Frontiers of Information Technology & Electronic Engineering
www.jzus.zju.edu.cn; engineering.cae.cn; www.springerlink.com
ISSN 2095-9184 (print); ISSN 2095-9230 (online)
E-mail: jzus@zju.edu.cn

Resilient distributed economic dispatch of a cyber-power system under DoS attack*

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Received Apr. 30, 2020; Revision accepted Oct. 10, 2020; Crosschecked Nov. 23, 2020

Abstract: The economic dispatch problem of a smart grid under vicious denial of service (DoS) is the main focus of this paper. Taking the actual situation of power generation as a starting point, a new distributed optimization model is established which takes the environmental pollution penalty into account. For saving the limited bandwidth, a novel distributed event-triggered scheme is proposed to keep the resilience and economy of a class of cyber-power systems when the communication network is subject to malicious DoS attack. Then an improved multi-agent consensus protocol based on the gradient descent idea is designed to solve the minimization problem, and the prerequisites to minimize the system power generation cost are analyzed from the aspects of optimality and stability. Finally, the theoretical results are verified through a single-area 10-generator unit simulation.

Key words: Economic dispatch; Denial of service (DoS) attack; Resilient event-triggered scheme; Distributed optimization

https://doi.org/10.1631/FITEE.2000201 **CLC number:** TP273

1 Introduction

The smart grid is a network for the production and consumption of electricity. Different dispatch strategies lead to different economic benefits. Hence, there should be a reasonable coordinated scheduling plan between power generation and electricity consumption. This would not only ensure the safe and stable operation of the power systems, but also pay attention to the economic benefits of the power market (Qin et al., 2020). Therefore, economic dispatch (ED) is suggested as a way forward. The essence of the ED problem is an optimization problem which minimizes the generation cost while maintaining the balance between supply and demand and at the same time satisfying the capacity constraint by coordinating the distributed energy output power of the smart grid. In Guo FH et al. (2019), an accelerated distributed gradient-based algorithm for constrained optimization was proposed for ED in a large-scale power system. As to the ED problem, the research from centralized to distributed dispatch has been carried out incrementally at home and abroad.

To solve the cost minimization problem, many methods have been suggested such as centralized dispatch approaches, including λ-iteration (Lin et al., 1992), Lagrange relaxation (Guo T et al., 1996), and quadratic programming (Fan and Zhang, 1998). However, the aforementioned methods usually result in higher communication and computation costs. Note that the security of private information is under greater threat when these methods are adopted. In addition, the plug-and-play requirements of

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* Project supported by the National Natural Science Foundation of China (No. 62073269), the China Postdoctoral Science Foundation (No. 2018M643661), and the Natural Science Basic Research Plan in Shaanxi Province of China (No. 2018JQ60330)

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renewable energy generation cannot be implemented (Elsayed and El-Saadany, 2015). As a result, distributed optimization methods have attracted more attention in recent years in order to overcome the aforementioned disadvantages and improve the resilience and privacy of the power system (Wu, 2019), such as the distributed bisection algorithm (Xing et al., 2015), approximate dynamic programming (Shuai et al., 2019), and multi-agent consensus (Qin et al., 2019).

Although there has been a lot of research on the ED problem, the security of the communication network and environmental pollution penalty caused by thermal generation have not been widely studied. As far as we know, there has been no work studying the ED optimization problem considering both issues. As far as the security of the communication network is concerned, most existing work proceeds on the assumption that the transmission is working well, and neglects the reality that it is vulnerable to malicious adversary. Just a few studies have mentioned the cyber-attacked power system ED problem at present. In Wang R et al. (2020), a gossip-based distributed algorithm was proposed for the smart grid with random communication link failures. Zhao et al. (2017) analyzed a stealthy attack through false data injection for both offline and online cases, and the necessary and sufficient conditions were provided to guarantee the convergence of the algorithm. Huang et al. (2019) studied mainly the ED problem with communication delays, and a delay-free-based distributed algorithm was presented to optimally assign the whole energy demand among local generation units with the objective of minimizing the agminated operation cost. A denial of service (DoS) attack robust dispatch strategy based on mixed integer linear programming was proposed in Zhang et al. (2019), in which a priority-based recovery process was used to reconfigure the load requirement of distributed generation units. Although cyber attack is taken into account in the above research, the main focus concentrates on the distributed algorithm design, and system stability analysis was not given.

Renewable energy generation has been widely studied in recent years as a means of overcoming the shortage of resources and environmental pollution caused by conventional thermal generation (TG). As is well known, wind power generation (WG) and photovoltaic power generation (PG) have been applied widely because of their free availability and environmental friendliness. Nevertheless, WG and PG have strong randomness and uncertainty; hence, the ED problem needs to be reformulated. A Lagrangian relaxation with incremental proximal method was proposed which is suitable for a large number of WG scenarios (Tan et al., 2019), and a finite-time average consensus algorithm was proposed in Guo FH et al. (2016) to overcome the drawbacks caused by random WG. However, all ignored the pollution penalty of thermal generation and the communication network security.

Thus, in this paper we focus on the distributed ED optimization problem of a cyber-attacked power grid which has hybrid generation. The cyber-power system under study is shown in Fig. 1. To reduce the frequency disturbance caused by intermittent wind power and photovoltaic power, energy storages are integrated with WG and PG. The main contributions of this work are as follows:

1. The distributed event-triggered scheme proposed in this study is resilient to data dropouts induced by DoS attacks, and can effectively reduce the communication bandwidth consumption. The analytically explicit relationship between the proposed resilient triggered threshold and a general one has been presented for the first time.

2. An improved multi-agent consensus protocol is designed, which eliminates the negative impacts of the centralized ED approaches despite the privacy preserving priorities of gradient information. The optimal active power output is also found by adopting a gradient descent algorithm, and a sufficient stability condition for the convergence to the equilibrium state is derived.

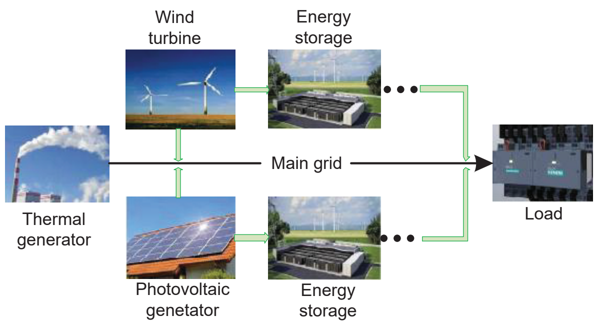

Fig. 1 Structure of the smart grid

2 Problem formulation

2.1 Graph theory

The communication network among agents can be modeled as an undirected graph $G = (V, E, \boldsymbol{A})$, where $V = \{1, 2, ..., N\}$ is the set of nodes with N being the total number of generators and $E \subseteq V \times V$ is the edge set. When $(v_i, v_j) \in E$, it means that the information can be exchanged between node i and node j. Define matrix $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ to be the adjacency matrix, whose elements satisfy

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E,\ i \neq j, \\ 0, & \text{otherwise.} \end{cases}$$

It is obvious that $\boldsymbol{A}$ is a symmetric matrix and its diagonal elements $a_{ii} = 0$. Define matrix $\boldsymbol{D}$ as the in-degree matrix, which is a diagonal matrix, and its diagonal element d_{ii} represents the total number of nodes connected to node i. Based on the above definitions, the Laplacian matrix of graph G can be obtained as $\tilde{\boldsymbol{L}} = \boldsymbol{D} - \boldsymbol{A}$. Denote $\boldsymbol{L} = \tilde{\boldsymbol{L}} \otimes \boldsymbol{I}_n$, where "$\otimes$" is the Kronecker product.

2.2 Distributed economic dispatch

The conventional ED model needs to be reformulated because of the integration of renewable energy power and the environmental pollution cost resulting from thermal generation. When WG and PG are connected to a smart grid, the environmental benefit of the generator group should be taken into account because of the high cost. Thus, an environmental pollution penalty (EPP) is introduced:

$$C_{\mathrm{EPP}i} = \eta_{\mathrm{EPP}i} \cdot M_{\mathrm{TG}i},$$

where

$$M_{\mathrm{TG}i} = a_i P_{\mathrm{TG}i}^2 + b_i P_{\mathrm{TG}i} + c_i,$$

in which $C_{\mathrm{EPP}i}$ and $\eta_{\mathrm{EPP}i}$ are environmental penalty cost and penalty coefficient respectively, a_i, b_i, and c_i are the blowdown characteristic coefficients, and $P_{\mathrm{TG}i}$ and $M_{\mathrm{TG}i}$ are the power output and sewage discharge of the i^{th} TG respectively. Then, the total cost of TG can be expressed as follows:

$$C_{\mathrm{TG}i} = \alpha_i + \beta_i P_{\mathrm{TG}i} + \gamma_i P_{\mathrm{TG}i}^2 + C_{\mathrm{EPP}i},$$

where α_i, β_i, and γ_i are the cost coefficients of the i^{th} TG.

As is well known, the WG and PG are random and uncontrollable. A system fluctuation or system out of control may occur when they are integrated into a smart grid directly such that we assume that every wind turbine or photovoltaic turbine is connected with an energy storage unit. Considering the plug-and-play characteristic of renewable energy generation, the cost functions of WG and PG, notated as $C_{\mathrm{WG}i}$ and $C_{\mathrm{PG}i}$ respectively, are given as follows:

$$\begin{aligned} C_{\mathrm{WG}i} &= a_{\mathrm{w}i} P_{\mathrm{WG}i}^2 + b_{\mathrm{w}i} P_{\mathrm{WG}i} + c_{\mathrm{w}i}, \\ C_{\mathrm{PG}i} &= a_{\mathrm{p}i} P_{\mathrm{PG}i}^2 + b_{\mathrm{p}i} P_{\mathrm{PG}i} + c_{\mathrm{p}i}, \end{aligned}$$

where $P_{\mathrm{WG}i}$ and $P_{\mathrm{PG}i}$ are the active power outputs, $a_{\mathrm{w}i}$, $b_{\mathrm{w}i}$, and $c_{\mathrm{w}i}$ are WG characteristic coefficients, and $a_{\mathrm{p}i}$, $b_{\mathrm{p}i}$, and $c_{\mathrm{p}i}$ are PG characteristic coefficients. Assume that there are N generator units, consisting of n_{T} TGs, n_{W} WGs, and n_{P} PGs. For simplicity, in the following analysis, we treat each generator unit as an agent assigned a unique serial number. Define the set $S_{\mathrm{T}} = \{1, 2, \ldots, n_{\mathrm{T}}\}$ to represent all the thermal power units; similarly, define $S_{\mathrm{W}} = \{n_{\mathrm{T}} + 1, n_{\mathrm{T}} + 2, \ldots, n_{\mathrm{T}} + n_{\mathrm{W}}\}$ and $S_{\mathrm{Pv}} = \{n_{\mathrm{T}} + n_{\mathrm{W}} + 1, n_{\mathrm{T}} + n_{\mathrm{W}} + 2, \ldots, N\}$. Based on the above analyses, the total generation cost of a smart grid with renewable energy penetration can be written as

$$\begin{aligned} & C(P_{\mathrm{TG}i}, P_{\mathrm{WG}i}, P_{\mathrm{PG}i}) \\ &= \sum_{i\in S_{\mathrm{T}}} C_{\mathrm{TG}i} + \sum_{i\in S_{\mathrm{W}}} C_{\mathrm{WG}i} + \sum_{i\in S_{\mathrm{Pv}}} C_{\mathrm{PG}i}. \end{aligned} \tag{1}$$

Taking both generator and demand-supply balance constraints into consideration, the ED problem can be formulated as follows:

$$\min_{P_{\mathrm{TG}i}, P_{\mathrm{WG}i}, P_{\mathrm{PG}i}} C(P_{\mathrm{TG}i}, P_{\mathrm{WG}i}, P_{\mathrm{PG}i})$$

$$\text{s.t.} \begin{cases} \sum\limits_{i\in S_{\mathrm{T}}} P_{\mathrm{TG}i} + \sum\limits_{i\in S_{\mathrm{W}}} P_{\mathrm{WG}i} + \sum\limits_{i\in S_{\mathrm{Pv}}} P_{\mathrm{PG}i} \\ \quad = \sum\limits_{i=1}^{N} P_{\mathrm{d}i} = P_{\mathrm{d}}, \\ P_{\mathrm{TG}i}^{\min} \leq P_{\mathrm{TG}i} \leq P_{\mathrm{TG}i}^{\max}, i \in S_{\mathrm{T}}, \\ 0 \leq P_{\mathrm{WG}i} \leq P_{\mathrm{WG}i}^{\max},\ \ i \in S_{\mathrm{W}}, \\ 0 \leq P_{\mathrm{PG}i} \leq P_{\mathrm{PG}i}^{\max},\ \ i \in S_{\mathrm{Pv}}, \end{cases} \tag{2}$$

where $P_{\mathrm{TG}i}^{\min}$ and $P_{\mathrm{TG}i}^{\max}$ are the lower and upper power output bounds on the i^{th} TG respectively, $P_{\mathrm{WG}i}^{\max}$ and $P_{\mathrm{PG}i}^{\max}$ are the upper bounds on the i^{th} WG and PG respectively, $P_{\mathrm{d}i}$ is the load demand corresponding to each generation unit, and P_{d} is the total load demand which is assumed to be a constant.

The active power output of any agent i can be represented as $x_i \in \mathbb{R}$, and $\boldsymbol{x} = (x_1, x_2, \ldots, x_N)^{\mathrm{T}}$. Thus, the cost function and constraints of each generator unit i can be listed as follows:

$$\begin{cases} c_i(\boldsymbol{x}) = \begin{cases} C_{\mathrm{TG}i}, i \in S_{\mathrm{T}}, \\ C_{\mathrm{WG}i}, i \in S_{\mathrm{W}}, \\ C_{\mathrm{PG}i}, i \in S_{\mathrm{Pv}}, \end{cases} \\ X_i = \begin{cases} P_{\mathrm{TG}i}^{\min} \leq x_i \leq P_{\mathrm{TG}i}^{\max},\ i \in S_{\mathrm{T}}, \\ 0 \leq x_i \leq P_{\mathrm{WG}i}^{\max},\ i \in S_{\mathrm{W}}, \\ 0 \leq x_i \leq P_{\mathrm{PG}i}^{\max},\ i \in S_{\mathrm{Pv}}, \\ \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} P_{\mathrm{d}i}. \end{cases} \end{cases} \tag{3}$$

However, Gharesifard and Cortés (2014) addressed that the centralized optimization problem (2) can be transformed into the following distributed optimization by adding an equality constraint:

$$\begin{aligned} &\min_{x_i} \sum_{i=1}^{N} c_i(x_i) \\ &\text{s.t. } \boldsymbol{L}\boldsymbol{x} = \boldsymbol{0}_{Nn},\ x_i \in \bigcap_{i=1}^{N} X_i. \end{aligned} \tag{4}$$

The optimization problem (2) has a unique optimal solution $\boldsymbol{x}^*$ by the Lagrange multiplier method, equivalent to the distributed optimization problem (4) which can be solved by adopting a gradient descent approach and a consensus protocol. That is, the minimum operating cost of the power system is obtained when the active power output of each generator unit reaches the consistent optimum point.

3 Distributed economic dispatch under denial of service

3.1 Multi-agent system model for distributed optimization

To solve the distributed optimization problem, an improved dynamic multi-agent model is given (Kia et al., 2015):

$$\begin{cases} \dot{x}_i(t) = -\hat{a}\nabla c_i(x_i(t)) - \hat{b}u_i(t), \\ \dot{z}_i(t) = \hat{c}\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)), \end{cases} \tag{5}$$

where $\hat{a}$, $\hat{b}$, and $\hat{c}$ are positive constants which indicate the system parameters. $u_i(t) = \sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t)) + z_i(t)$. $x_i(t)$ and $z_i(t)$ are the active power output and auxiliary state of agent i respectively, and N_i is the neighbor set of agent i. In terms of physical meaning, $u_i(t)$ is actually a proportional integral (PI) controller, and the auxiliary state $z_i(t)$ is an integral function of the controller. Since $u_i(t)$ contains the item $\sum_{j\in N_i} a_{ij}(z_i(t) - z_j(t))$ in Gharesifard and Cortés (2014), the information of auxiliary state $\boldsymbol{z}(t)$ needs additional interaction between $z_i(t)$ and $z_j(t)$. This improved multi-agent model can avoid the transmission of auxiliary state $z_i(t)$ and effectively reduce the communication cost.

3.2 Resilient event-triggered scheme for consensus protocol

Recent work on event triggering and attack on networked systems has achieved meaningful progress (Yan et al., 2019, 2020; Zhang et al., 2019). It is known that communication interruption usually appears when that communication network is subjected to malicious DoS attack. Combined with Fig. 2, we know that not all the triggered system states can be sent to the controller successfully. Here, we assume that there exist M missed system states $\boldsymbol{x}(j_r h)$ in the interval between two successful transmission instants $[t_k h, t_{k+1} h)$, where $j_r h = t_k h + rh$, h is the sampling period, $t_k h$ denotes the instant at which the triggered packets successfully broadcast to the neighbors of agent i, and $M \leq m_k$ with m_k indicating the number of sampled packets in interval $[t_k h, t_{k+1} h)$. To keep the power system stable under DoS attack, a new distributed resilient event-triggered scheme is presented, which not only makes the power system resilient to DoS attack, but also saves the limited bandwidth.

$$\begin{aligned} &|x_i(t_k h + rh) - x_i(t_k h)| \\ \leq &\sigma_{\mathrm{re}} \left| \sum_{j\in N_i} a_{ij}(x_i(t_k h) - x_j(t_k h)) \right|, \end{aligned} \tag{6}$$

where $|\cdot|$ stands for the infinite norm of a matrix or a vector, and σ_{re} is the resilient event-triggered threshold. From inequality (6), we can see that agent i updates its local state only when the triggered condition is violated. Then inequality (6) can be written as the following compact form:

$$|\boldsymbol{x}(t_k h + rh) - \boldsymbol{x}(t_k h)| \le \sigma_{\mathrm{re}} |\boldsymbol{L}_2 \boldsymbol{x}(t_k h)|, \quad (7)$$

where $\boldsymbol{L}_2$ is the communication topograph of the multi-agent system after the DoS attack.

In contrast to the conventional periodic event-triggered scheme, the sampled packets will be transmitted if and only if the triggered condition is violated. Therefore, fewer packets will be transmitted, and the communication consumption can be reduced. Compared with the traditional event-triggered threshold σ, the resilient triggered threshold σ_{re} is smaller, which means that more system states will be successfully triggered, and that parts of them are allowed to be abandoned so as to eliminate the effects of DoS attack.

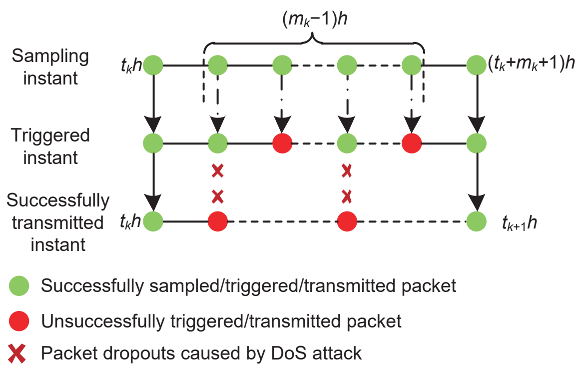

Fig. 2 Event-triggered sequence with DoS attack. m_k indicates the number of sampled packets in interval $[t_k h, t_{k+1} h)$

Proposition 1 The resilient event-triggered threshold σ_{re} designed in this study satisfies the following constraint:

$$\sigma_{\mathrm{re}} \le \frac{\sqrt[M+1]{\sigma |\boldsymbol{L}_1|} - 1}{|\boldsymbol{L}_2|}. \quad (8)$$

Here, $\boldsymbol{L}_1$ is the communication topograph of the multi-agent system before the DoS attack. The proof is provided in the Appendix.

Remark 1 It is clear that the resilient event-triggered condition (6) is based on the information of agent i and its neighbors. It depends on whether the difference between the current sampled system state $x_i(j_r h)$ and the latest triggered state $x_i(t_k h)$ exceeds the threshold decided by the latest triggered state of agent i and its neighbors.

A time delay $d(t)$ is introduced because of the application of the event-triggered scheme:

$$d(t) = t - rh - t_k h, \quad t \in [t_k h + rh, t_k h + (r+1)h),$$

where $r = 0, 1, \ldots, M$.

Then, $\forall t \in [t_k h, t_{k+1} h)$, define

$$e_{ki}(t) = x_i(t_k h) - x_i(t_k h + rh) = x_i(t_k h) - x_i(t - d(t)),$$

where $t \in [t_k h + rh, t_k h + (r+1)h)$, $r = 0, 1, \ldots, M$.

Combined with the definitions of $d(t)$ and $e_{ki}(t)$, we can transform the system state into the following form:

$$x_i(t_k h) = x_i(t - d(t)) + e_{ki}(t).$$

The distributed consensus protocol can be written as follows:

$$\begin{aligned} u_i(t) &= -\sum_{j \in N_i} a_{ij}(x_i(t_k h) - x_j(t_k h)) - z_i(t) \\ &= -\sum_{j \in N_i} a_{ij}(x_i(t - d(t)) - x_j(t - d(t))) \\ &\quad - \sum_{j \in N_i} a_{ij}(e_{ki}(t) - e_{kj}(t)) - z_i(t). \end{aligned} \quad (9)$$

Therefore, system (5) can be rewritten in a compact form:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = -\hat{\boldsymbol{a}} \nabla \boldsymbol{c}(\boldsymbol{x}(t)) - (\boldsymbol{L} \otimes \hat{\boldsymbol{b}})(\boldsymbol{x}(t - d(t)) \\ \qquad\quad + \boldsymbol{e}_k(t)) - \hat{\boldsymbol{b}} \boldsymbol{z}(t), \\ \dot{\boldsymbol{z}}(t) = \hat{\boldsymbol{c}} \boldsymbol{L} \boldsymbol{x}(t), \end{cases} \quad (10)$$

where $\hat{\boldsymbol{a}} = \boldsymbol{I}_N \otimes \hat{a}$, $\hat{\boldsymbol{b}} = \boldsymbol{I}_N \otimes \hat{b}$, $\hat{\boldsymbol{c}} = \boldsymbol{I}_N \otimes \hat{c}$, $\boldsymbol{e}_k(t) = (e_{k1}(t), e_{k2}(t), \ldots, e_{kN}(t))^{\mathrm{T}}$, $\nabla \boldsymbol{c}(\boldsymbol{x}(t)) = (\nabla c_1(x_1(t)), \nabla c_2(x_2(t)), \ldots, \nabla c_N(x_N(t)))^{\mathrm{T}}$.

4 Optimality and stability analyses

4.1 Optimality analysis

Assumption 1 The communication topograph is strongly connected and weight-balanced.

Lemma 1 (Olfati-Saber and Murray, 2004) $\boldsymbol{L}$ is defined as the Laplacian matrix of an undirected weighted graph, and the following properties hold:

(1) $\boldsymbol{L}$ exists with at least one zero eigenvalue, and other non-zero eigenvalues have positive real parts.

(2) $\boldsymbol{1}_n$ ($\boldsymbol{1}_n \in \mathbb{R}^n$ is a column vector with all elements being 1) is the left eigenvector of graph

Laplacian matrix $\boldsymbol{L}$ with the only zero eigenvalue, when the graph is connected.

(3) If $\mathbf{1}_n^{\mathrm{T}}\boldsymbol{L}=\mathbf{0}$ holds, the graph is said to be weight-balanced.

Theorem 1 For any $\boldsymbol{\varepsilon}\in\mathbb{R}^n$, there exists a positive invariant set $\Lambda(\boldsymbol{\varepsilon})=\{(\boldsymbol{z},\boldsymbol{x})|(\mathbf{1}_N\otimes\boldsymbol{I}_n)^{\mathrm{T}}\boldsymbol{z}=\boldsymbol{\varepsilon}\}$. When the optimal solution $(\boldsymbol{z}^*,\boldsymbol{x}^*)\in\Lambda(\mathbf{0}_{Nn})$ is the equilibrium of system (10), we can conclude that $\boldsymbol{x}^*$ is the optimal solution to problem (4).

Proof With Assumption 1 and Lemma 1, we know that $\mathbf{1}_N^{\mathrm{T}}\boldsymbol{L}=\mathbf{0}$. Then the following equality is derived:

$$(\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}})(\boldsymbol{L}\otimes\boldsymbol{I}_n)=(\mathbf{0}_N\otimes\boldsymbol{I}_n)^{\mathrm{T}}.$$

Left multiplying $\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}}$ to $\dot{\boldsymbol{z}}(t)$, we can obtain

$$(\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}})\dot{\boldsymbol{z}}(t)=(\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}})\hat{\boldsymbol{c}}\boldsymbol{L}\boldsymbol{x}(t)\equiv\mathbf{0}_{Nn},$$

which means that set $\Lambda(\boldsymbol{\varepsilon})$ is invariant, and $\boldsymbol{\varepsilon}\equiv(\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}})\boldsymbol{z}(0)$.

If there exists an optimal solution $\boldsymbol{x}^*=\mathbf{1}_N\otimes x^*$ satisfying $(\boldsymbol{z}^*,\boldsymbol{x}^*)\in\Lambda(\mathbf{0}_{Nn})$, and it is the equilibrium point of system (10), the following equality is established:

$$-\hat{\boldsymbol{a}}\nabla\boldsymbol{c}(\boldsymbol{x}^*)-(\boldsymbol{L}\otimes\hat{\boldsymbol{b}})\boldsymbol{x}(t_kh)-\hat{\boldsymbol{b}}\boldsymbol{z}^*=\mathbf{0}_{Nn}. \quad (11)$$

Left multiplying $\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}}$ to Eq. (11), and then combined with $\mathbf{1}_N^{\mathrm{T}}\boldsymbol{L}=\mathbf{0}$, we have

$$-(\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}})\hat{\boldsymbol{a}}\nabla\boldsymbol{c}(\boldsymbol{x}^*)-(\mathbf{1}_N^{\mathrm{T}}\otimes\boldsymbol{I}_n^{\mathrm{T}})\hat{\boldsymbol{b}}\boldsymbol{z}^*=\mathbf{0}_{Nn}. \quad (12)$$

Since $\boldsymbol{z}^*\in\Lambda(\mathbf{0}_{Nn})$, we can conclude that $\sum_{i=1}^N\nabla c_i(\boldsymbol{x}^*)=\mathbf{0}$. Then $\nabla c_i(\boldsymbol{x}^*)=\mathbf{0}$ is derived along with Eq. (12), which means that $\boldsymbol{x}^*$ is the optimal solution to problem (4), and $\boldsymbol{z}_i^*=-\hat{b}^{-1}\hat{a}\nabla c_i(\boldsymbol{x}^*)$.

Remark 2 In the above analysis, the system's initial value needs to be located in set $\Lambda(\mathbf{0}_{Nn})$. If the initial value is not located in set $\Lambda(\mathbf{0}_{Nn})$, the system might be convergent as well, but does not converge to the optimal value (Gharesifard and Cortés, 2014).

If there exists an optimal solution $(\boldsymbol{x}^*,\boldsymbol{z}^*)$, combined with Eq. (10), we can derive

$$\begin{cases}\mathbf{0}=-\hat{\boldsymbol{a}}\nabla\boldsymbol{c}(\boldsymbol{x}^*)-(\boldsymbol{L}\otimes\hat{\boldsymbol{b}})\boldsymbol{x}(t_kh)-\hat{\boldsymbol{b}}\boldsymbol{z}^*,\\ \mathbf{0}=\hat{\boldsymbol{c}}\boldsymbol{L}\boldsymbol{x}^*,\end{cases} \quad (13)$$

and further obtain

$$\begin{cases}(\boldsymbol{L}\otimes\hat{\boldsymbol{b}})\boldsymbol{x}(t_kh)=-\hat{\boldsymbol{b}}\boldsymbol{z}^*,\\ \boldsymbol{L}\boldsymbol{x}^*=\mathbf{0}.\end{cases} \quad (14)$$

4.2 Stability analysis

In this subsection, a sufficient condition is given to ensure that the algorithm converges to the optimal solution to the distributed ED optimization problem, and the theoretical proof is provided.

Theorem 2 The algorithm converges to the optimal value of the ED minimization problem if there exists

$$0<\boldsymbol{P}=\begin{bmatrix}\boldsymbol{P}_{11} & \boldsymbol{P}_{12}\\ * & \boldsymbol{P}_{22}\end{bmatrix}\in\mathbb{R}^{(2Nn)\times(2Nn)}$$

such that the following linear matrix inequality (LMI) holds:

$$\begin{aligned}&\mathrm{Sym}\{\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}_{11}\boldsymbol{e}_1+\boldsymbol{H}^{\mathrm{T}}\boldsymbol{P}_{12}\boldsymbol{e}_3\\ &+\boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{P}_{21}\boldsymbol{e}_1+\boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{P}_{22}\boldsymbol{e}_3\}<0,\end{aligned} \quad (15)$$

where $\boldsymbol{H}=-\hat{\boldsymbol{a}}\boldsymbol{e}_2-\hat{\boldsymbol{b}}\boldsymbol{e}_3$, $\boldsymbol{H}_1=\hat{\boldsymbol{c}}\boldsymbol{L}\boldsymbol{e}_1$, $\mathrm{Sym}\{(\cdot)\}=(\cdot)+(\cdot)^{\mathrm{T}}$, and $\boldsymbol{e}_i=[\mathbf{0}_{n\times[(i-1)n]}\quad\boldsymbol{I}_n\quad\mathbf{0}_{n\times[(3-i)n]}]$ $(i=1,2,3)$.

Proof Define the residual vectors:

$$\begin{cases}\tilde{\boldsymbol{x}}(t)=\boldsymbol{x}(t)-\boldsymbol{x}^*,\\ \tilde{\boldsymbol{z}}(t)=\boldsymbol{z}(t)-\boldsymbol{z}^*.\end{cases} \quad (16)$$

Hence, we have

$$\begin{cases}\boldsymbol{x}(t)=\tilde{\boldsymbol{x}}(t)+\boldsymbol{x}^*,\\ \boldsymbol{z}(t)=\tilde{\boldsymbol{z}}(t)+\boldsymbol{z}^*.\end{cases} \quad (17)$$

Substituting the above formula into Eq. (10), and combined with Eq. (14), we can obtain

$$\begin{cases}\dot{\tilde{\boldsymbol{x}}}(t)=-\hat{\boldsymbol{a}}\nabla\boldsymbol{c}(\tilde{\boldsymbol{x}}(t))-\hat{\boldsymbol{b}}\tilde{\boldsymbol{z}}(t),\\ \dot{\tilde{\boldsymbol{z}}}(t)=\hat{\boldsymbol{c}}\boldsymbol{L}\tilde{\boldsymbol{x}}(t).\end{cases} \quad (18)$$

This process transforms the optimal solution $(\boldsymbol{x}^*,\boldsymbol{z}^*)$ to the origin. Then we need only to prove the stability of residual system (18). We construct an augmented Lyapunov function as follows:

$$V(t)=(\boldsymbol{w}(t))^{\mathrm{T}}(\boldsymbol{P}\otimes\boldsymbol{I}_n)\boldsymbol{w}(t),$$

where $(\boldsymbol{w}(t))^{\mathrm{T}}=[(\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}\ (\tilde{\boldsymbol{z}}(t))^{\mathrm{T}}]$, and $\boldsymbol{P}$ is the matrix to be defined. The derivation of $V(t)$ with respect to time t is as follows:

$$\begin{aligned}
&\dot{V}(t)= \\
&-2(\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}(\boldsymbol{P}_{11}\otimes\boldsymbol{I}_n)(\hat{\boldsymbol{a}}\nabla\boldsymbol{c}(\tilde{\boldsymbol{x}}(t))-\hat{\boldsymbol{b}}\tilde{\boldsymbol{z}}(t)) \\
&-(\tilde{\boldsymbol{z}}(t))^{\mathrm{T}}(\boldsymbol{P}_{21}\otimes\boldsymbol{I}_n)(\hat{\boldsymbol{a}}\nabla\boldsymbol{c}(\tilde{\boldsymbol{x}}(t))-\hat{\boldsymbol{b}}\tilde{\boldsymbol{z}}(t)) \\
&+\left(-(\nabla\boldsymbol{c}(\tilde{\boldsymbol{x}}(t)))^{\mathrm{T}}\hat{\boldsymbol{a}}^{\mathrm{T}}-(\tilde{\boldsymbol{z}}(t))^{\mathrm{T}}\hat{\boldsymbol{b}}^{\mathrm{T}}\right)(\boldsymbol{P}_{12}\otimes\boldsymbol{I}_n)\tilde{\boldsymbol{z}}(t) \\
&+(\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}\boldsymbol{L}^{\mathrm{T}}\tilde{\boldsymbol{c}}^{\mathrm{T}}(\boldsymbol{P}_{21}\otimes\boldsymbol{I}_n)\tilde{\boldsymbol{x}}(t) \\
&+(\tilde{\boldsymbol{x}}(t))^{\mathrm{T}}(\boldsymbol{P}_{12}\otimes\boldsymbol{I}_n)\tilde{\boldsymbol{c}}\boldsymbol{L}\tilde{\boldsymbol{x}}(t) \\
&+2(\tilde{\boldsymbol{z}}(t))^{\mathrm{T}}(\boldsymbol{P}_{22}\otimes\boldsymbol{I}_n)\tilde{\boldsymbol{c}}\boldsymbol{L}\tilde{\boldsymbol{x}}(t).
\end{aligned}$$

To simplify the statement, denote $\tilde{\boldsymbol{P}}_{ij}=\boldsymbol{P}_{ij}\otimes\boldsymbol{I}_n$, and define an augmented vector $\boldsymbol{\zeta}(t)$ as follows:

$$\boldsymbol{\zeta}(t)=\mathrm{col}\{\tilde{\boldsymbol{x}}(t),\ \nabla\boldsymbol{c}(\tilde{\boldsymbol{x}}(t)),\ \tilde{\boldsymbol{z}}(t)\}.$$

Then $\dot{V}(t)$ can be written in the following compact form:

$$\begin{aligned}
\dot{V}(t)=&\boldsymbol{\zeta}^{\mathrm{T}}(t)[\mathrm{Sym}\{\boldsymbol{H}^{\mathrm{T}}\tilde{\boldsymbol{P}}_{11}\boldsymbol{e}_1+\boldsymbol{H}^{\mathrm{T}}\tilde{\boldsymbol{P}}_{12}\boldsymbol{e}_3 \\
&+\boldsymbol{H}_1^{\mathrm{T}}\tilde{\boldsymbol{P}}_{21}\boldsymbol{e}_1+\boldsymbol{H}_1^{\mathrm{T}}\tilde{\boldsymbol{P}}_{22}\boldsymbol{e}_3\}]\boldsymbol{\zeta}(t).
\end{aligned}$$

Hence, if inequality (15) has a feasible solution, we can conclude that $\dot{V}(t)<0$, and that system (18) is asymptotically stable, which completes the proof.

5 Simulation and discussion

A comparison simulation is performed to verify the effectiveness of the proposed resilient triggered scheme. Suppose that the single-area power system consists of four TGs, three WGs, and three PGs; the model parameters are given in Tables 1 and 2. The communication topograph is shown in Fig. 3, and the corresponding graph Laplacian matrices are given as follows:

$$\boldsymbol{L}_1=$$
$$\begin{bmatrix}
3 & -1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 4 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 3 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & -1 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & -1 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 3 & 0 & -1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2
\end{bmatrix},$$

Table 1 Model parameters of thermal generation (TG)

i	α_i	β_i	γ_i	$\eta_{\mathrm{EPP}i}$	a_i	b_i	c_i
1	12	1.4	2	0.5	3	0.92	3
3	20	−10	5	0.4	1	−4	8
5	14	5	2	0.6	0.8	0	−9
8	5	−5	1.5	0.8	0.4	2	1

Table 2 Model parameters of wind power generation (WG, left) and photovoltaic power generation (PG, right)

i	$a_{\mathrm{w}i}$	$b_{\mathrm{w}i}$	$c_{\mathrm{w}i}$	i	$a_{\mathrm{p}i}$	$b_{\mathrm{p}i}$	$c_{\mathrm{p}i}$
2	2	0	0	6	2.4	0	0
4	1.5	−0.4	−0.6	7	1.2	−0.3	−0.8
9	8	0	0	10	6	−3	−2

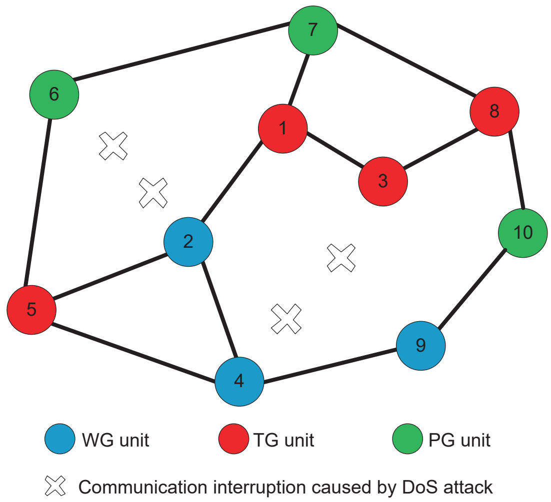

Fig. 3 Communication topograph with denial of service (DoS) attack. References to color refer to the online version of this figure

$$\boldsymbol{L}_2=$$
$$\begin{bmatrix}
3 & -1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
-1 & 3 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 2 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & -1 & 0 & 3 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & -1 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 3 & 0 & -1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2
\end{bmatrix}.$$

The distributed optimization problem can be written as follows:

$$\min\ \boldsymbol{c}(\boldsymbol{x})=\sum_{i=1}^{10}c_i(x_i), \tag{19}$$

where $c_1(x_1)=4x_1^2+1.62x_1+9$, $c_2(x_2)=2x_2^2$,

$c_3(x_3) = 3x_3^2 - 8x_3 + 16$, $c_4(x_4) = 1.5x_4^2 - 0.4x_4 - 0.6$, $c_5(x_5) = 2x_5^2 + 3x_5 + 7$, $c_6(x_6) = 2.4x_6^2$, $c_7(x_7) = 1.2x_7^2 - 0.3x_7 - 0.18$, $c_8(x_8) = 1.2x_8^2 - 2x_8 + 5$, $c_9(x_9) = 8x_9^2$, and $c_{10}(x_{10}) = 6x_{10}^2 - 3x_{10} - 2$.

For the case without attack, we choose $\hat{a} = 0.8$, $\hat{b} = 1$, $\hat{c} = 1.2$, the general triggered parameter $\sigma = 0.4$, and the upper bound of time delay $\bar{d} = 0.4$. The feasible solutions to matrix inequality (15) can be obtained based on the LMI toolbox. The simulation results are shown in Fig. 4, where we choose $\boldsymbol{x}(0) = \boldsymbol{0}$.

From the above results, we can conclude that the multi-agent system which leaves out the DoS attack eventually converges to the optimal solution $x^* = 0.1605$ and the minimum generation cost $c^* = 33.5793$. As for the case where the DoS attack occurs, we choose $\sigma_{\text{re}} = 0.03$ based on inequality (8), and the results are shown in Fig. 5.

Based on the above simulation results, it can be known that the multi-agent system can basically reach the optimal solution when the DoS attack is taken into account, and the optimal solution is $x^* = 0.1614$, $c^* = 33.5803$. The resilient event-triggered scheme designed in this study can effectively eliminate the impact of DoS attack, so that the results are basically the same as those without

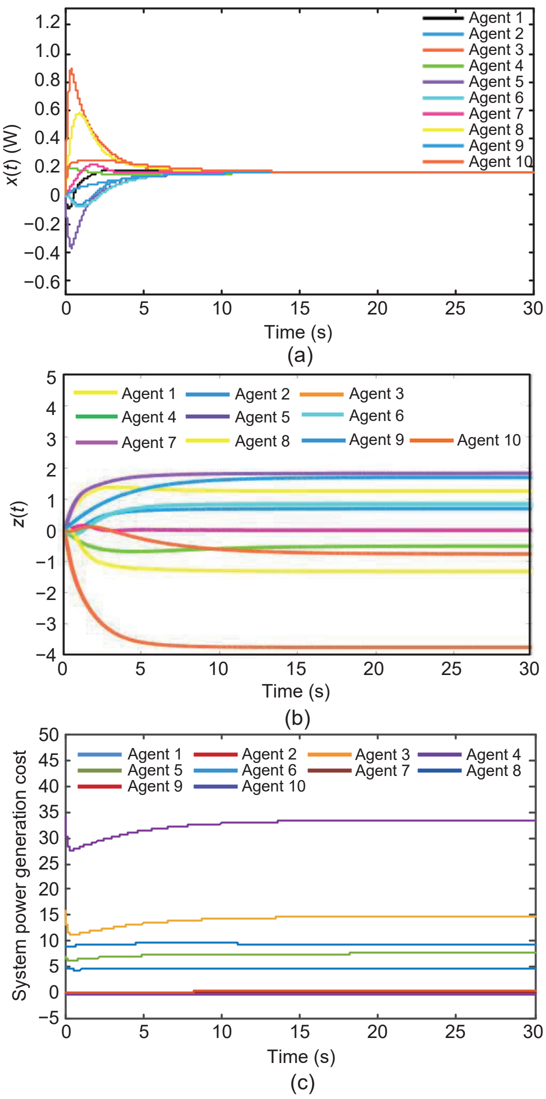

Fig. 4 Change trends of the multi-agent system state $x(t)$ (a), auxiliary state $z(t)$ (b), and system power generation cost (c) without DoS attack. References to color refer to the online version of this figure

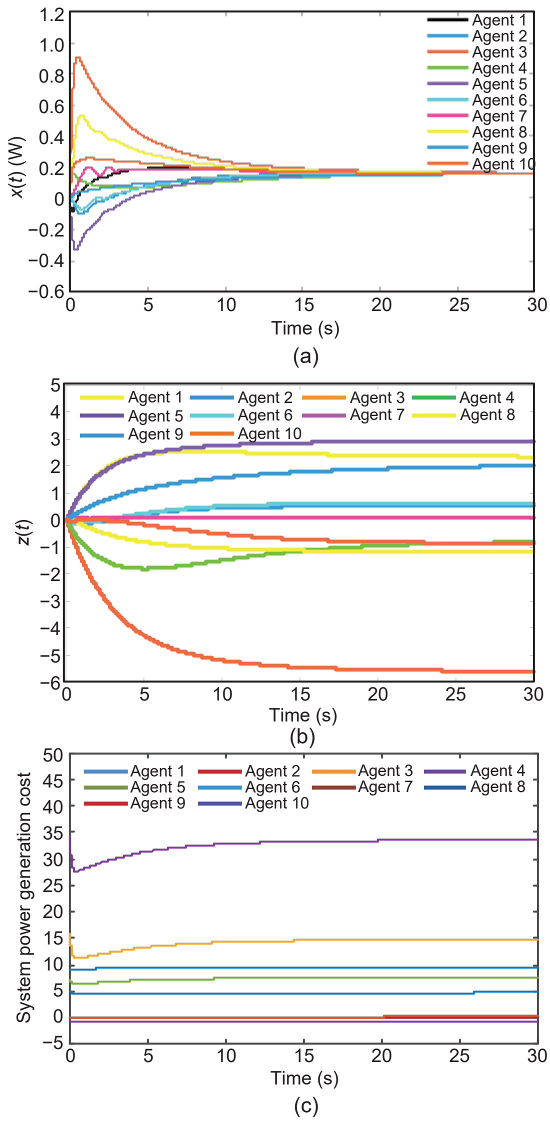

Fig. 5 Change trends of the multi-agent system state $x(t)$ (a), auxiliary state $z(t)$ (b), and system power generation cost (c) with DoS attack. References to color refer to the online version of this figure

attack.

A more intuitive simulation result to verify the effectiveness of the resilient event-triggered scheme is shown in Fig. 6, in which the general triggered parameter $\sigma = 0.4$ is adopted in the case where a DoS attack occurs.

To verify the effectiveness of the designed distributed resilient event-triggered mechanism in reducing communication bandwidth consumption, we make a simple comparison. We choose the time triggered sampling period $h = 0.01$ s and simulation time $T = 30$ s. Then we can obtain Table 3.

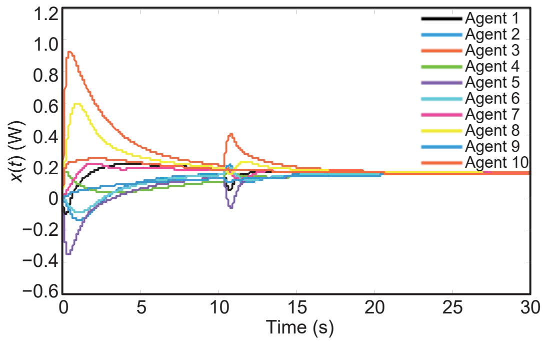

Fig. 6 Change trends of the multi-agent system state $x(t)$ with DoS attack when $\sigma = 0.4$. References to color refer to the online version of this figure

Table 3 Triggering times with different schemes

Scheme	Triggering threshold	Triggering times
TTS	0	3000
ETS	0.4	3
RETS	0.03	138

TTS: time-triggered scheme; ETS: event-triggered scheme; RETS: resilient event-triggered scheme

6 Conclusions

In this paper, we are concerned mainly with the distributed economic dispatch optimization problem of a smart grid in the presence of malicious attack on the communication network. To reduce the communication bandwidth consumption and eliminate the effects caused by DoS, a distributed resilient event-triggered scheme has been proposed, and the explicit relationship between the resilient triggered threshold $\sigma_{\rm re}$ and a general one has been established. Then, a modified multi-agent model has been put forward which transforms the minimization problem into a consensus protocol design problem, and a sufficient condition for reaching the optimal state has been derived. Finally, the effectiveness of the proposed method has been verified by simulations.

Contributors

Feisheng YANG designed the research. Feisheng YANG, Xiaohong GUAN, and Xuhui LIANG processed the data. Feisheng YANG drafted the manuscript. Feisheng YANG and Xuhui LIANG revised and finalized the paper.

Compliance with ethics guidelines

Feisheng YANG, Xuhui LIANG, and Xiaohong GUAN declare that they have no conflict of interest.

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Appendix: Proof of Proposition 1

The general distributed event-triggered scheme is shown as follows (Li et al., 2016; Wang AP et al., 2019):

$$\begin{aligned}&|x_i(t_kh+rh)-x_i(t_kh)|\\ &\leq\sigma\left|\sum_{j\in N_i}a_{ij}(x_i(t_kh)-x_j(t_kh))\right|,\end{aligned}\tag{A1}$$

where $\sigma\in(0,1)$. The compact form is given below:

$$|\boldsymbol{x}(t_kh+rh)-\boldsymbol{x}(t_kh)|\leq\sigma|\boldsymbol{L}_1\boldsymbol{x}(t_kh)|.\tag{A2}$$

Note that the infinite norm is used here.

We can divide the interval $[t_kh,t_{k+1}h)$ into $M+1$ subintervals, and any of them can be described as $t\in[r_l,r_{l+1})$, $l=0,1,\ldots,M$.

Define

$$\boldsymbol{e}(t)=(e_1(t),e_2(t),\ldots,e_N(t))^{\mathrm{T}}$$

and $e_i(t)=x_i(t)-x_i(t_kh)$. $\forall t\in[t_kh,t_{k+1}h)$, we can obtain

$$\begin{aligned}|e_i(t)|&=|x_i(t)-x_i(t_kh)|\\ &\leq\sum_{n=0}^{l-1}|x_i(r_{n+1})-x_i(r_n)|+|x_i(t)-x_i(r_l)|.\end{aligned}$$

However, there is no sampled data which satisfies the triggered condition (6) in $[r_l,r_{l+1})$. Hence, the following inequalities can be established:

$$\begin{cases}|x_i(t)-x_i(r_l)|\leq\sigma_{\mathrm{re}}\left|\sum\limits_{j\in N_i}a_{ij}(x_i(r_l)-x_j(r_l))\right|,\\ |x_i(r_{n+1})-x_i(r_n)|\leq\sigma_{\mathrm{re}}\left|\sum\limits_{j\in N_i}a_{ij}(x_i(r_n)-x_j(r_n))\right|.\end{cases}\tag{A3}$$

Then,

$$\begin{aligned}|e_i(t)|&=|x_i(t)-x_i(t_kh)|\\ &\leq\sum_{n=0}^{l}\sigma_{\mathrm{re}}\left|\sum_{j\in N_i}a_{ij}(x_i(r_n)-x_j(r_n))\right|,\end{aligned}$$

$$|\boldsymbol{e}(t)|=|\boldsymbol{x}(t)-\boldsymbol{x}(t_kh)|\leq\sum_{n=0}^{l}\sigma_{\mathrm{re}}|\boldsymbol{L}\boldsymbol{x}(r_n)|.\tag{A4}$$

The compact form of inequality (A3) is

$$|\boldsymbol{x}(r_{n+1})-\boldsymbol{x}(r_n)|\leq\sigma_{\mathrm{re}}|\boldsymbol{L}_2\boldsymbol{x}(r_n)|.$$

We can conclude

$$|\boldsymbol{x}(r_{n+1})|-|\boldsymbol{x}(r_n)| \leq |\boldsymbol{x}(r_{n+1})-\boldsymbol{x}(r_n)| \leq \sigma_{\rm re}|\boldsymbol{L}_2||\boldsymbol{x}(r_n)|$$

based on the vector norm triangle inequality and $|\boldsymbol{A}\boldsymbol{x}| \leq |\boldsymbol{A}||\boldsymbol{x}|$. $|\boldsymbol{x}(r_{n+1})| \leq (\sigma_{\rm re}|\boldsymbol{L}_2|+1)|\boldsymbol{x}(r_n)|$ is obtained. Combined with inequality (A4), we have

$$\begin{aligned}|\boldsymbol{e}(t)| &= |\boldsymbol{x}(t)-\boldsymbol{x}(t_k h)| \\ &\leq \sum_{n=0}^{l} \sigma_{\rm re}|\boldsymbol{L}_2|(\sigma_{\rm re}|\boldsymbol{L}_2|+1)^n|\boldsymbol{x}(t_k h)| \\ &\leq (\sigma_{\rm re}|\boldsymbol{L}_2|+1)^{M+1}|\boldsymbol{x}(t_k h)|. \qquad \text{(A5)}\end{aligned}$$

Nevertheless, the triggered packets are transmitted unsuccessfully when $t \in [r_l, r_{l+1})$ such that the following inequality can be obtained based on inequality (A2):

$$\begin{aligned}|\boldsymbol{e}(t)| &= |\boldsymbol{x}(t)-\boldsymbol{x}(t_k h)| \\ &\leq \sigma|\boldsymbol{L}_1\boldsymbol{x}(t_k h)| \leq \sigma|\boldsymbol{L}_1||\boldsymbol{x}(t_k h)|. \qquad \text{(A6)}\end{aligned}$$

Combining the above analyses, it is clear that the current data will be updated if and only if the triggered conditions are violated. However, the resilient event-triggered scheme needs to eliminate the packet loss caused by DoS attack, and hence the general triggered threshold should be greater than the resilient triggered one, such that inequality (8) can be derived based on inequalities (A5) and (A6). This completes the proof.