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Controllability of Boolean control networks with multiple time delays in both states and controls

Key words: Boolean control networks; Semi-tensor product of matrices; Controllability; Time delay

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Motivation

1. Boolean networks (BNs), are an effective tool in modeling, analyzing, and simulating genetic regulatory networks (GRNs).
2. The controllability of Boolean control networks (BCNs) with multiple time delays in both states and controls has not been studied yet.
3. How to construct controllability matrices for BCNs with multiple time delays in both states and controls is an interesting issue. Furthermore, whether there is a concise criterion for checking the controllability of BCNs with multiple time delays in both states and controls is an open issue.

Method

Based on the algebraic state space representation framework of BCNs, the controllability problem of BCNs with multiple time delays in controls is solved.

$$x(t+1) = Lu(t)u(t-1)\cdots u(t-\mu)x(t). \quad (6)$$

Theorem 1 BCN (6) is controllable if and only if $\mathcal{M} > 0$.

$$x(t+1) = Lu(t)x(t)x(t-1)\cdots x(t-\lambda). \quad (13)$$

Theorem 2 Consider BCN (13). Given the initial state $x_0 = \delta^{\beta}_{2^{(\lambda+1)n}}$ and destination state $x_{\mathrm{d}} = \delta^{\alpha}_{2^n}$, there exists a control sequence $\{u(t)\}_{t=0}^{s-1}$ such that $x(s) = x_{\mathrm{d}}$ can be reached from x_0 if and only if $[(I_{2^n} \otimes 1^{\mathrm{T}}_{2^{\lambda n}})\hat{\mathcal{C}}]_{\alpha\beta} > 0$.

Theorem 3 BCN (13) is controllable if and only if $(I_{2^n} \otimes 1^{\mathrm{T}}_{2^{\lambda n}})\hat{\mathcal{C}} > 0$.

Method

Based on the algebraic state space representation framework of BCNs constructed by the semi-tensor product (STP), the controllability problem of BCNs with multiple time delays in both states and controls is solved well.

$$\begin{aligned} \boldsymbol{x}(t+1) =& \boldsymbol{L}\boldsymbol{u}(t)\boldsymbol{u}(t-1)\cdots\boldsymbol{u}(t-\mu) \\ & \cdot \boldsymbol{x}(t)\boldsymbol{x}(t-1)\cdots\boldsymbol{x}(t-\lambda), \end{aligned}$$

where $\boldsymbol{L} \in \mathcal{L}_{2^n \times 2^{(\mu+1)m+(\lambda+1)n}}$.

Theorem 4 BCN (4) is controllable if and only if $\left(\boldsymbol{I}_{2^n} \otimes \mathbf{1}^{\mathrm{T}}_{2^{\lambda n}}\right) \hat{\boldsymbol{\mathcal{M}}} > 0$.

Conclusions

1. The controllability problem of BCNs with multiple time delays in both states and controls has been studied using the STP of matrices, which is transformed into two problems: the controllability problem of BCNs with multiple time delays in controls and the controllability problem of BCNs with multiple time delays in states.
2. For these two controllability problems mentioned above, necessary and sufficient conditions have been given, and subsequently, based on them, a controllability matrix and a necessary and sufficient condition have been proposed for the controllability of BCNs with multiple time delays in both states and controls.

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