



Supplementary materials for

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Proof of Theorem 1

Assumption S1 The TFO variable $\mathbf{s}[\mathbf{z}(k)]$ is Lipschitz continuous with a Lipschitz constant L_s . Since the uncertainty vector $\mathbf{D}(k)$ is bounded, over a finite prediction horizon, each term $\|\mathbf{D}(k+i|k)\|_2$ ($i = 0, 1, \dots, N_p$) satisfies $\|\mathbf{D}(k+i|k)\|_2 \leq \mathcal{Y}$.

Combined with Assumption , the norm of the TFO prediction errors between the nominal and true systems is bounded as follows:

$$\begin{cases} \|\mathbf{s}[\hat{\mathbf{z}}(k+1|k)] - \mathbf{s}[\mathbf{z}(k+1)]\|_2 \leq L_s \|\mathbf{D}\|_2 \leq L_s \mathcal{Y}, \\ \vdots \\ \|\mathbf{s}[\hat{\mathbf{z}}(k+i|k)] - \mathbf{s}[\mathbf{z}(k+i)]\|_2 \leq \frac{1-\|\mathbf{A}\|^i}{1-\|\mathbf{A}\|} L_s \mathcal{Y}. \end{cases} \quad (\text{S1})$$

This completes the proof of Theorem 1.

Proof of Theorem 2

Lemma S1 (Marruedo et al., 2002) Consider the system $\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{D}(k))$, if there exists a positive definite function $V(\mathbf{x}(k), \mathbf{D}(k))$, and $\mathcal{K}_\infty$ functions a_3, a_4, a_5, a_6 , such that $a_3(\|\mathbf{x}(k)\|_2) \leq V(\mathbf{x}(k), \mathbf{D}(k)) \leq a_4(\|\mathbf{x}(k)\|_2)$ and furthermore $V(\mathbf{x}(k+1), \mathbf{D}(k+1)) - V(\mathbf{x}(k), \mathbf{D}(k)) \leq -a_5(\|\mathbf{x}(k)\|_2) + a_6(\|\mathbf{w}(k)\|_2)$, then the system is input-to-state stable (ISS).

Assumption S2 The stage cost function $L\{\mathbf{s}[\hat{\mathbf{z}}(k+i|k)], \mathbf{u}(k+i|k)\}$ satisfies Lipschitz continuity with constant L_a . Additionally, there exists a $\mathcal{K}_\infty$ function a_1 , such that $L\{\mathbf{s}[\hat{\mathbf{z}}(k+i|k)], \mathbf{u}(k+i|k)\} \geq a_1(\|\mathbf{s}[\hat{\mathbf{z}}(k+i|k)]\|_2)$.

Assumption S3 The terminal cost function $L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k)]\}$ is Lipschitz continuous with constant L_b . There exists a $\mathcal{K}_\infty$ function a_2 , such that $L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k)]\} \geq a_2(\|\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k)]\|_2)$.

Assumption S4 There exists a local stabilizing controller $\boldsymbol{\mu}(k)$ for the terminal set $\mathcal{X}_f$, ensuring that $L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+1)]\} - L_f\{\mathbf{s}[\mathbf{z}(k)]\} \leq L\{\mathbf{s}[\mathbf{z}(k)], \boldsymbol{\mu}(k)\}$. Consider the following Lyapunov function definition:

$$V(k) = J^*(k), \quad (\text{S2})$$

where $J^*(k)$ represents the optimal cost function for the objective.

Let $\bar{J}(k+1)$ denote the cost function associated with a feasible trajectory at the subsequent time step $k+1$.

The discrepancy between $\bar{J}(k+1)$ and $J^*(k)$ can be expressed as

$$\begin{aligned}\Delta J(k) &= \bar{J}(k+1) - J^*(k) \\ &= \sum_{i=0}^{N_p-2} (L\{\mathbf{s}[\hat{\mathbf{z}}(k+i+1|k+1)], \mathbf{u}(k+i+1|k+1)\} - L\{\mathbf{s}[\hat{\mathbf{z}}(k+i+1|k)], \mathbf{u}^*(k+i+1|k)\}) \\ &\quad + L\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k+1)], \mathbf{u}(k+N_p|k+1)\} - L\{\mathbf{s}[\mathbf{z}(k)], \mathbf{u}^*(k)\} + L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p+1|k+1)]\} \\ &\quad - L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k)]\} + L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k+1)]\} - L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k+1)]\}.\end{aligned}\tag{S3}$$

Invoking Assumptions S2 and S3, we have

$$\begin{aligned}L\{\mathbf{s}[\hat{\mathbf{z}}(k+i+1|k+1)], \mathbf{u}(k+i+1|k+1)\} - L\{\mathbf{s}[\hat{\mathbf{z}}(k+i+1|k)], \mathbf{u}^*(k+i+1|k)\} &\leq L_a L_s \|\mathbf{A}\|^i \Upsilon \\ L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k+1)]\} - L_f\{\mathbf{s}[\hat{\mathbf{z}}(k+N_p|k)]\} &\leq L_b L_s \|\mathbf{A}\|^{N_p-1} \Upsilon.\end{aligned}\tag{S4}$$

Invoking Assumption S4, we have

$$\Delta J(k) \leq L_c \Upsilon - L\{\mathbf{s}[\hat{\mathbf{z}}(k)], \mathbf{u}^*(k)\},\tag{S5}$$

where $L_c = \left(L_b \|\mathbf{A}\|^{N_p-1} + L_a \frac{1 - \|\mathbf{A}\|^{N_p-1}}{1 - \|\mathbf{A}\|} \right) L_s$.

Invoking Assumption S2, $L\{\mathbf{s}[\hat{\mathbf{z}}(k)], \mathbf{u}^*(k)\} \geq a_1 \|\mathbf{s}[\mathbf{z}(k)]\|_2$, the first difference of (S2) is

$$\Delta V(k) = J^*(k+1) - J^*(k) \leq L_c \Upsilon - a_1 \|\mathbf{s}[\mathbf{z}(k)]\|_2.\tag{S6}$$

Invoking Lemma S1, the TFO variable $\mathbf{s}[\mathbf{z}(k)]$ is ISS.

This completes the proof of Theorem 2.

Proof of Theorem 3

Lemma S2 (Li et al., 2022) The relation between $\binom{\alpha}{j}$ and $\Gamma(\chi)$ can be established by

$$\sum_{j=0}^L (-1)^j \binom{\alpha}{j} = \frac{\Gamma(L+1-\alpha)}{\Gamma(1-\alpha)\Gamma(L+1)},\tag{S7}$$

where $\Gamma(\chi) = \int_0^\infty \tau^{\chi-1} e^{-\tau} d\tau$.

Considering the bounded sequence $y(k), k = 0, 1, \dots$ with $\max\{y(k-1)\} \leq \rho_y$, the corresponding fractional-order difference is bounded by

$$\frac{1}{(\Delta T)^\alpha} \sum_{j=1}^L (-1)^j \binom{\alpha}{j} y(k-j) \leq \frac{(\mathcal{N}-1)\rho_y}{(\Delta T)^\alpha},\tag{S8}$$

where $\mathcal{N} = \frac{\Gamma(L+1-\alpha)}{\Gamma(1-\alpha)\Gamma(L+1)} \geq 1$.

Lemma S3 (Li et al., 2013) Considering a dynamical system $y(k+1) - y(k) + \xi[y(k)]^\lambda = g(k)$ with $|g(k)| \leq \gamma$, there is a finite step to guarantee

$$|y(k)| \leq \Lambda \cdot \max \left\{ \left(\frac{\gamma}{\xi} \right)^{\frac{1}{\lambda}}, \xi^{\frac{1}{1-\lambda}} \right\},\tag{S9}$$

where $\Lambda = 1 + \lambda^{\frac{\lambda}{1-\lambda}} - \lambda^{\frac{1}{1-\lambda}}$.

Lemma S4 (Toth and Kelley, 2015) For a sequence of vectors $\{\mathbf{x}_k\}$ in the vector space $\mathbb{C}^n$ and a vector $\mathbf{x}_0$, where $\mathbf{x}_k = [a_1^{(k)}, \dots, a_n^{(k)}]^\top$ and $\mathbf{x}_0 = [a_1, \dots, a_n]^\top$, if the condition $\lim_{k \rightarrow \infty} a_j^{(k)} = a_j$ holds $\forall 1 \leq j \leq n$, then the sequence $\{\mathbf{x}_k\}$ converges to $\mathbf{x}_0$ with respect to the vector norm.

The first difference of $\mathbf{s}[\mathbf{z}(k)]$ is

$$\mathbf{s}[\mathbf{z}(k+1)] - \mathbf{s}[\mathbf{z}(k)] = \mathbf{e}(k+1) - \mathbf{e}(k) + \Delta T \boldsymbol{\xi} \mathcal{D}^\alpha [\mathbf{e}(k)]^\lambda. \quad (\text{S10})$$

By invoking the Grünwald-Letnikov fractional-order definition and setting $\boldsymbol{\varrho}(k) = \mathbf{s}[\mathbf{z}(k+1)] - \mathbf{s}[\mathbf{z}(k)]$, (S10) can be reformulated as

$$\boldsymbol{\varrho}(k) = \mathbf{e}(k+1) - \mathbf{e}(k) + \frac{\boldsymbol{\xi}}{(\Delta T)^{\alpha-1}} [\mathbf{e}(k)]^\lambda + \frac{\boldsymbol{\xi}}{(\Delta T)^{\alpha-1}} \sum_{j=1}^L (-1)^j \binom{\alpha}{j} [\mathbf{e}(k-j)]^\lambda. \quad (\text{S11})$$

Without loss of generality, for each element $e_i(k)$ ($i = 1, \dots, n$) in $\mathbf{e}(k)$, there exists

$$\varrho_i(k) = e_i(k+1) - e_i(k) + \frac{\xi_i}{(\Delta T)^{\alpha-1}} [e_i(k)]^\lambda + \frac{\xi_i}{(\Delta T)^{\alpha-1}} \sum_{j=1}^L (-1)^j \binom{\alpha}{j} [e_i(k-j)]^\lambda. \quad (\text{S12})$$

By applying Lemma S2 and considering the bounded sequence $e_i(k)$ with $\max\{e_i(k-1)\} \leq \rho_i$, we obtain

$$\frac{1}{(\Delta T)^{\alpha-1}} \sum_{j=1}^L (-1)^j \binom{\alpha}{j} [e_i(k-j)]^\lambda \leq \frac{(\mathcal{N}_i - 1)\rho_i}{(\Delta T)^{\alpha-1}}, \quad (\text{S13})$$

where $\mathcal{N}_i = \frac{\Gamma(L+1-\alpha)}{\Gamma(1-\alpha)\Gamma(L+1)} \geq 1$.

When $\mathbf{s}[\mathbf{z}(k)]$ converges, $\boldsymbol{\varrho}(k)$ also converges. For each element $\varrho_i(k)$ in $\boldsymbol{\varrho}(k)$, there exists a positive constant κ_i such that $|\varrho_i(k)| \leq \kappa_i$. Consequently, (S12) can be reformulated as

$$e_i(k+1) - e_i(k) + \frac{\xi_i}{(\Delta T)^{\alpha-1}} [e_i(k)]^\lambda \leq \kappa_i - \frac{(\mathcal{N}_i - 1)\xi_i \rho_i}{(\Delta T)^{\alpha-1}}. \quad (\text{S14})$$

By applying Lemma S3, $e_i(k)$ converges within a finite number of steps:

$$|e_i(k)| \leq \Lambda \cdot \max \left\{ \left[\frac{(\Delta T)^{\alpha-1} \kappa_i - (\mathcal{N}_i - 1)\xi_i \rho_i}{\xi_i} \right]^{\frac{1}{\lambda}}, ((\Delta T)^{1-\alpha} \xi_i)^{\frac{1}{1-\lambda}} \right\}. \quad (\text{S15})$$

where $\Lambda = 1 + \lambda^{\frac{\lambda}{1-\lambda}} - \lambda^{\frac{1}{1-\lambda}}$.

By applying Lemma S4, if the condition that $e_i(k)$ converges holds $\forall 1 \leq i \leq n$, then the error vector $\mathbf{e}(k)$ also converges. Therefore, $\mathbf{e}(k)$ converges whenever $\mathbf{s}[\mathbf{z}(k)]$ converges.

This completes the proof of Theorem 3.

Monte Carlo simulations

We conduct two sets of Monte Carlo simulations with different target-point ranges, each comprising 200 runs. During the simulations, the base satellite and each manipulator component are subject to 20% uncertainties in mass and inertia. In addition, each joint is subjected to an unknown disturbance torque $\Delta_m^i(t) = 0.01 \times (\sin(20t) + \cos(10t) - e^{-0.1t}) \text{ N} \cdot \text{m}$ ($i = 1, 2, \dots, 6$).

We define the DEN fitting error during the control process by

$$\mathbf{r}(k) = \mathbf{p}_e(k+1) - \mathbf{p}_e(k) - \boldsymbol{\Psi}(k)(\boldsymbol{\theta}(k+1) - \boldsymbol{\theta}(k)). \quad (\text{S16})$$

During the Monte Carlo simulations, we record the terminal end-effector tracking error $\|\mathbf{p}_{\text{ref}} - \mathbf{p}_e(T)\|_2$, as well as the maximum DEN fitting error over the simulation, defined by $r_{\text{max}} = \max \|\mathbf{r}(k)\|_2$.

In the first set of Monte Carlo simulations, the target point is specified as

$$\mathbf{p}_{\text{ref}} = \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix} \text{ m} + \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} \text{ m}, \quad (\text{S17})$$

where δx , δy , and δz are independently sampled from a uniform distribution over $(-0.1, 0.1)$.

In the second set of Monte Carlo simulations, the target point is defined in the same manner, except that δx , δy , and δz are independently sampled from a uniform distribution over $(-0.2, 0.2)$.

Figs. S1 and S2 present the results of the two sets of Monte Carlo simulations. Defining the success criterion as a terminal end-effector tracking error of less than 2×10^{-3} m, set 1 achieves a 100% success rate, whereas set 2 achieves 93.5%. An analysis of the maximum DEN fitting error reveals that substantial fitting inaccuracies during the simulation are the primary cause of failure. This issue stems from two main factors. First, the kinematic model of the FFSR is influenced by both its geometric configuration and dynamic factors, resulting in deviations when the system is subjected to external disturbances and model uncertainties. Second, in scenarios involving widely dispersed target points, the system is more likely to approach kinematic singularities, which degrades the DEN's fitting performance in out-of-distribution regions.

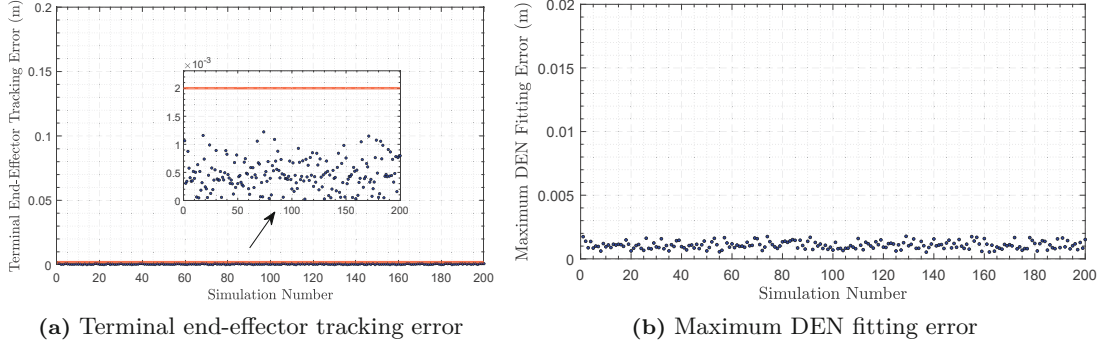


Fig. S1 Results of Monte Carlo simulation (set 1)

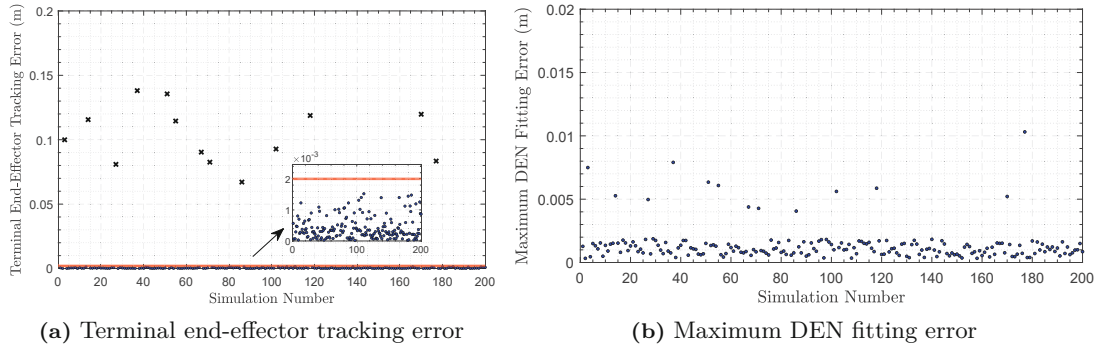


Fig. S2 Results of Monte Carlo simulation (set 2)

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