

Daquan LI, Weigang SUN, Hongxiang HU, 2023. Impact of distance between two hubs on the network coherence of tree networks. *Frontiers of Information Technology & Electronic Engineering*, 24(9):1349-1356.

<https://doi.org/10.1631/FITEE.2200400>

Impact of distance between two hubs on the network coherence of tree networks

Key words: Consensus; Coherence; Distance; Average path length

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Motivation

- Propose a method of obtaining exact solutions of network coherence quantified by the Laplacian spectrum for a family of trees with two hubs.
- Study the impact of distance and difference in the degrees between two hubs on network coherence.
- Reveal the relationship between network coherence and average path length.

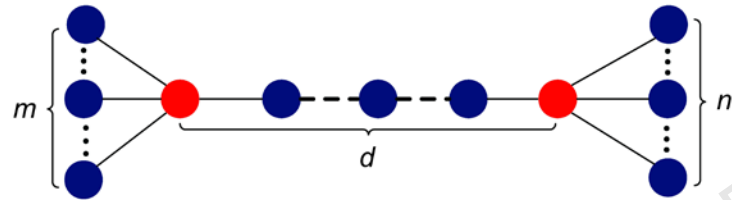
Main idea

- We choose a family of tree networks with two hubs controlled by two parameters.
- We obtain exact expressions of network coherence regarding the distance to investigate the role of this distance.
- We apply the distance to bridge network coherence and average path length and reveal their relationship.

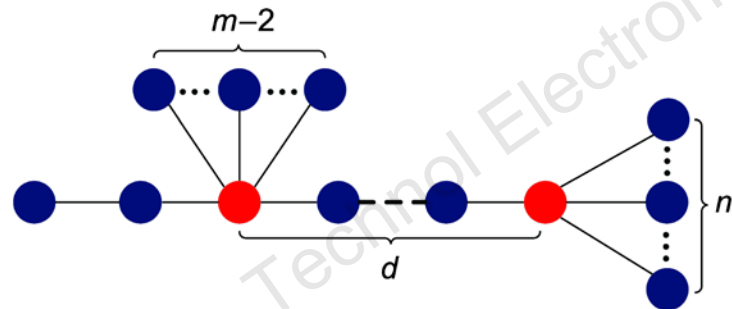
Method

- We use the tree's regular structure and matrix theory to obtain the analytic expressions of network coherence.
- We apply theoretical analysis and Matlab simulation to discuss the influence of distance between two hubs and network parameters on coherence.
- We numerically calculate network coherence and average path length to find the relationship between them.

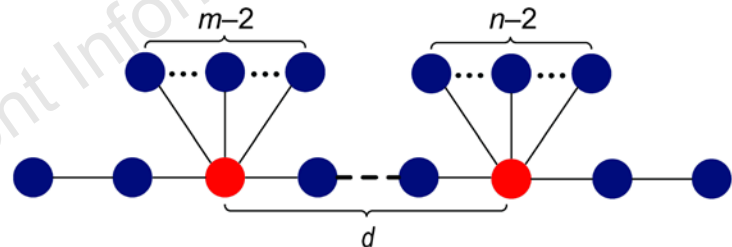
Method



(a)



(b)



(c)

Fig. 1 A family of tree networks with the same distance d between two hubs: (a) T_1 ; (b) T_2 ; (c) T_3 . The parameters m and n control the tree's asymmetry, and the network size is $N = m + n + d + 1$

Major results

Exact expression of network coherence regarding distance d :

$$\begin{aligned} H_1 &= \frac{1}{2N} \sum_{i=2}^N \frac{1}{\lambda_i} = -\frac{1}{2N} \frac{\overline{P}_1(1)}{\overline{P}_1(0)} \\ &= \frac{\phi_1 - d(m - \frac{N-d-1}{2})^2}{2N^2}, \end{aligned}$$

where

$$\begin{aligned} \phi_1 &= \frac{1}{6}(d-2)(d-1)d + \frac{1}{2}(d-1)d(N-d+1) \\ &\quad + \frac{1}{4}d(N-d-1)^2 \\ &\quad + (N-d)(N+d-1) - (N-1). \end{aligned}$$

Major results

1. The network coherence is better with a smaller distance.

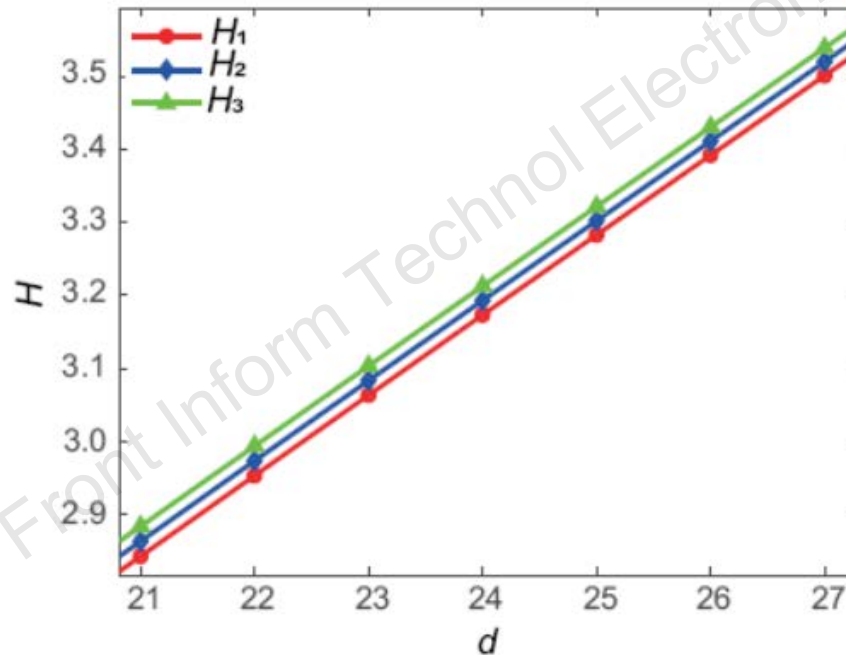


Fig. 2 Distributions of coherences H_1 , H_2 , and H_3 against distance d with $m = 30$ and $n = 20$

Major results

2. The larger the difference between the degrees of the two hubs, the higher the coherence.

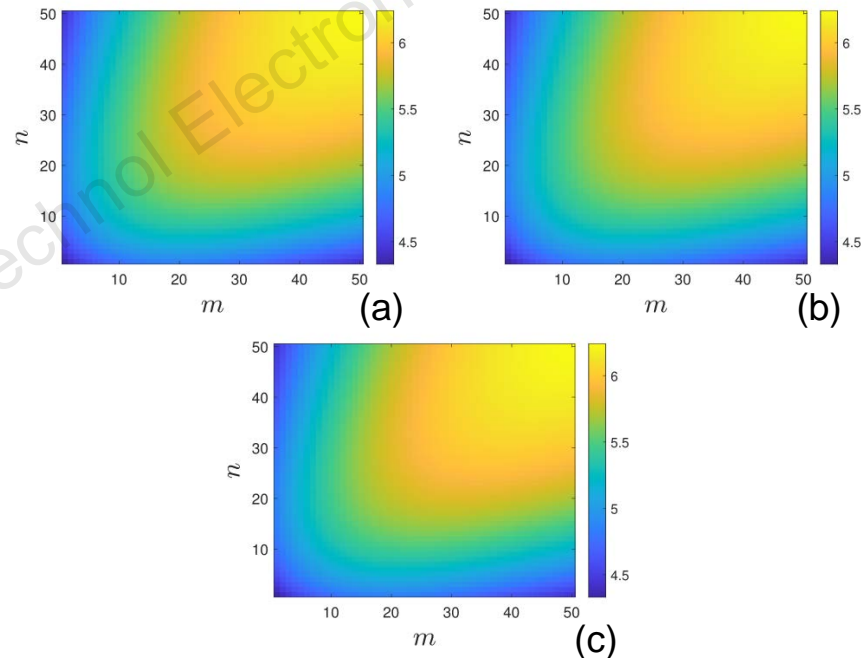
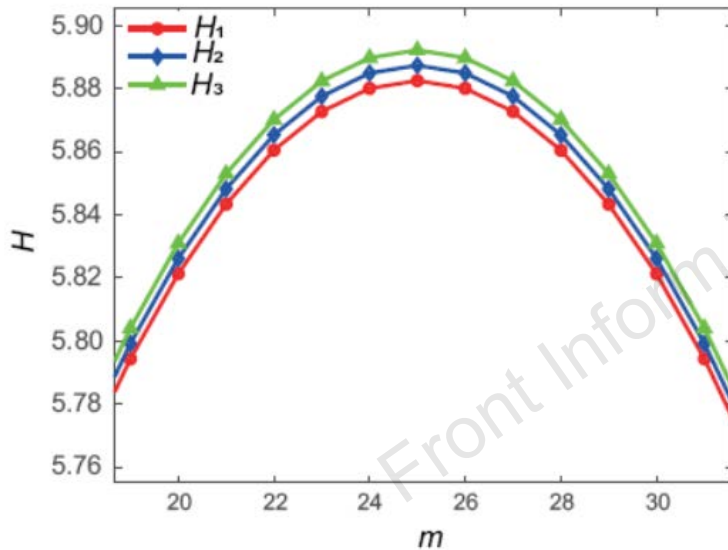


Fig. 3 Distributions of coherences H_1 , H_2 , and H_3 against parameter m with $N = 100$, $d = 49$, and $n = 50 - m$

Fig. 4 Phase diagrams for coherences H_1 (a), H_2 (b), and H_3 (c) in a two-dimensional space (m, n) with $d = 49$

Major results

3. Network coherence and average path length behave linearly. When the average path length becomes smaller, the tree displays better coherence.

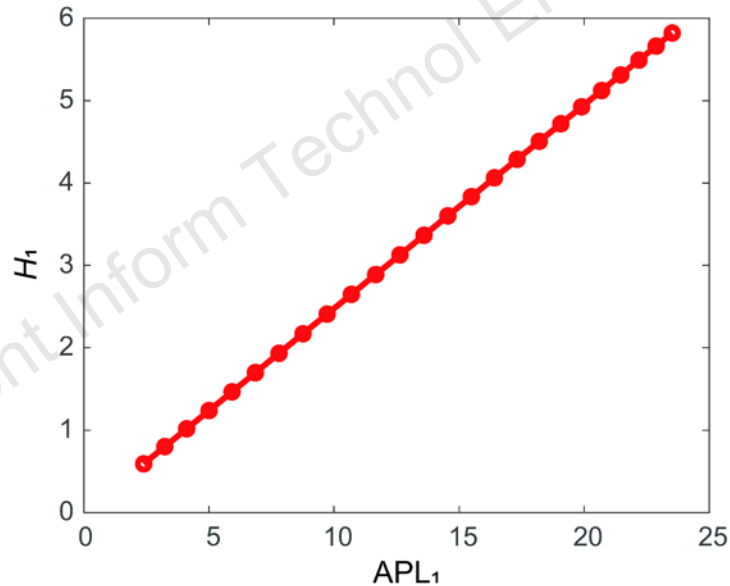


Fig. 5 Relationship between coherence H_1 and average path length APL_1 with $N = 100, m = 30$, and $n = 69 - d$

Conclusions

- The analytical solutions of network coherence are obtained based on the tree's structure.
- A smaller distance between two hubs and a large difference between the degrees will result in higher coherence.
- The tree displays better coherence when the average path length is smaller.



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