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Outlier-resistant distributed fusion filtering for nonlinear discrete-time singular systems under a dynamic event-triggered scheme

Key words: Distributed fusion filtering; Multi-sensor nonlinear singular systems; Dynamic event-triggered scheme; Outlier-resistant filter; Uniform boundedness

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Motivation

- The estimation/filtering issue with regard to sensor networks has received much attention, and many results have been reported for normal systems over sensor networks. However, studies of handling singular system estimation issues are still limited, and deserve further investigation.
- The distributed fusion filtering problem has seldom been addressed for multi-sensor nonlinear singular systems in the presence of measurement outliers, which motivates our current study.
- For multi-sensor nonlinear normal systems with the dynamic event-triggered scheme, the distributed filtering and distributed fusion filtering issues have been tackled. Nevertheless, the distributed fusion filtering problem has not been completely handled for multi-sensor nonlinear singular systems under the dynamic event-triggered scheme, which further motivates our investigation.

Method

We consider the following class of multi-sensor discrete time-varying nonlinear singular systems:

$$
\begin{aligned}
Kx_{f+1} &= A'_f x_f + h'(x_f) + B'_f \varpi_f, \\
y_{i,f} &= C_{i,f} x_f + \mu_{i,f}, \ i = 1, 2, \ldots, \kappa,
\end{aligned}
$$

The i^{th} filter is established as follows:

$$
\hat{x}_{i,f+1|f} = A_f \hat{x}_{i,f|f} + h(\hat{x}_{i,f|f}),
$$

$$
\begin{aligned}
\hat{x}_{i,f+1|f+1} = \hat{x}_{i,f+1|f} &+ G_{i,f+1} S_{\pi_{i,f+1}}(y_{i,t^i_{l+1}} \\
&- C_{i,f+1}\hat{x}_{i,f+1|f}),
\end{aligned}
$$

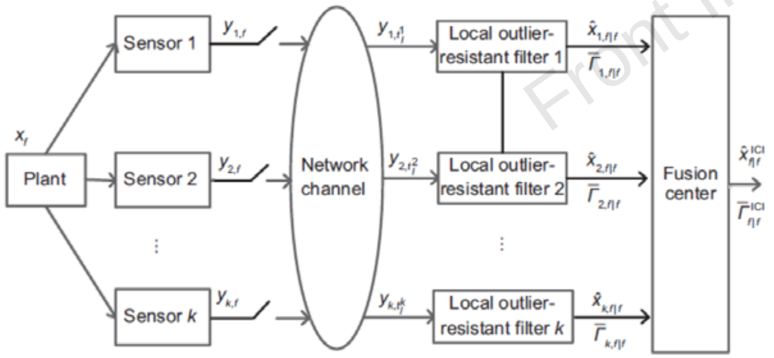

Fig. 1 Diagram of the system framework

Theorem 2 For each sensor subsystem, let scalars $\varphi_{q_1} > 0$ $(q_1 = 0, 1, \ldots, 13)$ be given. If the following two difference equations

$$
\begin{aligned}
&\bar{\Gamma}_{i,f+1|f} \\
=&(1+\varphi_1+\varphi_2)A_f\bar{\Gamma}_{i,f|f}A_f^{\mathrm{T}} + (1+\varphi_1^{-1}+\varphi_3)\alpha^2 \\
&\cdot \mathrm{tr}(\bar{\Gamma}_{i,f|f})UU^{\mathrm{T}} + (1+\varphi_2^{-1}+\varphi_3^{-1}+\varphi_4+\varphi_5) \\
&\cdot V_{i,f+1}\Theta_{i,f+1}V_{i,f+1}^{\mathrm{T}} + (1+\varphi_4^{-1})B_fQ_fB_f^{\mathrm{T}} \\
&+ (1+\varphi_5^{-1})V_{i,f+1}R_{i,f+1}V_{i,f+1}^{\mathrm{T}}
\end{aligned}
$$

$$
\begin{aligned}
&\bar{\Gamma}_{i,f+1|f+1} \\
=&(1+\varphi_6+\varphi_7+\varphi_8)(I - G_{i,f+1}\Lambda_{i,f+1}C_{i,f+1}) \\
&\cdot \bar{\Gamma}_{i,f+1|f}(I - G_{i,f+1}\Lambda_{i,f+1}C_{i,f+1})^{\mathrm{T}} \\
&+ (1+\varphi_6^{-1}+\varphi_9+\varphi_{10})\Delta_{i,f+1}^{(1)}G_{i,f+1}\Lambda_{i,f+1} \\
&\cdot \Lambda_{i,f+1}^{\mathrm{T}}G_{i,f+1}^{\mathrm{T}} + (1+\varphi_7^{-1}+\varphi_9^{-1})\Delta_{i,f+1}^{(3)}G_{i,f+1} \\
&\cdot (\Lambda_{i,f+1}-I)(\Lambda_{i,f+1}-I)^{\mathrm{T}}G_{i,f+1}^{\mathrm{T}} \\
&+ (1+\varphi_8^{-1}+\varphi_{10}^{-1})G_{i,f+1}\Lambda_{i,f+1}R_{i,f+1} \\
&\cdot \Lambda_{i,f+1}^{\mathrm{T}}G_{i,f+1}^{\mathrm{T}}
\end{aligned}
$$

Method

Fusion filtering method

In accordance with the local filter $\hat{x}_{i,f|f}$ and its UB $\bar{\varGamma}_{i,f|f}$ $(i = 1, 2, \ldots, \kappa)$, the recursive forms concerning the ICI-based FF $\hat{x}^{\text{ICI}}_{f|f}$ and its covariance $\bar{\varGamma}^{\text{ICI}}_{f|f}$ are shown as follows:

$$\hat{x}^{\text{ICI}}_{f|f} = \sum_{i=1}^{\kappa} \varPi^{\text{ICI}}_{i,f|f} \hat{x}_{i,f|f},$$

$$\bar{\varGamma}^{\text{ICI}}_{f|f} = \left[\sum_{i=1}^{\kappa} \bar{\varGamma}^{-1}_{i,f|f} - \left(\sum_{i=1}^{\kappa} d_i \bar{\varGamma}_{i,f|f} \right)^{-1} \right]^{-1},$$

where $d_i \in [0,\ 1]$ denote the weighted coefficients and satisfy $\sum_{i=1}^{\kappa} d_i = 1$. The fusion parameters $\varPi^{\text{ICI}}_{i,f|f}$ are computed by

$$\varPi^{\text{ICI}}_{i,f|f} = \bar{\varGamma}^{\text{ICI}}_{f|f} \left[\bar{\varGamma}^{-1}_{i,f|f} - d_i \left(\sum_{i=1}^{\kappa} d_i \bar{\varGamma}_{i,f|f} \right)^{-1} \right].$$

In addition, the DFF algorithm can be transformed into the following optimization problem:

$$\min_{d_i} \left\{ \text{tr}(\bar{\varGamma}^{\text{ICI}}_{f|f}) \right\} \ \text{ s.t. } \sum_{i=1}^{\kappa} d_i = 1, \ \ d_i \in [0,\ 1].$$

Method

To facilitate implementation, the procedure of the DETS-based DFF (DETSBDFF) algorithm is given in Algorithm 1.

Algorithm 1 DETSBDFF
Step 1 : Let $f=0$ and initialize the parameters
Step 2 : Compute $\hat{x}_{i,f+1\|f}$ according to Eq. (10)
Step 3 : Compute $\bar{\Gamma}_{i,f+1\|f}$ by Eq. (18)
Step 4 : Derive the filter gain $G_{i,f+1}$ by Eq. (21)
Step 5 : Compute $\hat{x}_{i,f+1\|f+1}$ based on Eq. (11)
Step 6 : Obtain $\bar{\Gamma}_{i,f+1\|f+1}$ via Eq. (19)
Step 7 : Obtain the fusion estimate $\hat{x}^{\mathrm{ICI}}_{f\|f}$ by Eq. (32)
Step 8 : Compute the fusion FEC $\bar{\Gamma}^{\mathrm{ICI}}_{f\|f}$ from Eq. (33)
Step 9 : Let $f=f+1$ and return to step 2

$$\begin{aligned}
\theta' &\triangleq (1+\varphi_0)w_1\bar{c}+(1+\varphi_0^{-1})\bar{r},\\
\breve{\chi} &\triangleq (1+\varphi_1+\varphi_2)\bar{\chi}\bar{a}+(1+\varphi_1^{-1}+\varphi_3)\\
&\quad \cdot n\bar{\chi}\alpha^2\bar{u}+(1+\varphi_2^{-1}+\varphi_3^{-1}+\varphi_4\\
&\quad +\varphi_5)\theta'\bar{n}+(1+\varphi_4^{-1})\bar{q}\bar{b}+(1+\varphi_5^{-1})\bar{r}\bar{n},\\
\bar{\Delta}^{(2)} &\triangleq (1+\varphi_{11}+\varphi_{12})\breve{\chi}\bar{c}+(1+\varphi_{11}^{-1}+\varphi_{13})\bar{\delta}\\
&\quad +(1+\varphi_{12}^{-1}+\varphi_{13}^{-1})\bar{r},\\
\bar{\Delta}^{(3)} &\triangleq m[\bar{\pi}_{i,0}+(f+1)m\bar{z}\bar{\Delta}^{(2)}]/\bar{\rho},
\end{aligned}$$

Boundedness analysis

Assumption 6 For any sensor i $(i=1,2,\ldots,\kappa)$ and time f, there exist positive numbers $\bar{a}$, $\bar{b}$, $\bar{c}$, $\underline{r}$, $\bar{r}$, $\bar{u}$, $\bar{n}$, $\bar{q}$, $\underline{\delta}$, $\bar{\delta}$, $\underline{\lambda}$, $\bar{\lambda}$, $\bar{z}$, and $\bar{\rho}$ such that the following conditions hold:

$$\begin{gathered}
A_fA_f^{\mathrm{T}}\le\bar{a}I,\ B_fB_f^{\mathrm{T}}\le\bar{b}I,\ C_{i,f}C_{i,f}^{\mathrm{T}}\le\bar{c}I,\\
\underline{r}I\le R_{i,f}\le\bar{r}I,\ UU^{\mathrm{T}}\le\bar{u}I,\ Q_f\le\bar{q}I,\\
V_{i,f}V_{i,f}^{\mathrm{T}}\le\bar{n}I,\ \underline{\delta}\le\Delta_{i,f}^{(1)}\le\bar{\delta},\ \underline{\lambda}\le\Lambda_{i,f}\Lambda_{i,f}^{\mathrm{T}}\le\bar{\lambda},\\
\gamma_i<1,\ Z_i\le\bar{z}I,\ \rho_i^{(j)}\ge\bar{\rho}.
\end{gathered}$$

Theorem 4 Consider the time-varying multi-sensor SSs depicted in Eqs. (1) and (2). Under Assumption 6 and notations (37), if the inequality condition $\chi'\le\bar{\chi}$ holds with the initial condition $\bar{\Gamma}_{i,0|0}\le\bar{\chi}I$, then $\bar{\Gamma}_{i,f+1|f+1}\le\bar{\chi}I$ holds for any f.

$$\begin{aligned}
\bar{g} &\triangleq \frac{(1+\varphi_6+\varphi_7+\varphi_8)^2\breve{\chi}^2\bar{\lambda}\bar{c}}{[(1+\varphi_8^{-1}+\varphi_{10}^{-1})\underline{r}\,\underline{\lambda}]^2},\\
\chi' &\triangleq 2(1+\varphi_6+\varphi_7+\varphi_8)\breve{\chi}(1+\bar{c}\bar{\lambda}\bar{g})\\
&\quad +(1+\varphi_6^{-1}+\varphi_9+\varphi_{10})\bar{\delta}\bar{\lambda}\bar{g}\\
&\quad +(1+\varphi_7^{-1}+\varphi_9^{-1})\bar{\Delta}^{(3)}(1+\bar{\lambda})^2\bar{g}\\
&\quad +(1+\varphi_8^{-1}+\varphi_{10}^{-1})\bar{r}\bar{\lambda}\bar{g}.
\end{aligned}$$

Method

An illustrative example

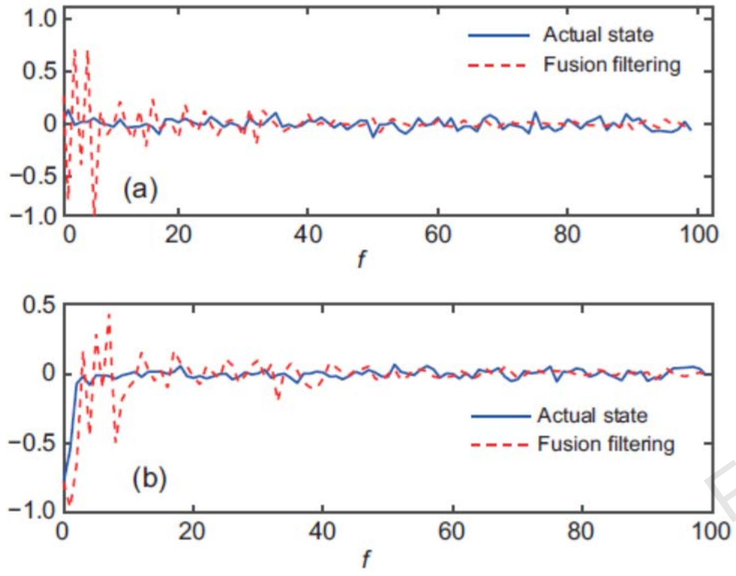

Fig. 2 Actual state and fusion filtering: (a) the first state component; (b) the second state component

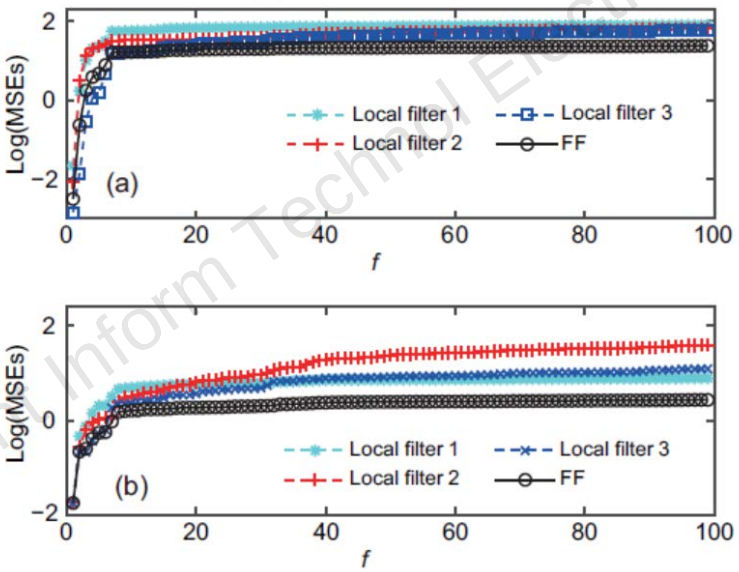

Fig. 3 Log(MSEs) of the fusion filter and three local filters: (a) the first state component; (b) the second state component

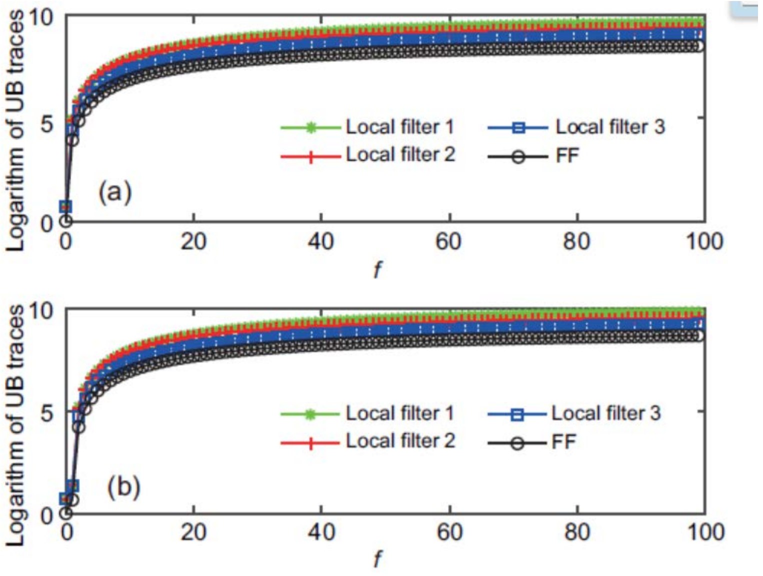

Fig. 4 Logarithm of UB traces of the fusion filter and three local filters: (a) the first state component; (b) the second state component

Method

An illustrative example

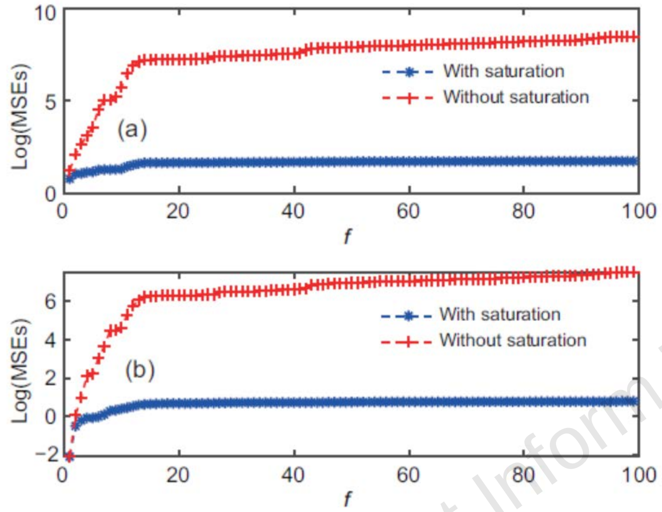

Fig. 5 Log(MSEs) of the fusion filter with or without adaptive saturation condition: (a) the first state component; (b) the second state component

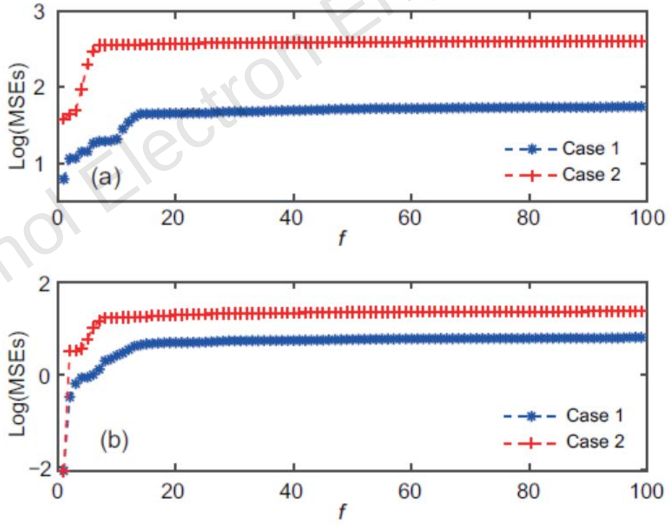

Fig. 6 Log(MSEs) of the fusion filter in two cases: (a) the first state component; (b) the second state component

Method

An illustrative example

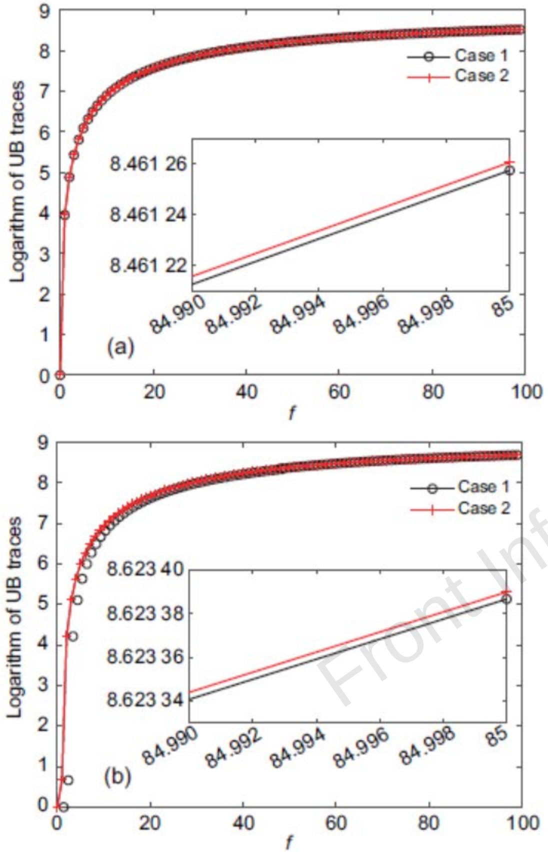

Fig. 7 Logarithm of UB traces of the fusion filter in two cases: (a) the first state component; (b) the second state component

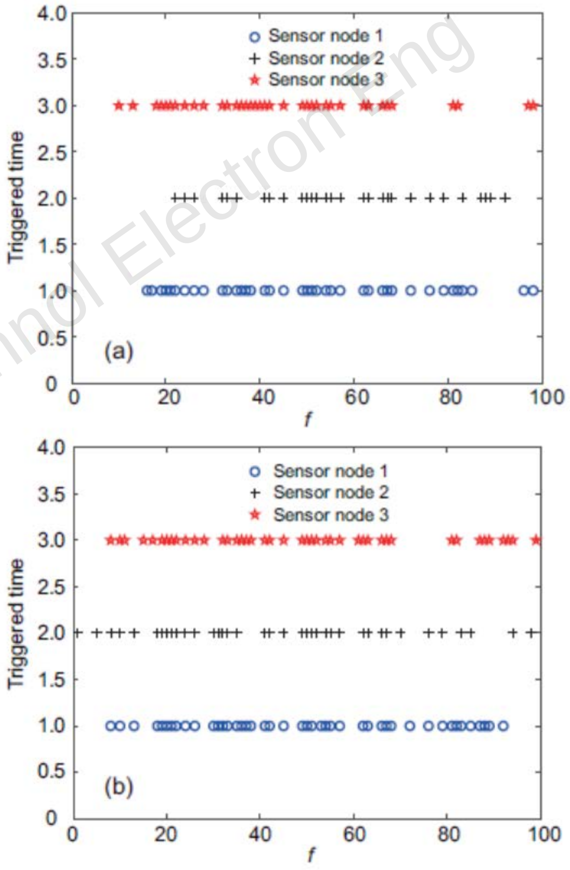

Fig. 8 Event-triggered time of each node under the DETS (a) and SETS (b)

Conclusions

- The outlier-resistant distributed fusion filtering issue has been tackled for a class of multi-sensor nonlinear singular systems with measurement outliers and a dynamic event-triggered scheme.
- In view of the stochastic analysis method, a local upper bound with respect to the filtering error covariance has been deduced, and the filter gain has been computed by minimizing the trace of the obtained upper bound.
- The outlier-resistant fusion filtering approach has been implemented using the inverse covariance intersection fusion.

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