

Alain Soup Tewa KAMMOGNE, Michaux Noubé KOUNTCHOU, Romanic KENGNE, Ahmad Taher AZAR, Hilaire Bertrand FOTSIN, Soup Teoua Michael OUAGNI, 2020. Polynomial robust observer implementation based passive synchronization of nonlinear fractional-order systems with structural disturbances *Frontiers of Information Technology & Electronic Engineering*, 21(9):1369-1386. <https://doi.org/10.1631/FITEE.1900430>

# Polynomial robust observer implementation based passive synchronization of nonlinear fractional-order systems with structural disturbances

**Key words:** Robust passive observer; Adaptive synchronization; Lyapunov theory; Fractional-order; Polynomial observer; Uncertain parameters;  $H_\infty$ -performance

Corresponding author: Alain Soup Tewa KAMMOGNE

E-mail: kouaneteoua@yahoo.fr



ORCID: <https://orcid.org/0000-0003-0234-8652>

# Motivation

- In the controlled systems, the complex environment is often omitted. The orders of the proposed systems are integers. However, this is impossible in real applications, so the robustness is not satisfactory and steady state fluctuation is inevitable.
- Many methods developed in the literature are based on the linear matrix inequality (LMI) to the best of our knowledge. It is very difficult and unpredictable to control the LMI with unknown parameters (for example, the loop gain or the Lipchitz constant cannot be infinite in spite of the perturbations or the uncertainties in the system).
- Controlling the nonlinear effect in the control scheme will make the observer structure considerably more complicated, and requires extra control effort.
- The implementation of the passivity concept with fractional order has not yet been achieved to prove the feasibility.

# Main idea

- Evaluate the robustness in terms of the control effort, which enables the drive response system to have a coupling speed cost.
- Propose a method to attenuate unmatched disturbances in fractional systems.
- Implement the robust control scheme in PSpice.

# Method

**Definition 2** View to the external nonlinear input/output respectively  $u_f(t)/y_e(t)$ , the system (7) subjected to disturbances and uncertainties is said to be passive if for any two nonnegative constant  $\hbar$  and  $\hat{\lambda}$ , the following relation hold:

$$\int_0^t (u_f^T(t)y_e(t))dt + \hbar \geq \int_0^t s(x(t))dt - \hat{\lambda}^2 \int_0^t r(t)dt, \quad \forall t \geq 0 \quad (13)$$

## Dynamical error

$$D^q e(t) = (A - LC)e(t) + (\varphi(x(t)) - \varphi(\hat{x}(t)) + \bar{G}\zeta(t) - \bar{G}d(t) + \Delta Ae(t) - u_f(t) \quad (17)$$

where  $e(t) = [e_1(t), e_2(t), \dots, e_n(t)]^T \in R^n$  are the state vectors of error system (17)

# Control design

$$u_f(t) = -\sum_{j=1}^p H(y - C\hat{x}(t))^{2j-1} + \xi(t) \quad (18)$$

where  $p \in \mathbb{Z}^+$  is considered as an odd number and  $j > 1$ .  $\xi(t)$  denotes the external input signal to be determined later.  $H$  represents the adaptive control strength gain matrix.

$$\dot{k}_{ii}(t) = k_{ii}(0) \sum_{j=1}^n e_j(t) P_{ij} e_i(t) \quad (19)$$

with  $k_i(0)$  are positive constants.

## Observation error stability

**Assumption 3.** Let select an appropriate matrix  $L \in \mathbb{R}^{3 \times 3}$  and two others matrices such that  $P = P^T > 0$  and  $Q = Q^T$  satisfying the following conditions:

$$P(A - LC + H_1 C) + (A - LC + H_1 C)^T P - PH - H^T P + PDD^T P + \bar{D}^T D - \gamma^{-2} I = -Q \quad (24)$$

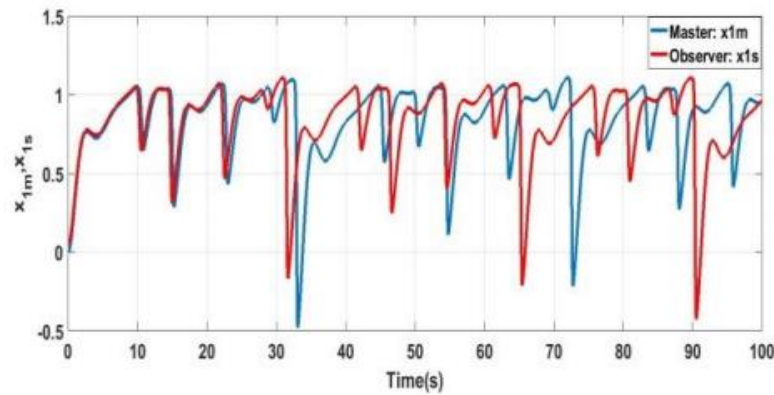
and

$$\lambda_{\min}(E_i + E_i^T) \geq 0, \quad i = 2, \dots, p \quad (25)$$

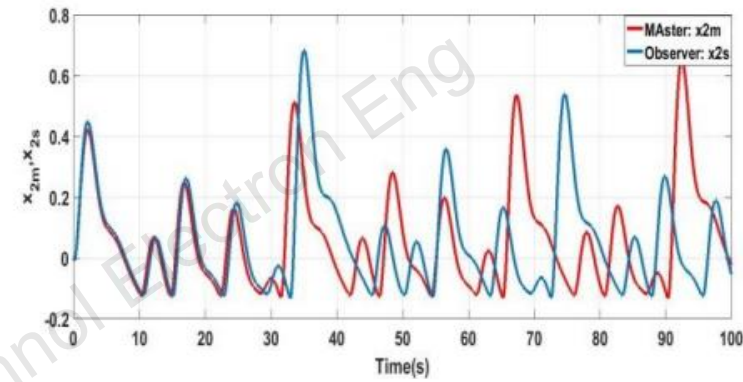
with  $E_i = PH_i C$

Then, the robust passitivity synchronization can be obtained under the controller (17) with adaptive law (18).

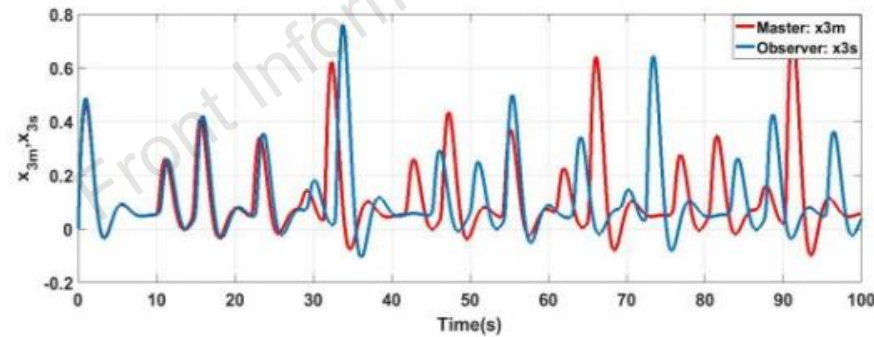
# Major results



(a)



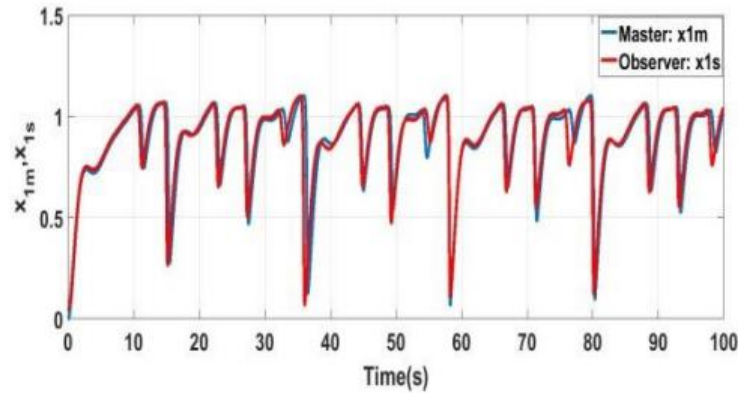
(b)



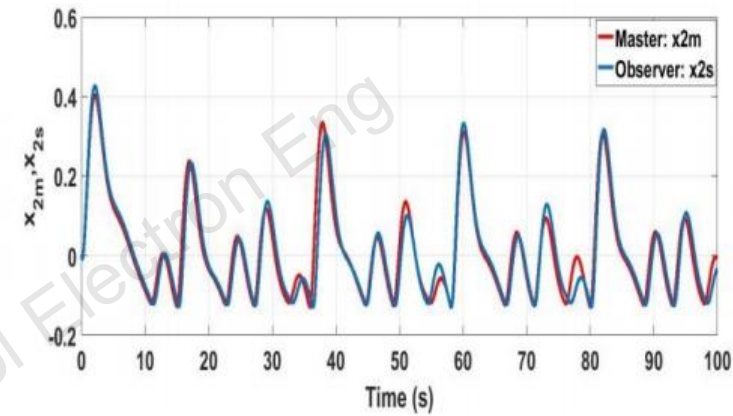
(c)

**Fig. 4** Evolutions of the trajectories when the controller is deactivated, (a):  $x_1(t), \hat{x}_1(t)$ , (b):  $x_2(t), \hat{x}_2(t)$ , (c):  $x_3(t), \hat{x}_3(t)$

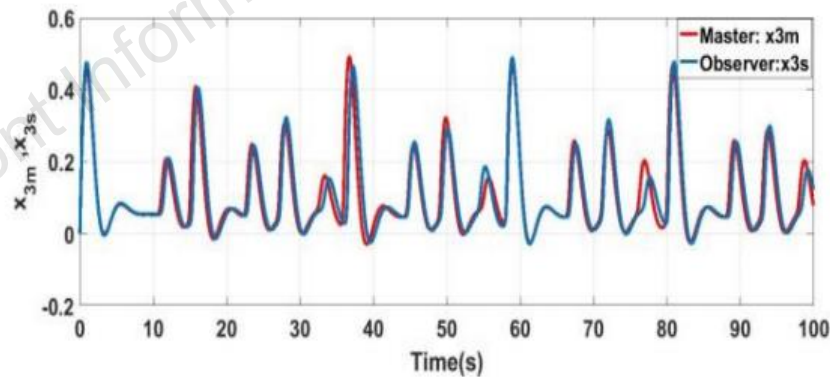
# Major results



(a)



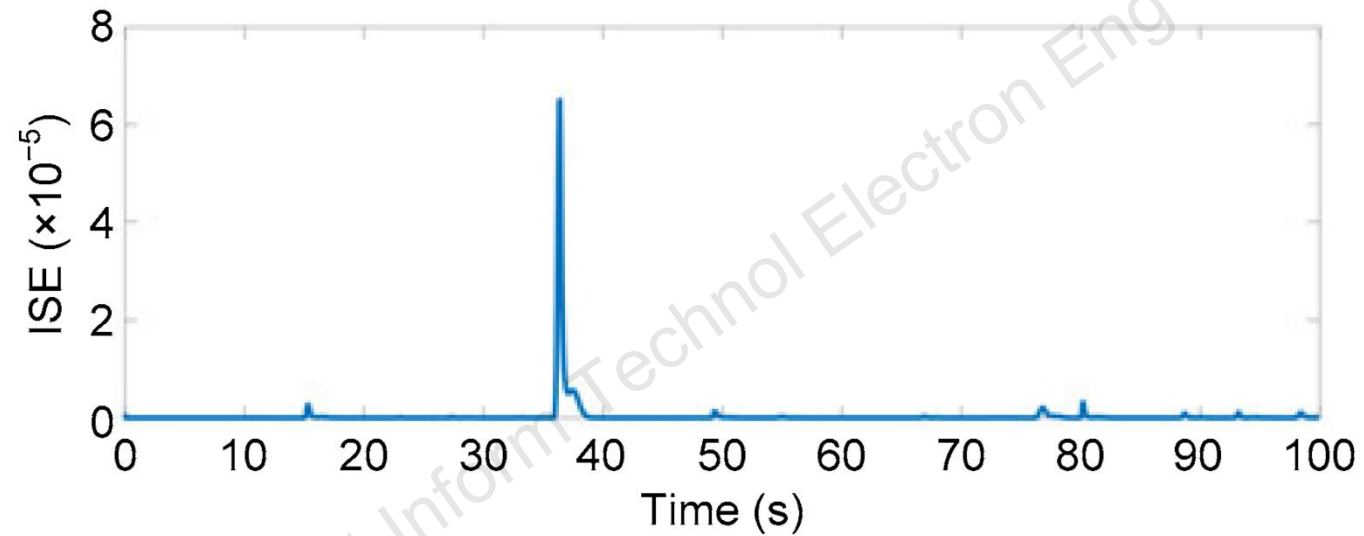
(b)



(c)

Fig. 5. Evolutions of the trajectories when the controller is activated, (a):  $x_1(t), \hat{x}_1(t)$ , (b):  $x_2(t), \hat{x}_2(t)$ , (c):  $x_3(t), \hat{x}_3(t)$

## Performance index



**Fig. 7 Performance index of the synchronization (ISE: integral of a squared error)**

# PSpice diagram

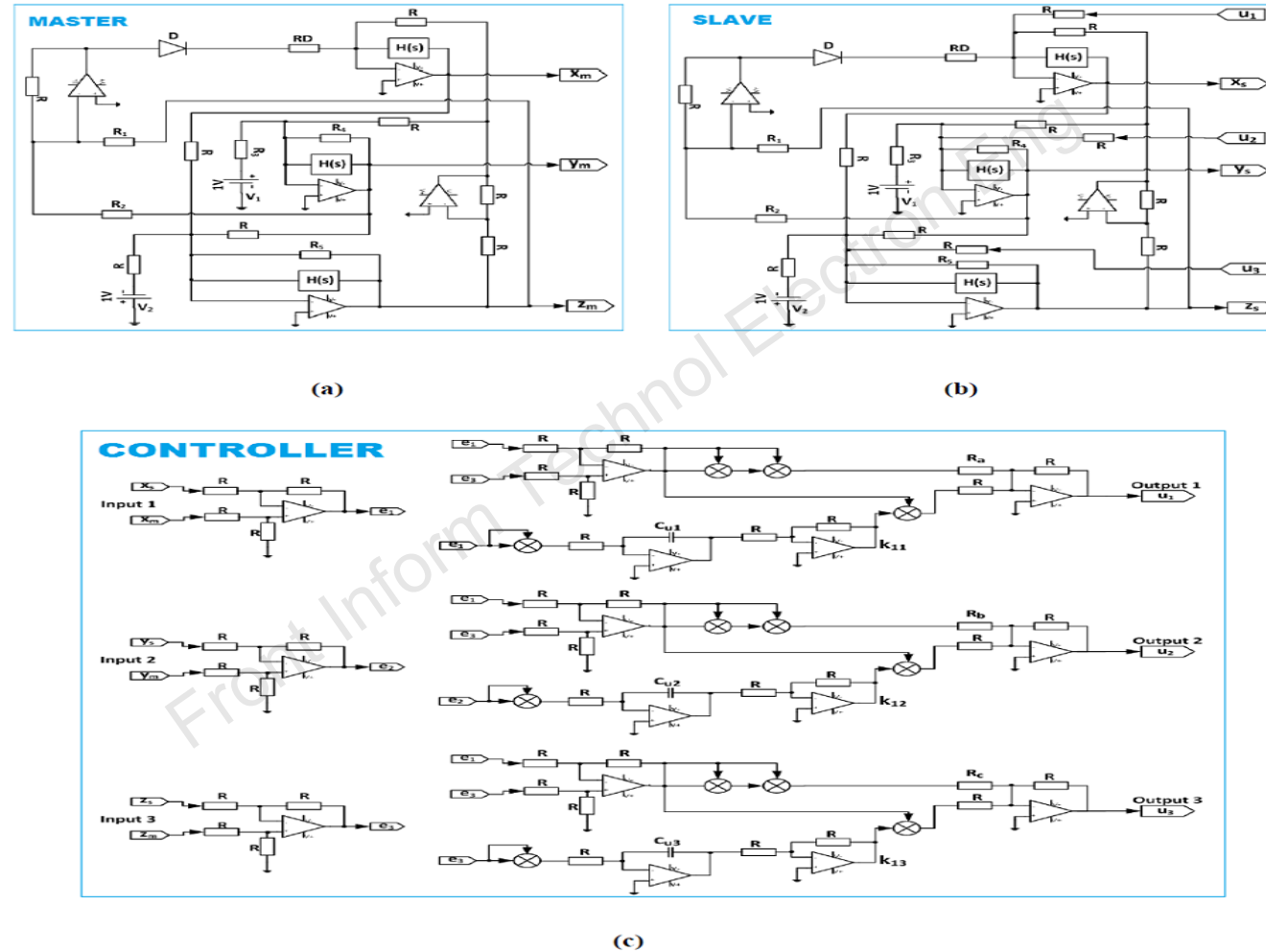


Fig. 9 Equivalent model of the fractional-order modified Colpitts oscillator (FMCO) view as a master system (a), circuit implementation of the slave system (b), and circuit implementation of the feedback coupling controller given by Eqs. (56) and (57) (c)

# PSpice results

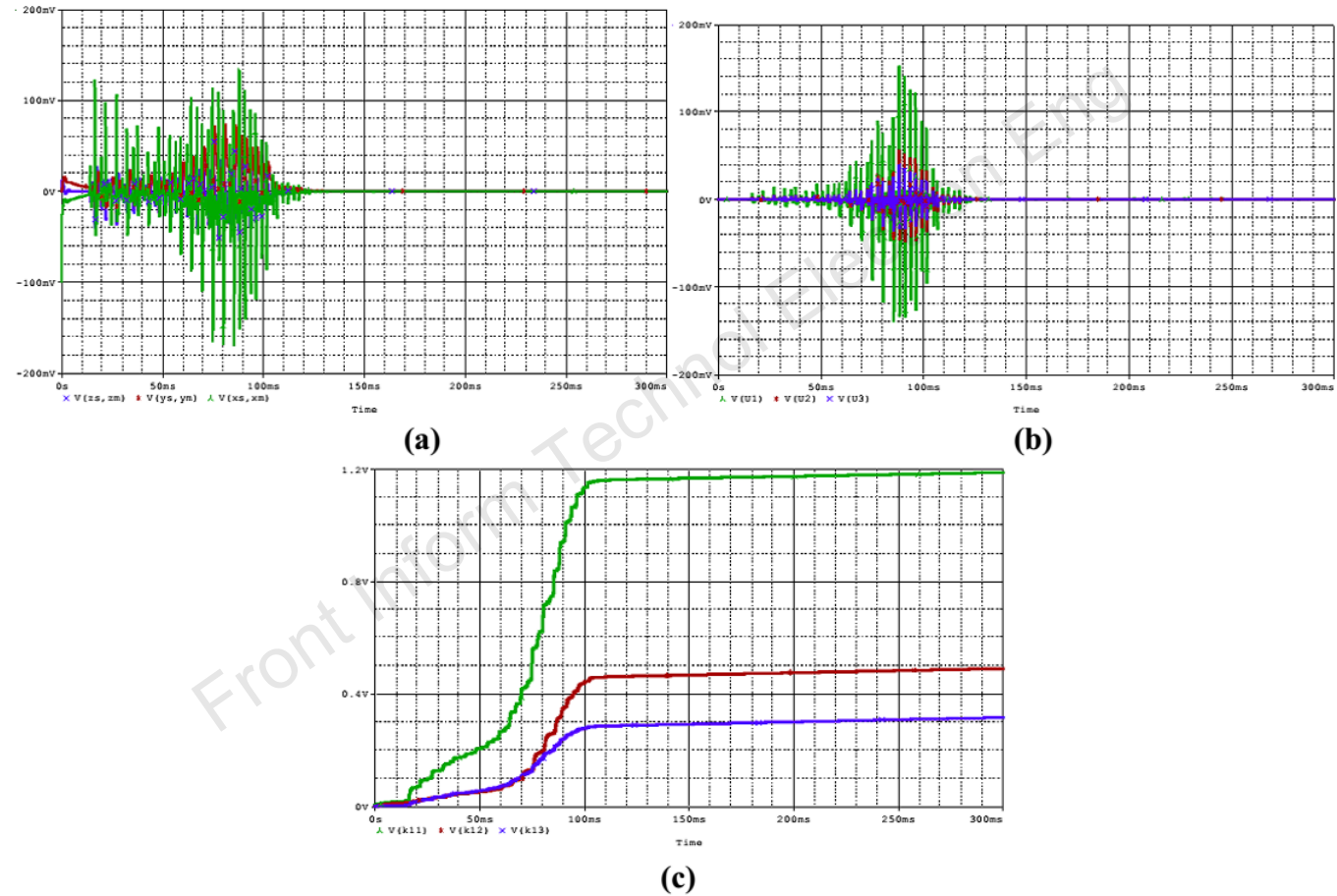


Fig. 12. (a) time evolution of the synchronization errors, (b) Time evolution of the feedback coupling controllers  $u_1$ ,  $u_2$  and  $u_3$ , (c) Time evolution of the feedback gains  $k_{11}$ ,  $k_{12}$  and  $k_{13}$

# Case of the reduced passive feedback couplings controller on PSpice

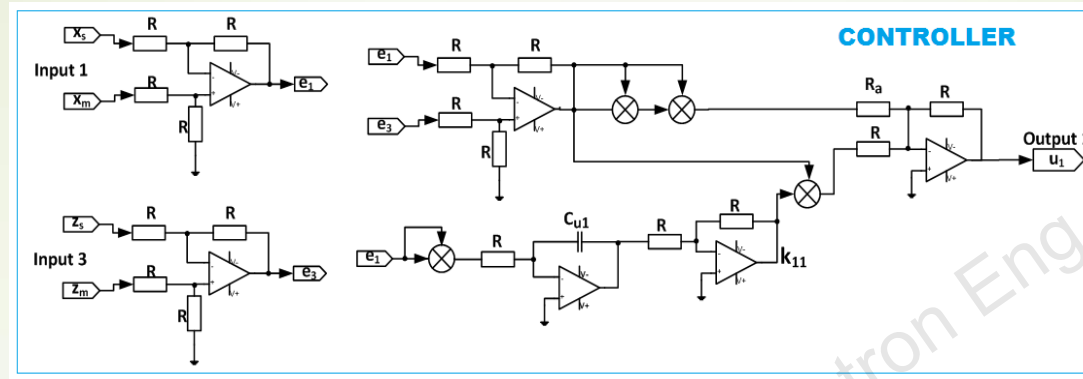


Fig. 14 Circuit implementation of the feedback coupling controller Eqs. (65) and (66)

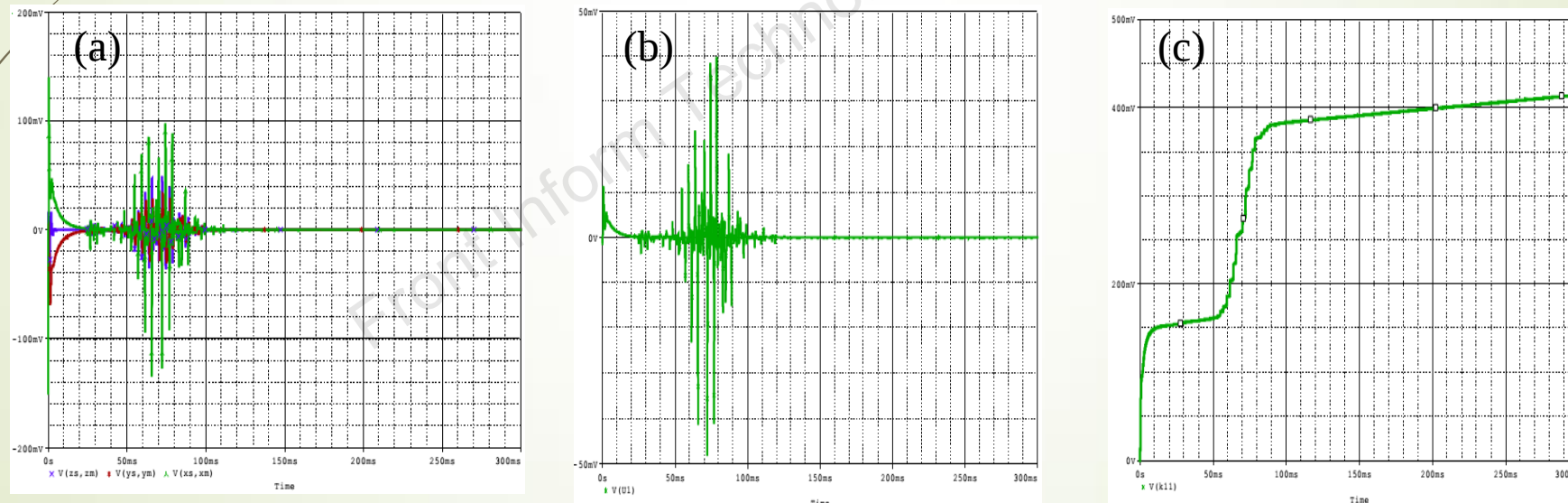


Fig. 15 Time evolution of synchronization errors (a), feedback coupling controller  $u_1$  (b), and feedback gain  $k_{11}$  (c)

# Conclusions

- In term of adaptive control theory and passivity concept, a polynomial robust observer has been proposed.
- Based on Lyapunov stability theory and Finsler's lemma, the sufficient conditions for the stability of the closed-loop systems are derived.
- A special case based on the minimum phase properties of the Colpitts oscillators is considered and the practicable robust observer is derived.
- The proposed approach allows us to establish a typical analysis procedure of the stability in chaotic FOS.
- PSpice implementation demonstrated that our method is not limited to numerical simulations, but provides strong experimental tools in designing the passive controller in real time.
- An illustrative example is used to judge the performance of the suggested approach, which presents good robustness properties against structural uncertainties and unmodeled dynamics.