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Supplementary materials for

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Proof S1 Proof of Proposition 1

According to Eq.(34), we choose $J(\tau)=\sum_{i\in\mathcal{N}}\omega_i D_{\mathrm{KL}}^{(\tau_i)}\left(\boldsymbol{\pi}_i \parallel \sum_{j\in\mathcal{N}}\omega_j\boldsymbol{\pi}_j\right)$ as the cost function. Taking $\boldsymbol{\pi}_i(\mathbf{X})=h_i(L_i)g_i(\mathbf{X})$, $\boldsymbol{\pi}(\mathbf{X})=h(L)g(\mathbf{X})$ and label matching τ into consideration, the KLD for two LMB densities $\boldsymbol{\pi}_i(\mathbf{X})$ and $\boldsymbol{\pi}(\mathbf{X})$ is derived as follows

$$\begin{aligned}
D_{\mathrm{KL}}^{(\tau_i)}(\boldsymbol{\pi}_i \parallel \boldsymbol{\pi}) &= \int \boldsymbol{\pi}_i(\mathbf{X})\log\frac{\boldsymbol{\pi}_i(\mathbf{X})}{\boldsymbol{\pi}(\mathbf{X})}\delta\mathbf{X} \\
&= \int h_i(L_i)g_i(\mathbf{X})\log\frac{h_i(L_i)g_i(\mathbf{X})}{h(L)g(\mathbf{X})}\delta\mathbf{X} \\
&= \sum_{n=0}^{\infty}\frac{1}{n!}\sum_{L_{i,n}\subseteq\mathbb{L}_i}\int h_i(L_{i,n})\prod_{k=1}^{n}p_i^{(\ell_{i,k})}(x_k)\log\frac{h_i(L_{i,n})\prod_{k=1}^{n}p_i^{(\ell_{i,k})}(x_k)}{h(L_n)\prod_{k=1}^{n}p^{(\ell_k)}(x_k)}d(x_1,\ldots,x_n) \\
&= \sum_{n=0}^{\infty}\frac{1}{n!}\sum_{L_{i,n}\subseteq\mathbb{L}_i}h_i(L_{i,n})\log\frac{h_i(L_{i,n})}{h(\tau(L_{i,n}))} \\
&\quad + \sum_{n=0}^{\infty}\frac{1}{n!}\sum_{L_{i,n}\subseteq\mathbb{L}_i}h_i(L_{i,n})\int\prod_{k=1}^{n}p_i^{(\ell_{i,k})}(x_k)\sum_{k=1}^{n}\log\frac{p_i^{(\ell_{i,k})}(x_k)}{p^{(\tau(\ell_{i,k}))}(x_k)}d(x_1,...,x_n) \\
&= D_{\mathrm{KL}}(h_i\|h)+\sum_{L_i\subseteq\mathbb{L}_i}h_i(L_i)\sum_{\ell_i\in L_i}D_{\mathrm{KL}}\left(p_i^{(\ell_i)}\|p^{(\tau(\ell_i))}\right),
\end{aligned} \tag{S1}$$

where

$$\begin{aligned}
D_{\mathrm{KL}}(h_i\|h) &= \sum_{L_i\subseteq\mathbb{L}_i}\prod_{\ell_i\in L_i}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\setminus L_i}\left(1-r_i^{(\ell_i')}\right)\log\frac{\prod_{\ell_i\in L_i}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\setminus L_i}\left(1-r_i^{(\ell_i')}\right)}{\prod_{\ell_i\in L_i}r^{(\tau(\ell_i))}\prod_{\ell_i'\in\mathbb{L}_i\setminus L_i}\left(1-r^{(\tau(\ell_i'))}\right)} \\
&= \sum_{L_i\subseteq\mathbb{L}_i}\prod_{\ell_i\in L_i}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\setminus L_i}\left(1-r_i^{(\ell_i')}\right)\left[\sum_{\ell_i\in L_i}\log\frac{r_i^{(\ell_i)}}{r^{(\tau(\ell_i))}}+\sum_{\ell_i'\in\mathbb{L}_i\setminus L_i}\log\frac{1-r_i^{(\ell_i')}}{1-r^{(\tau(\ell_i'))}}\right] \\
&= \sum_{L_i\subseteq\mathbb{L}_i}\prod_{\ell_i\in L_i}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\setminus L_i}\left(1-r_i^{(\ell_i')}\right)\sum_{\ell_i\in L_i}\log\frac{r_i^{(\ell_i)}}{r^{(\tau(\ell_i))}} \\
&\quad + \sum_{L_i\subseteq\mathbb{L}_i}\prod_{\ell_i\in L_i}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\setminus L_i}\left(1-r_i^{(\ell_i')}\right)\sum_{\ell_i'\in\mathbb{L}_i\setminus L_i}\log\frac{1-r_i^{(\ell_i')}}{1-r^{(\tau(\ell_i'))}} \\
&= \sum_{\ell_i\in\mathbb{L}_i}\left\{\left[\sum_{L_i\subseteq\mathbb{L}_i\setminus\{\ell_i\}}\prod_{\ell_i\in L_i\cup\{\ell_i\}}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\setminus L_i\cup\{\ell_i\}}\left(1-r_i^{(\ell_i')}\right)\right]\log\frac{r_i^{(\ell_i)}}{r^{(\tau(\ell_i))}}\right\}
\end{aligned} \tag{S2-1}$$

$$
\begin{aligned}
&+\sum_{\ell_i\in\mathbb{L}_i}\left\{\left[\sum_{L_i\subseteq\mathbb{L}_i\backslash\{\ell_i\}}\prod_{\ell_i\in L_i}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\backslash L_i}\left(1-r_i^{(\ell_i')}\right)\right]\log\frac{1-r_i^{(\ell_i)}}{1-r^{(\tau(\ell_i))}}\right\}\\
&=\sum_{\ell_i\in\mathbb{L}_i}\left\{\left[r_i^{(\ell_i)}\log\frac{r_i^{(\ell_i)}}{r^{(\tau(\ell_i))}}+\left(1-r_i^{(\ell_i)}\right)\log\frac{\left(1-r_i^{(\ell_i)}\right)}{\left(1-r^{(\tau(\ell_i))}\right)}\right]\right\}\\
&=\sum_{\ell_i\in\mathbb{L}_i}D_{\mathrm{KL}}\left(r_i^{(\ell_i)}\parallel r^{(\tau(\ell_i))}\right),
\end{aligned}
\tag{S2-2}
$$

$$
\begin{aligned}
&\sum_{L_i\subseteq\mathbb{L}_i}h_i\left(L_i\right)\sum_{\ell_i\in L_i}D_{\mathrm{KL}}\left(p_i^{(\ell_i)}\|p^{(\tau(\ell_i))}\right)\\
&=\sum_{L_i\subseteq\mathbb{L}_i}\prod_{\ell_i\in L_i}r_i^{(\ell_i)}\prod_{\ell_i'\in\mathbb{L}_i\backslash L_i}\left(1-r_i^{(\ell_i')}\right)\sum_{\ell_i\in L_i}D_{\mathrm{KL}}\left(p_i^{(\ell_i)}\middle\|p^{(\tau(\ell_i))}\right)\\
&=\sum_{\ell_i\in\mathbb{L}_i}r_i^{(\ell_i)}D_{\mathrm{KL}}\left(p_i^{(\ell_i)}\middle\|p^{(\tau(\ell_i))}\right).
\end{aligned}
\tag{S3}
$$

Then,(S1) can be formulated as

$$
D_{\mathrm{KL}}^{(\tau_i)}\left(\boldsymbol{\pi_i}\parallel\boldsymbol{\pi}\right)=\sum_{\ell_i\in\mathbb{L}_i}\left(D_{\mathrm{KL}}\left(r_i^{(\ell_i)}\parallel r^{(\tau(\ell_i))}\right)+r_i^{(\ell_i)}D_{\mathrm{KL}}\left(p_i^{(\ell_i)}\parallel p^{(\tau(\ell_i))}\right)\right).
\tag{S4}
$$

Consequently, the cost function of these two LMB densities is given as follows

$$
\begin{aligned}
J\left(\tau\right)&=\sum_{i\in\{a,b\}}\omega_iD_{\mathrm{KL}}^{(\tau_i)}\left(\boldsymbol{\pi}_i\parallel\sum_{j\in\{a,b\}}\boldsymbol{\omega}_j\boldsymbol{\pi}_j\right)\\
&=\sum_{i\in\{a,b\}}\omega_iD_{\mathrm{KL}}^{(\tau_i)}\left(\boldsymbol{\pi}_i\parallel\omega_a\boldsymbol{\pi}_a+\omega_b\boldsymbol{\pi}_b\right)\\
&=\sum_{i\in\{\alpha,b\}}\omega_i\left(\sum_{\ell\in\mathbb{L}_i}\left(D_{\mathrm{KL}}\left(r_i^{(\ell)}\parallel r^{(\ell)}\right)+r_i^{(\ell)}D_{\mathrm{KL}}\left(p_i^{(\ell)}\middle\|p^{(\ell)}\right)\right)\right)\\
&=\omega_a\sum_{\ell\in\mathbb{L}_a}\left[D_{\mathrm{KL}}\left(r_a^{(\ell)}\|r^{(\ell)}\right)+r_a^{(\ell)}D_{\mathrm{KL}}\left(p_a^{(\ell)}\|p^{(\ell)}\right)\right]\\
&\quad+\omega_b\sum_{\tau(\ell)\in\mathbb{L}_b}\left[D_{\mathrm{KL}}\left(r_b^{(\tau(\ell))}\parallel r^{(\ell)}\right)+r_b^{(\tau(\ell))}D_{\mathrm{KL}}\left(p_b^{(\tau(\ell))}\parallel p^{(\ell)}\right)\right]\\
&=\sum_{\ell\in\mathbb{L}_a}\left[\omega_a\left(D_{\mathrm{KL}}\left(r_a^{(\ell)}\parallel r^{(\ell)}\right)+r_a^{(\ell)}D_{\mathrm{KL}}\left(p_a^{(\ell)}\middle\|p^{(\ell)}\right)\right)\right.\\
&\qquad\left.+\omega_b\left(D_{\mathrm{KL}}\left(r_b^{(\tau(\ell))}\parallel r^{(\ell)}\right)+r_b^{(\tau(\ell))}D_{\mathrm{KL}}\left(p_b^{(\tau(\ell))}\parallel p^{(\ell)}\right)\right)\right].
\end{aligned}
\tag{S5}
$$

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