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Passive Source Localization Using Importance Sampling Based on TOA and FOA Measurements

Key words: Passive source localization; time of arrival (TOA); frequency of arrival (FOA); Monte Carlo importance sampling (MCIS); maximum likelihood (ML)

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Introduction

- Source localization has been widely applied in signal processing fields.
- Variable localization system has been researched in recent years and the corresponding localization algorithms are proposed.
- TOA and FOA localization system can achieve high accuracy when synchronization is achieved, but involves expensive calculations for nonlinearity and nonconvex.
- A Monte Carlo importance sampling (MCIS) algorithm, which achieve the global solution to the ML problem in a computationally efficient manner has been presented in this work.

TOA and FOA localization model

$$\begin{cases} z_i = \left\| \boldsymbol{x} - \boldsymbol{s}_i \right\| + e_i = d_i + e_i \\ \dot{z}_i = \dfrac{\left(\dot{\boldsymbol{x}} - \dot{\boldsymbol{s}}_i \right)^{\mathrm{T}} \left(\boldsymbol{x} - \boldsymbol{s}_i \right)}{\left\| \boldsymbol{x} - \boldsymbol{s}_i \right\|} + \dot{e}_i = \dot{d}_i + \dot{e}_i \end{cases}, \quad i = 1, \cdots, N,$$

ML estimation of position and velocity

$$\min_{\boldsymbol{\alpha}} \boldsymbol{f}\left(\boldsymbol{\theta}\right) = \tilde{\boldsymbol{e}}^{\mathrm{T}} \boldsymbol{Q}^{-1} \tilde{\boldsymbol{e}} = \left(\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{d}}\right)^{\mathrm{T}} \boldsymbol{Q}^{-1} \left(\tilde{\boldsymbol{z}} - \tilde{\boldsymbol{d}}\right),$$

Steps of MCIS method

- Compute the mean value and covariance matrix to construct the importance function.

$$q(\boldsymbol{\theta})=\frac{\exp\left\{-\lambda_1\left(\boldsymbol{\theta}-\overline{\boldsymbol{\theta}}\right)^{\mathrm{T}}\boldsymbol{R}^{-1}\left(\boldsymbol{\theta}-\overline{\boldsymbol{\theta}}\right)\right\}}{\left(2\pi\right)^{N/2}\left|\boldsymbol{R}/2\lambda_1\right|^{N/2}}.$$

- Conduct importance sampling to obtain M samples $\boldsymbol{\theta}_1,\boldsymbol{\theta}_2,\ldots,\boldsymbol{\theta}_M$.
- Compute importance weight and obtain the position and velocity.

$$\hat{\boldsymbol{\theta}}=\sum_{m=1}^{M}\boldsymbol{\theta}_m\tilde{w}\left(\boldsymbol{\theta}_m\right),\quad \tilde{w}\left(\boldsymbol{\theta}_m\right)=\frac{w\left(\boldsymbol{\theta}_m\right)}{\sum_{m=1}^{M}w\left(\boldsymbol{\theta}_m\right)}.$$

Measurement results (1)

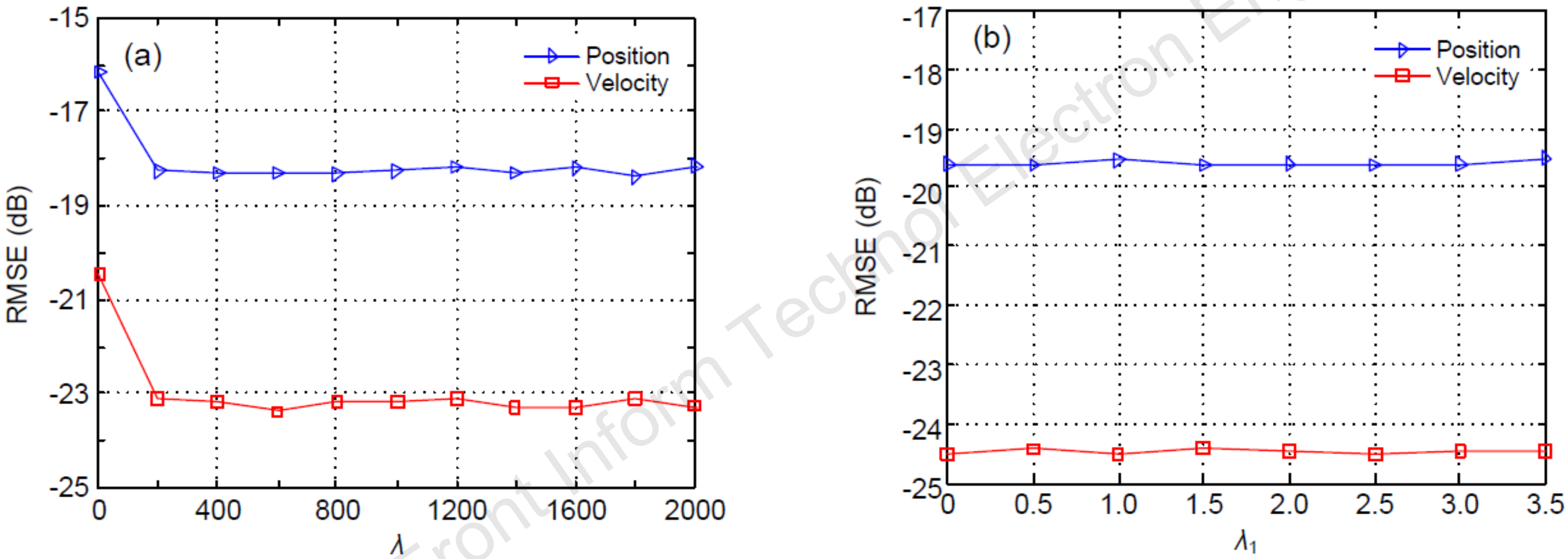

Fig. 1 Sensitivity of RMSEs to λ with λ_1=0.5 (a) and sensitivity of RMSEs to λ_1 with λ=2000 (b)

The passive source is located at (600, −50) with velocity (30, −15)

Measurement results (2)

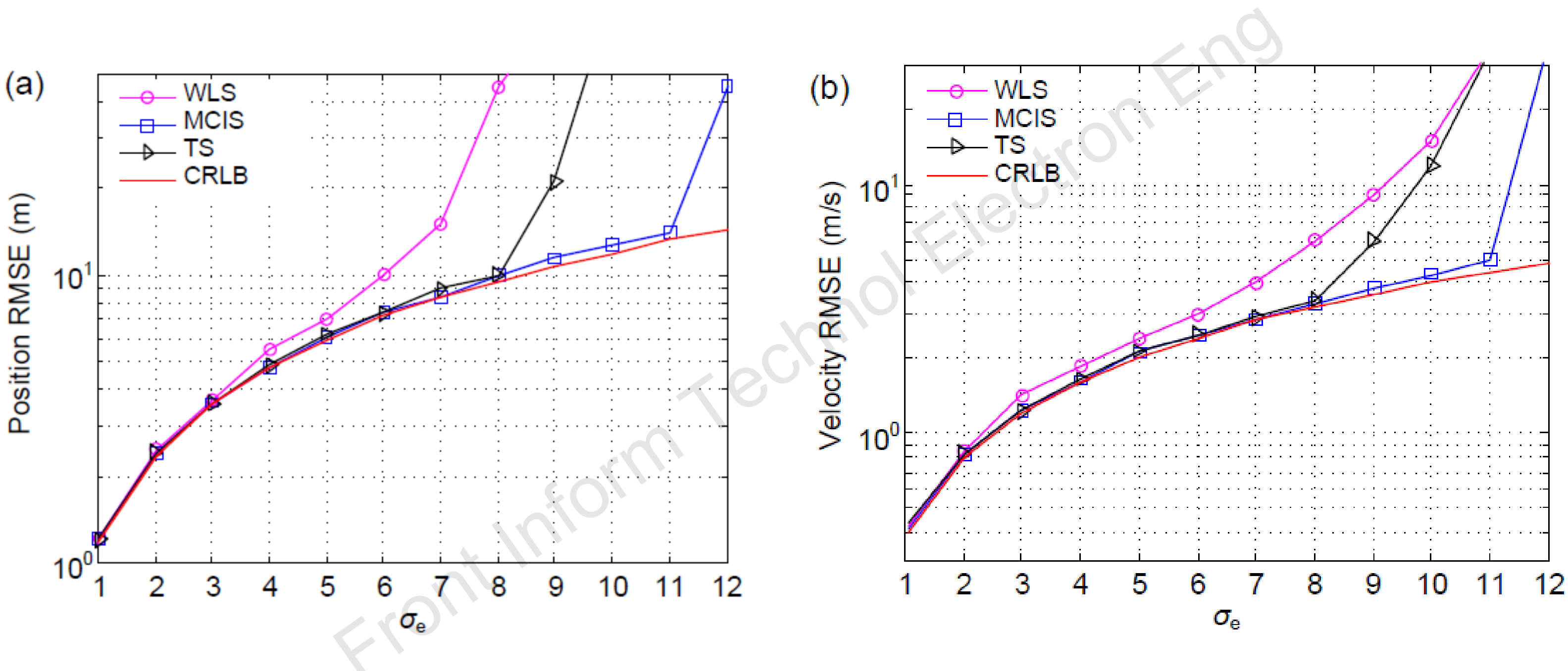

Fig. 2 **RMSEs of the localization methods compared with the CRLB using four sensors in the 2D scenario when the position and velocity of the source are fixed: (a) RMSEs of source position estimates; (b) RMSEs of source velocity estimates**

Measurement results (3)

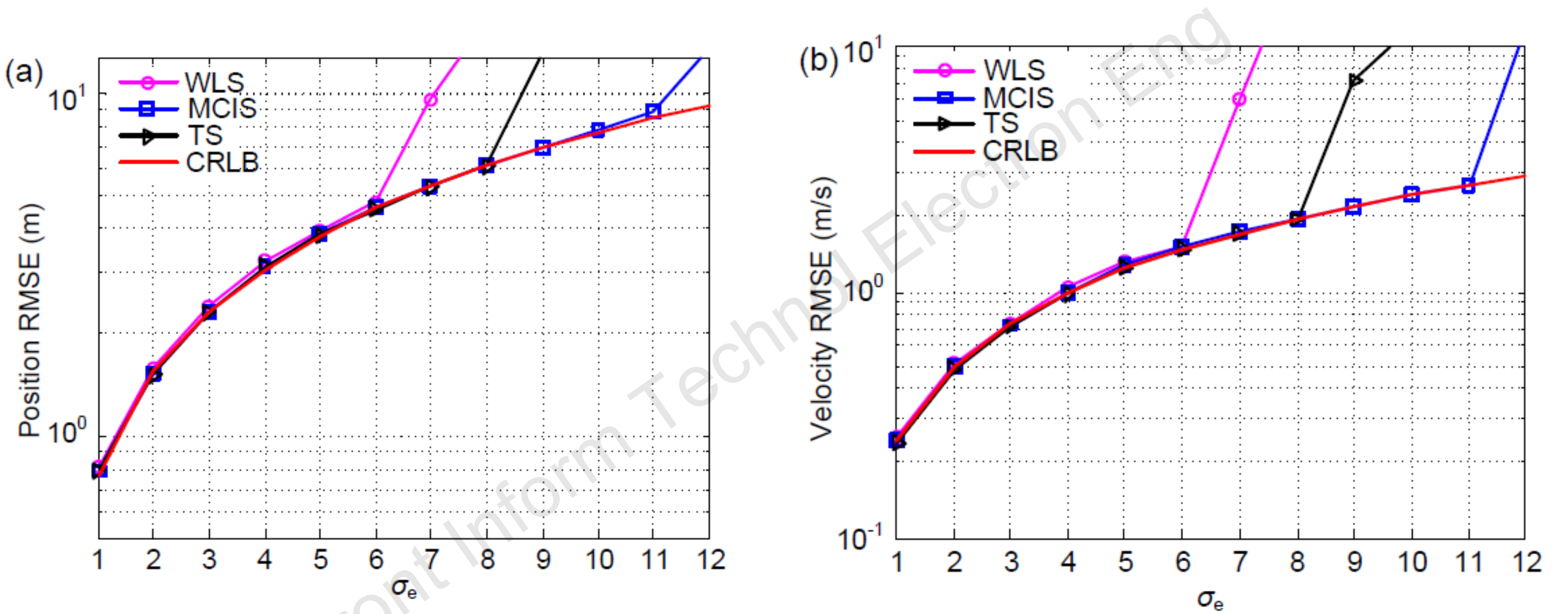

Fig.3. RMSEs of the localization methods compared with the CRLB using four sensors in the 2D scenario when the source is located at a determinate square: (a) RMSEs of source position estimates; (b) RMSEs of source velocity estimates

Measurement results (4)

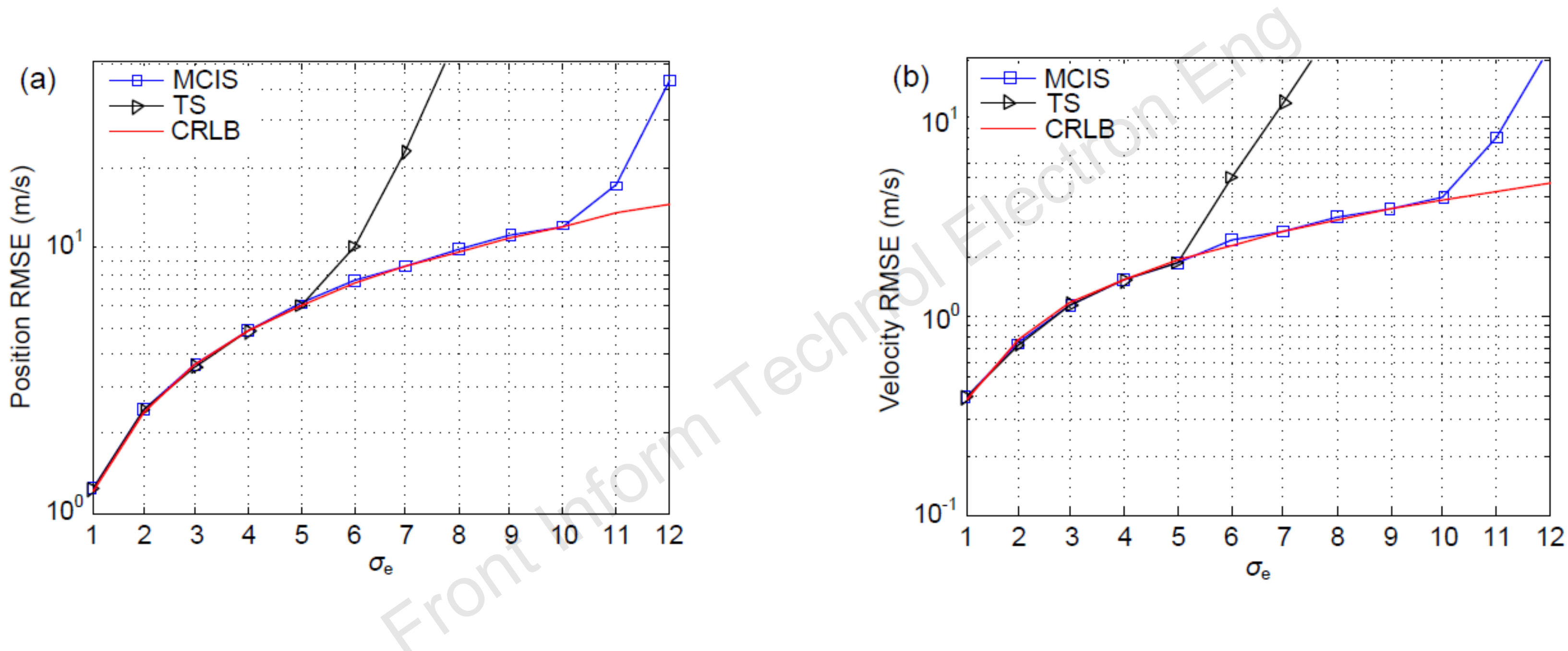

Fig. 4 RMSEs of the localization methods compared with the CRLB using four sensors located in a line in the 2D scenario: (a) RMSEs of source position estimates; (b) RMSEs of source velocity estimates

Measurement results (5)

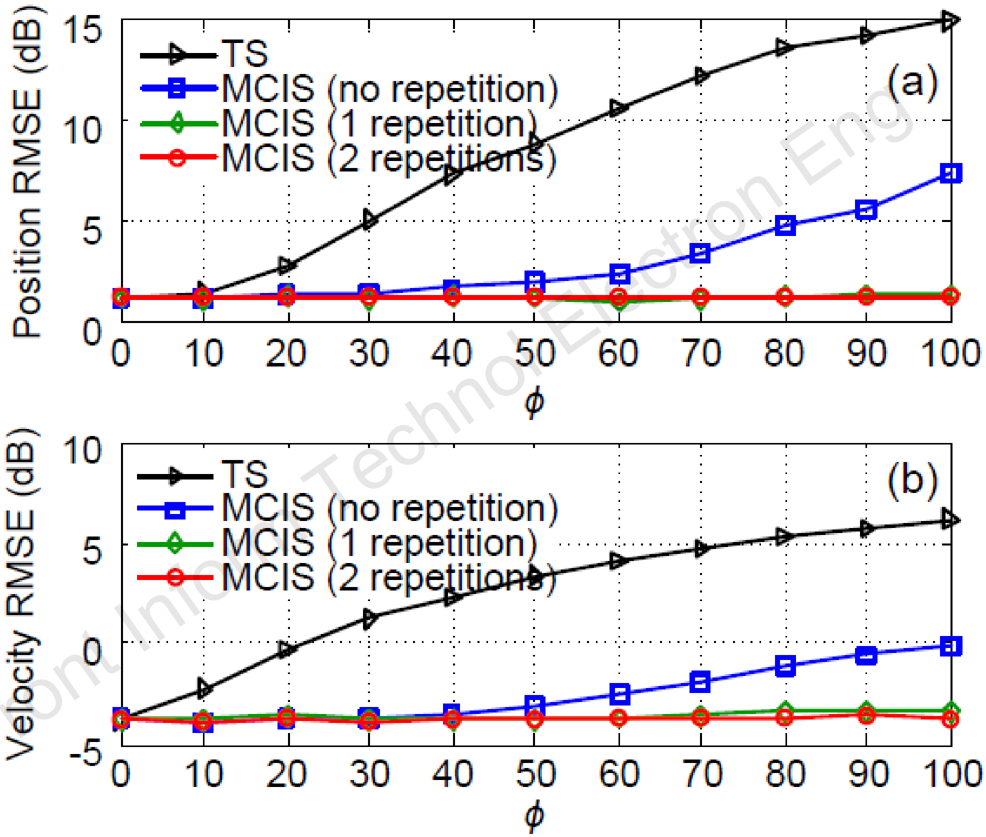

Fig.5. Sensitivities of the estimation methods to the initial estimate errors in the 2D scenario when the position and velocity of the source are fixed: (a) RMSEs of the position estimates; (b) RMSEs of the velocity estimates

Measurement results (6)

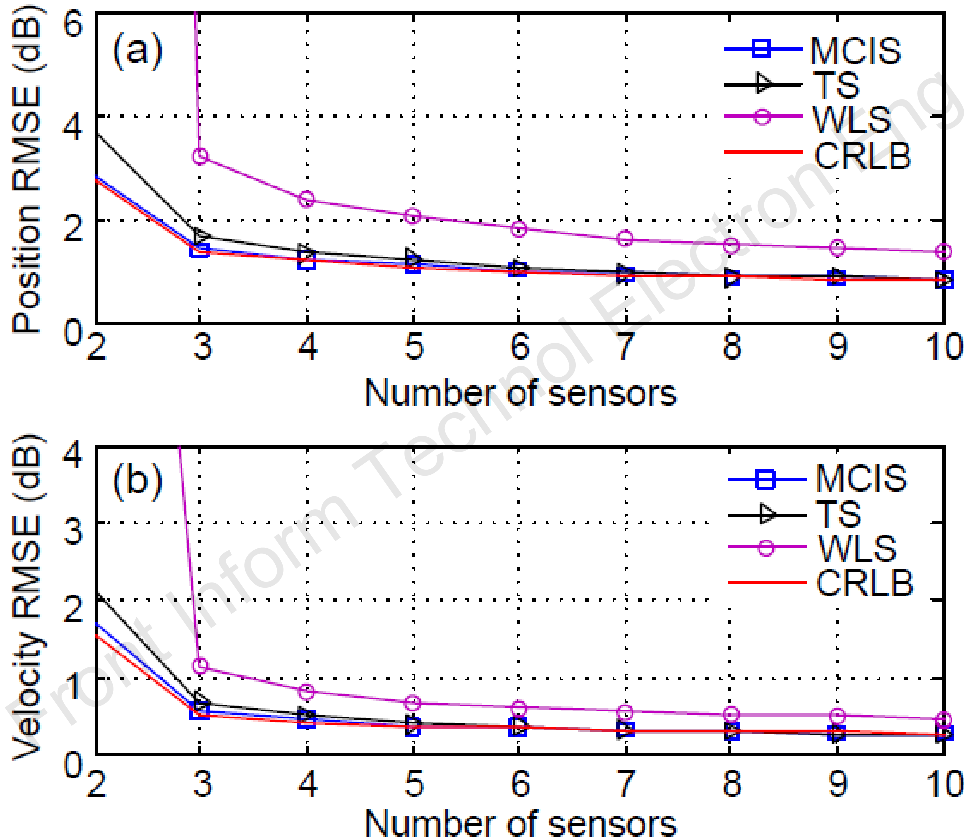

Fig.6. Sensitivities of the estimation methods to the number of sensors used in the 2D scenario when the position and velocity of the source are fixed: (a) RMSEs of the position estimates; (b) RMSEs of the velocity estimates

Complexity comparison

Table 1 Complexity assessment of the existing algorithms

Algorithm	Complexity	CR	RT (s)
MCIS	$O(20NK+10K^2+2MK+MN^2)$	1.0000	1.012
WLS	$O(20NK+2K^2+N^2)$	0.1916	0.656
TS	$O((20NK+4K^2+N^2)N_{\mathrm{Itr}})$	4.0839	1.690
GS	$O((2NK+4K^2)N_{\mathrm{grid}}^{2K})$	6559.0000	53.631

TS: Taylor series; GS: grid search. CR: complexity ratio; RT: running time. N: number of sensors; K: dimensionality of the estimation; M: number of sampling points; N_{Itr}: number of iterations in the iterative method; N_{grid}: number of grids used in the grid search method

Conclusions

- The proposed method can achieve high-accuracy and low-complexity based on TOA and FOA measurements.
- Global optimum of ML estimate can be obtained based on Pincus theorem.
- Importance sampling method significantly decreases the computational cost and outperforms the existing method with fewer available sensors.