Nan Jiang, Chi Huang, Yao Chen, Jürgen Kurths, 2019. Bisimulation-based stabilization of probabilistic Boolean control networks with state feedback control. *Frontiers of Information Technology & Electronic Engineering*, 21(2):268-280.
https://doi.org/10.1631/FITEE.1900447

Bisimulation-based stabilization of probabilistic Boolean control networks with state feedback control

Key words: Probabilistic Boolean control network; Bisimulation; Stabilization with probability one; State feedback control

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Motivation

- The Boolean control network (BCN) is a promising architecture for the bio-medical domain, and controllers play a critical role in BCN.
- BCN is vulnerable to random errors occurring in the data collecting process and cannot emulate the uncertainty in a real genetic regulation process.
- Studying the stabilization of PBCN is worthwhile since stabilizing to achieve the desirable goals is of vital importance in practice.
- Analyzing the stabilization of a large PBCN could lead to high computational complexity.

Novelty

- The concept of bisimulation relation between PBCNs is proposed.
- Necessary and sufficient conditions involving the skeleton matrices of PBCNs and a certain logical matrix are given to test bisimulation relations between two PBCNs.
- Using bisimulation relations, the stabilization of PBCNs with probability one is then propagated.
- If two PBCNs are matched by a bisimulation relation, then they share the same transient period.

Major results

- Necessary and sufficient conditions for two PBCNs to be bisimilar:

Theorem 1 Consider PBCNs Σ and $\widetilde{\Sigma}$ as in Eqs. (7) and (8) with their state feedback control systems as given in Eqs. (4) and (5), respectively. Let $\boldsymbol{Q}_F$ and $\widetilde{\boldsymbol{Q}}_{\widetilde{F}}$ be the skeleton matrices and $\boldsymbol{C} \in \mathcal{L}_{\widetilde{N}\times N}$. Then the relation R defined in Eq. (6) is a bisimulation relation between Σ and $\widetilde{\Sigma}$ if and only if

$$\boldsymbol{C} \odot \boldsymbol{Q}_F = \boldsymbol{Q}_{\widetilde{F}} \odot \boldsymbol{C}. \qquad (11)$$

Major results (Cont'd)

- Bisimilar PBCNs exhibit a uniform stabilization property.

Theorem 2 Given PBCNs Σ and $\widetilde{\Sigma}$ as in Eqs. (2) and (3) with their state feedback control systems as given in Eqs. (4) and (5), respectively. Let $\boldsymbol{z}^* \in \Delta_{\widetilde{N}}$ and $\mathcal{W} = \{\boldsymbol{x} \in \Delta_N : \boldsymbol{C} \ltimes \boldsymbol{x} = \boldsymbol{z}^*\}$. Suppose that the relation R defined in Eq. (6) is a bisimulation relation between Σ and $\widetilde{\Sigma}$. $\boldsymbol{C} \in \mathcal{L}_{\widetilde{N} \times N}$ is a surjection. Then, Σ is stabilizable to $\mathcal{W}$ with probability one if and only if $\widetilde{\Sigma}$ is stabilizable to $\boldsymbol{z}^*$ with probability one.

Conclusions

- Definition of bisimulation relation of PBCNs has been proposed.
- A necessity and sufficiency has been proposed to examine bisimulation relations.
- If two PBCNs are coupled with a bisimulation relation, then one can achieve stabilization to a state set with probability one if and only if the other can realize stabilization to a fixed point with probability one.