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# Finite-time coordinated path-following control of leader-following multi-agent systems

**Key words:** Finite-time; Coordinated path-following; Multi-agent systems; Barrier function

Corresponding author: Yangyang CHEN

E-mail: [yychen@seu.edu.cn](mailto:yychen@seu.edu.cn)

 ORCID: <https://orcid.org/0000-0003-0136-0174>

# Motivation

1. In the application scenarios such as oceanic and planetary explorations, to achieve better performance of multi-agent systems, unmanned systems are required to simultaneously follow a set of given orbits with a desired formation, that is, the coordinated path-following control problem.
2. In the area of coordinated path-following control, most scholars focused on the asymptotic stability of the resulting multi-agent systems, and the coordinated path-following control problem within a finite settling time is still unsolved.

# Main idea

1. Decouple the whole system as a path-following subsystem and a formation subsystem;
2. Design the control law by backstepping to achieve finite-time path-following along the given orbits;
3. Design the control law by backstepping to achieve finite-time formation along the given orbits;
4. According to steps 1 and 2, give the control law and show the finite-time convergence of the coordinated path-following control system.

# Method

## Path-following controller design

1. Consider the path-following subsystem consisting of the following equations:

$$\dot{\lambda}_i = \|\nabla \lambda_i\| v_{N_i}$$

$$\dot{v}_{N_i} = u_{N_i} + \Delta_{N_i}$$

2. Consider the path-following candidate Lyapunov function as

$$V_P = \sum_{i=1}^n \Psi_i(\lambda_i) + \gamma_1 \sum_{i=1}^n \int_{\hat{v}_{N_i}}^{v_{N_i}} (\tau^\alpha - \hat{v}_{N_i}^\alpha)^{2-\frac{1}{\alpha}} d\tau.$$

3. Let the virtual control  $\hat{v}_{N_i}$  be  $\hat{v}_{N_i} = -k_1 (\nabla \Psi_i)^{\frac{1}{\alpha}}$ .

4. Design  $u_{N_i}$  by the backstepping method.

# Method (Cont'd)

## Coordinated formation controller design

1. Consider the formation subsystem consisting of the following equations:

$$\begin{aligned}\dot{v}_{T_i} &= u_{T_i} + \Delta_{T_i} \\ \dot{\varsigma}_i &= \sum_{j=0}^n a_{ij} \left( \frac{\partial \xi_i}{\partial s_i} v_{T_i} + \Delta_{\xi_i} - \frac{\partial \xi_j}{\partial s_j} v_{T_j} - \Delta_{\xi_j} \right)\end{aligned}$$

2. Consider the coordinated formation candidate Lyapunov function as

$$V_F = \frac{1}{2} \sum_{i=1}^n \varsigma_i^2 + \gamma_2 \sum_{i=1}^n \int_{\hat{v}_{T_i}}^{v_{T_i}} (\tau^\alpha - \hat{v}_{T_i}^\alpha)^{2-\frac{1}{\alpha}} d\tau,$$

3. Let the virtual control  $\hat{v}_{T_i}$  be  $\hat{v}_{T_i} = -\left(\frac{\partial \xi_i}{\partial s_i}\right)^{-1} k_3 \varsigma_i^{\frac{1}{\alpha}}.$
4. Design  $u_{T_i}$  by the backstepping method.

# Major results

**Theorem 3** Suppose that the initial positions of vehicles are such that  $p_i(0) \in \Omega_i$ . Assume moreover that Assumption 1 holds. For  $i = 1, 2, \dots, n$ , the finite-time coordinated path-following control problem is solved by the coordinated path-following control:

$$u_i = \begin{bmatrix} N_i^T \\ T_i^T \end{bmatrix}^{-1} \begin{bmatrix} u_{N_i} \\ u_{T_i} \end{bmatrix}, \quad (47)$$

where  $u_{N_i}$  and  $u_{T_i}$  are as given in Eqs. (19) and (41), respectively.

# Major results (static case)

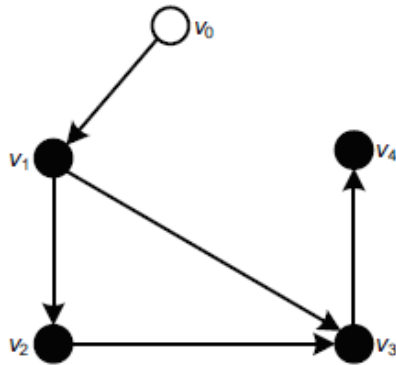


Fig. 2 A directed topology

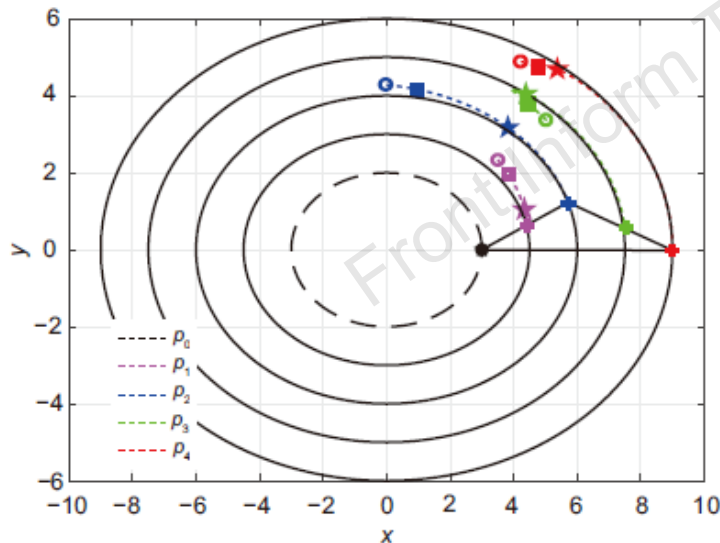


Fig. 3 Motion of the agents in the static virtual leader case

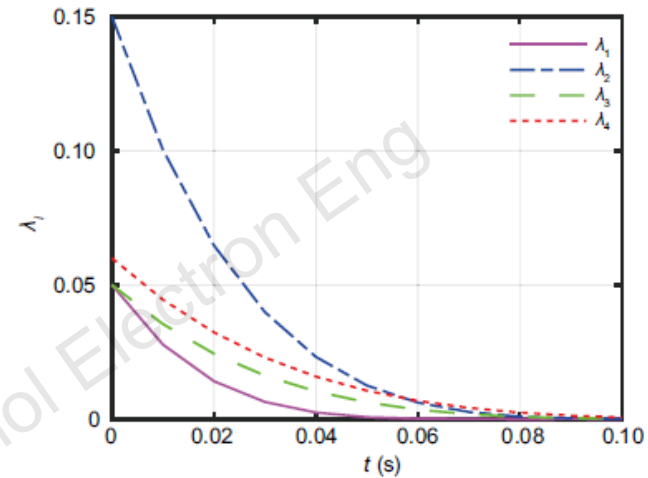


Fig. 4 Path-following errors in the static virtual leader case

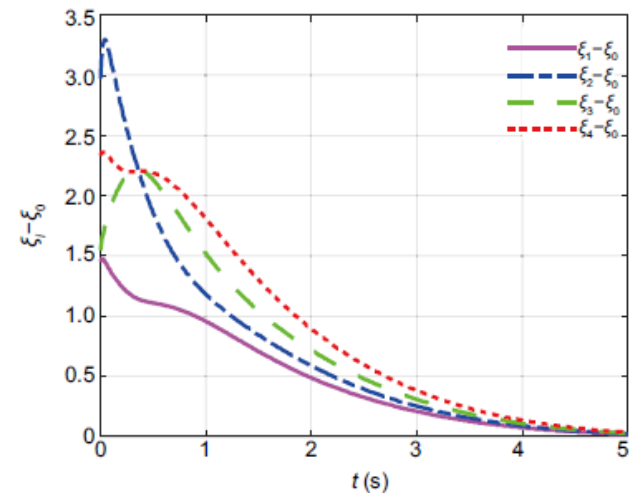


Fig. 5 Formation errors in the static virtual leader case

# Major results (dynamic case)

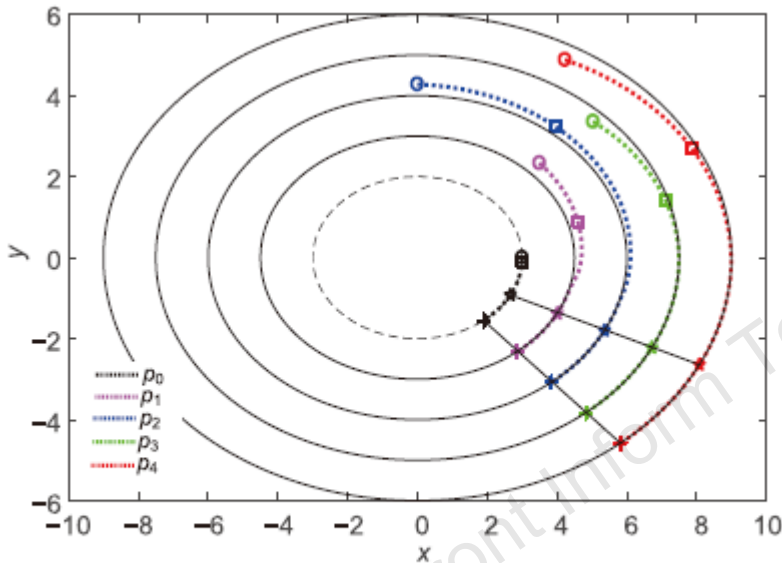


Fig. 6 Motion of the agents in the dynamic virtual leader case

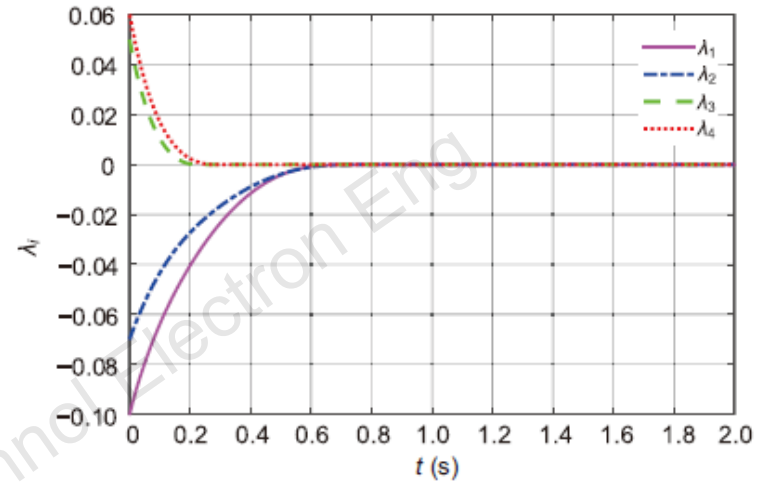


Fig. 7 Path-following errors in the dynamic virtual leader case

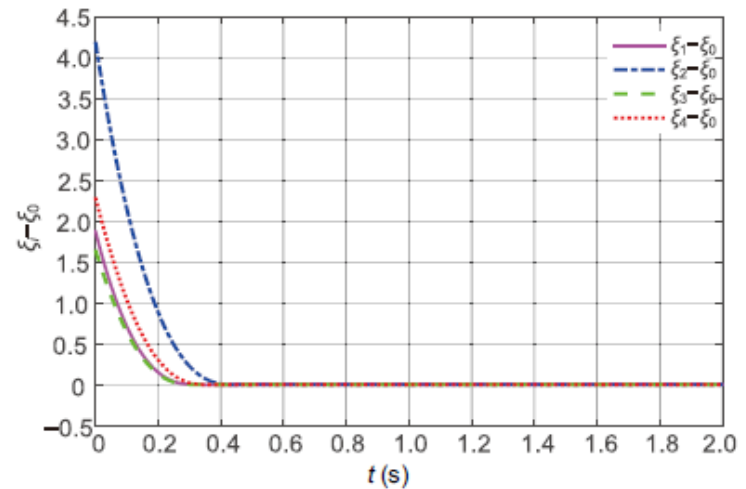


Fig. 8 Formation errors in the dynamic virtual leader case

# Conclusions

1. A continuous feedback method to solve the finite-time coordinated path-following control problem is presented, where the topology for the virtual leader and followers is directed.
2. Conditions on the control gains to guarantee that the path-following errors and the formation errors converge to zeros in finite time are presented.



Weibin CHEN received his BS and MS degrees from Shandong University, China, in 2009 and 2012, respectively. He is currently pursuing his Eng.D degree in the School of Automation, Southeast University, Nanjing, China. He is also a senior engineer in the Jiangsu Automation Research Institute. His research interests include robot control and multi-agent systems.



Yangyang CHEN (Senior Member, IEEE) received his BS and MS degrees from Hefei University of Technology, Hefei, China, in 2003 and 2006, respectively, and his PhD degree from Southeast University, Nanjing, China, in 2010. Since 2010, he has been with the Southeast University, where he is currently an associated professor with the School of Automation. His research interests include nonlinear formation control and coordinated path-following control.



Ya ZHANG (Senior Member, IEEE) received her BS degree in applied mathematics from the China University of Mining and Technology, Beijing, China, in 2004, and her PhD degree in control engineering from Southeast University, Nanjing, China, in 2010. Since 2010, she has been with the Southeast University, where she is currently a professor. Her research interests include cooperative control and estimation, multi-agent systems, and network security.

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