Junwei WU, Qiong HUA, Hui XU, Hanqing YANG, Zhengxing WANG, Qiang CHENG, Tie Jun CUI, 2023. Realizing complex beams via amplitude-phase digital coding metasurfaces and semidefinite relaxation optimization. *Frontiers of Information Technology & Electronic Engineering*, 24(12):1708-1716. https://doi.org/10.1631/FITEE.2300146



# Realizing complex beams via amplitude-phase digital coding metasurfaces and semidefinite relaxation optimization

**Key words:** Antenna beams; Amplitude-phase coding metasurface; Far-field synthesis; Semidefinite relaxation

Corresponding author: Qiang CHENG, Tie Jun CUI
E-mail: qiangcheng@seu.edu.cn (Qiang CHENG), tjcui@seu.edu.cn (Tie Jun CUI)
ORCID: https://orcid.org/0000-0002-2442-8357 (Qiang CHENG), https://orcid.org/0000-0002-5862-1497 (Tie Jun CUI)

# Motivation

1. Complex beams play important roles in wireless communications, radar, and satellites, and have attracted great interest in recent years.
2. The design of complex beams is an array synthesis problem, which aims to find the excitation vector of the array.
3. Analytical approaches have fast speed, but they have limited applications, for example, suitable only for linear arrays with equal spacing and few elements.
4. Stochastic optimization methods have been widely used, but they feature slow convergence, extensive computations, or even prohibitive costs.

# Method

The pattern of array can be written as

$$|G(\theta)|^2 = \boldsymbol{x}^{\mathrm{T}}\overline{\boldsymbol{U}}^{\mathrm{T}}\overline{\boldsymbol{U}}\boldsymbol{x},$$

where

$$\overline{\boldsymbol{U}} = [\boldsymbol{a} \quad \boldsymbol{b}]^{\mathrm{T}}.$$

$$\begin{aligned}\boldsymbol{a}(\theta) =& [R(v_1), R(v_2), \ldots, R(v_N),\\ & -I(v_1), -I(v_2), \ldots, -I(v_N)],\end{aligned} \qquad \begin{aligned}\boldsymbol{b}(\theta) =& [I(v_1), I(v_2), \ldots, I(v_N),\\ & R(v_1), R(v_2), \ldots, R(v_N)],\end{aligned}$$

and

$$\begin{aligned}\boldsymbol{x} =& [R(w_1), R(w_2), \ldots, R(w_N),\\ & I(w_1), I(w_2), \ldots, I(w_N)]^{\mathrm{T}},\end{aligned}$$

$$\boldsymbol{v}(\theta) = \left[\mathrm{e}^{\mathrm{j}kx_1\sin\theta}, \mathrm{e}^{\mathrm{j}kx_2\sin\theta}, \ldots, \mathrm{e}^{\mathrm{j}kx_N\sin\theta}\right]^{\mathrm{T}},$$

where $w_n$ represents the weighting factor of the $n^{\text{th}}$ elements, and *R* and *I* stand for the real and image parts of a number, respectively.

# Method

Taking the cosecant and flat-top beams as examples, the problems are

$$
\begin{aligned}
&\text{find } \boldsymbol{w} \\
&\text{s.t. } l(\theta_i) \le |G(\theta_i)|^2 \le u(\theta_i), \\
&\qquad\qquad\qquad \theta_i \in \mathrm{ML}\ (i = 1, 2, \ldots, I), \\
&|G(\theta_j)|^2 \le \rho(\theta_j), \quad \theta_j \in \mathrm{SL}\ (j = 1, 2, \ldots, J).
\end{aligned}
$$

We transform the above problem into the following form:

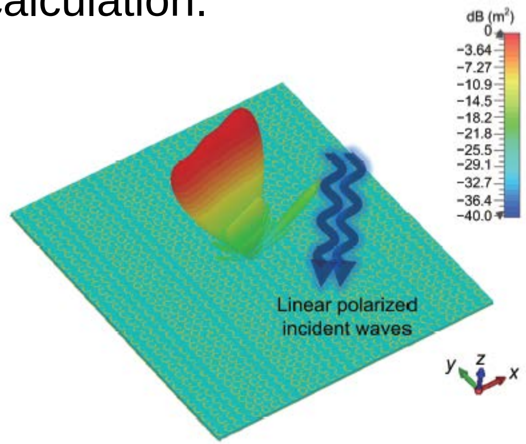
$$
\begin{aligned}
&\min_{\overline{\boldsymbol{X}}^k \in S^M} \zeta \operatorname{tr}\left(\left(\overline{\boldsymbol{X}}^{k-1} + \delta \overline{\boldsymbol{I}}\right)^{-1} \overline{\boldsymbol{X}}^k\right) \\
&\text{s.t. } l(\theta_i) \le \operatorname{tr}\left(\overline{\boldsymbol{P}}_i \overline{\boldsymbol{X}}^k\right) \le u(\theta_i), \\
&\qquad\qquad\qquad \theta_i \in \mathrm{ML}\ (i = 1, 2, \ldots, I), \\
&\operatorname{tr}\left(\overline{\boldsymbol{P}}_j \overline{\boldsymbol{X}}^k\right) \le \rho(\theta_j), \theta_j \in \mathrm{SL}\ (j = 1, 2, \ldots, J), \\
&\overline{\boldsymbol{X}}^k \ge 0.
\end{aligned}
$$

It is a semidefinite relaxation (SDR), and the rank-one constraint is realized by the rank minimization solution. The above form can be solved by convex optimization, which is fast and efficient.

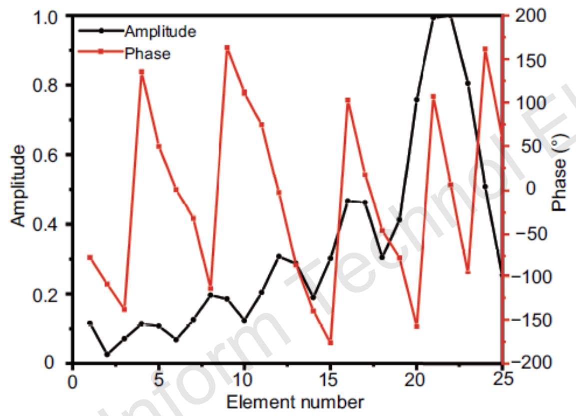
# Method

We design a 25*25 element array whose weighting factors are determined by the SDR method to give cosecant beam. The optimization time is 27.7 s and the memory consumption is 4.8 MB, being fast and efficient.

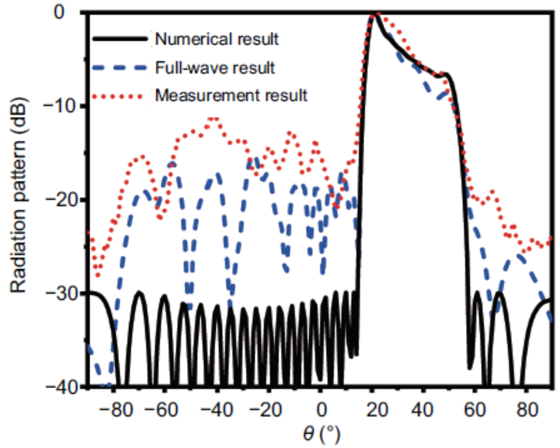
We further implement the array by using an amplitude-and-phase coding metasurface composed of C-shaped elements, and measured the sample in microwave anechoic chamber. The results are in good agreement with the calculation.



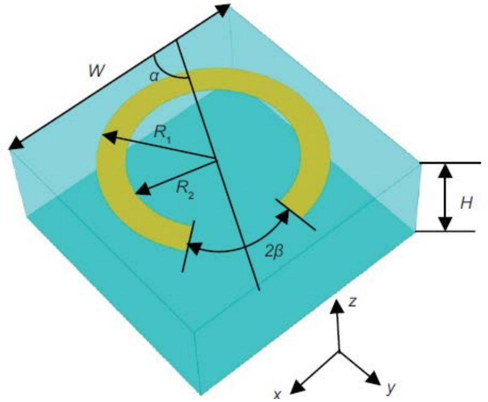

Schematic illustration



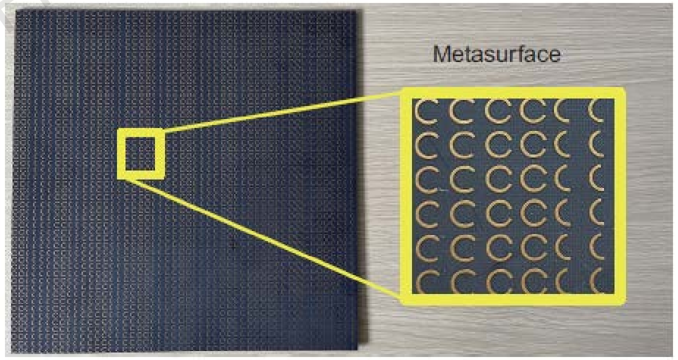

Weighting factors of the array



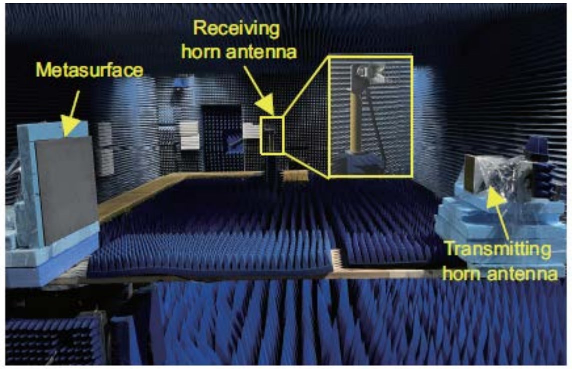

The results




The C-shaped element



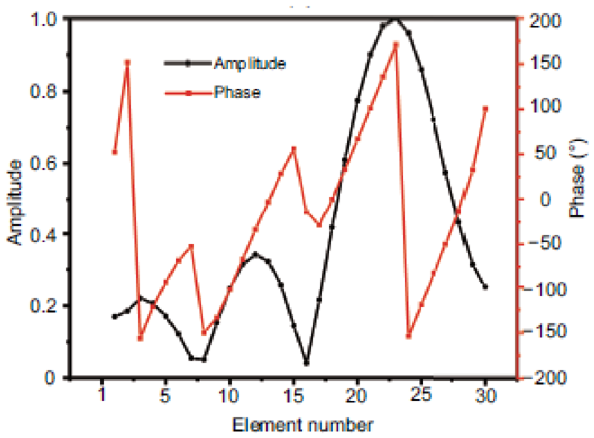

The fabricated metasurface



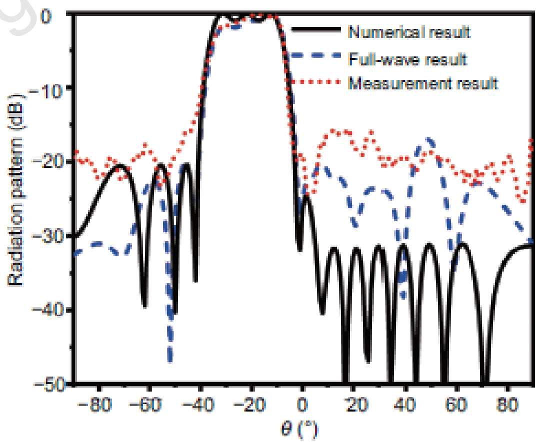

Measurement scenario

# Method

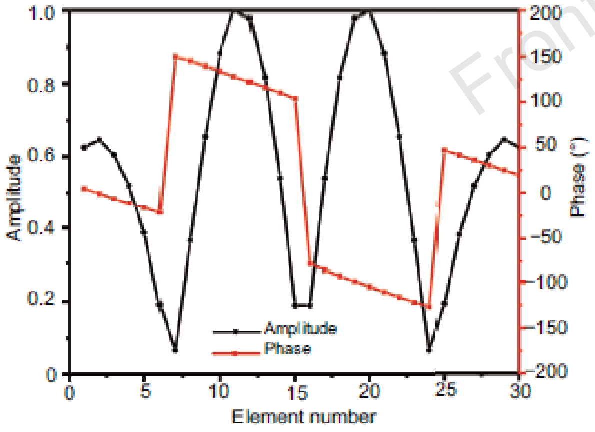
We further use the method to design arrays giving flat-top and double beams, and verify them by experimental measurements. The optimization times are 31.9 and 15.3 s, respectively, and the memory consumptions are 46.2 and 7.5MB, respectively. The consumptions are quite low and attractive for embedded implementation.



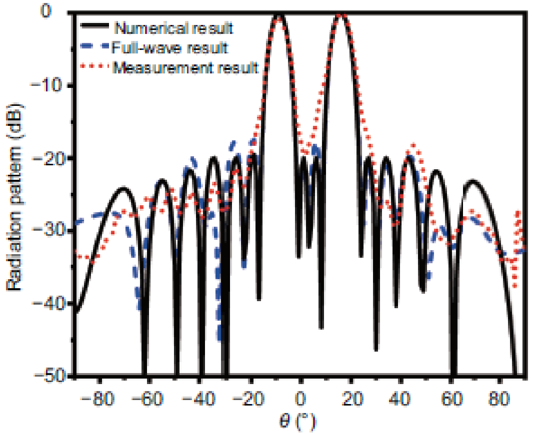

Weighting factors of the array for giving flat-top beam




The results of flat-top beam




Weighting factors of the array for giving double beams




The results of double beams

# Conclusions

1. We presented a fast and efficient approach to realize complex beams by using semidefinite relaxation (SDR) optimization.
2. We applied the approach in designing complex beam patterns with cosecant, flat-top, and double shapes based on amplitude-phase coding metasurfaces.
3. We verified the designs by both full-wave simulations and experimental measurements in microwave anechoic.

Junwei WU received the B.S. degree in electrical engineering from Shandong University, Jinan, China, in 2008, and the Ph.D. degree in radio physics from Wuhan University, Wuhan, China, in 2015. From 2012 to 2014, he was a visiting scholar at the Center for Computational Electromagnetics, University of Illinois at Urbana–Champaign, Champaign, IL, USA. He is currently an associate professor with Southeast University, Nanjing, China, where he is also with the State Key Laboratory of Millimeter Waves. His research interests include numerical methods in electromagnetics, antennas and metamaterials, and electromagnetic information theory.

Qiang CHENG received the B.S. and M.S. degrees from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2001 and 2004, respectively, and the Ph.D. degree from Southeast University, Nanjing, in 2008. In 2008, he joined the State Key Laboratory of Millimeter Waves, Southeast University, where he was involved in the development of metamaterials and metadevices. He is currently a full professor with the Radio Department, Southeast University. He leads a group of Ph.D. students and master’s students in the areas of metamaterials, tunable microwaves circuits, microwave imaging, and terahertz systems. He has authored or coauthored more than 100 publications, with citation over 2000 times.