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Output feedback stabilizer design of Boolean networks based on network structure

Key words: Boolean networks; Output feedback stabilizer; Network structure; Semi-tensor product of matrices

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Motivation

1. Considering the traditional analysis on Boolean networks, the information of network structure is missing, such as nodes' connection and feedback loops.
2. Note that the dimension of the state transition matrix of Boolean networks will grow exponentially with the size of the network, which leads to a high computational complexity.
3. The existing pinning control methods for Boolean networks are difficult to implement for some large-scale Boolean networks.

Main idea

1. An output feedback pinning control is designed, which is partially imposed on the nodes of networks and is much easier to implement than in traditional design.
2. The computational complexity is dramatically reduced by analyzing the acyclic network structure (NS).
3. All the proposed algorithms are validated by biological simulations.

Method

1. The semi-tensor product of matrices is used to transform the logical dynamics of controlled nodes into algebraic representations.
2. The feedback arc set of the network structure of Boolean networks is used to achieve an acyclic structure, which will guarantee global stability.
3. Based on the feedback arc set, pinning controlled nodes and an output feedback stabilizer are designed.

Major results

Output feedback stabilizer design:

Original Boolean networks

$$\begin{cases} x_i(t+1) = f_i\left(x_1(t), x_2(t), \ldots, x_n(t)\right), & i \in [1, n], \\ y_j(t) = g_j\left(x_1(t), x_2(t), \ldots, x_n(t)\right), & j \in [1, p], \end{cases} \tag{1}$$

Network structure of Boolean networks

Definition 2 A logical function $f(x_1, x_2, \ldots, x_n) : \mathcal{D}^n \to \mathcal{D}$ is said to be dependent on variable x_i if there exists a tuple $\bar{x} \in \mathcal{D}^{n-1}$ such that $f(\bar{x}, x_i) \neq f(\bar{x}, \neg x_i)$, where $\bar{x} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$.

Definition 3 Consider a BN associated with a sequence of logical functions $f = (f_1, f_2, \ldots, f_n)$. The wiring digraph of a BN is a digraph denoted by $G = (V, E)$ with n vertices; that is, $V = \{x_1, x_2, \ldots, x_n\}$, where $x_1, x_2, \ldots, x_n$ are nodes of the network. An edge $x_i \to x_j$ exists in the wiring digraph $G = (V, E)$ if and only if function f_j is dependent on x_i.

Major results (Cont'd)

Find pinning controlled nodes using the feedback arc set:

Definition 6 (FAS (Bang-Jensen and Gutin, 2008)) An FAS is defined as a subset of edges containing at least one edge of every cycle in a directed network, and it is called a minimum FAS if its cardinality is the minimum. Therefore, the removal of the FAS renders the network acyclic.

Solve equations:

$$\boldsymbol{M}_{\oplus_i}\hat{\boldsymbol{H}}_i(\boldsymbol{I}_{2^n}\otimes\check{\boldsymbol{F}}_i)\Phi_{2^n}=\tilde{\boldsymbol{F}}_i,\quad i\in\Sigma_\tau, \tag{9a}$$

$$\hat{\boldsymbol{H}}_i=\boldsymbol{H}_i\boldsymbol{G}_i,\quad i\in\Sigma_\tau. \tag{9b}$$

An output feedback stabilizer design:

$$\begin{cases}\boldsymbol{x}_i(t+1)=\boldsymbol{u}_i(t)\oplus_i f_i\Big(\boldsymbol{x}_1(t),\boldsymbol{x}_2(t),\ldots,\boldsymbol{x}_n(t)\Big),\\ \qquad\qquad i\in\Sigma_\tau,\\ \boldsymbol{x}_j(t+1)=f_j\Big(\boldsymbol{x}_1(t),\boldsymbol{x}_2(t),\ldots,\boldsymbol{x}_n(t)\Big),\\ \qquad\qquad j\in N\backslash\Sigma_\tau,\\ \boldsymbol{u}_i(t)=h_i\Big(\boldsymbol{y}_1(t),\boldsymbol{y}_2(t),\ldots,\boldsymbol{y}_p(t)\Big),i\in\Sigma_\tau.\end{cases} \tag{14}$$

Major results (Cont'd)

Simulation results:

Pinning control on nodes x_1 and x_4:

$$\begin{cases} \boldsymbol{x}_1(t+1) = \boldsymbol{u}_1(t) \vee [\neg \boldsymbol{x}_2(t) \wedge \neg \boldsymbol{x}_4(t)], \\ \boldsymbol{u}_1(t) = \neg \boldsymbol{y}_2(t), \\ \boldsymbol{x}_4(t+1) = \boldsymbol{u}_4(t) \wedge \boldsymbol{x}_2(t), \\ \boldsymbol{u}_4(t) = \neg \boldsymbol{y}_1(t). \end{cases} \tag{22}$$

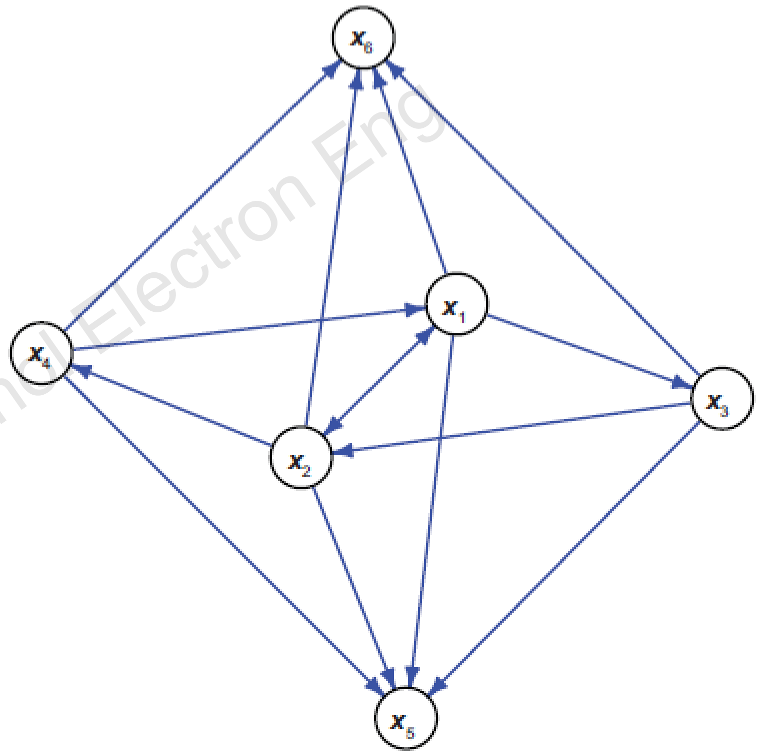

Fig. 2 Wiring digraph $G = (V, E)$ of Eq. (19)

Major results (Cont'd)

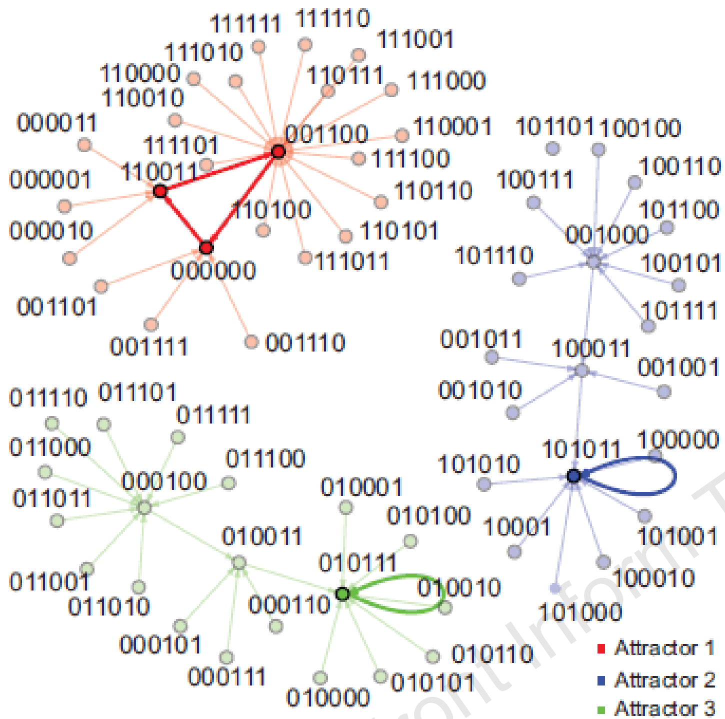

Fig. 3 State transition digraph of Eq. (19), which has three attractors including two steady states and one cyclic attractor with length three. Each node represents a state, and each arrow is a state transition. The label beside each node is the logical state variable encoded in the order $x_1, x_2, \ldots, x_6$. Colors mark different basins of attraction. Attractors are highlighted using bold lines. References to color refer to the online version of this figure

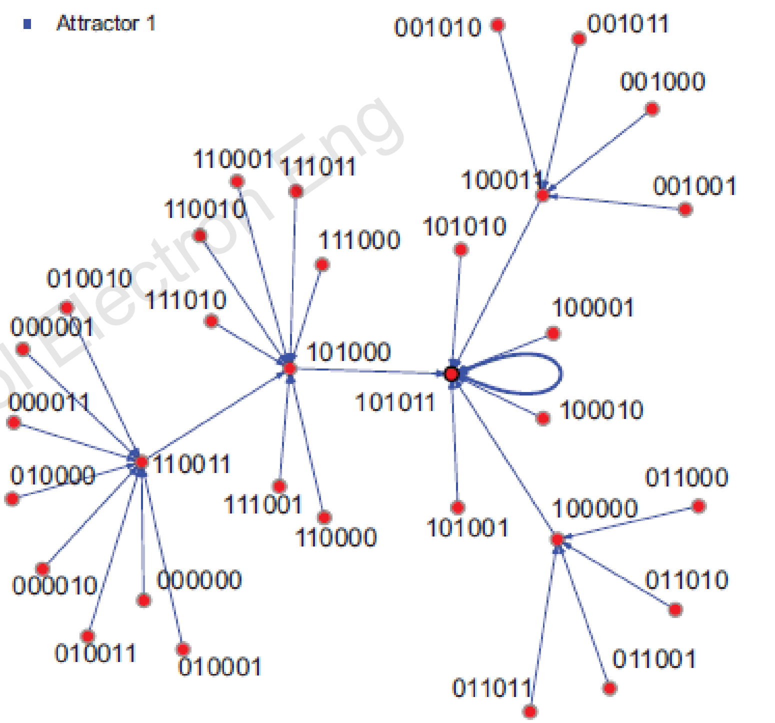

Fig. 4 State transition digraph of system (19) under the output feedback stabilizer (22), which has a unique steady state. Each node represents a state of the network, and each arrow is a state transition. The label beside each node is the logical state variable encoded in the order $x_1, x_2, \ldots, x_6$. Attractor is highlighted using bold lines

Conclusions

1. The network structure of Boolean networks has been used to analyze the global stability.
2. A novel output feedback pinning control has been proposed, where the controllers are imposed on partial nodes and are not designed by the state transition matrix of Boolean networks.
3. The output feedback stabilizer design presented provides a new way to design pinning control with low dimension and less calculation.