Mao-bin LU, Lu LIU, 2019. Leader-following consensus of second-order nonlinear multi-agent systems subject to disturbances. *Frontiers of Information Technology & Electronic Engineering*, 20(1):88-94. https://doi.org/10.1631/FITEE.1800611

Leader-following consensus of second-order nonlinear multi-agent systems subject to disturbances

Key words: Multi-agent systems; Leader-following consensus; Distributed control

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Motivation

1. Multi-agent systems have received considerable attention due to the broad applications of cooperative control in engineering.

2. Various cooperative control problems have been studied, such as consensus, formation, flocking, connectivity preservation, containment control, and cooperative output regulation.

3. Consensus is one of the most fundamental problems in cooperative control.

4. In some cases, the relative state of the systems, instead of the relative internal state, is available for feedback control.

Challenges of the problem

1. The nonlinear systems do not satisfy the global Lipschitz condition, and the initial condition of the systems is not in some prescribed compact set.

2. The nonlinear systems have different dynamics.

3. The nonlinear systems contain uncertainties.

4. Communication of the internal state cannot be conducted.

Problem formulation

Followers:

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = f_i(q_i, p_i, t) + d_i + u_i, \ i = 1, 2, \ldots, N, \end{cases} \quad (1)$$

where

$$f_i(q_i, p_i, t) = g_{i0}(q_i, p_i) + g_i(q_i, p_i)\theta_i$$

Leaders:

$$\begin{cases} \dot{v} = Sv, \\ d_i = D_i v, \\ q_0 = Fv, \end{cases} \quad (2)$$

Problem formulation

Digraph: $G = (V, E)$ where $V = \{0, 1, \ldots, N\}$ and $E \subseteq V \times V$.

Consensus problem:

Design a class of distributed control law of form

$$\begin{cases} u_i = \psi_i(\xi_i, p_i, q_i, \dot{q}_i, q_j - q_i, j \in N_i), \\ \dot{\xi}_i = \varphi_i(\xi_i, p_i, q_i, \dot{q}_i, q_j - q_i, j \in N_i), i = 1, 2, \ldots, N, \end{cases} \tag{3}$$

such that $\lim_{t \to \infty}(q_i(t) - q_0(t)) = 0, \lim_{t \to \infty}(p_i(t) - \dot{q}_0(t)) = 0.$ (4)

Main results

Adaptive distributed control law:

$$\begin{cases} \dot{\boldsymbol{q}}_i = \boldsymbol{p}_i, \\ \dot{\boldsymbol{p}}_i = -\boldsymbol{K}_i \boldsymbol{s}_i - \boldsymbol{g}_i(\boldsymbol{q}_i, \boldsymbol{p}_i)\tilde{\boldsymbol{\theta}}_i - \boldsymbol{D}_i \tilde{\boldsymbol{\eta}}_i + \ddot{\boldsymbol{q}}_{ri}, \\ \dot{\hat{\boldsymbol{\theta}}}_i = -\boldsymbol{\Lambda}_i \boldsymbol{g}_i^{\mathrm{T}}(\boldsymbol{q}_i, \boldsymbol{p}_i)\boldsymbol{s}_i, \\ \dot{\boldsymbol{\eta}}_i = \boldsymbol{S}\boldsymbol{\eta}_i + \boldsymbol{L}\sum_{j \in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i), \ i = 1, 2, \ldots, N, \end{cases} \tag{10}$$

where $\tilde{\boldsymbol{\eta}}_i = \boldsymbol{\eta}_i - \boldsymbol{v}$ and $\tilde{\boldsymbol{\theta}}_i = \hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i$, $i = 1, 2, \ldots, N$.

Main results

Assumption 1 Matrix $\boldsymbol{S}$ is marginally stable.
Assumption 2 Digraph G contains a directed spanning tree with node 0 as its root.

Theorem 1 Given Assumptions 1 and 2, leader-following consensus for the multi-agent system consisting of Eqs. (1) and (2) can be achieved by the adaptive distributed control law of the form (9).

Conclusions

The leader-following consensus problem for a class of heterogeneous second-order nonlinear uncertain multi-agent systems subject to disturbances has been studied.

An adaptive distributed control law, which depends on the relative state of the systems, has been proposed to solve the problem. The control law can be implemented when communication of the internal state is not allowed.