

Bin-bin HE, Hua-cheng ZHOU, Chun-hai KOU, 2020. Controllability of fractional-order damped systems with time-varying delays in control. *Frontiers of Information Technology & Electronic Engineering*, 21(6):844-855.

<https://doi.org/10.1631/FITEE.1900736>

Controllability of fractional-order damped systems with time-varying delays in control

Key words: Controllability; Fractional-order damped systems; Time-varying delays; Gramian matrix

Corresponding author: Hua-cheng ZHOU

E-mail: hczhou@amss.ac.cn

 ORCID: Bin-bin HE, <https://orcid.org/0000-0002-8860-6263>;

Hua-cheng ZHOU, <https://orcid.org/0000-0001-6856-2358>;

Chun-hai KOU, <https://orcid.org/0000-0002-0958-5725>

Motivation

- Fractional-order calculus is a useful tool for describing a process with hereditary and memory properties and has attracted more and more attention. Fractional control theory is a basic topic of application-oriented fractional-order calculus in information science.
- The fractional damped system is an extension of the integer-order damped system, and it is a meaningful and interesting theory. Considering that the delay often occurs in practical systems, we investigate the fractional damped system with time-varying delays.

Main idea

- The controllability of a linear fractional-order damped system can be described by its coefficient matrix.
- The small disturbance shall not change the controllability of the controllable linear system.

Method

1. Find the solution expression of the fractional-order damped system with time-varying delays;
2. Define the controllability Gramian matrix;
3. Construct the control input function.

Major results

- The linear fractional damped system (1) is controllable on $[0, T]$ if and only if the controllability Gramian matrix W is positive definite.

- System

$$\begin{cases} {}^C_0D_t^p \mathbf{y}(t) - \mathbf{A}_0^C {}^C_0D_t^p \mathbf{y}(t) = \sum_{i=1}^M \mathbf{B}_i \mathbf{u}[t - \tau_i], t \geq 0, \\ \mathbf{y}(0) = \mathbf{y}_0, \mathbf{y}'(0) = \mathbf{y}'_0, \\ \mathbf{u}(t) = \boldsymbol{\psi}(t), -\tau_M \leq t \leq 0. \end{cases}$$

is controllable on $[0, T]$ if and only if

$$\begin{aligned} \text{rank}[\mathbf{B}_0, \mathbf{A}\mathbf{B}_0, \mathbf{A}^2\mathbf{B}_0, \dots, \mathbf{A}^{n-1}\mathbf{B}_0, \\ \mathbf{B}_1, \mathbf{A}\mathbf{B}_1, \mathbf{A}^2\mathbf{B}_1, \dots, \mathbf{A}^{n-1}\mathbf{B}_1, \\ \mathbf{B}_M, \mathbf{A}\mathbf{B}_M, \mathbf{A}^2\mathbf{B}_M, \dots, \mathbf{A}^{n-1}\mathbf{B}_M] = n \end{aligned}$$

Major results

- Let f be a continuous function satisfying

$$\lim_{\|(\mathbf{y}, \mathbf{u})\| \rightarrow \infty} \frac{\|f(t, \mathbf{y}, \mathbf{u})\|}{\|(\mathbf{y}, \mathbf{u})\|} = 0$$

uniformly on $t \in [0, T]$. Let the linear fractional system (1) be controllable. Then the nonlinear system

$$\begin{cases} {}^C D_t^p \mathbf{y}(t) - \mathbf{A}_0^C D_t^p \mathbf{y}(t) = \sum_{i=1}^M \mathbf{B}_i \mathbf{u}[t - \tau_i] + f(t, \mathbf{y}, \mathbf{u}), t \geq 0, \\ \mathbf{y}(0) = \mathbf{y}_0, \mathbf{y}'(0) = \mathbf{y}'_0, \\ \mathbf{u}(t) = \boldsymbol{\psi}(t), -\tau_M(0) \leq t \leq 0 \end{cases}$$

is controllable on $[0, T]$.

Conclusions

- The linear fractional damped system is controllable on $[0, T]$ if and only if the controllability Gramian matrix is positive definite.
- When the delay is independent of time, the controllability can be equivalently characterized by the rank of a matrix.
- For the nonlinear system, under the controllability of its corresponding linear system, the controllability can be ensured by a restriction on the nonlinear term.