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Unified construction of two *n*-order circuit networks with diodes

Key words: Complex networks; Equivalent transform; Nonlinear difference equation; Equivalent resistance

Corresponding author: Zhizhong TAN
E-mail: tanz@ntu.edu.cn
ORCID: https://orcid.org/0000-0001-6068-3112

Motivation

1. The method of simulating problems by constructing circuit network models, which is widely used in physics, material science and other fields, has gradually become an effective method to solve theoretical and engineering technical problems.

2. The recursion-transform (RT) method is based on recursive technology and variable transformation technology, which is commonly used for studying all kinds of finite and infinite circuit networks.

3. Two different resistor networks are considered to be unified into a single network model. It is an interesting and novel aspect of the relevant research.

Method

The RT method is used to study the unified network model. It can be divided into four steps:

Step 1 is to build an equivalent circuit model according to equivalent idea.

Step 2 is to create a nonlinear difference equation model using Kirchhoff's laws.

Step 3 is to construct the method of equivalent transformation to obtain the general solution.

Step 4 is to discuss special situations and verify the correctness of the conclusion.

A unified model

The unified structure (Fig. 2) is divided into two structures (Fig. 3, Fig. 4)

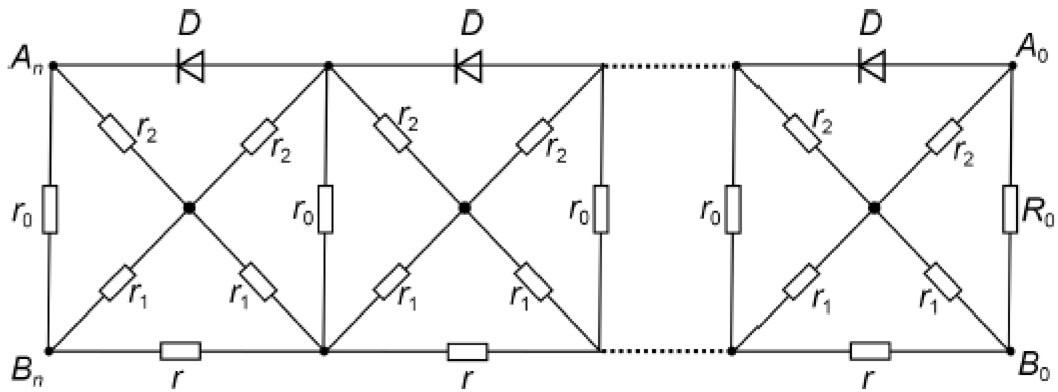

Fig. 2 A multi-functional n-order resistor network with X circuits and diodes, which contains five different resistor elements (including R_0) and ideal diodes in the upper boundary

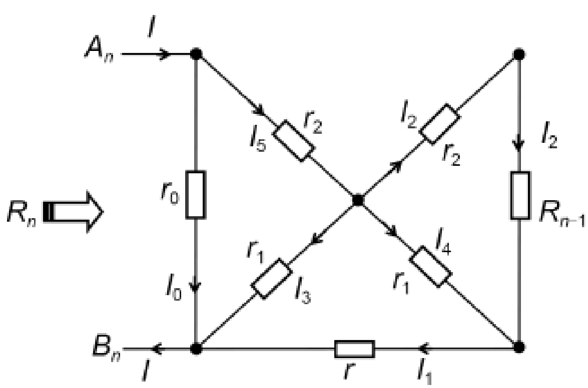

Fig. 3 Equivalent model of a two-terminal circuit network with triangular structure

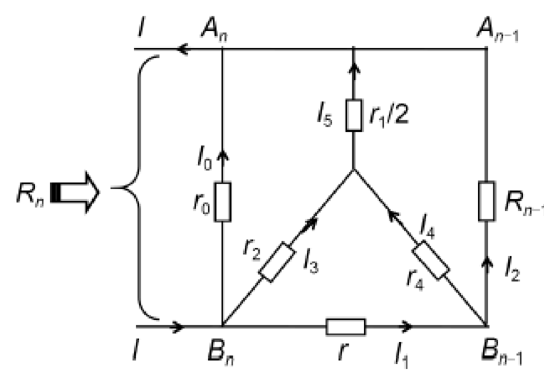

Fig. 4 Equivalent model of a two-terminal circuit network with Y circuits

The equivalent resistance values

Structure 1

$$R_n(A_n \to B_n) = \frac{(R_0-\lambda_1)F_{n+1}^{(1)} + F_{n+2}^{(1)}}{(R_0-\lambda_1)F_n^{(1)} + F_{n+1}^{(1)}} - \frac{a_1+r_0}{b_1}$$

$$F_n^{(1)} = \frac{\alpha_1^n - \beta_1^n}{\alpha_1 - \beta_1} \qquad \lambda_1 = \frac{r_0}{b_1}$$

$$\alpha_1 = \frac{1}{2b_1}\left(a_1 + 2r_0 + \sqrt{(a_1+4r_0b_1)a_1}\right)$$

$$\beta_1 = \frac{1}{2b_1}\left(a_1 + 2r_0 - \sqrt{(a_1+4r_0b_1)a_1}\right)$$

$$a_1 = \frac{r(r_2+r_1)+2r_1r_2}{(r+r_1)(r_1+r_2)+r_1r_2}(r_1+r_2)$$

$$b_1 = 1 + \frac{(r+2r_1)r_0}{(r+r_1)(r_1+r_2)+r_1r_2}$$

Structure 2

$$R_n(B_n \to A_n) = \frac{(R_0-\lambda_2)F_{n+1}^{(2)} + F_{n+2}^{(2)}}{(R_0-\lambda_2)F_n^{(2)} + F_{n+1}^{(2)}} - \frac{a_2+r_0}{b_2}$$

$$F_n^{(2)} = \frac{\alpha_2^n - \beta_2^n}{\alpha_2 - \beta_2} \qquad \lambda_2 = \frac{r_0}{b_2}$$

$$\alpha_2 = \frac{1}{2b_2}\left(a_2 + 2r_0 + \sqrt{(a_2+4r_0b_2)a_2}\right)$$

$$\beta_2 = \frac{1}{2b_2}\left(a_2 + 2r_0 - \sqrt{(a_2+4r_0b_2)a_2}\right)$$

$$a_2 = \frac{2rr_1(r_1+r_2)}{2(r_1+r_2)(r_1+r)-r_2r}$$

$$b_2 = 1 + \frac{2r_0(r+2r_1)}{2(r_1+r_2)(r_1+r)-r_2r}$$

The two strutures have a unified form of equivalent resistance values.

Special cases

Case 1. In the network of Fig. 2, when $r_1 = \infty$

$$R(A_n \to B_n) = \frac{(R_0 - r_0)F_{n+1}^{(1)} + F_{n+2}^{(1)}}{(R_0 - r_0)F_n^{(1)} + F_{n+1}^{(1)}} - \left(r + r_0 + 2r_2\right)$$

$$R(B_n \to A_n) = \frac{(R_0 - r_0)F_{n+1}^{(2)} + F_{n+2}^{(2)}}{(R_0 - r_0)F_n^{(2)} + F_{n+1}^{(2)}} - (r + r_0)$$

Case 2. In the network of Fig. 2, when $r_1 = 0$

$$R(A_n \to B_n) = \alpha_1 - r_2 = \frac{r_2 r_0}{r_2 + r_0} \qquad R(B_n \to A_n) = \frac{R_0 \alpha_2}{nR_0 + \alpha_2}$$

Case 3. In the network of Fig. 2, when $n = 0$

$$R_0(A_0 \to B_0) = (R_0 - \lambda_1) + F_2^{(1)} - \frac{a_1 + r_0}{b_1} = R_0$$

$$R_0(B_0 \to A_0) = (R_0 - \lambda_2) + F_2^{(2)} - \frac{a_2 + r_0}{b_2} = R_0$$

Case 4. In the network of Fig. 2, when $n = 1$

$$R_1(A_1 \to B_1) = \frac{R_0 + a_1}{b_1 R_0 + a_1 + r_0} r_0 \qquad R_1(B_1 \to A_1) = \frac{R_0 + a_2}{b_2 R_0 + a_2 + r_0} r_0$$

Conclusions

1. A unified *n*-order resistor network model with X circuits and diodes is proposed and is studied for the first.

2. The recursion-transform (RT) method is used to evaluate the equivalent resistance of this new resistor network. A unified difference model is established. The general solution of the unified difference equation is given.

3. Based on the general equivalent resistance, a series of special equivalent resistance conclusions are derived. It is also applicable to the study of complex impedance networks.

Xiaoyan LIN received her PhD degree in optical engineering from Southeast University, China, in 2014. Currently, She is an associate professor in Nantong University, China. Her research interests include solar cells and resistor network.

Zhizhong TAN received his Bachelor degree in physics from Yangzhou University, China, in 1991. Currently, He is a professor in Nantong University, China. His research interests include educational psychology, theoretical physics, matrix transform, and circuit networks. He won the IOP Royal Society Award for Top Cited Chinese Authors three times. He was included in the list of the world's top 2% scientists jointly published by Elsevier and Stanford University for three consecutive years (2020, 2021, 2022).