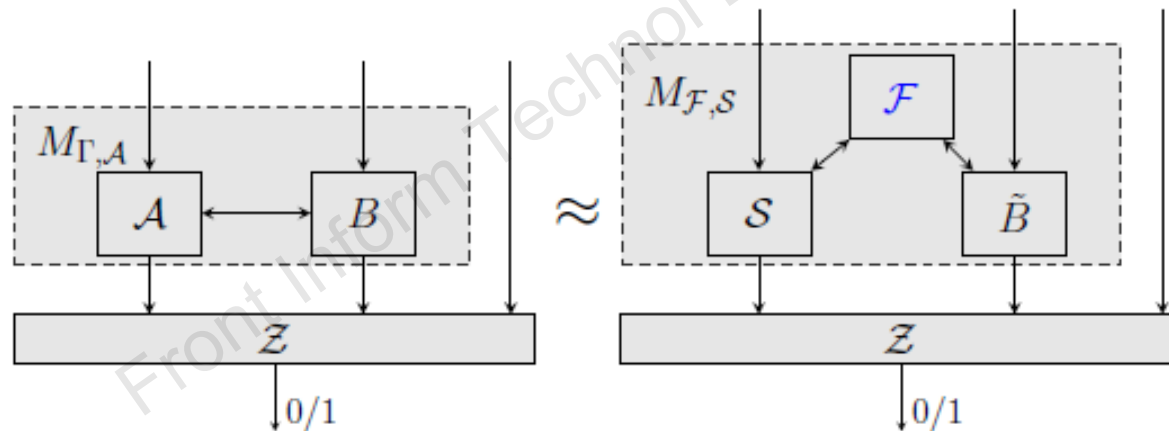
**Computational**: **Prove (computational)QPPT classical UC security?**

$$\pi^c \cup \{Z', Adv'\} \approx^c \rho^c \cup \{Z', Sim'\}$$

Lattice- based OT is quantum UC secure.

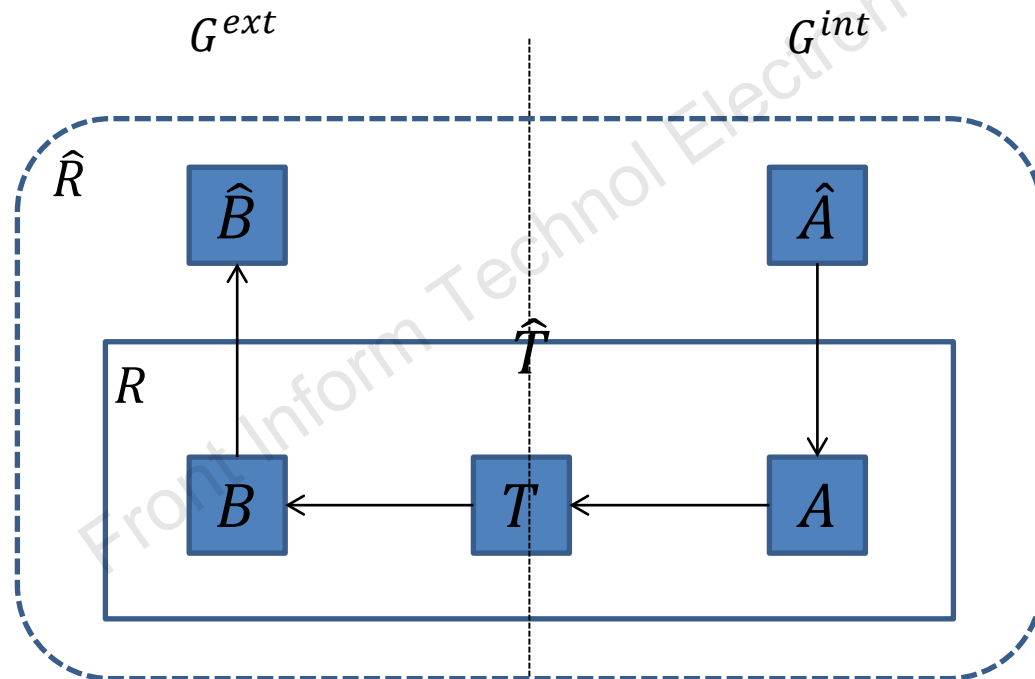
# A Support from [HSS11]

**Definition 4.4 (SHA).** Two machines  $M_0$  and  $M_\ell$  are related by a  $(t, \varepsilon)$ -SHA of length  $\ell$  if there is a sequence of intermediate machines  $M_1, M_2, \dots, M_{\ell-1}$  such that each adjacent pair  $M_{i-1}, M_i$  of machines,  $i = 1, \dots, \ell$ , is  $(t, \frac{\varepsilon}{\ell})$ -simply related.



# Quantum Security Analysis Framework [Son14]

- Game-preserving reduction:



# Quantum Security Analysis Framework [Son14]

Game-preserving reduction: (Class-respectful reduction)

$R$  is  $\beta$ - $(\hat{A}, \hat{B})$ -respectful ( $\hat{A}, \hat{B} \in Q$ )

1.  $(\beta, \hat{A})$ -extendable

2.  $(\hat{A}, \hat{B})$ -closed

(1)  $R$  is black-box

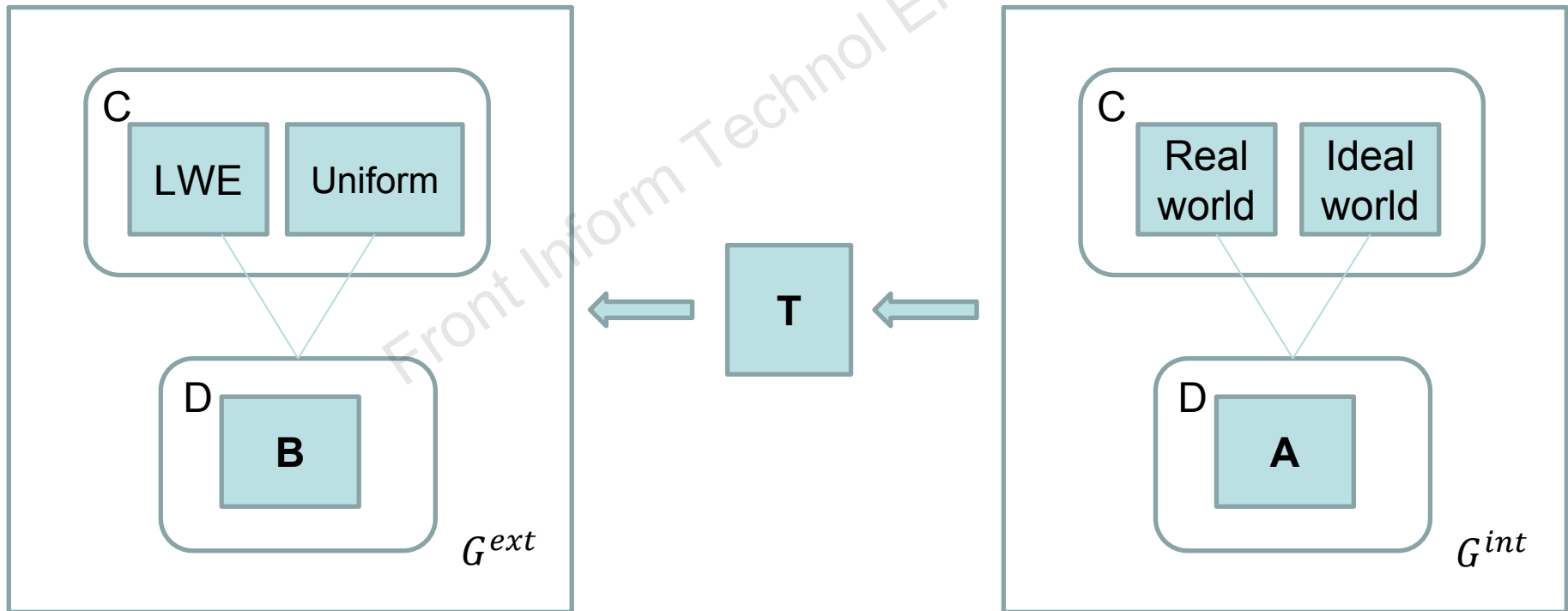
(2)  $R$  is straight line

(3)  $R$  is  $\hat{A}$ -compatible

(4)  $R$  is value-dominating

# Quantum Security Analysis Framework [Son14]

1. Define  $G^{int}$  : i.e. Distinguish (real world & ideal world)
2. Define  $G^{ext}$  : solve decisional LWE problem. i.e. Distinguish (A, b) & (A, b=As+e)



# Conclusion & Future work

- Apply three tools [Unr10,HSS11,Son14] to the lattice-based OT protocol [PVW08] to show a comprehensive quantum security analysis of this OT protocol.
- Explore the post-quantum OT protocols with quantum security proofs.