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Supplementary materials for

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1 Proof of Theorem 1

Before the formal derivation process of Eq. (18), we first prove a mathematical conclusion required for the subsequent part.

Lemma S1 For deterministic matrices $\boldsymbol{A}, \boldsymbol{B} \in \mathbb{C}^{M\times M}$ and $\boldsymbol{W} \in \mathbb{C}^{N\times N}$, while $\boldsymbol{A}$ and $\boldsymbol{B}$ are both unitary matrices, there is

$$\mathbb{E}\{\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{A}\widetilde{\boldsymbol{H}}_2\boldsymbol{W}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{B}\widetilde{\boldsymbol{H}}_2\} = \mathrm{Tr}\{\boldsymbol{W}\}\mathrm{Tr}\{\boldsymbol{A}\boldsymbol{B}\}\boldsymbol{I}_N + \mathrm{Tr}\{\boldsymbol{A}\}\mathrm{Tr}\{\boldsymbol{B}\}\boldsymbol{W}. \tag{S1}$$

Proof We define $\widetilde{\boldsymbol{H}}_2 = [\boldsymbol{J}_1, \boldsymbol{J}_2, \ldots, \boldsymbol{J}_N]$, where $\boldsymbol{J}_n \in \mathbb{C}^{M\times 1}$, $1 \le n \le N$, and each component is independent and satisfies $\boldsymbol{J}_n \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_M)$. Therefore, the $(i,j)^{\mathrm{th}}$ element in $\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{A}\widetilde{\boldsymbol{H}}_2\boldsymbol{W}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{B}\widetilde{\boldsymbol{H}}_2$ can be written as

$$\left[\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{A}\widetilde{\boldsymbol{H}}_2\boldsymbol{W}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{B}\widetilde{\boldsymbol{H}}_2\right]_{i,j} = \sum_{h=1}^{N}\sum_{m=1}^{N}\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{J}_m w_{mh}\boldsymbol{J}_h^{\mathrm{H}}\boldsymbol{B}\boldsymbol{J}_j, \tag{S2}$$

where w_{mh} is the $(m,h)^{\mathrm{th}}$ element of $\boldsymbol{W}$. Since $\mathbb{E}\{\boldsymbol{J}_k^{\mathrm{H}}\boldsymbol{A}\boldsymbol{J}_k\boldsymbol{J}_k^{\mathrm{H}}\boldsymbol{B}\boldsymbol{J}_k\} = \mathrm{Tr}\{\boldsymbol{A}\boldsymbol{B}\} + \mathrm{Tr}\{\boldsymbol{A}\}\mathrm{Tr}\{\boldsymbol{B}\}$, the expectation of diagonal elements in $\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{A}\widetilde{\boldsymbol{H}}_2\boldsymbol{W}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{B}\widetilde{\boldsymbol{H}}_2$ can be calculated as follows:

$$\begin{aligned}
\left[\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{A}\widetilde{\boldsymbol{H}}_2\boldsymbol{W}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{B}\widetilde{\boldsymbol{H}}_2\right]_{i,i} =& \mathbb{E}\{\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{J}_i w_{ii}\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{B}\boldsymbol{J}_i\} + \mathbb{E}\left\{\sum_{m=1,m\ne i}^{N}\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{J}_m w_{mm}\boldsymbol{J}_m^{\mathrm{H}}\boldsymbol{B}\boldsymbol{J}_i\right\} \\
=& w_{ii}\mathbb{E}\{\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{J}_i\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{B}\boldsymbol{J}_i\} + \mathbb{E}\left\{\sum_{m=1,m\ne i}^{N} w_{mm}\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{A}\mathbb{E}\{\boldsymbol{J}_m\boldsymbol{J}_m^{\mathrm{H}}\}\boldsymbol{B}\boldsymbol{J}_i\right\} \\
=& w_{ii}(\mathrm{Tr}\{\boldsymbol{A}\boldsymbol{B}\} + \mathrm{Tr}\{\boldsymbol{A}\}\mathrm{Tr}\{\boldsymbol{B}\}) + \sum_{m=1,m\ne i}^{N} w_{mm}\mathrm{Tr}\{\boldsymbol{A}\boldsymbol{B}\} \\
=& w_{ii}\,\mathrm{Tr}\{\boldsymbol{A}\}\mathrm{Tr}\{\boldsymbol{B}\} + \mathrm{Tr}\{\boldsymbol{A}\boldsymbol{B}\}\mathrm{Tr}\{\boldsymbol{W}\}.
\end{aligned} \tag{S3}$$

The expectation of non-diagonal elements in $\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{A}\widetilde{\boldsymbol{H}}_2\boldsymbol{W}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{B}\widetilde{\boldsymbol{H}}_2$ can be derived through the following process:

$$\begin{aligned}
\left[\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{A}\widetilde{\boldsymbol{H}}_2\boldsymbol{W}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\boldsymbol{B}\widetilde{\boldsymbol{H}}_2\right]_{i,j} &= \mathbb{E}\{\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{J}_i w_{ij}\boldsymbol{J}_j^{\mathrm{H}}\boldsymbol{B}\boldsymbol{J}_j\} \\
&= w_{ij}\mathbb{E}\{\boldsymbol{J}_i^{\mathrm{H}}\boldsymbol{A}\boldsymbol{J}_i\}\mathbb{E}\{\boldsymbol{J}_j^{\mathrm{H}}\boldsymbol{B}\boldsymbol{J}_j\} \\
&= w_{ij}\mathrm{Tr}\{\boldsymbol{A}\}\mathrm{Tr}\{\boldsymbol{B}\}.
\end{aligned} \tag{S4}$$

Therefore, by combining the above two equations, the conclusion in Lemma S1 can be obtained.

1.1 Derivation of the noise term

As $\boldsymbol{q}_k$ can be written in the form of Eq. (16), we divide it into four components:

$$\begin{cases} \boldsymbol{q}_k^1 = \sqrt{c_k\delta\varepsilon_k}\left(\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{\frac{1}{2}}\bar{\boldsymbol{h}}_k\right), & \boldsymbol{q}_k^2 = \sqrt{c_k\delta}\left(\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{\frac{1}{2}}\tilde{\boldsymbol{h}}_k\right), \\ \boldsymbol{q}_k^3 = \sqrt{c_k\varepsilon_k}\left(\widetilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{\frac{1}{2}}\boldsymbol{\Phi}\boldsymbol{D}_k^{\frac{1}{2}}\bar{\boldsymbol{h}}_k\right), & \boldsymbol{q}_k^4 = \sqrt{c_k}\left(\widetilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{\frac{1}{2}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{\frac{1}{2}}\tilde{\boldsymbol{h}}_k\right), \end{cases} \tag{S5}$$

where $\boldsymbol{q}_k = \boldsymbol{q}_k^1 + \boldsymbol{q}_k^2 + \boldsymbol{q}_k^3 + \boldsymbol{q}_k^4$. The expansion of $E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi})$ can be written as

$$E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi}) = \mathbb{E}\|\boldsymbol{q}_k\|^2 = \mathbb{E}\left\{\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{q}_k\right\} = \sum_{\omega=1}^{4}\sum_{\psi=1}^{4}\mathbb{E}\left\{(\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right\}. \tag{S6}$$

Due to the assumption that the components of $\tilde{\boldsymbol{h}}_k$ and $\widetilde{\boldsymbol{H}}_2$ are i.i.d. complex Gaussian random variables with zero mean and unit variance, $E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi})$ can be simplified as follows:

$$E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi}) = \sum_{\omega=1}^{4}\mathbb{E}\left\{(\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\omega}\right\}, \tag{S7}$$

where

$$\begin{aligned} \mathbb{E}\left\{\left(\boldsymbol{q}_k^1\right)^{\mathrm{H}}\boldsymbol{q}_k^1\right\} &= c_k\delta\varepsilon_k\mathbb{E}\left\{\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\} \\ &= c_k\delta\varepsilon_k M\left(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{a}_N\boldsymbol{a}_N^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right) \\ &= c_k\delta\varepsilon_k M|f_k(\boldsymbol{\Phi})|^2, \end{aligned} \tag{S8}$$

$$\begin{aligned} \mathbb{E}\left\{\left(\boldsymbol{q}_k^2\right)^{\mathrm{H}}\boldsymbol{q}_k^2\right\} &= c_k\delta\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\} \\ &= c_k\delta M\mathrm{Tr}\left(\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\right) \\ &= c_k\delta M f_{k,1,1}(\boldsymbol{\Phi}), \end{aligned} \tag{S9}$$

$$\begin{aligned} \mathbb{E}\left\{\left(\boldsymbol{q}_k^3\right)^{\mathrm{H}}\boldsymbol{q}_k^3\right\} &= c_k\varepsilon_k\mathbb{E}\left\{\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\widetilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\} \\ &= c_k\varepsilon_k M\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k \\ &= c_k\varepsilon_k M f_{kk,2}(\boldsymbol{\Phi}), \end{aligned} \tag{S10}$$

$$\begin{aligned} \mathbb{E}\left\{\left(\boldsymbol{q}_k^4\right)^{\mathrm{H}}\boldsymbol{q}_k^4\right\} &= c_k\,\mathrm{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\widetilde{\boldsymbol{H}}_2^{\mathrm{H}}\widetilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\} \\ &= c_k M\mathrm{Tr}\left(\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\right) \\ &= c_k M f_{k,3,1}(\boldsymbol{\Phi}). \end{aligned} \tag{S11}$$

The values of $f_k(\boldsymbol{\Phi})$, $f_{k,1,1}(\boldsymbol{\Phi})$, $f_{kk,2}(\boldsymbol{\Phi})$, and $f_{k,3,1}(\boldsymbol{\Phi})$ are given in Eq. (22). In summary, the result of the noise term $E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi})$ is

$$E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi}) = c_k\left\{\delta\varepsilon_k M|f_k(\boldsymbol{\Phi})|^2 + \delta M f_{k,1,1}(\boldsymbol{\Phi}) + \varepsilon_k M f_{kk,2}(\boldsymbol{\Phi}) + M f_{k,3,1}(\boldsymbol{\Phi})\right\}. \tag{S12}$$

1.2 Derivation of the signal term

Due to $E_{\mathrm{VR},k}^{\mathrm{signal}}(\boldsymbol{\Phi}) = \mathbb{E}\|\boldsymbol{q}_k\|^4 = \mathbb{E}\left\{\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{q}_k\right\}$, we have

$$\begin{aligned}E_{\mathrm{VR},k}^{\mathrm{signal}}(\boldsymbol{\Phi}) &= \sum_{\omega_1,\psi_1}^{4}\sum_{\omega_2,\psi_2}^{4} \mathrm{E}\left\{\left((\boldsymbol{q}_k^{\omega_1})^{\mathrm{H}}\boldsymbol{q}_k^{\psi_1}\right)\left((\boldsymbol{q}_k^{\omega_2})^{\mathrm{H}}\boldsymbol{q}_k^{\psi_2}\right)^{\mathrm{H}}\right\} \\ &= \sum_{\omega=1}^{4}\sum_{\psi=1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)^{\mathrm{H}}\right\} + 2\sum_{\omega_1=1}^{4}\sum_{\psi_1=\omega_1+1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega_1})^{\mathrm{H}}\boldsymbol{q}_k^{\omega_1}\right)\left((\boldsymbol{q}_k^{\psi_1})^{\mathrm{H}}\boldsymbol{q}_k^{\psi_1}\right)^{\mathrm{H}}\right\} \\ &\quad + 2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^{1})^{\mathrm{H}}\boldsymbol{q}_k^{2}\right)\left((\boldsymbol{q}_k^{3})^{\mathrm{H}}\boldsymbol{q}_k^{4}\right)^{\mathrm{H}}\right\}\right\} + 2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^{1})^{\mathrm{H}}\boldsymbol{q}_k^{3}\right)\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_k^{4}\right)^{\mathrm{H}}\right\}\right\} \\ &\quad + 2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_k^{1}\right)\left((\boldsymbol{q}_k^{4})^{\mathrm{H}}\boldsymbol{q}_k^{3}\right)^{\mathrm{H}}\right\}\right\} + 2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_k^{4}\right)\left((\boldsymbol{q}_k^{1})^{\mathrm{H}}\boldsymbol{q}_k^{3}\right)^{\mathrm{H}}\right\}\right\}.\end{aligned} \tag{S13}$$

Then we classify different values of ω_1, ψ_1, ω, ψ, and first calculate the result of

$$\sum_{\omega=1}^{4}\sum_{\psi=1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)^{\mathrm{H}}\right\}.$$

The formula can be further simplified into the following form:

$$\begin{aligned}&\sum_{\omega=1}^{4}\sum_{\psi=1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)^{\mathrm{H}}\right\} \\ &= \sum_{\omega=1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\omega}\right)\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\omega}\right)^{\mathrm{H}}\right\} + 2\sum_{\omega=1}^{4}\sum_{\psi=\omega+1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\psi}\right)^{\mathrm{H}}\right\}.\end{aligned} \tag{S14}$$

The specific calculation of the first part $\sum_{\omega=1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\omega}\right)\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_k^{\omega}\right)^{\mathrm{H}}\right\}$ is as follows. When $\omega = 1$, we have

$$\begin{aligned}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{1})^{\mathrm{H}}\boldsymbol{q}_k^{1}\right)\left((\boldsymbol{q}_k^{1})^{\mathrm{H}}\boldsymbol{q}_k^{1}\right)^{\mathrm{H}}\right\} &= (c_k\delta\varepsilon_k)^2\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right|^2\right\} \\ &= (c_k\delta\varepsilon_k)^2\mathbb{E}\left\{\left|\left(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{a}_N\right)\boldsymbol{a}_M^{\mathrm{H}}\boldsymbol{a}_M(\boldsymbol{a}_N^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)\right|^2\right\} \\ &= M^2(c_k\delta\varepsilon_k)^2|f_k(\boldsymbol{\Phi})|^4.\end{aligned} \tag{S15}$$

The value of $f_k(\boldsymbol{\Phi})$ is given in Eq. (22). When $\omega = 2$, we have

$$\begin{aligned}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_k^{2}\right)\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_k^{2}\right)^{\mathrm{H}}\right\} =&(c_k\delta)^2\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right|^2\right\} \\ =&(c_k\delta)^2\left\{\mathrm{Tr}\left(\left(\boldsymbol{R}_{\mathrm{VR,k}}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR,k}}^{1/2}\right)^2\right)\right. \\ &\left.+\left|\mathrm{Tr}(\boldsymbol{R}_{\mathrm{VR,k}}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR,k}}^{1/2})\right|^2\right\} \\ =&(c_k\delta)^2\left(f_{kk,1,2}(\boldsymbol{\Phi}) + |f_{k,1,1}(\boldsymbol{\Phi})|^2\right).\end{aligned} \tag{S16}$$

The values of $f_{kk,1,2}(\boldsymbol{\Phi})$ and $f_{k,1,1}(\boldsymbol{\Phi})$ are given in Eq. (22). When $\omega=3$, we have

$$\begin{aligned}\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_k^3\right)\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_k^3\right)^{\mathrm{H}}\right\} =&(c_k\varepsilon_k)^2\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right|^2\right\}\\ =&(c_k\varepsilon_k)^2\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\left(M\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\boldsymbol{I}_N\right.\\ &\left.+M^2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\right)\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\ =&(c_k\varepsilon_k)^2M(M+1)\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right|^2\\ =&(c_k\varepsilon_k)^2M(M+1)|f_{kk,2}(\boldsymbol{\Phi})|^2.\end{aligned}\tag{S17}$$

This derivation uses Eq. (30), where $\boldsymbol{A}=\boldsymbol{B}=\boldsymbol{I}_N$ and

$$\boldsymbol{W}=\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}.$$

The value of $f_{kk,2}(\boldsymbol{\Phi})$ is given in Eq. (22). When $\omega=4$, we have

$$\begin{aligned}\mathbb{E}\left\{\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_k^4\right)\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_k^4\right)^{\mathrm{H}}\right\} =&c_k^2\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right|^2\right\}\\ =&c_k^2\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\left(M\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\boldsymbol{I}_N\right.\right.\\ &\left.\left.+M^2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{h}_k^1\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\right)\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\ =&c_k^2M(M+1)\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\ =&c_k^2M(M+1)\left\{\mathrm{Tr}\left(\left(\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\right)^2\right)+\left|\mathrm{Tr}(\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k})\right|^2\right\}\\ =&c_k^2M(M+1)\left(f_{kk,3,2}(\boldsymbol{\Phi})+|f_{k,3,1}(\boldsymbol{\Phi})|^2\right).\end{aligned}\tag{S18}$$

The values of $f_{kk,3,2}(\boldsymbol{\Phi})$ and $f_{k,3,1}(\boldsymbol{\Phi})$ are given in Eq. (22). Then we calculate the result of the second part $2\sum_{\omega_1=1}^{4}\sum_{\psi_1=\omega_1+1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega_1})^{\mathrm{H}}\boldsymbol{q}_k^{\omega_1}\right)\left((\boldsymbol{q}_k^{\psi_1})^{\mathrm{H}}\boldsymbol{q}_k^{\psi_1}\right)^{\mathrm{H}}\right\}$. First, we consider the terms with $\omega=1$. When $\psi=2$, we have

$$\begin{aligned}2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_k^2\right)\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_k^2\right)^{\mathrm{H}}\right\}&=2(c_k\delta)^2\varepsilon_k\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right|^2\right\}\\ &=2(c_k\delta)^2\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\ &=2(c_k\delta)^2\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\ &=2M(c_k\delta)^2\varepsilon_k|f_k(\boldsymbol{\Phi})|^2f_{k,1,1}(\boldsymbol{\Phi}).\end{aligned}\tag{S19}$$

When $\psi=3$, we have

$$\begin{aligned}&2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_k^3\right)\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_k^3\right)^{\mathrm{H}}\right\}\\ =&2(c_k\varepsilon_k)^2\delta\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right|^2\right\}\\ =&2(c_k\varepsilon_k)^2\delta\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\mathbb{E}\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k^{\mathrm{H}}\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\ =&2(c_k\varepsilon_k)^2\delta\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\\ =&2(c_k\varepsilon_k)^2\delta(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)\\ =&2M(c_k\varepsilon_k)^2\delta|f_k(\boldsymbol{\Phi})|^2f_{kk,2}(\boldsymbol{\Phi}).\end{aligned}\tag{S20}$$

When $\psi = 4$, we have

$$\begin{aligned}
&2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_k^4\right)\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_k^4\right)^{\mathrm{H}}\right\}\\
=&2c_k^2\delta\varepsilon_k\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right|^2\right\}\\
=&2c_k^2\delta\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\mathbb{E}\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&2c_k^2\delta\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\mathbb{E}\{\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\}\\
=&2c_k^2\delta\varepsilon_k(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)\mathbb{E}\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\}\\
=&2Mc_k^2\delta\varepsilon_k|f_k(\boldsymbol{\Phi})|^2f_{k,3,1}(\boldsymbol{\Phi}).
\end{aligned} \tag{S21}$$

Second, we consider the terms with $\omega = 2$. When $\psi = 3$, we have

$$\begin{aligned}
&2\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_k^3\right)\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_k^3\right)^{\mathrm{H}}\right\}\\
=&2c_k^2\delta\varepsilon_k\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right|^2\right\}\\
=&2c_k^2\delta\varepsilon_k\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\mathbb{E}\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta\varepsilon_k\mathbb{E}\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\}\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k^{\mathrm{H}}\bar{\boldsymbol{h}}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\\
=&2c_k^2\delta\varepsilon_k\mathrm{Tr}(\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}})(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)\\
=&2c_k^2\delta\varepsilon_kf_{k,1,1}(\boldsymbol{\Phi})f_{kk,2}(\boldsymbol{\Phi}).
\end{aligned} \tag{S22}$$

When $\psi = 4$, we have

$$\begin{aligned}
&2\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_k^4\right)\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_k^4\right)^{\mathrm{H}}\right\}\\
=&2c_k^2\delta\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right|^2\right\}\\
=&2c_k^2\delta\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\mathbb{E}\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\overline{\boldsymbol{H}}_2^{\mathrm{H}}\overline{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\right\}\\
=&2c_k^2\delta\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta\left\{\mathrm{Tr}\big(\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\big)\mathrm{Tr}\big(\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\big)\right.\\
&\left.+\mathrm{Tr}\big(\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\big)\right\}\\
=&2c_k^2\delta\left(f_{k,1,1}(\boldsymbol{\Phi})f_{k,3,1}(\boldsymbol{\Phi})+f_{kk,5}(\boldsymbol{\Phi})\right).
\end{aligned} \tag{S23}$$

The value of $f_{kk,5}(\boldsymbol{\Phi})$ is given in Eq. (22). Third, we consider the terms with $\omega = 3$. When $\psi = 4$, we have

$$\begin{aligned}
&2\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^4\right)\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^4\right)^{\rm H}\right\}\\
=&2c_k^2\varepsilon_k\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right|^2\right\}\\
=&2c_k^2\varepsilon_k\mathbb{E}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_{\rm ris}\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\varepsilon_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\left(M{\rm Tr}\big(\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\big)\boldsymbol{I}_N+M^2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\right)\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&2c_k^2\varepsilon_k\left(M{\rm Tr}\big(\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\big)\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k+M^2\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right)\\
=&2c_k^2\varepsilon_kM\left(f_{kk,2}(\boldsymbol{\Phi})f_{k,3,1}(\boldsymbol{\Phi})+Mf_{kk,4}(\boldsymbol{\Phi})\right).
\end{aligned}\tag{S24}$$

The value of $f_{kk,4}(\boldsymbol{\Phi})$ is given in Eq. (22). Next, we calculate the result of

$$2\sum_{\omega_1=1}^{4}\sum_{\psi_1=\omega_1+1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega_1})^{\rm H}\boldsymbol{q}_k^{\omega_1}\right)\left((\boldsymbol{q}_k^{\psi_1})^{\rm H}\boldsymbol{q}_k^{\psi_1}\right)^{\rm H}\right\}$$

in Eq. (S13).

First, we consider the terms with $\omega_1=1$. When $\psi_1=2$, we have

$$2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^1\right)\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^2\right)^{\rm H}\right\}=2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^1\right)\right\}\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^2\right)\right\}=2M(c_k\delta)^2\varepsilon_k|f_k(\boldsymbol{\Phi})|^2f_{k,1,1}(\boldsymbol{\Phi}).\tag{S25}$$

When $\psi_1=3$, we have

$$2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^1\right)\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^3\right)^{\rm H}\right\}=2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^1\right)\right\}\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^3\right)\right\}=2M^2(c_k\varepsilon_k)^2\delta|f_k(\boldsymbol{\Phi})|^2f_{kk,2}(\boldsymbol{\Phi}).\tag{S26}$$

When $\psi_1=4$, we have

$$2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^1\right)\left((\boldsymbol{q}_k^4)^{\rm H}\boldsymbol{q}_k^4\right)^{\rm H}\right\}=2\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^1\right)\right\}\mathbb{E}\left\{\left((\boldsymbol{q}_k^4)^{\rm H}\boldsymbol{q}_k^4\right)\right\}=2M^2c_k^2\delta\varepsilon_k|f_k(\boldsymbol{\Phi})|^2f_{k,3,1}(\boldsymbol{\Phi}).\tag{S27}$$

Second, we consider the terms with $\omega_1=2$. When $\psi_1=3$, we have

$$2\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^2\right)\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^3\right)^{\rm H}\right\}=2\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^2\right)\right\}\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^3\right)\right\}=2Mc_k^2\delta\varepsilon_kf_{k,1,1}(\boldsymbol{\Phi})f_{kk,2}(\boldsymbol{\Phi}).\tag{S28}$$

When $\psi_1=4$, we have

$$\begin{aligned}
&2\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^2\right)\left((\boldsymbol{q}_k^4)^{\rm H}\boldsymbol{q}_k^4\right)^{\rm H}\right\}\\
=&2c_k^2\delta\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\}\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta M\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta M\left\{{\rm Tr}\big(\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\big){\rm Tr}\big(\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\big)\right.\\
&\left.+{\rm Tr}\big(\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\big)\right\}\\
=&2c_k^2\delta M\left(f_{k,1,1}(\boldsymbol{\Phi})f_{k,3,1}(\boldsymbol{\Phi})+f_{kk,5}(\boldsymbol{\Phi})\right).
\end{aligned}\tag{S29}$$

Third, we consider the terms with $\omega_1 = 3$. When $\psi_1 = 4$, we have

$$
\begin{aligned}
&2\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^3\right)\left((\boldsymbol{q}_k^4)^{\rm H}\boldsymbol{q}_k^4\right)^{\rm H}\right\}\\
=&2c_k^2\varepsilon_k\mathbb{E}\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k^{\rm H}\tilde{\boldsymbol{h}}_k\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\}\\
=&2c_k^2\varepsilon_k\mathbb{E}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\left(M{\rm Tr}\big(\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\big)\boldsymbol{I}_N\right.\right.\\
&\left.\left.+M^2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\right)\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\varepsilon_k M\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
&+2c_k^2\varepsilon_k M^2\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\mathbb{E}\{\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\}\\
=&2c_k^2\varepsilon_k M\left(Mf_{kk,2}(\boldsymbol{\Phi})f_{k,3,1}(\boldsymbol{\Phi})+f_{kk,4}(\boldsymbol{\Phi})\right).
\end{aligned}
\tag{S30}
$$

Finally, we focus on the remaining terms in Eq. (S13):

$$
\begin{aligned}
&2{\rm Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^2\right)\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_k^4\right)^{\rm H}\right\}\right\}\\
=&2{\rm Re}\left\{c_k^2\delta\varepsilon_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\}\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2Mc_k^2\delta\varepsilon_k{\rm Re}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2Mc_k^2\delta\varepsilon_k{\rm Re}\{f_{kk,6}(\boldsymbol{\Phi})\},
\end{aligned}
\tag{S31}
$$

$$
\begin{aligned}
&2{\rm Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^3\right)\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^4\right)^{\rm H}\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\left\{\mathbb{E}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\{f_{kk,6}(\boldsymbol{\Phi})\},
\end{aligned}
\tag{S32}
$$

$$
\begin{aligned}
&2{\rm Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^1\right)\left((\boldsymbol{q}_k^4)^{\rm H}\boldsymbol{q}_k^3\right)^{\rm H}\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\left\{{\rm E}\left\{\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\}\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k M{\rm Re}\left\{{\rm E}\left\{\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k M{\rm Re}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2Mc_k^2\delta\varepsilon_k{\rm Re}\{f_{kk,6}(\boldsymbol{\Phi})\},
\end{aligned}
\tag{S33}
$$

$$
\begin{aligned}
&2{\rm Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\rm H}\boldsymbol{q}_k^4\right)\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_k^3\right)^{\rm H}\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\left\{\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2^{1/2}\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\left\{\mathbb{E}\left\{{\rm Tr}(\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2})\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\left\{\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_k\tilde{\boldsymbol{h}}_k^{\rm H}\}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2c_k^2\delta\varepsilon_k{\rm Re}\{f_{kk,6}(\boldsymbol{\Phi})\}.
\end{aligned}
\tag{S34}
$$

The value of $f_{kk,6}(\boldsymbol{\Phi})$ is given in Eq. (22). Therefore, by combining the above parts, the result of $E_{{\rm VR},k}^{\rm signal}(\boldsymbol{\Phi})$ can be obtained as shown in Eq. (20).

1.3 Derivation of the interference term

Due to $I_{\mathrm{VR},ki}(\boldsymbol{\Phi}) = \mathbb{E}\{|\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{q}_i|^2\} = \mathbb{E}\{\boldsymbol{q}_k^{\mathrm{H}}\boldsymbol{q}_i\boldsymbol{q}_i^{\mathrm{H}}\boldsymbol{q}_k\}$, we have

$$\begin{aligned} I_{\mathrm{VR},ki}(\boldsymbol{\Phi}) &= \sum_{\omega_1,\psi_1}^{4}\sum_{\omega_2,\psi_2}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega_1})^{\mathrm{H}}\boldsymbol{q}_i^{\psi_1}\right)\left((\boldsymbol{q}_k^{\omega_2})^{\mathrm{H}}\boldsymbol{q}_i^{\psi_2}\right)^{\mathrm{H}}\right\} \\ &= \sum_{\omega=1}^{4}\sum_{\psi=1}^{4}\mathbb{E}\left\{((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi})((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi})^{\mathrm{H}}\right\} + 2\sum_{\omega_1=1}^{4}\sum_{\psi_1=\omega_1+1}^{4}\mathbb{E}\left\{((\boldsymbol{q}_k^{\omega_1})^{\mathrm{H}}\boldsymbol{q}_i^{\omega_1})\left((\boldsymbol{q}_k^{\psi_1})^{\mathrm{H}}\boldsymbol{q}_i^{\psi_1}\right)^{\mathrm{H}}\right\} \\ &\quad + 2\mathrm{Re}\left\{\mathbb{E}\left\{\left(\boldsymbol{q}_k^{1\mathrm{H}}\boldsymbol{q}_i^{2}\right)\left(\boldsymbol{q}_k^{3\mathrm{H}}\boldsymbol{q}_i^{4}\right)^{\mathrm{H}}\right\}\right\} + 2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_i^{1}\right)\left((\boldsymbol{q}_k^{4})^{\mathrm{H}}\boldsymbol{q}_i^{3}\right)^{\mathrm{H}}\right\}\right\}. \end{aligned} \tag{S35}$$

As the forms of $I_{\mathrm{VR},ki}(\boldsymbol{\Phi})$ and $E_{\mathrm{VR},k}^{\mathrm{signal}}(\boldsymbol{\Phi})$ are similar in Eq. (S35), we adopt the same processing method. Therefore, we divide $I_{\mathrm{VR},ki}(\boldsymbol{\Phi})$ into different parts and calculate them sequentially. In the following, we first calculate the result of $\sum_{\omega=1}^{4}\sum_{\psi=1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi}\right)\left((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi}\right)^{\mathrm{H}}\right\}$. The formula can be further rewritten into the following form:

$$\sum_{\omega=1}^{4}\sum_{\psi=1}^{4}\mathbb{E}\left\{((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi})((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi})^{\mathrm{H}}\right\} = \sum_{\omega=1}^{4}\mathbb{E}\left\{((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\omega})((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\omega})^{\mathrm{H}}\right\} + \sum_{\omega=1}^{4}\sum_{\psi\neq\omega}^{4}\mathbb{E}\left\{((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi})((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\psi})^{\mathrm{H}}\right\}. \tag{S36}$$

The specific calculation of the first part $\sum_{\omega=1}^{4}\mathbb{E}\left\{((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\omega})((\boldsymbol{q}_k^{\omega})^{\mathrm{H}}\boldsymbol{q}_i^{\omega})^{\mathrm{H}}\right\}$ is as follows. When $\omega = 1$, we have

$$\begin{aligned} &\mathbb{E}\left\{\left((\boldsymbol{q}_k^{1})^{\mathrm{H}}\boldsymbol{q}_i^{1}\right)\left((\boldsymbol{q}_k^{1})^{\mathrm{H}}\boldsymbol{q}_i^{1}\right)^{\mathrm{H}}\right\} \\ =&c_kc_i\delta^2\varepsilon_k\varepsilon_i\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\right|^2\right\} \\ =&c_kc_i\delta^2\varepsilon_k\varepsilon_i\mathbb{E}\left\{\left|(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{a}_N)\boldsymbol{a}_M^{\mathrm{H}}\boldsymbol{a}_M(\boldsymbol{a}_N^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i)\right|^2\right\} \\ =&c_kc_i\delta^2\varepsilon_k\varepsilon_iM^2\mathbb{E}\left\{(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{a}_N)(\boldsymbol{a}_N^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i)(\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{a}_N)(\boldsymbol{a}_N^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)\right\} \\ =&M^2c_kc_i\delta^2\varepsilon_k\varepsilon_i|f_k(\boldsymbol{\Phi})|^2|f_i(\boldsymbol{\Phi})|^2. \end{aligned} \tag{S37}$$

When $\omega = 2$, we have

$$\begin{aligned} &\mathbb{E}\left\{\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_i^{2}\right)\left((\boldsymbol{q}_k^{2})^{\mathrm{H}}\boldsymbol{q}_i^{2}\right)^{\mathrm{H}}\right\} \\ =&c_kc_i\delta^2\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\right|^2\right\} \\ =&c_kc_i\delta^2\mathrm{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\mathrm{H}}\}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\} \\ =&c_kc_i\delta^2\mathrm{Tr}\{\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\} \\ =&c_kc_i\delta^2f_{ki,1,2}(\boldsymbol{\Phi}). \end{aligned} \tag{S38}$$

When $\omega=3$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_i^3\right)\left((\boldsymbol{q}_k^3)^{\rm H}\boldsymbol{q}_i^3\right)^{\rm H}\right\}\\
=&c_kc_i\varepsilon_k\varepsilon_i\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\varepsilon_k\varepsilon_i\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\Big(M{\rm Tr}\big(\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\rm H}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\big)\boldsymbol{I}_N\\
&+M^2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\rm H}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\Big)\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&c_kc_i\varepsilon_k\varepsilon_iM\left(\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\bar{\boldsymbol{h}}_i^{\rm H}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i+M\left|\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\right|^2\right)\\
=&c_kc_i\varepsilon_k\varepsilon_iM\left(f_{kk,2}(\boldsymbol{\Phi})f_{ii,2}(\boldsymbol{\Phi})+M\left|f_{ki,2}(\boldsymbol{\Phi})\right|^2\right).
\end{aligned}\tag{S39}$$

This derivation uses Eq. (30), where $\boldsymbol{A}=\boldsymbol{B}=\boldsymbol{I}_N$ and

$$\boldsymbol{W}=\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\rm H}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}.$$

When $\omega=4$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^4)^{\rm H}\boldsymbol{q}_i^4\right)\left((\boldsymbol{q}_k^4)^{\rm H}\boldsymbol{q}_i^4\right)^{\rm H}\right\}\\
=&c_kc_i\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\widetilde{\boldsymbol{H}}_2^{\rm H}\widetilde{\boldsymbol{H}}_2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\mathbb{E}\Big\{\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\Big(M{\rm Tr}\big(\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\rm H}\}\boldsymbol{R}_{{\rm VR},i}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\big)\boldsymbol{I}_N\\
&+M^2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\rm H}\}\boldsymbol{R}_{{\rm VR},i}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\Big)\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\Big\}\\
=&c_kc_iM\mathbb{E}\Big(\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k{\rm Tr}(\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}\boldsymbol{\Phi}^{\rm H})\\
&+M\tilde{\boldsymbol{h}}_k^{\rm H}\boldsymbol{R}_{{\rm VR},k}^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\Big)\\
=&c_kc_iM\left(f_{k,3,1}(\boldsymbol{\Phi})f_{i,3,1}(\boldsymbol{\Phi})+Mf_{ki,3,2}(\boldsymbol{\Phi})\right).
\end{aligned}\tag{S40}$$

Then we calculate the result of the second part $\sum_{\omega=1}^4\sum_{\psi\neq\omega}^4\mathbb{E}\left\{((\boldsymbol{q}_k^\omega)^{\rm H}\boldsymbol{q}_i^\psi)((\boldsymbol{q}_k^\omega)^{\rm H}\boldsymbol{q}_i^\psi)^{\rm H}\right\}$. First, we consider the terms with $\omega=1$. When $\psi=2$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_i^2\right)\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_i^2\right)^{\rm H}\right\}\\
=&c_kc_i\delta^2\varepsilon_k\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\delta^2\varepsilon_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\rm H}\}\boldsymbol{R}_{{\rm VR},i}^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&c_kc_i\delta^2\varepsilon_k\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{{\rm VR},i}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&Mc_kc_i\delta^2\varepsilon_k|f_k(\boldsymbol{\Phi})|^2f_{i,1,1}(\boldsymbol{\Phi}).
\end{aligned}\tag{S41}$$

When $\psi=3$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_i^3\right)\left((\boldsymbol{q}_k^1)^{\rm H}\boldsymbol{q}_i^3\right)^{\rm H}\right\}\\
=&c_kc_i\delta\varepsilon_k\varepsilon_i\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\delta\varepsilon_k\varepsilon_i\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\mathbb{E}\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\rm H}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2}\tilde{\boldsymbol{H}}_2^{\rm H}\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&c_kc_i\delta\varepsilon_k\varepsilon_i\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k{\rm Tr}(\boldsymbol{R}_{\rm ris}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\rm H}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}^{1/2})\\
=&c_kc_i\delta\varepsilon_k\varepsilon_i(\bar{\boldsymbol{h}}_k^{\rm H}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\rm H}\bar{\boldsymbol{H}}_2^{\rm H}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)(\bar{\boldsymbol{h}}_i^{\rm H}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\rm H}\boldsymbol{R}_{\rm ris}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i)\\
=&Mc_kc_i\delta\varepsilon_k\varepsilon_i|f_k(\Phi)|^2f_{ii,2}(\boldsymbol{\Phi}).
\end{aligned}\tag{S42}$$

When $\psi=4$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_i^4\right)\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_i^4\right)^{\mathrm{H}}\right\}\\
=&c_kc_i\delta\varepsilon_k\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\delta\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\mathbb{E}\left\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\right\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&c_kc_i\delta\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\mathbb{E}\{\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\\
=&c_kc_i\delta\varepsilon_k(\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k)\mathbb{E}\{\tilde{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\}\\
=&Mc_kc_i\delta\varepsilon_k|f_k(\boldsymbol{\Phi})|^2f_{i,3,1}(\boldsymbol{\Phi}).
\end{aligned}\tag{S43}$$

Second, we consider the terms with $\omega=2$. When $\psi=1$, we have

$$\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^1\right)\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^1\right)^{\mathrm{H}}\right\}=\mathbb{E}\left\{\left((\boldsymbol{q}_i^1)^{\mathrm{H}}\boldsymbol{q}_k^2\right)\left((\boldsymbol{q}_i^1)^{\mathrm{H}}\boldsymbol{q}_k^2\right)^{\mathrm{H}}\right\}=Mc_kc_i\delta^2\varepsilon_i|f_i(\boldsymbol{\Phi})|^2f_{k,1,1}(\boldsymbol{\Phi}).\tag{S44}$$

When $\psi=3$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^3\right)\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^3\right)^{\mathrm{H}}\right\}\\
=&c_kc_i\delta\varepsilon_i\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\widetilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\delta\varepsilon_i\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\mathbb{E}\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&c_kc_i\delta\varepsilon_i\mathbb{E}\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\}\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\\
=&c_kc_i\delta\varepsilon_i\,\mathrm{Tr}(\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}})(\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i)\\
=&c_kc_i\delta\varepsilon_if_{k,1,1}(\boldsymbol{\Phi})f_{ii,2}(\boldsymbol{\Phi}).
\end{aligned}\tag{S45}$$

When $\psi=4$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^4\right)\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^4\right)^{\mathrm{H}}\right\}\\
=&c_kc_i\delta\mathbb{E}\left\{\left|\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\delta\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\mathbb{E}\left\{\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\mathrm{H}}\}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\right\}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\\
=&c_kc_i\delta\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\,\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\right\}\\
=&c_kc_i\delta\mathrm{Tr}(\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2})\mathrm{Tr}(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2})\\
=&c_kc_i\delta f_{k,1,1}(\boldsymbol{\Phi})f_{i,3,1}(\boldsymbol{\Phi}).
\end{aligned}\tag{S46}$$

Third, we consider the terms with $\omega=3$. When $\psi=1$, we have

$$\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^1\right)\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^1\right)^{\mathrm{H}}\right\}=Mc_kc_i\delta\varepsilon_k\varepsilon_i|f_i(\boldsymbol{\Phi})|^2f_{kk,2}(\boldsymbol{\Phi}).\tag{S47}$$

When $\psi=2$, we have

$$\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^2\right)\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^2\right)^{\mathrm{H}}\right\}=c_kc_i\delta\varepsilon_kf_{i,1,1}(\boldsymbol{\Phi})f_{kk,2}(\boldsymbol{\Phi}).\tag{S48}$$

When $\psi=4$, we have

$$\begin{aligned}
&\mathbb{E}\left\{\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^4\right)\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^4\right)^{\mathrm{H}}\right\}\\
=&c_kc_i\varepsilon_k\mathbb{E}\left\{\left|\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\tilde{\boldsymbol{h}}_i\right|^2\right\}\\
=&c_kc_i\varepsilon_k\mathbb{E}\left\{\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&c_kc_i\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\left(M\mathrm{Tr}\big(\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\big)\boldsymbol{I}_N\right.\\
&\left.+M^2\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\right)\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\\
=&c_kc_i\varepsilon_k\left(M\mathrm{Tr}\big(\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\big)\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right.\\
&\left.+M^2\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right)\\
=&c_kc_i\varepsilon_kM\left(f_{kk,2}(\boldsymbol{\Phi})f_{i,3,1}(\boldsymbol{\Phi})+Mf_{ki,4}(\boldsymbol{\Phi})\right).
\end{aligned}\tag{S49}$$

Fourth, we consider the terms with $\omega=4$. When $\psi=1$, we have

$$\mathbb{E}\left\{\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^1\right)\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^1\right)^{\mathrm{H}}\right\}=Mc_kc_i\delta\varepsilon_i|f_i(\boldsymbol{\Phi})|^2f_{k,3,1}(\boldsymbol{\Phi}).\tag{S50}$$

When $\psi=2$, we have

$$\mathbb{E}\left\{\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^2\right)\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^2\right)^{\mathrm{H}}\right\}=c_kc_i\delta f_{i,1,1}(\boldsymbol{\Phi})f_{k,3,1}(\boldsymbol{\Phi}).\tag{S51}$$

When $\psi=3$, we have

$$\mathbb{E}\left\{\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^3\right)\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^3\right)^{\mathrm{H}}\right\}=c_kc_i\varepsilon_iM\left(f_{ii,2}(\boldsymbol{\Phi})f_{k,3,1}(\boldsymbol{\Phi})+Mf_{ik,4}(\boldsymbol{\Phi})\right).\tag{S52}$$

Next, we calculate the result of $2\sum_{\omega_1=1}^4\sum_{\psi_1=\omega_1+1}^4\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega_1})^{\mathrm{H}}\boldsymbol{q}_i^{\omega_1}\right)\left((\boldsymbol{q}_k^{\psi_1})^{\mathrm{H}}\boldsymbol{q}_i^{\psi_1}\right)^{\mathrm{H}}\right\}$ in Eq. (S35). Due to the fact that $\tilde{\boldsymbol{h}}_i$, $\tilde{\boldsymbol{h}}_k$, and $\tilde{\boldsymbol{H}}_2$ are independent of each other, the expression can be further simplified as follows after removing the terms with zero expectation:

$$\begin{aligned}
&2\sum_{\omega_1=1}^{4}\sum_{\psi_1=\omega_1+1}^{4}\mathbb{E}\left\{\left((\boldsymbol{q}_k^{\omega_1})^{\mathrm{H}}\boldsymbol{q}_i^{\omega_1}\right)\left((\boldsymbol{q}_k^{\psi_1})^{\mathrm{H}}\boldsymbol{q}_i^{\psi_1}\right)^{\mathrm{H}}\right\}\\
=&2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_i^1\right)\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^3\right)^{\mathrm{H}}\right\}\right\}+2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^2\right)\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^4\right)^{\mathrm{H}}\right\}\right\}.
\end{aligned}\tag{S53}$$

Therefore, we calculate the two items separately.

$$\begin{aligned}
&2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_i^1\right)\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^3\right)^{\mathrm{H}}\right\}\right\}\\
=&2c_kc_i\delta\varepsilon_k\varepsilon_i\mathrm{Re}\left\{\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2Mc_kc_i\delta\varepsilon_k\varepsilon_i\mathrm{Re}\left\{\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2M^2c_kc_i\delta\varepsilon_k\varepsilon_i\mathrm{Re}\left\{\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{a}_N\boldsymbol{a}_N^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2M^2c_kc_i\delta\varepsilon_k\varepsilon_i\mathrm{Re}\left\{f_{ki,7}(\boldsymbol{\Phi})f_{ik,2}(\boldsymbol{\Phi})\right\}.
\end{aligned}\tag{S54}$$

$$\begin{aligned}
&2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^2\right)\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^4\right)^{\mathrm{H}}\right\}\right\}\\
=&2c_kc_i\delta\varepsilon_k\varepsilon_i\mathrm{Re}\left\{\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\tilde{\boldsymbol{h}}_k\right\}\right\}\\
=&2Mc_kc_i\delta\mathrm{Re}\left\{\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\right\}\\
=&2Mc_kc_i\delta\mathrm{Re}\left\{\mathrm{Tr}\left\{\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\right\}\right\}\\
=&2c_kc_i\delta\mathrm{Re}\{f_{ki,5}(\boldsymbol{\Phi})\}.
\end{aligned}\tag{S55}$$

Finally, we focus on the remaining terms in Eq. (S35). The derivation process of the first item is as follows:

$$\begin{aligned}
&2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^1)^{\mathrm{H}}\boldsymbol{q}_i^2\right)\left((\boldsymbol{q}_k^3)^{\mathrm{H}}\boldsymbol{q}_i^4\right)^{\mathrm{H}}\right\}\right\}\\
=&2\mathrm{Re}\left\{c_kc_i\delta\varepsilon_k\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{h}}_i\tilde{\boldsymbol{h}}_i^{\mathrm{H}}\}\boldsymbol{R}_{\mathrm{VR},i}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\right\}\\
=&2Mc_kc_i\delta\varepsilon_k\mathrm{Re}\{\bar{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{D}_k^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},i}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\}\\
=&2Mc_kc_i\delta\varepsilon_k\mathrm{Re}\{f_{ki,6}(\boldsymbol{\Phi})\}.
\end{aligned}\tag{S56}$$

The derivation process of the second item is as follows:

$$\begin{aligned}
&2\mathrm{Re}\left\{\mathbb{E}\left\{\left((\boldsymbol{q}_k^2)^{\mathrm{H}}\boldsymbol{q}_i^1\right)\left((\boldsymbol{q}_k^4)^{\mathrm{H}}\boldsymbol{q}_i^3\right)^{\mathrm{H}}\right\}\right\}\\
=&2\mathrm{Re}\left\{\mathbb{E}\left\{c_kc_i\delta\varepsilon_i\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\mathbb{E}\{\tilde{\boldsymbol{H}}_2^{\mathrm{H}}\tilde{\boldsymbol{H}}_2\}\boldsymbol{R}_{\mathrm{ris}}^{1/2}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\right\}\\
=&2c_kc_i\delta\varepsilon_i\boldsymbol{M}\mathrm{Re}\left\{\mathbb{E}\left\{\tilde{\boldsymbol{h}}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_i^{1/2}\bar{\boldsymbol{h}}_i\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}^{1/2}\tilde{\boldsymbol{h}}_k\right\}\right\}\\
=&2c_kc_i\delta\varepsilon_iM\mathrm{Re}\{\bar{\boldsymbol{h}}_i^{\mathrm{H}}\boldsymbol{D}_i^{1/2}\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{ris}}\boldsymbol{\Phi}\boldsymbol{R}_{\mathrm{VR},k}\boldsymbol{\Phi}^{\mathrm{H}}\bar{\boldsymbol{H}}_2^{\mathrm{H}}\bar{\boldsymbol{H}}_2\boldsymbol{\Phi}\boldsymbol{D}_k^{1/2}\bar{\boldsymbol{h}}_k\}\\
=&2Mc_kc_i\delta\varepsilon_i\mathrm{Re}\{f_{ik,6}(\boldsymbol{\Phi})\}.
\end{aligned}\tag{S57}$$

Therefore, by combining the above parts, the result of $I_{\mathrm{VR},ki}(\boldsymbol{\Phi})$ can be obtained as shown in Eq. (21). Based on the value of i and the superposition of various terms, the final result is $\sum_{i=1,i\neq k}^{K} I_{\mathrm{VR},ki}(\boldsymbol{\Phi})$. Then, the uplink approximate achievable rate of user k can be obtained by substituting the results of $E_{\mathrm{VR},k}^{\mathrm{noise}}(\boldsymbol{\Phi})$, $E_{\mathrm{VR},k}^{\mathrm{signal}}(\boldsymbol{\Phi})$, and $\sum_{i=1,i\neq k}^{K} I_{\mathrm{VR},ki}(\boldsymbol{\Phi})$ into the original Eq. (18).