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Finite-time formation control for first-order multi-agent systems with region constraints*

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Abstract: In this study, the finite-time formation control of multi-agent systems with region constraints is studied. Multiple agents have first-order dynamics and a common target area. A novel control algorithm is proposed using local information and interaction. If the communication graph is undirected and connected and the desired framework is rigid, it is proved that the controller can be used to solve the formation problem with a target area. That is, all agents can enter the desired region in finite time while reaching and maintaining the desired formation shapes. Finally, a numerical example is given to illustrate the results.

Key words: Finite-time formation; Multi-agent system; Asymptotic convergence; Set constraint; Lyapunov theorem

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1 Introduction

Research on flight formation and consensus control of multiple spacecraft has been an interesting topic for decades. In addition, the explosion of computing and the rapid improvement of communication have increased activities in many research fields. In particular, there has been increasing interest in the problem of formation control of multiple agents in recent years (Porfiri et al., 2007; Oh and Ahn, 2013; Oh et al., 2015; Dong and Hu, 2016). The formation of a multi-agent system (MAS) is designed to perform multitasking and achieve high precision, slow displacement, and stable automatic control. The task of the agents is to realize and keep formation shape, where each agent depends on internal communication and perception with neighbors. The multi-agent formation with a large effective coverage area can solve problems that a single agent cannot do (Fax and Murray, 2004; Lafferriere et al., 2005).

Earlier works on the control maneuver problem of MAS focus mostly on the consistent control of agents and achieving the goal of formation. Porfiri et al. (2007) and Dong and Hu (2016) investigated the time-varying and time-invariant formation control problems of an MAS, considering general linear dynamics. Xia et al. (2016) studied the collision avoidance problem of multiple agents in the formation process. Under the assumption that network topologies are fixed and undirected, Xie and Wang (2009) provided the decentralized formation control maneuvers for a first-order MAS. The formation problem in complex situations has attracted much attention, and numerous results have been reported. Li et al. (2013) investigated the formation problem for a nonlinear MAS with time-varying communication delay. The formation of multiple agents is applied to not only single-integration MAS, but also high-order integral MAS.

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Rezaee and Abdollahi (2015) considered the formation of an MAS with second-order dynamics. Dong et al. (2014) introduced time-invariant formation control for linear MAS, and considered multiple agents with high-order dynamics and communication time delays. In reality, we may need to consider some constraints, including space and angle-only constraints (Egerstedt and Hu, 2001; Hong et al., 2006; Basiri et al., 2010; Ge et al., 2016). Basiri et al. (2010) designed a distributed coupled controller with specified angle constraints, and provided a new tool for trajectory tracking control. Yang et al. (2019) studied the formation problem for an MAS with region constraints. The majority of works mentioned above studied only the stability of formation and few concerned the finite-time stability. Xiao et al. (2009) developed the framework to deal with consensus and finite-time formation control of an MAS. Focusing on first- and second-order dynamics of multiple agents, Sun et al. (2014) proposed an algorithm that can reach formation stabilization in finite time.

In practical applications, we need to consider some constraints and timeliness, such as the need to implement a formation into a specific area within a specific time. So, it is necessary to consider the formation problem with both finite time and regional constraints. In this study, we design a controller which controls all agents to enter the desired region and to track a desired shape in finite time. An MAS with first-order dynamics is considered. Compared with Xiao et al. (2009) and Sun et al. (2014), we consider the regional constraints. Compared with Basiri et al. (2010) and Yang et al. (2019), we consider the finite time. In fact, we study a more complex situation. For the desired region, the previous results often lead to a leader-following formation controller. Unlike the previous results, our algorithm does not depend on the leader. It has good robustness. In comparison with the controller in Yang et al. (2019), the proposed algorithm handles the finite-time formation problem in two steps successively. First, it makes all the agents enter the desired area in finite time. Second, it enables all the agents to reach the desired formation shapes in finite time while keeping the shapes.

2 Preliminaries

Notations are described as follows: Let $\boldsymbol{x}$ be a two-dimensional (2D) column vector; its transpose is denoted as $\boldsymbol{x}^{\mathrm{T}}$. $\mathbb{R}^2$ is the set of all 2D real vectors. Ω is the closed convex set in $\mathbb{R}^2$. $\|\boldsymbol{x}\|_1$ is the 1-norm of $\boldsymbol{x}$. $\|\boldsymbol{x}\|$ and $P_\Omega(\boldsymbol{x}) = \arg\min\limits_{\bar{\boldsymbol{x}}\in\Omega} \|\boldsymbol{x}-\bar{\boldsymbol{x}}\|$ denote the 2-norm of $\boldsymbol{x}$ and the projection of vector $\boldsymbol{x}$ in Ω, respectively. The sign function of $\boldsymbol{x}$ is defined as $\mathrm{sgn}(\boldsymbol{x})$. If the function $h(\boldsymbol{x})$ is reversible, then its inverse is denoted as $h^{-1}(\boldsymbol{x})$. $\partial/\partial\boldsymbol{x}$ is the partial differential with respect to $\boldsymbol{x}$. $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $\boldsymbol{A}\in\mathbb{R}^{n\times m}$ is a matrix, and its transpose is denoted as $\boldsymbol{A}^{\mathrm{T}}$.

In this study, we consider each node in the graph as an agent. The mobile agents are regarded as the vertices of a graph, and interactions among agents are considered as edges of the graph. We consider that the communication graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ is composed of vertex set $\mathcal{V}=\{1,2,\ldots,N\}$ and edge set $\mathcal{E}$. If $(i,j)\in\mathcal{E}$, then nodes i and j are neighbors, and node i can obtain the information of node j. The neighbors of vertex i are represented as $N_i=\{j: j\in\mathcal{V},(i,j)\in\mathcal{E}\}$. The adjacency matrix of graph $\mathcal{G}$ is $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$, where $a_{ii}=0$, $a_{ij}=a_{ji}=1$ if $(i,j)\in\mathcal{E}$; otherwise, $a_{ij}=0$.

We consider that the system is composed of N agents (indexed as $1,2,\ldots,N$). The dynamics of the agents can be described as

$$\dot{\boldsymbol{x}}_i(t)=\boldsymbol{u}_i(t), \tag{1}$$

where $\boldsymbol{x}_i(t)\in\mathbb{R}^2$ and $\boldsymbol{u}_i(t)\in\mathbb{R}^2$ represent the position vector and control input acting on agent i, respectively.

We employ a rigid framework $(\mathcal{G},\boldsymbol{X})$ to describe the target formation shape. Each agent is viewed as a vertex of the framework. First, we need the following definition:

Definition 1 The pair $(\mathcal{G},\boldsymbol{X})$ is a framework of $\mathcal{G}$ in $\mathbb{R}$, where $\boldsymbol{X}=[\boldsymbol{x}_1^{\mathrm{T}},\boldsymbol{x}_2^{\mathrm{T}},\ldots,\boldsymbol{x}_N^{\mathrm{T}}]^{\mathrm{T}}\in\mathbb{R}^{2N}$. Ordering edges $\mathcal{E}$ in some way, the rigidity function $h_G(\boldsymbol{X}):\mathbb{R}^{2N}\rightarrow\mathbb{R}^{|\mathcal{E}|}$ related to the pair $(\mathcal{G},\boldsymbol{X})$ is defined as (Oh et al., 2015)

$$\begin{aligned}h_G(\boldsymbol{X})=&\frac{1}{2}\big[\|\boldsymbol{x}_i-\boldsymbol{x}_{j1}\|^2,\|\boldsymbol{x}_i-\boldsymbol{x}_{j2}\|^2,\ldots,\\&\|\boldsymbol{x}_i-\boldsymbol{x}_{jk}\|^2\big]^{\mathrm{T}},\ k=|\mathcal{E}|,\end{aligned} \tag{2}$$

where $|\mathcal{E}|$ denotes the cardinality of $\mathcal{E}$ and $(i,j)\in\mathcal{E}$.

Second, the rigidity framework is defined as follows:

Definition 2 Let $\mathcal{G}$ and $\mathcal{K}$ be a graph and the complete graph of N vertices, respectively. A framework

$(\mathcal{G}, \boldsymbol{X})$ in $\mathbb{R}^2$ is said to be rigid if there exists a neighborhood $\mathcal{U}$ in $\mathbb{R}^{2N}$ such that $h_G^{-1}(h_G(\boldsymbol{X})) \cap \mathcal{U} = h_{\mathcal{K}}^{-1}(h_{\mathcal{K}}(\boldsymbol{X})) \cap \mathcal{U}$ (Asimow and Roth, 1979). The complete graph $\mathcal{K}$ means that for any two vertices in $\mathcal{V}$, there is an edge between them.

Remark 1 The rigidity framework is a graph, in which the lengths of some edges are sufficiently limited to maintain the entire configuration. In other words, in a rigid graph, the motion of any vertex will not destroy the shape of the whole graph. We can see that Fig. 1a represents a nonrigid framework and that Fig. 1b shows a rigid one.

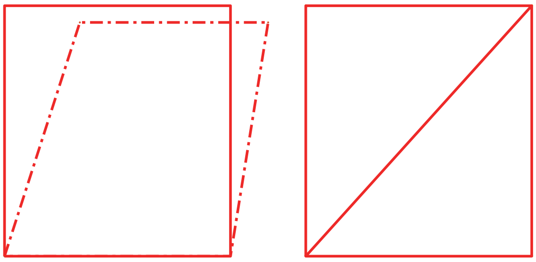

Fig. 1 Nonrigid (a) and rigid (b) frameworks

In this study, we consider the finite-time formation of a multi-agent problem. That is, all agents are given a target region $\Omega \in \mathbb{R}^2$ which limits all the agents to enter the desired area and reach formation in finite time. To deal with the problem, given that a vector $\boldsymbol{X}^* = [\boldsymbol{x}_1^{*\mathrm{T}}, \boldsymbol{x}_2^{*\mathrm{T}}, \ldots, \boldsymbol{x}_N^{*\mathrm{T}}]^{\mathrm{T}}$ ($\boldsymbol{x}_i^* \in \Omega$) is the final desired state of the formation, $(\mathcal{G}, \boldsymbol{X}^*)$ is rigid. The target formation is given as follows:

$$E_{\boldsymbol{X}} := \{\boldsymbol{X} : \boldsymbol{x}_i - \boldsymbol{x}_j = \boldsymbol{x}_i^* - \boldsymbol{x}_j^*, j \in N_i, i \in \mathcal{K}\}. \quad (3)$$

That is, $E_{\boldsymbol{X}}$ is the set of all formations congruent to $\boldsymbol{X}^*$.

The following two lemmas will be used in this study:

Lemma 1 The system is defined as

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = f(\boldsymbol{x}),\ f(0) = 0,\ \boldsymbol{x} \in \mathbb{R}^2. \quad (4)$$

Let a continuous function $V(\boldsymbol{x}) : \boldsymbol{U} \longrightarrow \mathbb{R}$ be positive definite. If there exist an open neighborhood $\boldsymbol{U}_0 \subset \boldsymbol{U}$ of the origin, $c > 0$, and $\alpha \in (0, 1)$, such that $\dot{V}(\boldsymbol{x}) + c(V(\boldsymbol{x}))^\alpha \le 0$, $\boldsymbol{x} \in \boldsymbol{U}_0 \backslash \{0\}$, then $V(\boldsymbol{x}(t)) \longrightarrow 0$ for all $t > T(\boldsymbol{x}_0)$, where $T(\boldsymbol{x}_0) \le \dfrac{(V(\boldsymbol{x}_0))^{1-\alpha}}{c(1-\alpha)}$ and the initial state $\boldsymbol{x}(t_0) = \boldsymbol{x}_0 \in \boldsymbol{U}_0 \setminus \{0\}$. This implies that $V(\boldsymbol{x})$ would converge to zero in finite time. Therefore, the stability of system (4) is proved (Bhat and Bernstein, 2000).

Lemma 2 Assume that $Y \ne \varnothing$ is a closed convex set in $\mathbb{R}^2$. Then, we have $\nabla \frac{1}{2}\|\boldsymbol{y} - P_Y(\boldsymbol{y})\|^2 = \boldsymbol{y} - P_Y(\boldsymbol{y})$, and $\|\boldsymbol{y} - P_Y(\boldsymbol{y})\|$ is continuous with respect to $\boldsymbol{y}$ ($\forall \boldsymbol{y} \in \mathbb{R}^2$) (Facchinei and Pang, 2003).

To facilitate the later analysis, the following assumptions are necessary:

Assumption 1 In this paper, the communication graph $\mathcal{G}$ is undirected and connected for all t.

Assumption 2 The desired framework $(\mathcal{G}, \boldsymbol{X}^*)$ is rigid.

Remark 2 The rigidity of the framework $(\mathcal{G}, \boldsymbol{X}^*)$ ensures that the target formation can be achieved.

3 Asymptotic formation control and entry into the desired area in finite time

In this section, we introduce a novel maneuver to make the agents enter the target convex region and achieve target formation in finite time. An MAS with first-order dynamics is considered. The algorithm consists of a target coordination part and a sign function of the projection of $\boldsymbol{x}_i(t)$ in Ω. The formation controller is given as follows:

$$\begin{aligned} \boldsymbol{u}_i(t) = &- \sum_{j \in N_i} a_{ij} \mathrm{sgn}(\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t) - \boldsymbol{\gamma}_{ij}^*) \\ &- \beta \mathrm{sgn}(\boldsymbol{x}_i(t) - P_\Omega(\boldsymbol{x}_i(t))), \end{aligned} \quad (5)$$

where β is a positive coefficient, $\boldsymbol{u}_i(t)$ is the control input of agent i, $\boldsymbol{\gamma}_{ij}^*$ is the state vector of the desired formation, $P_\Omega(\boldsymbol{x}_i(t))$ is the convex projection of $\boldsymbol{x}_i(t)$ in Ω, and $\mathrm{sgn}(\cdot)$ is the sign function. For simplification, let $\boldsymbol{\gamma}_{ij}^* = \boldsymbol{x}_i^* - \boldsymbol{x}_j^*$. Then, $\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t) - \boldsymbol{\gamma}_{ij}^* = \boldsymbol{x}_i(t) - \boldsymbol{x}_i^* - (\boldsymbol{x}_j(t) - \boldsymbol{x}_j^*) = \tilde{\boldsymbol{x}}_i(t) - \tilde{\boldsymbol{x}}_j(t)$, where $\tilde{\boldsymbol{x}}_i(t) = \boldsymbol{x}_i(t) - \boldsymbol{x}_i^*$ and $\tilde{\boldsymbol{x}}_j(t) = \boldsymbol{x}_j(t) - \boldsymbol{x}_j^*$. Then, we will give the main results of this study.

Theorem 1 Under Assumptions 1 and 2, if $\beta > N$, for the first-order MAS (1) with controller (5), all the agents can realize the asymptotic convergence of the formation shape in finite time and $\boldsymbol{x}_i(t) \in \Omega$ in finite time.

Then, Theorem 1 will be proved. First, we will prove that all the agents can enter the target convex region in finite time. Second, we will verify that all the agents achieve finite-time formation. For simplification, $\boldsymbol{x}_i(t)$ is written as $\boldsymbol{x}_i$ and $\boldsymbol{x}_j(t)$ is written as $\boldsymbol{x}_j$ for any $i, j = 1, 2, \ldots, N$.

Proof First, to prove that all the agents have access to the target area in a limited time, we take the Lyapunov candidate function of system (1) as

$$V_i(t) = \frac{1}{2}\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\|^2. \tag{6}$$

Taking the derivative of $V_i(t)$ with respect to t along system (1) with controller (5), it follows from Lemma 2 that

$$\begin{aligned}\dot{V}_i(t) =& [\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)]^\mathrm{T}\dot{\boldsymbol{x}}_i \\ =& -\sum_{j\in N_i} a_{ij}[\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)]^\mathrm{T}\mathrm{sgn}(\boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{\gamma}_{ij}^*) \\ &- \beta[\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)]^\mathrm{T}\mathrm{sgn}(\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)) \\ \leq& \sum_{j=1}^{N} a_{ij}\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\|\|\mathrm{sgn}(\boldsymbol{x}_i - \boldsymbol{x}_j - \boldsymbol{\gamma}_{ij}^*)\| \\ &- \beta[\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)]^\mathrm{T}\mathrm{sgn}(\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)) \\ \leq& N\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\| - \beta[\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)]^\mathrm{T} \\ &\cdot \mathrm{sgn}(\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)).\end{aligned} \tag{7}$$

Because $[\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)]^\mathrm{T}\mathrm{sgn}(\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)) = \|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\|_1$ and $\|\boldsymbol{x}\|_1 \geq \|\boldsymbol{x}\|$ ($\boldsymbol{x} \in \mathbb{R}^2$), we have

$$\begin{aligned}&-[\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)]^\mathrm{T}\mathrm{sgn}(\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)) \\ &\quad = -\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\|_1 \\ &\quad \leq -\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\|.\end{aligned}$$

Then, if $\beta > N$, inequality (7) can be rewritten as

$$\begin{aligned}\dot{V}_i(t) \leq& N\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\| - \beta\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\| \\ =& (N-\beta)\|\boldsymbol{x}_i - P_\Omega(\boldsymbol{x}_i)\| \\ \leq& 0.\end{aligned} \tag{8}$$

This implies that $V_i(t)$ is bounded. By Eq. (6), we have

$$\dot{V}_i(t) \leq \sqrt{2}(N-\beta)\sqrt{V_i(t)}. \tag{9}$$

Then inequality (9) can be rewritten as

$$\frac{\dot{V}_i(t)}{\sqrt{V_i(t)}} \leq -\sqrt{2}(\beta - N) < 0. \tag{10}$$

There exist $t_0 \geq 0$ and $t \geq t_0$. Integrating on both sides of inequality (10) starting from t_0 to t, we can obtain

$$\sqrt{V_i(t)} - \sqrt{V_i(t_0)} \leq -\sqrt{2}(\beta - N)(t - t_0). \tag{11}$$

It is clear that $V_i(t)$ asymptotically converges to zero in finite time. That is, there exists a constant $t_1 > t_0$ such that $\boldsymbol{x}_i(t) \in \Omega$ for all $t > t_1$ and for all i. So, system (1) is asymptotically stable by Lemma 1. Namely, agents can access the desired region in finite time.

Next, we will prove that all the agents can achieve an asymptotic target formation shape in limited time. Define the Lyapunov candidate function of system (1) as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}\left\|\tilde{\boldsymbol{x}}_i - \frac{1}{N}\sum_{k=1}^{N}\tilde{\boldsymbol{x}}_k\right\|^2. \tag{12}$$

Taking the time derivative of $V(t)$, we obtain

$$\dot{V}(t) = \sum_{i=1}^{N}\left(\tilde{\boldsymbol{x}}_i - \frac{1}{N}\sum_{k=1}^{N}\tilde{\boldsymbol{x}}_k\right)^\mathrm{T}\left(\dot{\tilde{\boldsymbol{x}}}_i - \frac{1}{N}\sum_{k=1}^{N}\dot{\tilde{\boldsymbol{x}}}_k\right). \tag{13}$$

For all $t > t_1$, according to $\tilde{\boldsymbol{x}}_i = \boldsymbol{x}_i - \boldsymbol{x}_i^*$, $\tilde{\boldsymbol{x}}_j = \boldsymbol{x}_j - \boldsymbol{x}_j^*$, and $\boldsymbol{x}_i \in \Omega$, system (1) with controller (5) can be rewritten by

$$\dot{\tilde{\boldsymbol{x}}}_i = -\sum_{j=1}^{N} a_{ij}\mathrm{sgn}(\tilde{\boldsymbol{x}}_i - \tilde{\boldsymbol{x}}_j). \tag{14}$$

Because the undirected network is considered in this study, then $a_{ij} = a_{ji}$. Thus,

$$\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\mathrm{sgn}(\tilde{\boldsymbol{x}}_i - \tilde{\boldsymbol{x}}_j) = \boldsymbol{0}. \tag{15}$$

By Eqs. (14) and (15), Eq. (13) can be rewritten by

$$\begin{aligned}\dot{V}(t) =& \sum_{i=1}^{N}\left(\tilde{\boldsymbol{x}}_i - \frac{1}{N}\sum_{k=1}^{N}\tilde{\boldsymbol{x}}_k\right)^\mathrm{T}\dot{\tilde{\boldsymbol{x}}}_i \\ =& -\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\left(\tilde{\boldsymbol{x}}_i - \frac{1}{N}\sum_{k=1}^{N}\tilde{\boldsymbol{x}}_k\right)^\mathrm{T}\mathrm{sgn}(\tilde{\boldsymbol{x}}_i - \tilde{\boldsymbol{x}}_j) \\ =& -\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\tilde{\boldsymbol{x}}_i^\mathrm{T}\mathrm{sgn}(\tilde{\boldsymbol{x}}_i - \tilde{\boldsymbol{x}}_j) \\ &+ \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} a_{ij}\tilde{\boldsymbol{x}}_k^\mathrm{T}\mathrm{sgn}(\tilde{\boldsymbol{x}}_i - \tilde{\boldsymbol{x}}_j) \\ =& -\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\tilde{\boldsymbol{x}}_i^\mathrm{T}\mathrm{sgn}(\tilde{\boldsymbol{x}}_i - \tilde{\boldsymbol{x}}_j).\end{aligned} \tag{16}$$

Because $a_{ij} = a_{ji}$, we have

$$
\begin{aligned}
& -\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\operatorname{sgn}(\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j) \\
= & -\frac{1}{2}\Bigg(\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\tilde{\boldsymbol{x}}_i^{\mathrm{T}}\operatorname{sgn}(\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j) \\
& +\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ji}\tilde{\boldsymbol{x}}_j^{\mathrm{T}}\operatorname{sgn}(\tilde{\boldsymbol{x}}_j-\tilde{\boldsymbol{x}}_i)\Bigg) \\
= & -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j)^{\mathrm{T}}\operatorname{sgn}(\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j) \\
= & -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\|_1.
\end{aligned} \tag{17}
$$

Then Eq. (16) can be rewritten as

$$
\dot{V}(t) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\|_1. \tag{18}
$$

Let $\max_{i,j\in\mathcal{V}}\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\| = \|\tilde{\boldsymbol{x}}_{i_0}-\tilde{\boldsymbol{x}}_{j_0}\|$, $(i_0,j_0)\in\mathcal{E}$. Since graph $\mathcal{G}$ is connected, then we have $\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\| \geq \|\tilde{\boldsymbol{x}}_{i_0}-\tilde{\boldsymbol{x}}_{j_0}\|$. Note that $\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\| \leq \|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\|_1$ for all $i,j\in\mathcal{V}$. We have

$$
\begin{aligned}
\dot{V}(t) = & -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\|_1 \\
\leq & -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_j\| \\
\leq & -\frac{1}{2}\|\tilde{\boldsymbol{x}}_{i_0}-\tilde{\boldsymbol{x}}_{j_0}\|.
\end{aligned}
$$

We also have

$$
\begin{aligned}
\sqrt{V(t)} = & \sqrt{\frac{1}{2}\sum_{i=1}^{N}\left\|\tilde{\boldsymbol{x}}_i-\frac{1}{N}\sum_{k=1}^{N}\tilde{\boldsymbol{x}}_k\right\|^2} \\
\leq & \frac{\sqrt{2}}{2}\sum_{i=1}^{N}\left\|\tilde{\boldsymbol{x}}_i-\frac{1}{N}\sum_{k=1}^{N}\tilde{\boldsymbol{x}}_k\right\| \\
\leq & \frac{\sqrt{2}}{2N}\sum_{i=1}^{N}\sum_{k=1}^{N}\|\tilde{\boldsymbol{x}}_i-\tilde{\boldsymbol{x}}_k\| \\
\leq & \frac{\sqrt{2}N}{2}\|\tilde{\boldsymbol{x}}_{i_0}-\tilde{\boldsymbol{x}}_{j_0}\|.
\end{aligned} \tag{19}
$$

By Eq. (18) and inequality (19), we can obtain

$$
\dot{V}(t) \leq -\frac{1}{2}\|\tilde{\boldsymbol{x}}_{i_0}-\tilde{\boldsymbol{x}}_{j_0}\| \leq -\frac{1}{\sqrt{2}N}\sqrt{V(t)}. \tag{20}
$$

Then inequality (20) can be rewritten as

$$
\frac{\dot{V}(t)}{\sqrt{V(t)}} \leq -\frac{\sqrt{2}}{2N}. \tag{21}
$$

Integrating on both sides of inequality (21), we have

$$
2\sqrt{V(t)} - 2\sqrt{V(t_0)} \leq -\frac{\sqrt{2}}{2N}(t-t_0). \tag{22}
$$

Clearly, there exists $t_2 \geq t_1$ such that $\sqrt{V(t)} \to 0$ when $t > t_2$, which means that

$$
\lim_{t\to+\infty}\left\|\tilde{\boldsymbol{x}}_i-\frac{1}{N}\sum_{k=1}^{N}\tilde{\boldsymbol{x}}_k\right\| = 0.
$$

Thus, we can obtain $\tilde{\boldsymbol{x}}_1 = \tilde{\boldsymbol{x}}_2 = \ldots = \tilde{\boldsymbol{x}}_N$. That is, $\boldsymbol{x}_1-\boldsymbol{x}_1^* = \boldsymbol{x}_2-\boldsymbol{x}_2^* = \ldots = \boldsymbol{x}_N-\boldsymbol{x}_N^*$, and then $\boldsymbol{x}_i-\boldsymbol{x}_j = \boldsymbol{x}_i^*-\boldsymbol{x}_j^*$ $(i,j=1,2,\ldots,N)$. This implies that the group can reach an asymptotic formation shape in limited time. In summary, according to Lemma 1, there exists $t > t_2$ such that all the agents can achieve an asymptotic target formation shape in finite time and can access the desired area in finite time.

Remark 3 Because the symmetry of an undirected graph is needed in the proof of the theorem, whether the control algorithm is suitable for the digraph or the time-varying graph remains to be further studied.

4 Simulation results

In this section, simulation is performed to illustrate the validity of the theoretical results.

In the simulation, we consider an MAS with 92 agents in $\mathbb{R}^2$. In controller (5), parameter β is taken as 100 according to $N = 92$ and $\beta > N$. We choose the initial relative positions randomly in the unit square. We choose the random network (Erdös and Rényi, 1959) which satisfies Assumption 1 as the communication topology network of the MAS and the adjacency matrix of the random network as $\boldsymbol{A} = [a_{ij}]$. The random network is defined as follows: start with a set of N isolated vertices and add successive edges among them according to probability p. Here, we choose $N = 92$ and $p = 0.1$. Fig. 2 shows the initial positions of the 92 agents. Fig. 2 shows that the edge describes the information exchange among agents, and that the node represents the initial relative position in the unit square. We choose the target formation shape $(\mathcal{G}, \boldsymbol{X}^*)$ of the group as “2019” (Fig. 3).

The desired area is a rectangle, where $\Omega = \{(x,y) : 3 \leq x \leq 12,\ 2 \leq y \leq 6\}$. By controller (5), we have the motion trajectories of 92 agents in

Fig. 4. Fig. 5 shows the final configuration of the group. Note that the multi-agent group will move into the desired region and achieve the target formation shape.

Errors between the position and projection of agents $\boldsymbol{x}_i(t)$ $(i=1,2,\ldots,92)$ in Ω are shown in Fig. 6 (we plotted the graph in 0.50 s). Note that the errors will asymptotically converge to zero in 0.05 s. This implies that all the agents would access the desired region in finite time. Finally, the control inputs of 92 agents are shown in Fig. 7. It can be seen that the finite-time formation for the first-order multi-agent system (1) is achieved in 3.0 s. Therefore, simulation results demonstrate the validity of our theoretical results. That is, all the agents will enter the target region in finite time and realize the target formation shape in limited time.

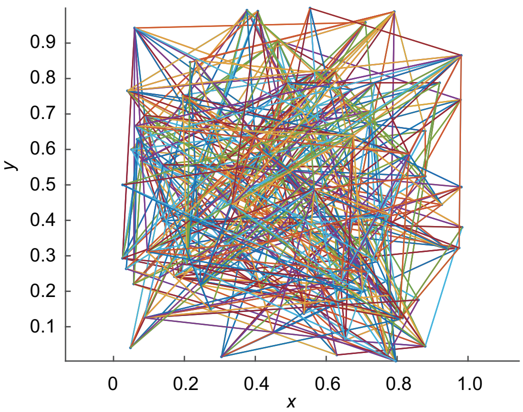

Fig. 2 Initial configuration of 92 agents

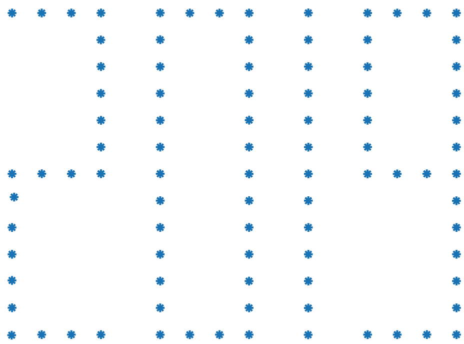

Fig. 3 Desired formation shape of 92 agents

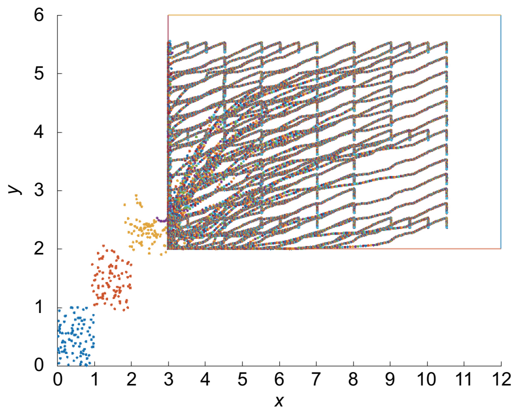

Fig. 4 Simulation process of a 92-agent target formation shape

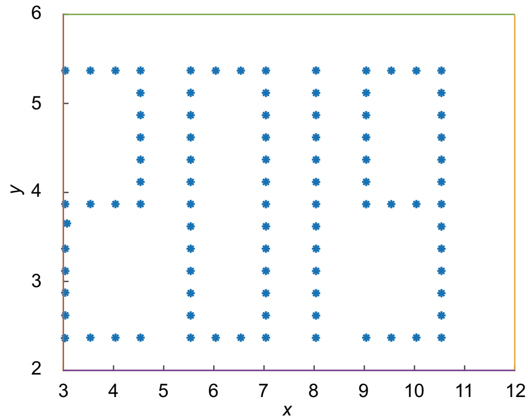

Fig. 5 Final configuration of 92 agents

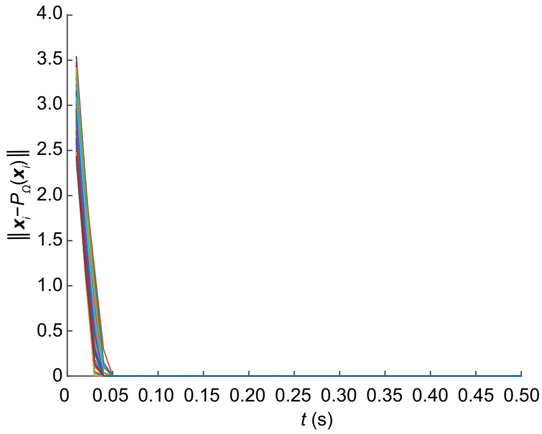

Fig. 6 Error between the convergence positions and projection of 92 agents

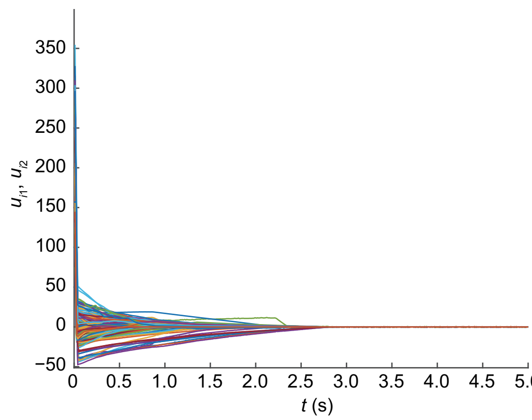

Fig. 7 Control inputs u_i of 92 agents

5 Conclusions and future work

In this study, a finite-time formation problem has been studied with the consideration of a convex constraint area. A formation controller has been proposed by the convex set and local information among the agents. Based on the rigorous undirected network topology, it is shown that all the agents would enter the global desired area and reach formation in finite time. Finally, the results have been illustrated by simulation. In the future, we will consider how to use the control algorithm to solve the constrained formation problem with nonlinear systems, directed communication networks, or time-varying communication networks.

Contributors

Zhengquan YANG and Xiaofang PAN designed the research and processed the data. Zhengquan YANG drafted the manuscript. Qing ZHANG and Zengqiang CHEN helped organize the manuscript. Zhengquan YANG and Xiaofang PAN revised and finalized the paper.

Compliance with ethics guidelines

Zhengquan YANG, Xiaofang PAN, Qing ZHANG, and Zengqiang CHEN declare that they have no conflict of interest.

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