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Causality fields in nonlinear causal effect analysis

Key words: Nonlinear causal effect; Causality field; z-specific causal effect; Positive causality; Negative causality; Null causality

Corresponding authors: Li LIU, Jiaoyun YANG

E-mail: dcsluli@cqu.edu.cn; jiaoyun@hfut.edu.cn;

 ORCID: <https://orcid.org/0000-0002-4776-5292>;
<https://orcid.org/0000-0002-0233-590X>

Motivation

1. Given the cause variable X , effect variable Y , variable Z that influences X and Y , and exogenous variable U , we can obtain the structural causal equation (SCE):
$$Y = f(X, Z, U).$$
2. The average causal effect (ACE) for discrete variables is:

$$\begin{aligned} \text{ACE}(X \rightarrow Y) &= \{E(Y|\text{do}(X=x)) - E(Y|\text{do}(X=x'))\} / (x - x') \\ &= E\{E_Z[f(x, Z, U) - f(x', Z, U)]\} / (x - x'). \end{aligned}$$

Motivation (Cont'd)

3. An Example

Table 1 The effect of a drug considering the time

Time	With or without drug	Effect
Daytime ($Z=1$)	With ($X=1$)	Recover ($Y=1$)
	Without ($X=0$)	Unchanged ($Y=0$)
Night ($Z=0$)	With ($X=1$)	Worsen ($Y=-1$)
	Without ($X=0$)	Unchanged ($Y=0$)

for $Z = 0$, $ACE|_{Z=1} = E(Y|do(X = 1), Z = 1) - E(Y|do(X = 0), Z = 1) = 1$

for $Z = 1$, $ACE|_{Z=0} = E(Y|do(X = 1), Z = 0) - E(Y|do(X = 0), Z = 0) = -1$

➤ The causal effect of X on Y is fluctuant and related to Z

4. Nonlinear causality

if X does not appear in the derivative $\frac{\partial}{\partial X} f(X, Z, U)$, the causal relation is quasi-linear.

if X appears in the derivative $\frac{\partial}{\partial X} f(X, Z, U)$, the causal relation is strong nonlinear.

Causality fields

1. Consists of the following three types of causal effect of X on Y

□ **Positive causality:** X causes a homodromous change in Y , denoted as Ω^+

□ **Negative causality:** X causes an antagonistic change in Y , denoted as Ω^-

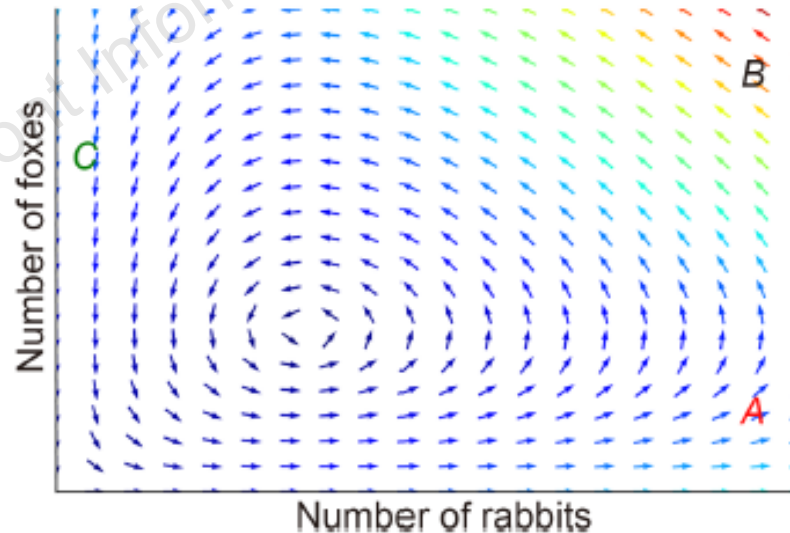
□ **Null causality:** X causes no change in Y , denoted as Ω^0

$$\left\{ \begin{array}{l} \Omega^+ = \left\{ (x, z, u) \mid \frac{\partial}{\partial X} f(X, Z, U) > 0 \right\}, \\ \Omega^- = \left\{ (x, z, u) \mid \frac{\partial}{\partial X} f(X, Z, U) < 0 \right\}, \\ \Omega^0 = \left\{ (x, z, u) \mid \frac{\partial}{\partial X} f(X, Z, U) = 0 \right\}, \end{array} \right.$$

Causality fields (Cont'd)

2. An Example: the competition-symbiotic relationships between rabbits and foxes

- ❑ We can observe that rabbits and foxes have competitive as well as symbiotic relationships
- ❑ area A , ecological mutualism model
- ❑ area B , ecological competition model
- ❑ area C , ecological total-loss model



Nonlinear causality computation

1. Three key assumptions that enable causal effect estimation

- Stable unit treatment value assumption (SUTVA)
- Ignorability
- Positivity

2. With the above assumptions, we can calculate the ACE

$$ACE = E[E_z(Y^{X=x}) - E_z(Y^{X=x'})]$$

3. Two ways for nonlinear causality computation when there are cases where the assumptions do not hold.

- SCE is more intuitive while considering estimating functions rather than probability distributions

$$X_i = f_i(PA_i, U_i)$$

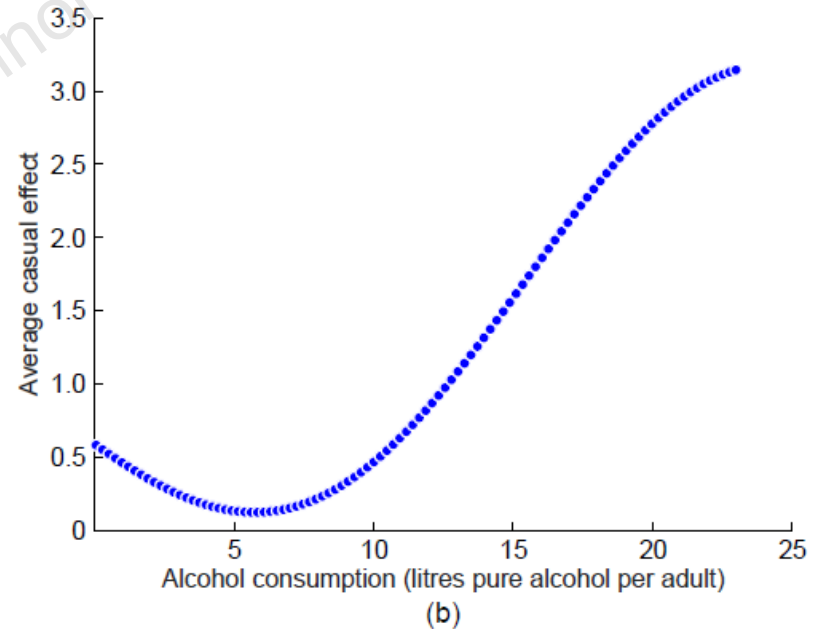
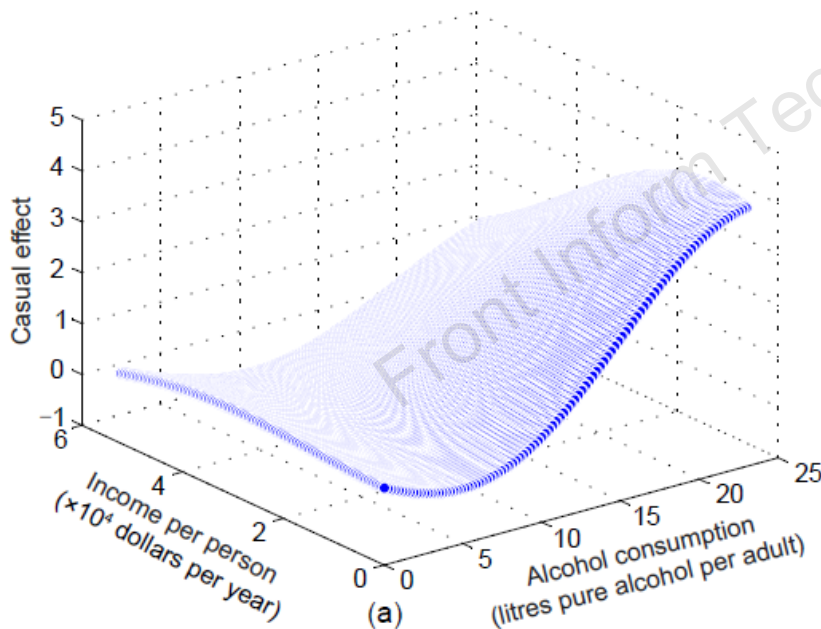
- Causal representation learns a latent disentangled representation

$$g: \{x_1, x_2, \dots, x_n\} \rightarrow \{w_1, w_2, \dots, w_n\}$$

Causality field examples

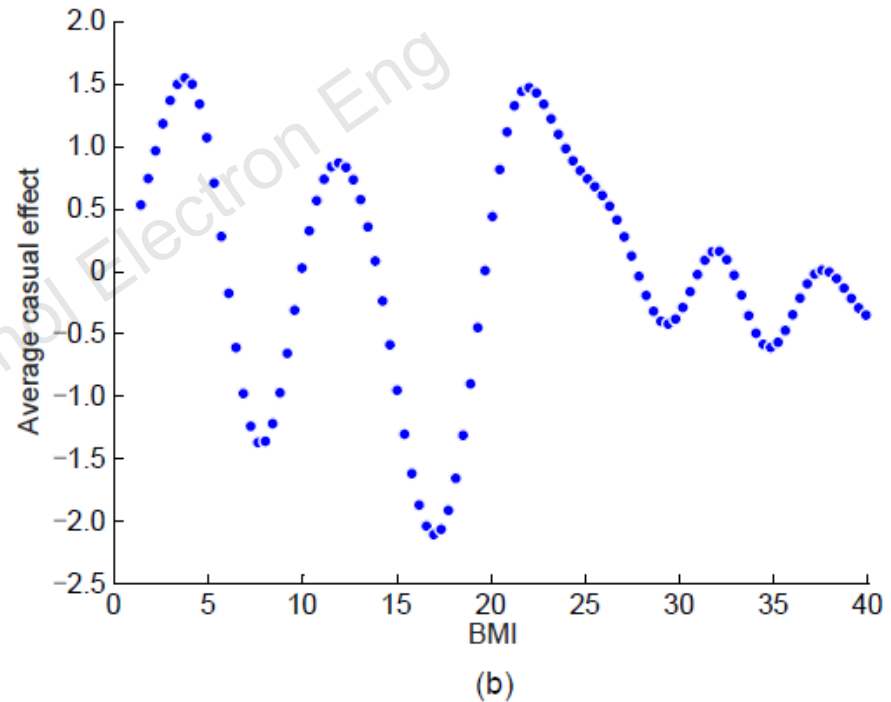
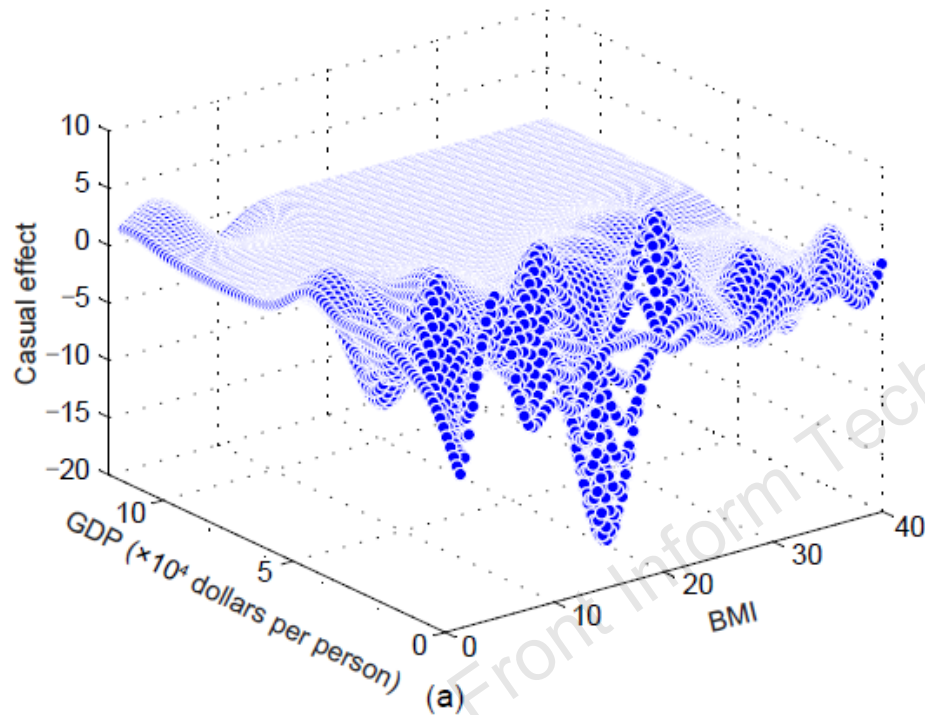
1. three datasets from Kaggle (<https://www.kaggle.com/>)

- The alcohol consumption dataset,
- The life expectancy (WHO) dataset,
- The diabetes dataset.



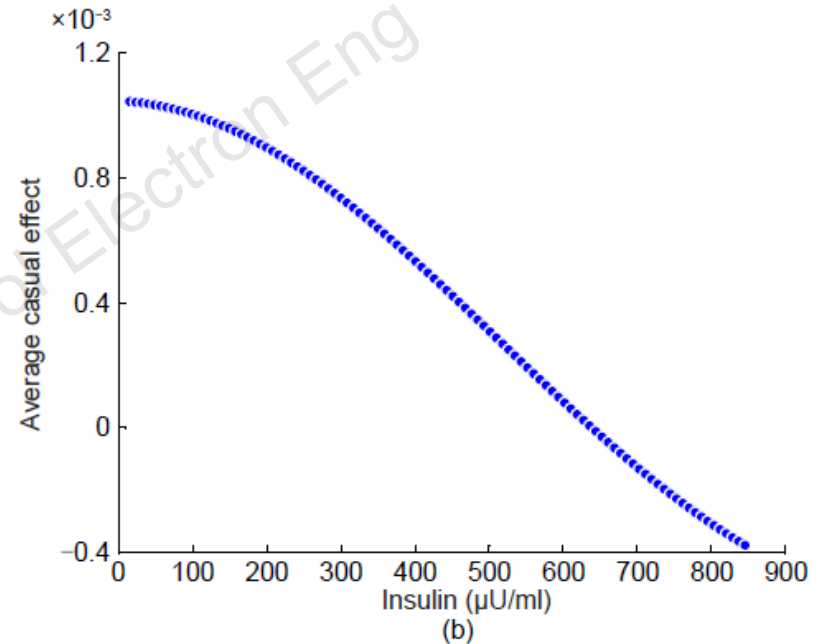
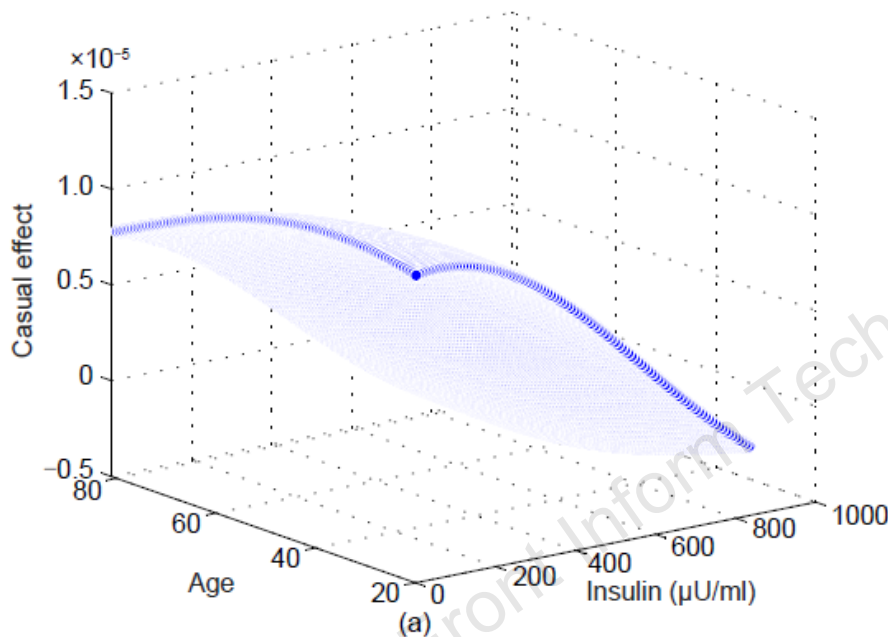
The causal effect of alcohol consumption (X) on suicide rate (Y) with income per person (Z) as the confounding variable

Causality field examples (Cont'd)



The causal effect of BMI (X) on life expectancy (Y) with GDP (Z) as the confounding variable

Causality field examples (Cont'd)



The causal effect of insulin (X) on diabetes (Y) with age (Z) as the confounding variable

Conclusions

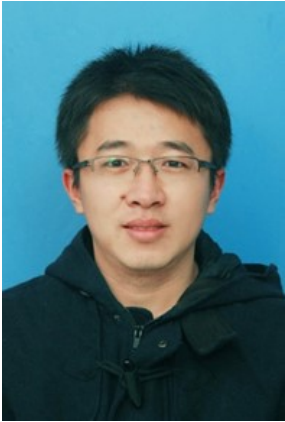
1. The problems of nonlinear causal effect calculation and the causality field are discussed.
2. Causality fields are formulated.
3. Methods for calculating nonlinear causal effects are introduced.
4. Three examples are used to illustrate causality fields.



Aiguo WANG is currently a Distinguished Researcher at Foshan University. Wang received his PhD degree in Computer Applied Technology from the Hefei University of Technology in 2015. His current research interests are in machine learning, data mining, and pervasive computing. Wang has published widely in conferences and journals with more than 50 peer-reviewed publications.



Li LIU is a professor at Chongqing University. He is also serving as a Senior Research Fellow of School of Computing at the National University of Singapore. Liu received his PhD degree in Computer Science from the Université Paris-Sud XI in 2008. His research interests are in pattern recognition, data analysis, and their applications on human behaviors. He aims to contribute in interdisciplinary research of computer science and human related disciplines. Li has published widely in conferences and journals with more than 150 peer-reviewed publications. Li has been the Principal Investigator of several funded projects from government and industry.



Jiaoyun YANG received his BS degree and PhD degree of computer science from University of Science and Technology of China, Hefei, in 2009 and 2014, respectively. He is an associate professor in the School of Computer Science and Information Engineering, Hefei University of Technology, Hefei, China. His research interests include heuristic search, machine learning and health computing. He is a member of CCF.



Lian LI received his BS degree of mathematics from Northwestern normal University, Lanzhou, in 1976, and his Master degree of mathematics from Lanzhou University, Lanzhou, in 1982. He is a professor in the School of Computer Science and Information Engineering, Hefei University of Technology, Hefei, China. His research interests include computational learning theory, and machine learning. He is a member of CCF.