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A fractional-order multifunctional *n*-step honeycomb RLC circuit network

Key words: Honeycomb network; Equivalent transformation; Fractional differential equation; Impedance characteristics

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Motivation

- Resistor networks are important models in the fields of natural science and engineering technology. How to calculate the equivalent resistance between two nodes has been a classical problem in circuit theory and graph theory.

- A multifunctional *n*-step honeycomb network has not been studied before. We derive two new formulae for equivalent resistance in the resistor network and equivalent impedance in the LC network, which are in the fractional-order domain.

Main idea

- Propose a general model of a multifunctional *n*-step honeycomb network.
- Establish a fractional difference equation and construct an equivalent transformation method.
- Apply the resistance result to study the equivalent impedance of an *n*-step impedance LC network.

Method

1. Establish a fractional difference equation by Kirchhoff's law:

$$R_n = \frac{2rr_0 + br_0R_{n\text{-}1}}{(2r + r_0b) + dR_{n\text{-}1}}$$

2. Obtain the solution of the nonlinear difference equation using the variable substitution method:

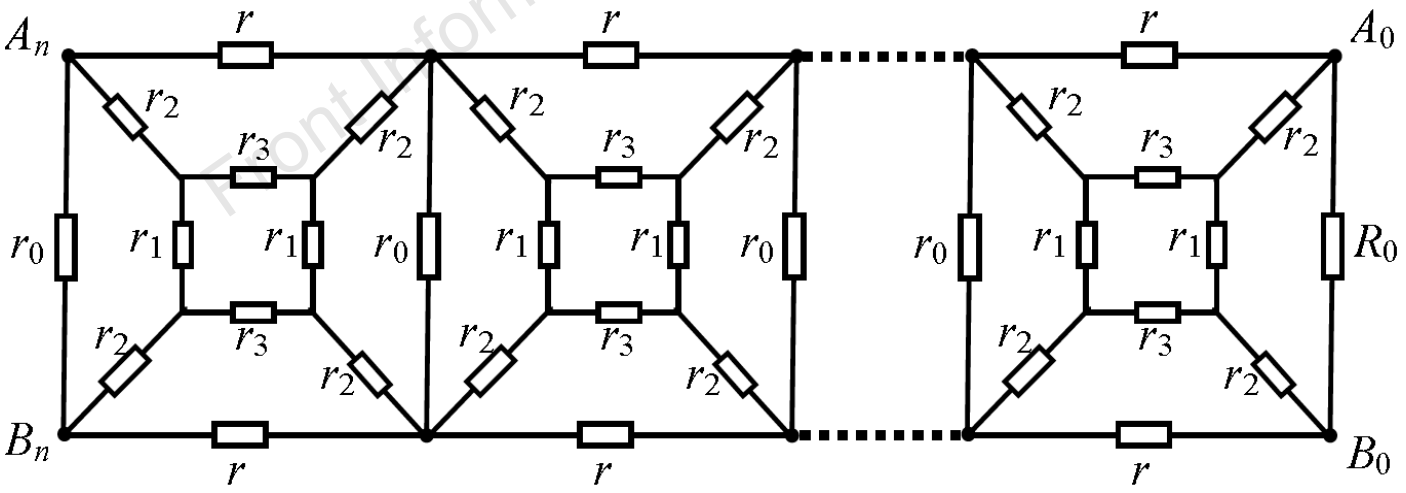
$$R_n = \frac{x_{n+1}}{x_n} - \frac{2r + r_0b}{d}$$

3. Apply the resistance result to study the equivalent impedance of an *n*-step impedance LC network.

Major results-1

- The general equivalent resistance between the two nodes can be expressed in a fractional form as the following:

$$R_{A_nB_n}(n) = \lambda r_0 - \alpha\beta\left(\frac{F_n + (R_0 - \lambda r_0)F_{n-1}}{F_{n+1} + (R_0 - \lambda r_0)F_n}\right)$$



Major results-2 (LC network)

Equivalent impedance of the LC network is

$$\frac{Z_{A_nB_n}}{r_0} = \lambda - \left(\frac{5x-3}{4x^2-11x+5}\right)^2 \left(\frac{\delta^n - \rho^n + (q-\lambda)(\delta^{n-1} - \rho^{n-1})}{\delta^{n+1} - \rho^{n+1} + (q-\lambda)(\delta^n - \rho^n)}\right)$$

where

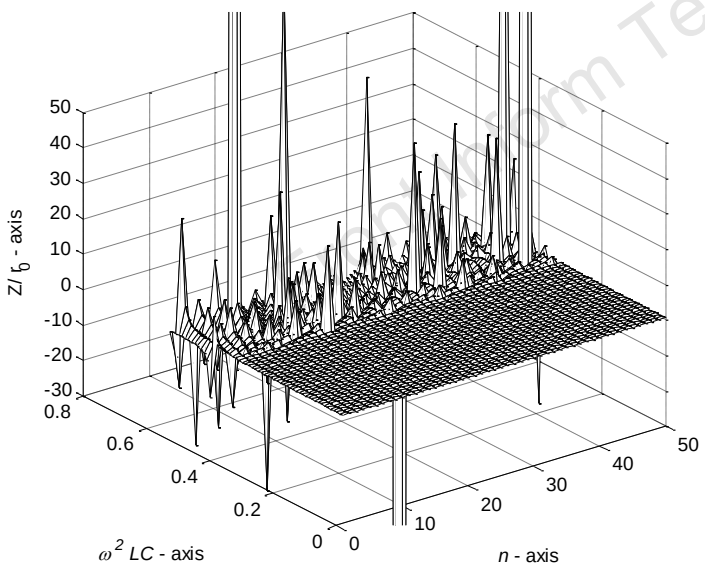
$$\delta = \frac{15x^2 - 20x + 6 - \sqrt{15x(3x-2)(5x^2-10x+4)}}{2(4x^2-11x+5)}$$

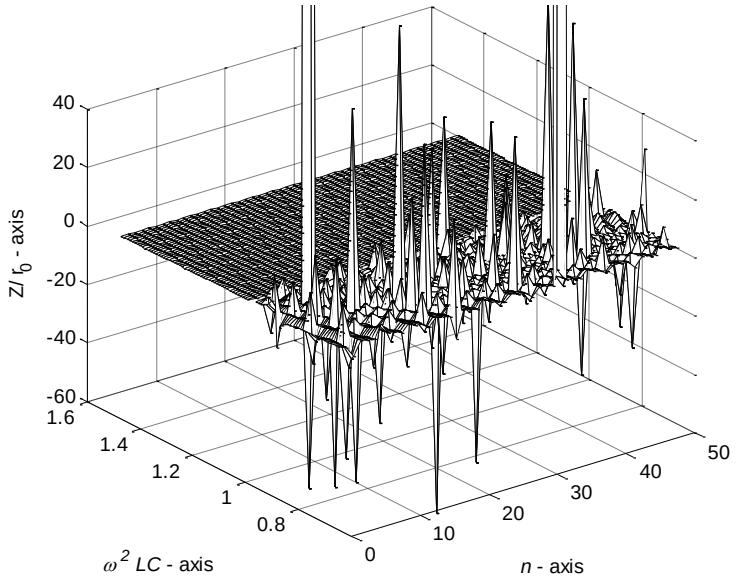
$$\rho = \frac{15x^2 - 20x + 6 + \sqrt{15x(3x-2)(5x^2-10x+4)}}{2(4x^2-11x+5)}$$

Major results-3 (figures)

- The 3D graphs showing the impedance characteristic

$$\frac{Z_{A_nB_n}}{r_0} = \lambda - \mu\left(\frac{\mu\sin(n\theta)+(q-\lambda)\sin(n-1)\theta}{\mu\sin(n+1)\theta+(q-\lambda)\sin(n\theta)}\right)$$





Conclusions

- We investigate a multifunctional *n*-step honeycomb network which has not been studied before. By adjusting the circuit parameters, such a network can be transformed into several different networks with a variety of functions.
- We derive two new formulae for equivalent resistance in the resistor network and equivalent impedance in the LC network, which are in the fractional-order domain.
- In practical applications, several interesting, special results are obtained. In particular, an *n*-step impedance LC network is discussed and many new characteristics of complex impedance have been found.