



Efficient coherent detection of maneuvering targets based on location rotation transform and non-uniform fast Fourier transform*

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Abstract: Long-term coherent integration can remarkably improve the ability of detection and motion parameter estimation of radar for maneuvering targets. However, the linear range migration, quadratic range migration (QRM), and Doppler frequency migration within the coherent processing interval seriously degrade the detection and estimation performance. Therefore, an efficient and noise-resistant coherent integration method based on location rotation transform (LRT) and non-uniform fast Fourier transform (NuFFT) is proposed. QRM is corrected by the second-order keystone transform. Using the relationship between the rotation angle and Doppler frequency, a novel phase compensation function is constructed. Motion parameters can be rapidly estimated by LRT and NuFFT. Compared with several representative algorithms, the proposed method achieves a nearly ideal detection performance with low computational cost. Finally, experiments based on measured radar data are conducted to verify the proposed algorithm.

Key words: Coherent integration; Maneuvering target; Parameter estimation; Location rotation transform (LRT); Non-uniform fast Fourier transform (NuFFT)

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1 Introduction

Rapid advancements in stealth aircraft and unmanned aerial vehicles (UAVs) have placed increasing demands on radar detection for weak maneuvering targets (Su et al., 2010; Chen et al., 2014; Rao et al., 2014; Zhu SQ et al., 2014; Huang PH et al., 2016; Li et al., 2016c, 2016e; Jin et al., 2017; Wu et al., 2018). To detect such low radar cross section (RCS) targets, the first choice is the long-term co-

herent integration (Huang X et al., 2019; Li et al., 2019a, 2019b). Unfortunately, due to the high maneuverability of the targets, the effects of linear range migration (LRM), quadratic range migration (QRM), and Doppler frequency migration (DFM) will inevitably occur during the observation period (Xing et al., 2011; Tian et al., 2013, 2014). These unfavorable effects seriously deteriorate the detection performance of conventional integration methods, e.g., moving target detection (MTD). Consequently, finding ways to robustly detect weak maneuvering targets becomes a hot topic in the field of radar signal processing.

LRM makes a target's envelope move across range units after pulse compression. To eliminate the effects of LRM, many successful detection methods

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have been proposed, such as scaled inverse Fourier transform (SCIFT) (Zheng et al., 2015; Niu et al., 2017), keystone transform (KT) (Perry et al., 1999; Zhu DY et al., 2007; Pignol et al., 2018), modified location rotation transform (MLRT) (Sun et al., 2018), and Radon-Fourier transform (RFT) (Xu et al., 2011a, 2011b; Yu et al., 2012). These algorithms achieve satisfactory detection performances with reasonable computational cost. However, they may suffer from integration performance loss because they ignore the QRM and DFM caused by the target's acceleration.

Researchers have shown considerable interest in addressing these issues and developed many methods. In general, these methods can be roughly divided into two categories, i.e., search based algorithms and non-search algorithms. Search based algorithms include the generalized Radon-Fourier transform (GRFT) (Xu et al., 2012), KT and Lv's distribution (KT-LVD) (Lv et al., 2011; Luo et al., 2013; Li et al., 2016a), KT and linear canonical transform (KT-LCT) (Huang X et al., 2018), modified axis rotation transform and Lv's transform (MART-LVT) (Li et al., 2019a), improved axis rotation fractional Fourier transform (IAR-FRFT) (Rao et al., 2015), and Radon-Lv's distribution (RLVD) (Li et al., 2015). These methods can achieve superior detection performances at a low signal-to-noise ratio (SNR) by parameter search. However, high computational complexity makes them impractical in applications. Non-search algorithms include the symmetric autocorrelation function and scaled Fourier transform (SAF-SFT) (Li et al., 2018), three-dimensional scaled transform (TDST) (Zheng et al., 2018), frequency-domain second-order phase difference (FD-SoPD) (Jin et al., 2019a), and adjacent cross correlation function (ACCF) (Li et al., 2014, 2016d). These methods use correlation operations to reduce the coupling order, which greatly eases the computational burden. However, the nonlinear correlation brings cross terms of multiple targets and anti-noise performance loss (Jin et al., 2019b).

In this study, we continue our previous work (Jin et al., 2019a). To alleviate the competing interests of computational complexity and detection performance, an efficient and robust coherent detection method based on location rotation transform (LRT) and non-uniform fast Fourier transform (NuFFT), i.e., LRT-NuFFT, is proposed. First, the second-order keystone transform (SoKT) (Kirkland, 2011) is

adopted to remove the effects of QRM. Then, using the relationship between the rotation angle and Doppler frequency, the effects of LRM and DFM are eliminated by LRT and NuFFT (Zheng et al., 2014; Qu et al., 2018). Comparison results of several representative methods show that the proposed method achieves a nearly ideal detection performance with low computational cost.

2 Signal model and problem formulation

Assume that a radar transmits a linear frequency-modulated signal as

$$s_T(\hat{t}) = \text{rect}\left(\frac{\hat{t}}{T_p}\right) \exp(j2\pi f_c \hat{t} + j\pi\gamma \hat{t}^2) \quad (1)$$

with

$$\text{rect}\left(\frac{\hat{t}}{T_p}\right) = \begin{cases} 1, & |\hat{t}| \leq T_p/2, \\ 0, & |\hat{t}| > T_p/2, \end{cases}$$

where T_p is the pulse duration, f_c the carrier frequency, \hat{t} the fast time variable, and γ the chirp rate.

Suppose that there is a maneuvering target moving with a constant acceleration, of which the instantaneous slant range $R(t_m)$ satisfies

$$R(t_m) = R_0 + vt_m + \frac{1}{2}at_m^2, \quad (2)$$

where R_0 , v , and a are the initial slant range, radial velocity, and acceleration of the target, respectively, and $t_m = mT$ ($m=1, 2, \dots, N_a$) the slow time variable (T is the pulse repetition time and N_a the number of integrated pulses).

Ignoring the influence of noise, the signal received after pulse compression can be expressed as

$$s_c(\hat{t}, t_m) = A_c \text{sinc}\left\{B\left[\hat{t} - \frac{2R(t_m)}{c}\right]\right\} \exp\left[-j\frac{4\pi f_c R(t_m)}{c}\right], \quad (3)$$

where A_c and B denote the amplitude and bandwidth of the signal, respectively, f_c the carrier frequency, and c the light speed.

Substituting Eq. (2) into Eq. (3) yields

$$s_c(\hat{t}, t_m) = A_c \operatorname{sinc} \left\{ B \left[\hat{t} - \frac{2(R_0 + vt_m + 0.5at_m^2)}{c} \right] \right\} \cdot \exp \left(-j4\pi \frac{R_0 + vt_m + 0.5at_m^2}{\lambda} \right), \quad (4)$$

where $\lambda=c/f_c$ is the wavelength.

Due to radar's low pulse repetition frequency (PRF) and target's high speed, Doppler ambiguity often occurs. Therefore, the radial velocity of the target can be expressed as (Zhang et al., 2017)

$$v = N_b v_b + v_0, \quad (5)$$

where $v_b=\lambda f_b/2$ is the blind speed (f_b is the PRF), N_b the Doppler ambiguity integer of the target, and v_0 the ambiguous velocity satisfying $-v_b/2 \leq v_0 < v_b/2$.

We usually have $\exp(-j2\pi f_c N_b v_b t_m/c)=1$. Thus, substituting Eq. (5) into Eq. (4), we obtain

$$s_c(\hat{t}, t_m) = A_c \operatorname{sinc} \left\{ B \left[\hat{t} - \frac{2(R_0 + vt_m + 0.5at_m^2)}{c} \right] \right\} \cdot \exp \left(-j4\pi \frac{R_0 + v_0 t_m + 0.5at_m^2}{\lambda} \right), \quad (6)$$

showing that the signal envelope changes nonlinearly over slow time. When the offset caused by the target's velocity during the observation exceeds the range resolution $\Delta r=c/(2B)$, LRM will occur. If the target is highly maneuvering (i.e., with a large acceleration), QRM could occur.

Performing the Fourier transform (FT) on Eq. (6) along the fast time axis, we have

$$s(f_r, t_m) = A_r \operatorname{rect} \left(\frac{f_r}{B} \right) \exp \left(-j4\pi f_r \frac{N_b v_b t_m}{c} \right) \cdot \exp \left[-j4\pi (f_r + f_c) \frac{R_0 + v_0 t_m + 0.5at_m^2}{c} \right], \quad (7)$$

where f_r denotes the range frequency with respect to \hat{t} . Eq. (7) indicates that the coupling between f_r and t_m is the essential cause of LRM and QRM. The Doppler frequency of the target, f_{d0} , is defined as

$$f_{d0} = \frac{2}{\lambda} \frac{d(R_0 + v_0 t_m + 0.5at_m^2)}{dt_m} = \frac{2(v_0 + at_m)}{\lambda}. \quad (8)$$

Due to the target's acceleration, the Doppler offset is $2at_m/\lambda$. When this value exceeds the Doppler resolution $1/(N_a T)$, DFM happens and leads to energy dispersion in the Doppler domain. Therefore, to detect a maneuvering target with a low SNR, the effects of LRM, QRM, and DFM should be eliminated by an effective detection algorithm.

3 Principle of the proposed coherent integration method

3.1 QRM elimination via SoKT

Employ SoKT to eliminate the quadratic coupling caused by the target's acceleration, which re-scales the slow time for each range frequency and can be expressed as

$$t_m = [f_c / (f_r + f_c)]^{1/2} t_a, \quad (9)$$

where t_a denotes the scaled slow time.

Inserting Eq. (9) into Eq. (7) yields

$$s_{\text{SoKT}}(f_r, t_a) = A_r \operatorname{rect} \left(\frac{f_r}{B} \right) \exp \left(-j2\pi \frac{at_a^2}{\lambda} \right) \cdot \exp \left(-j4\pi \frac{(f_r + f_c)R_0}{c} \right) \cdot \exp \left[-j4\pi \left(1 + \frac{f_r}{f_c} \right)^{1/2} f_c \frac{v_0 t_a}{c} \right] \cdot \exp \left[-j4\pi f_r \frac{N_b v_b t_a}{c} \left(\frac{f_c}{f_c + f_r} \right)^{1/2} \right]. \quad (10)$$

For a narrowband radar system, the first-order Taylor series expansion is used, expressed as

$$\begin{cases} \frac{f_c}{f_c + f_r} \approx 1, \\ \left(1 + \frac{f_r}{f_c} \right)^{1/2} \approx 1 + \frac{f_r}{2f_c}. \end{cases} \quad (11)$$

Thus, Eq. (10) can be simplified as

$$s_{\text{SoKT}}(f_r, t_a) = A_f \text{rect}\left(\frac{f_r}{B}\right) \exp\left(-j4\pi f_r \frac{R_0 + V_e t_a}{c}\right) \cdot \exp\left(-j4\pi \frac{R_0 + v_0 t_a + 0.5at_a^2}{\lambda}\right), \quad (12)$$

where $V_e = N_b v_b + v_0/2$ is called the equivalent velocity of the target.

Performing the inverse Fourier transform (IFT) on Eq. (12) along f_r , we have

$$s_{\text{SoKT}}(r, t_a) = A_c \text{sinc}\left[\frac{2B}{c}(r - R_0 - V_e t_a)\right] \cdot \exp\left(-j4\pi \frac{R_0 + v_0 t_a + 0.5at_a^2}{\lambda}\right), \quad (13)$$

where $r = ct/2$ represents the range variable corresponding to \hat{t} .

From Eqs. (12) and (13), it is easy to see that SoKT eliminates the quadratic coupling between f_r and t_m . Therefore, QRM caused by the target's acceleration is accordingly eliminated.

3.2 LRM elimination and parameter estimation

The residual LRM and DFM in Eq. (13) bring significant challenges for coherent integration. In this subsection, we propose a novel efficient elimination method by jointly employing LRT and NuFFT.

Note that $t_a = t_m$ and $f_s = KB$, where f_s is the sampling frequency and K the oversampling rate. Thus, we have $r = \rho n$ and $R_0 = \rho n_{R_0}$, where $\rho = c/(2f_s)$ denotes the range cell and n and n_{R_0} the range cell indices of r and R_0 , respectively. In this case, Eq. (13) can be rewritten in the (n, m) coordinate system as

$$s_{\text{SoKT}}(n, m) = A_c \text{sinc}\left[\frac{1}{K}(n - n_{R_0}) - \frac{1}{\Delta r} V_e m T\right] \cdot \exp\left(-j2\pi \frac{c}{\lambda KB} n_{R_0}\right) \exp\left(-j\frac{4\pi}{\lambda} v_0 m T\right) \cdot \exp\left[-j\frac{2\pi}{\lambda} a(mT)^2\right]. \quad (14)$$

To compensate for the LRM caused by equivalent velocity, LRT is introduced to perform the location rotation as

$$\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \tan \varphi & 1 \end{bmatrix} \begin{bmatrix} m' \\ n' \end{bmatrix}, \quad \varphi \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \quad (15)$$

where $[m' \ n']^T$ represents the coordinates after rotation and φ the rotation angle.

Substituting Eq. (15) into Eq. (14) yields

$$s_{\text{rot}}(n', m') = A_c \exp\left(-j2\pi \frac{c}{\lambda KB} n_{R_0}\right) \cdot \exp\left(-j\frac{4\pi}{\lambda} v_0 m' T\right) \exp\left(-j\frac{2\pi}{\lambda} a(m' T)^2\right) \cdot \text{sinc}\left[\frac{1}{K}(n' - n_{R_0}) + m' \left(\frac{\tan \varphi}{K} - \frac{V_e T}{\Delta r}\right)\right]. \quad (16)$$

When $\tan \varphi = KV_e T / \Delta r$ or equivalently $\varphi = \arctan(KV_e T / \Delta r)$, LRM in Eq. (16) will be eliminated, i.e.,

$$s_{\text{rot}}(n', m') = A_c \text{sinc}\left[\frac{1}{K}(n' - n_{R_0})\right] \cdot \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_0}\right) \exp\left(-j\frac{4\pi}{\lambda} v_0 m' T\right) \cdot \exp\left(-j\frac{2\pi}{\lambda} a(m' T)^2\right). \quad (17)$$

As shown in Eq. (17), the target's energy is located in the same range cell. Then, we can extract the signal along the slow time. For a certain range cell, the extracted signal can be expressed as

$$s_{\text{azi}}(m') = A_{\text{azi}} \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_0}\right) \cdot \exp\left(-j\frac{4\pi}{\lambda} v_0 m' T\right) \cdot \exp\left[-j\frac{2\pi}{\lambda} a(m' T)^2\right], \quad (18)$$

where A_{azi} is the amplitude.

It is evident that $s_{\text{azi}}(m')$ is a chirp signal with respect to the slow time $m' T$. In Li et al. (2019a), LVT was used to complete coherent integration and motion parameter estimation, with satisfactory performance in integration and anti-noise. However, high computational cost remains daunting.

From Eqs. (17) and (18), note that V_e , v_0 , and v can be obtained simultaneously from the rotation angle, expressed as

$$\hat{V}_e = \Delta r \frac{\tan \varphi}{KT}, \quad (19)$$

where \hat{V}_e is the estimate of V_e .

According to Eq. (12), v_0 can be estimated as

$$\hat{v}_0 = 2 \left[\hat{V}_e - \text{round} \left(\frac{\hat{V}_e}{v_b} \right) \cdot v_b \right], \quad (20)$$

where $\text{round}(\cdot)$ denotes the round-up operation and $\text{round}(\hat{V}_e/v_b)$ is the estimated Doppler ambiguity integer of the target. Then, the target's velocity is estimated as

$$\hat{v} = \text{round}(\hat{V}_e/v_b) \cdot v_b + \hat{v}_0. \quad (21)$$

With the estimated \hat{v}_0 , a novel phase compensation function is constructed as

$$H_1(m') = \exp \left(j \frac{4\pi}{\lambda} \hat{v}_0 m' T \right). \quad (22)$$

Multiplying Eq. (22) with Eq. (18), the linear phase term will be compensated for. Therefore, an efficient NuFFT can be performed to realize energy accumulation and parameter estimation, i.e.,

$$\begin{aligned} & S_{\text{NuFFT}} \left(f_{(m'T)^2} \right) \\ &= \text{NuFFT} \left(s_{\text{azi}}(m') H_1(m') \right) \\ &= \sum_{m'=-N_a/2}^{N_a/2-1} s_{\text{azi}}(m') H_1(m') \exp \left[-j 2\pi f_{(m'T)^2} (m'T)^2 \right] \quad (23) \\ &= A_{\text{NuFFT}} \exp \left(-j \frac{2\pi c}{\lambda KB} n_{R_0} \right) p \left(f_{(m'T)^2} + \frac{a}{\lambda} \right), \end{aligned}$$

where $\text{NuFFT}(\cdot)$ presents the NuFFT operation, $p(\cdot)$ the impulse response function of NuFFT, and $f_{(m'T)^2}$ the frequency variable corresponding to $(m'T)^2$.

According to Eq. (23), it is clear that the target's energy is concentrated into a single peak, whose lo-

cation indicates the target's acceleration, i.e.,

$$\begin{cases} \hat{f}_{(m'T)^2} = \arg \max_{f_{(m'T)^2}} \left\{ |S_{\text{NuFFT}}(f_{(m'T)^2})| \right\}, \\ \hat{a} = \lambda \hat{f}_{(m'T)^2}. \end{cases} \quad (24)$$

3.3 Coherent integration and target detection

With the estimated velocity and acceleration, another phase compensation function is constructed to compensate for the effects of LRM, QRM, and DFM, which can be written as

$$\begin{aligned} H_2(f_r, t_m) &= \exp \left(j 4\pi f_r \frac{\hat{v} t_m + 0.5 \hat{a} t_m^2}{c} \right) \\ &\cdot \exp \left(j 4\pi f_c \frac{0.5 \hat{a} t_m^2}{c} \right). \end{aligned} \quad (25)$$

Finally, long-term coherent integration is achieved via slow time FT and spatial frequency IFT with respect to Eq. (7), expressed as

$$\begin{aligned} s_{\text{CI}}(\hat{t}, f_d) &= \text{FT}_{t_m} \left\{ \text{IFT}_{f_r} \left[S(f_r, t_m) H_2(f_r, t_m) \right] \right\} \\ &= A_{\text{CI}} \text{sinc} \left[B \left(\hat{t} - \frac{2R_0}{c} \right) \right] \\ &\cdot \text{sinc} \left[N_a T \left(f_d + \frac{2v_0}{\lambda} \right) \right], \end{aligned} \quad (26)$$

where A_{CI} denotes the complex amplitude after coherent integration and f_d the Doppler frequency corresponding to t_m .

In Eq. (26), the received signal of a maneuvering target is integrated into a single peak at the position of $(2R_0/c, -2v_0/\lambda)$, and the peak value is $|s_{\text{CI}}(2R_0/c, -2v_0/\lambda)|$. Therefore, the constant false alarm rate (CFAR) (Guida et al., 1993) can be used for target detection, expressed as

$$\left| s_{\text{CI}} \left(\frac{2R_0}{c}, \frac{-2v_0}{\lambda} \right) \right|_{H_0} \stackrel{H_1}{\geq} \eta, \quad (27)$$

where η is the detection threshold. The adaptive threshold is obtained by the reference unit after coherent integration via the integration method. Then,

the test statistic in the detecting unit is compared with the threshold to confirm the presence or absence of a target (Li et al., 2016a). If the test statistic is smaller than the threshold, there is no moving target (or a target is missed); then, go on to the next detecting unit. Otherwise, if the test statistic is larger than the threshold, target detection is declared.

3.4 Detailed procedure of the LRT-NuFFT method

The detailed implementation procedure of the LRT-NuFFT method is summarized as follows:

Step 1: Perform pulse compression on the radar echoes and obtain $s_c(\hat{t}, t_m)$.

Step 2: Perform SoKT to correct QRM and use range IFT to obtain $s_{\text{SoKT}}(n, m)$.

Step 3: According to the interested scope of velocity, determine the rotation angle search scope $[\varphi_{\min}, \varphi_{\max}]$. The search interval $\Delta\varphi$ should be smaller than $\arctan[\lambda/(2N_a\rho)]$.

Step 4: With a certain rotation angle, perform LRT on $s_{\text{SoKT}}(n, m)$. Then we can obtain the signal after rotation $s_{\text{rot}}(n', m')$.

Step 5: Construct the phase compensation function $H_1(m')$ according to the search rotation angle (Eqs. (19)–(22)), and extract the data along the slow time and obtain $s_{\text{azi}}(m')$.

Step 6: Perform the NuFFT operation on $s_{\text{azi}}(m') \cdot H_1(m')$ to realize target energy accumulation and acceleration estimation (Eq. (23)).

Step 7: When the initial range and search rotation angle match the target's true value, the output of NuFFT will reach its maximum value. Then, estimate the target's velocity and acceleration via Eqs. (21) and (24), respectively.

Step 8: Construct another phase compensation function $H_2(f_r, t_m)$ with the estimated motion parameters and complete coherent integration via Eqs. (25) and (26).

The flowchart of the LRT-NuFFT method is shown in Fig. 1.

Remark 1 Compared with MART-LVT (Li et al., 2019a), improvements and advantages of LRT-NuFFT are as follows: (1) In MART-LVT, the QRM effects caused by the target's acceleration are neglected, which causes integration performance loss in some specific situations. However, SoKT is first used to correct QRM in LRT-NuFFT, which makes it better

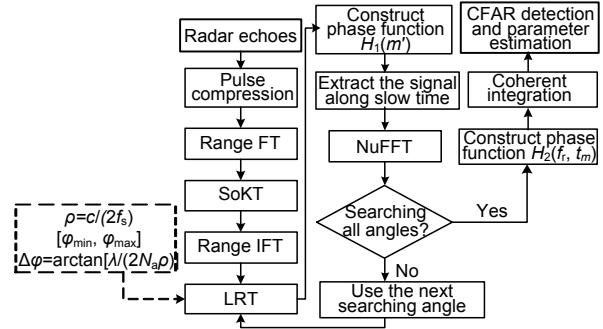


Fig. 1 Flowchart of the proposed method

suitable for detecting highly maneuvering targets. (2) Different from MART-LVT, the relationship between φ and v_0 is employed to construct a phase compensation function. Thus, the subsequent energy accumulation and acceleration estimation can be achieved via NuFFT, greatly alleviating the computational burden of LVD. Detailed computational complexity comparisons will be given in Section 4.3.

4 Analysis of the proposed method

4.1 Multi-target analysis

Consider that Q targets are observed during the integration time. Similar to Eq. (14), the result of SoKT can be expressed as

$$\begin{aligned}
 & s_{\text{SoKT}}(n, m) \\
 &= \sum_{q=1}^Q A_{c,q} \exp\left(-j \frac{2\pi c}{\lambda KB} n_{R_{0,q}}\right) \\
 & \quad \cdot \exp\left(-j \frac{4\pi}{\lambda} v_{0,q} m T\right) \exp\left[-j \frac{2\pi}{\lambda} a_q (m T)^2\right] \\
 & \quad \cdot \text{sinc}\left[\frac{1}{K}(n - n_{R_{0,q}}) - \frac{1}{\Delta r} V_{e,q} m T\right].
 \end{aligned} \quad (28)$$

Performing LRT on Eq. (28), we obtain

$$\begin{aligned}
 & s_{\text{rot}}(n', m') \\
 &= \sum_{q=1}^Q A_{c,q} \exp\left(-j \frac{2\pi c}{\lambda KB} n_{R_{0,q}}\right) \\
 & \quad \cdot \exp\left(-j \frac{4\pi}{\lambda} v_{0,q} m' T\right) \exp\left[-j \frac{2\pi}{\lambda} a_q (m' T)^2\right] \\
 & \quad \cdot \text{sinc}\left[\frac{1}{K}(n' - n_{R_{0,q}}) + m' \left(\frac{\tan \varphi}{K} - \frac{V_{e,q} T}{\Delta r}\right)\right].
 \end{aligned} \quad (29)$$

When $\tan \varphi = KV_{e,i}T/\Delta r$, LRM of the i^{th} target is corrected. However, LRM of other $(q-1)$ targets still exists, i.e.,

$$\begin{aligned}
 & s_{\text{rot}}(n', m') \\
 = & A_{c,i} \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_{0,i}}\right) \exp\left(-j\frac{4\pi}{\lambda} v_{0,i} m'T\right) \\
 & \cdot \exp\left[-j\frac{2\pi}{\lambda} a_i (m'T)^2\right] \text{sinc}\left[\frac{1}{K}(n' - n_{R_{0,i}})\right] \\
 & + \sum_{q=1, q \neq i}^Q A_{c,q} \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_{0,q}}\right) \\
 & \cdot \exp\left(-j\frac{4\pi}{\lambda} v_{0,q} m'T\right) \exp\left[-j\frac{2\pi}{\lambda} a_q (m'T)^2\right] \\
 & \cdot \text{sinc}\left[\frac{1}{K}(n' - n_{R_{0,q}}) + \frac{m'T}{\Delta r}(V_{e,i} - V_{e,q})\right].
 \end{aligned} \tag{30}$$

Then, extract the signal along the slow time of range cell $n_{R_{0,i}}$, and we obtain

$$\begin{aligned}
 & s_{\text{azi}}(m'; n_{R_{0,i}}) \\
 = & A_{c,i} \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_{0,i}}\right) \exp\left(-j\frac{4\pi}{\lambda} v_{0,i} m'T\right) \\
 & \cdot \exp\left[-j\frac{2\pi}{\lambda} a_i (m'T)^2\right] + s_{\text{other}}(m'; n_{R_{0,i}}),
 \end{aligned} \tag{31}$$

where

$$\begin{aligned}
 & s_{\text{other}}(m'; n_{R_{0,i}}) \\
 = & \sum_{q=1, q \neq i}^Q A_{c,q} \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_{0,q}}\right) \\
 & \cdot \exp\left(-j\frac{4\pi}{\lambda} v_{0,q} m'T\right) \exp\left[-j\frac{2\pi}{\lambda} a_q (m'T)^2\right] \\
 & \cdot \text{sinc}\left[\frac{1}{K}(n_{R_{0,i}} - n_{R_{0,q}}) + \frac{m'T}{\Delta r}(V_{e,i} - V_{e,q})\right].
 \end{aligned} \tag{32}$$

Similar to Eqs. (19)–(22), construct the phase compensation function according to the rotation angle, i.e.,

$$H_1(m') = \exp\left(j\frac{4\pi}{\lambda} \hat{v}_{0,i} m'T\right). \tag{33}$$

Multiply Eq. (31) with Eq. (33), perform NuFFT

to concentrate the target energy, and estimate the acceleration, so that we have

$$\begin{aligned}
 & s_{\text{NuFFT}}(f_{(m'T)^2}) \\
 = & \text{NuFFT}\left[s_{\text{azi}}(m'; n_{R_{0,i}}) H_1(m')\right] \\
 = & s_{\text{nu},i}(f_{(m'T)^2}) + s_{\text{other}}(f_{(m'T)^2}) \\
 = & A_{\text{NuFFT},i} \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_{0,i}}\right) p\left(f_{(m'T)^2} + \frac{a_i}{\lambda}\right) \\
 & + s_{\text{other}}(f_{(m'T)^2}).
 \end{aligned} \tag{34}$$

Note that only the i^{th} target's energy is concentrated into a single peak, while other targets' energy is dispersed due to two reasons: the envelopes of other targets are distributed in different range cells and the linear phase term $\exp(-j4\pi v_{0,q} m'T/\lambda)$ in Eq. (32) could not be well compensated for by $H_1(m')$.

Finally, the motion parameter estimation and coherent integration of the i^{th} target can be achieved. However, other targets' energy will not be focused using the i^{th} target's parameters.

Remark 2 Compared with MART-LVT, KT-LVD, TDST, SAF-SFT, and ACCF, LRT-NuFFT has no nonlinear operations. Thus, no cross terms appear, and the linearity property holds. Although LVD, TDST, SAF-SFT, and ACCF have satisfactory cross-term suppression ability, weak targets are still easily submerged in the residual cross-terms generated by strong targets.

4.2 Equivalent implementation via velocity search

The LRT-NuFFT method corrects LRM by searching for the rotation angle. However, the implementation of LRT requires extensive interpolation operations, which inevitably cause numerical errors. In addition, the rotation angle is not a natural representation of the target motion parameter. To solve this problem, we propose the equivalent interpolation-free implementation of LRT-NuFFT, namely, the velocity search NuFFT (VS-NuFFT) method.

Considering the LRT operation, Eq. (16) can be rewritten as

$$\begin{aligned}
& s_{\text{rot}}(n', m'; V_s) \\
&= A_c \exp\left(-j \frac{2\pi c}{\lambda KB} n_{R_0}\right) \\
&\quad \cdot \exp\left(-j \frac{4\pi}{\lambda} v_0 m' T\right) \exp\left[-j \frac{2\pi}{\lambda} a(m' T)^2\right] \\
&\quad \cdot \text{sinc}\left[\frac{1}{K}(n' - n_{R_0}) + \frac{m' T}{\Delta r}(V_s - V_e)\right], \quad (35)
\end{aligned}$$

where $V_s = (\Delta r \tan \varphi) / (KT)$ denotes the search velocity corresponding to the search rotation angle.

Performing FT on Eq. (35) along the fast time axis, we have

$$\begin{aligned}
& s_{\text{rot}}(f_r, m'; V_s) \\
&= A_c \exp\left[-j 4\pi \frac{f_r}{c}(R_0 + V_e m' T)\right] \\
&\quad \cdot \exp\left(j 4\pi \frac{f_r}{c} V_s m' T\right) \exp\left(-j 4\pi \frac{R_0}{\lambda}\right) \\
&\quad \cdot \exp\left(-j \frac{4\pi}{\lambda} v_0 m' T\right) \exp\left[-j \frac{2\pi}{\lambda} a(m' T)^2\right]. \quad (36)
\end{aligned}$$

From Eq. (36) we can see that the LRT operation can be replaced by the phase compensation in the spatial frequency domain. The compensation function is expressed as

$$H_{\text{vs}}(f_r, m'; V_s) = \exp\left(j 4\pi \frac{f_r}{c} V_s m' T\right). \quad (37)$$

When $V_s = V_e$, the linear coupling term between f_r and m' will be eliminated and LRM is accordingly removed. Performing IFT on Eq. (36) along the spatial frequency axis, we obtain

$$\begin{aligned}
& s_{\text{rot}}(n', m'; V_e) \\
&= A_c \text{sinc}\left[\frac{1}{K}(n' - n_{R_0})\right] \exp\left(-j \frac{2\pi c}{\lambda KB} n_{R_0}\right) \\
&\quad \cdot \exp\left(-j \frac{4\pi}{\lambda} v_0 m' T\right) \exp\left[-j \frac{2\pi}{\lambda} a(m' T)^2\right]. \quad (38)
\end{aligned}$$

Notably, the velocity search interval should be smaller than $\Delta v = \lambda / (2N_a T)$ according to the Doppler frequency resolution.

Remark 3 Rotation angle search is equivalent to

velocity search. The interpolation can be replaced by FT, phase compensation, and IFT, which avoid the numerical errors. In addition, velocity search is more intuitive than rotation angle search, and the result is a natural expression of motion parameters. As to the search interval, the uniform sampling in the rotation angle corresponds to the non-uniform sampling in velocity, which are approximately the same at small angles but have significant differences at large angles.

4.3 Computational cost analysis

In this subsection, computational cost of the proposed LRT-NuFFT will be analyzed in detail. The representative MART-LVT, KT-LVD, TDST, SAF-SFT, and MLRT methods are selected for comparison.

Suppose that N_φ , N_{KT} , N_r , and N_a represent the numbers of search angles, search fold factors, range cells, and pulses, respectively. When the velocity search interval equals the velocity resolution, we have $N_\varphi = N_{KT} N_a$.

Main steps in MART-LVT are the MART operation $O(N_{KT} N_a N_r)$ and LVT operation $O(3N_a^2 \log_2 N_a)$ (Lv et al., 2011). Thus, the total computational complexity is on the order of $O(3N_{KT} N_r N_a^3 \log_2 N_a)$.

As for the KT-LVD method, its main steps contain fold factor search operation $O(N_{KT} N_r)$ and LVD operation $O(3N_a^2 \log_2 N_a)$ for each range cell. Therefore, the computational cost of KT-LVD is about $O(3N_{KT} N_r N_a^2 \log_2 N_a)$.

For TDST, $O(3N_r N_a^2 \log_2 N_a)$ and $O(3N_a^2 N_r \log_2 N_r)$ are needed to perform two steps of the scaled Fourier transform (SFT). Thus, the total computational complexity is on the order of $O(3N_r N_a^2 \log_2(N_a N_r))$.

The main steps of SAF-SFT include the two steps of SFT, whose computational complexities are $O(3N_a N_r \log_2 N_a)$ and $O(3N_a^2 \log_2 N_a)$. Hence, its computational cost is on the order of $O(3N_a(N_a + N_r) \log_2 N_a)$.

It is easy to find that the computational cost of MLRT is on the order of $O(N_{KT} N_r N_a^2 \log_2 N_a)$.

Different from MART-LVT, phase compensation and NuFFT are used to decrease the computational burden, where the computational cost of an N -point

NuFFT is $O(2M \log_2 N)$. Thus, the overall computational cost of LRT-NuFFT is about $O(2N_{KT}N_rN_a^2 \log_2 N_a)$.

Computational complexities of the above methods are listed in Table 1. Under the assumption that $N_r=N_a$ and $N_{KT}=10$, Fig. 2 intuitively presents the computational cost curves. It is obvious that the LRT-NuFFT method has much lower computational complexity than MART-LVT and KT-LVD, but higher computational cost than TDST, MLRT, and SAF-SFT. However, when considering the detection performance, the proposed method has great advantages in a low-SNR environment.

5 Simulations and real-data processing results

5.1 Coherent integration ability

Radar parameters in the simulations are given in Table 2.

A single maneuvering target is used to evaluate the coherent integration ability of the proposed method. Motion parameters of the target are as follows: $R_0=150$ km, $v=150$ m/s, and $a=8$ m/s². Fig. 3a shows the results of the pulse compression with an SNR being -10 dB. It is obvious that the target trajectory is submerged by the serious noise and that no target can be observed. Fig. 3b shows the integration

Table 1 Computational complexity of different methods

Method	Computational complexity
MART-LVT	$O(3N_{KT}N_rN_a^3 \log_2 N_a)$
KT-LVD	$O(3N_{KT}N_rN_a^2 \log_2 N_a)$
SAF-SFT	$O(3N_a(N_a + N_r) \log_2 N_a)$
TDST	$O(3N_rN_a^2 \log_2(N_aN_r))$
MLRT	$O(N_{KT}N_rN_a^2 \log_2 N_a)$
Ours	$O(2N_{KT}N_rN_a^2 \log_2 N_a)$

N_{KT} , N_r , and N_a represent the numbers of search fold factors, range cells, and pulses, respectively

Table 2 Radar parameters in the simulations

Parameter	Value	Parameter	Value
Carrier frequency (GHz)	1	Bandwidth (MHz)	20
Sample frequency (MHz)	30	Pulse width (μ s)	10
PRF (Hz)	256	Number of pulses	512

PRF: pulse repetition frequency

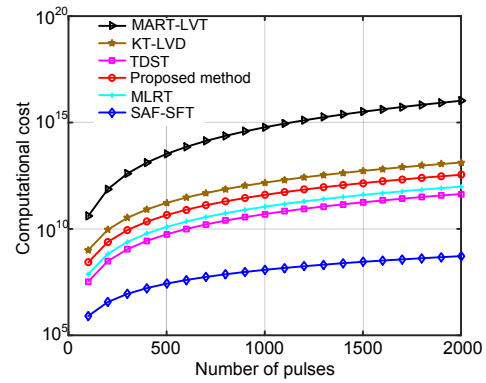


Fig. 2 Comparison results of computational complexity

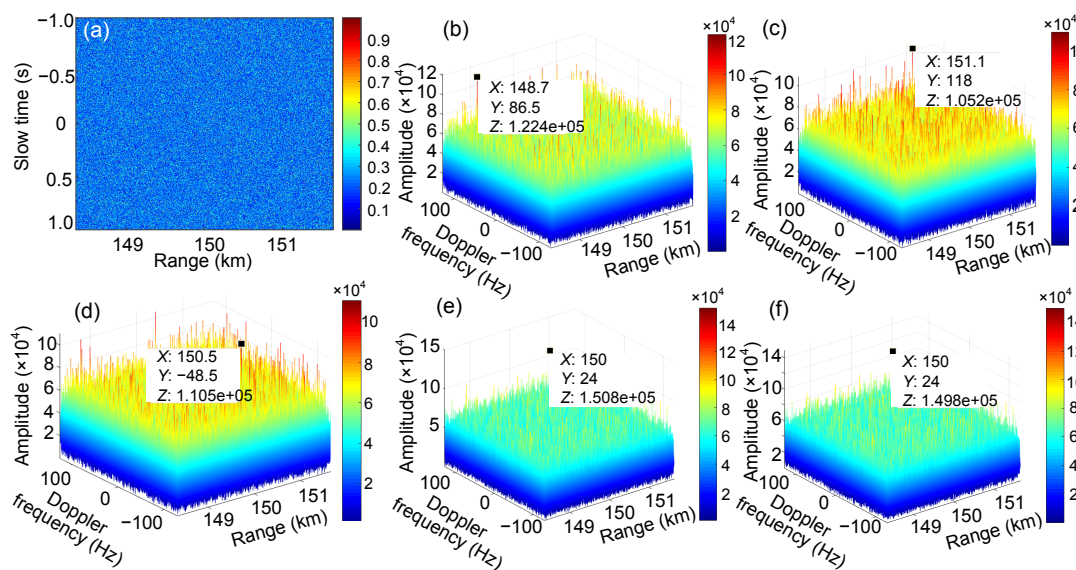


Fig. 3 Coherent integration simulation results of a maneuvering target: (a) pulse compression; (b) MLRT; (c) SAF-SFT; (d) TDST; (e) MART-LVT; (f) LRT-NuFFT

results of MLRT. Without eliminating the effects of QRM and DFM, MLRT fails in detecting the maneuvering target. Figs. 3c and 3d present the integration results of SAF-SFT and TDST, respectively. As the bilinear operation sacrifices some target energy, their outputs are still buried in the noise. For comparison, MART-LVT and LRT-NuFFT integrate the target as a peak at the expected position of the range-Doppler frequency domain (Figs. 3e and 3f). Experimental results demonstrate the integration capability of the proposed method with a low SNR.

In the simulations, the integration performances of two targets are analyzed. Motion parameters of the two targets (Tr1 and Tr2) are given in Table 3. Fig. 4a shows the results of pulse compression. Fig. 4b shows the velocity search results, where two obvious peaks indicate the equivalent velocities of Tr1 and Tr2. Thus, the velocities of Tr1 and Tr2 are estimated as $\hat{v}_{Tr1} = 150$ m/s and $\hat{v}_{Tr2} = -200$ m/s, respectively. With the corresponding velocity, LRMs of Tr1 and Tr2 are corrected, as shown in Figs. 4c and 4d, respectively. Via NuFFT, accelerations of Tr1 and Tr2 are estimated at the corresponding range cells, i.e., $\hat{a}_{Tr1} = 7.95$ m/s² and $\hat{a}_{Tr2} = -4.05$ m/s², as shown in Figs. 4e and 4f, respectively. Figs. 4g and 4h show the integration results of the two targets. Simulation results demonstrate the integration capability of the proposed method for multiple targets.

Table 3 Parameters of two maneuvering targets in the simulations

Target	SNR (dB)	Initial slant range (km)	Velocity (m/s)	Acceleration (m/s ²)
Tr1	10	150.0	150	8
Tr2	10	150.2	-200	-4

SNR: signal-to-noise ratio

Note that there are several false peaks in Fig. 4b and that the interval between these peaks is exactly half of the blind speed. Thus, denote these false peaks as half-blind-speed sidelobes (HBSSLs). The internal cause of the HBSSLs is analyzed in the Appendix. In real applications, weak targets may be submerged in the HBSSLs caused by strong targets. Therefore, the CLEAN technique (Li et al., 2016b) is suggested.

5.2 Detection performance

We evaluate the detection performance of the

proposed method via Monte-Carlo trials. Complex white Gaussian noise with zero mean is added to the signal after pulse compression to yield SNRs varying from -25 dB to 5 dB. SNR is defined as

$$\text{SNR} = 10 \lg \frac{A_c^2}{\sigma^2}, \quad (39)$$

where σ^2 is the variance of the Gaussian noise.

For each SNR, 500 independent simulations are performed. The false alarm ratio is set as $P_{fa} = 10^{-6}$. Five representative coherent integration methods, i.e., MART-LVT, KT-LVD, TDST, SAF-SFT, and MLRT, are selected for comparison.

Detection probability curves are shown in Fig. 5. It is clear that LRT-NuFFT obtains nearly the same detection ability as the ideal processing, because LRT-NuFFT effectively eliminates the effects of LRM, QRM, and DFM and realizes coherent integration of the target energy. MART-LVT and KT-LVD suffer from some integration performance losses because they ignore the QRM effects caused by the target's acceleration. SNRs for TDST and SAF-SFT are 3 dB and 6 dB higher than those of the ideal situation, respectively, indicating that the bilinear operations lose much signal energy and deteriorate the anti-noise performance. As expected, the detection performance of MLRT is poor due to its poor ability to deal with the effects of QRM and DFM.

5.3 Real-data processing results

In this subsection, we adopt the measured data of a DJI Phantom 3 commercial UAV to demonstrate the proposed method. The data was collected in March 2017 by the National University of Defense Technology, Hunan Province, China. Figs. 6a and 6b present the experimental scene and frequency modulated continuous wave (FMCW) radar system, respectively. Radar parameters are given in Table 4. Note that to obtain LRM and Doppler ambiguity, we artificially enlarge the bandwidth and reduce PRF.

Fig. 6c shows the target trajectory after pulse compression. The coherent time is 0.92 s, during which the UAV moves across several range cells and causes serious range migration. Fig. 6d shows the equivalent velocity estimation results. The velocity search scope is set as $[-10, 10]$ m/s, and the search interval is 0.01 m/s. From the peak position, we can

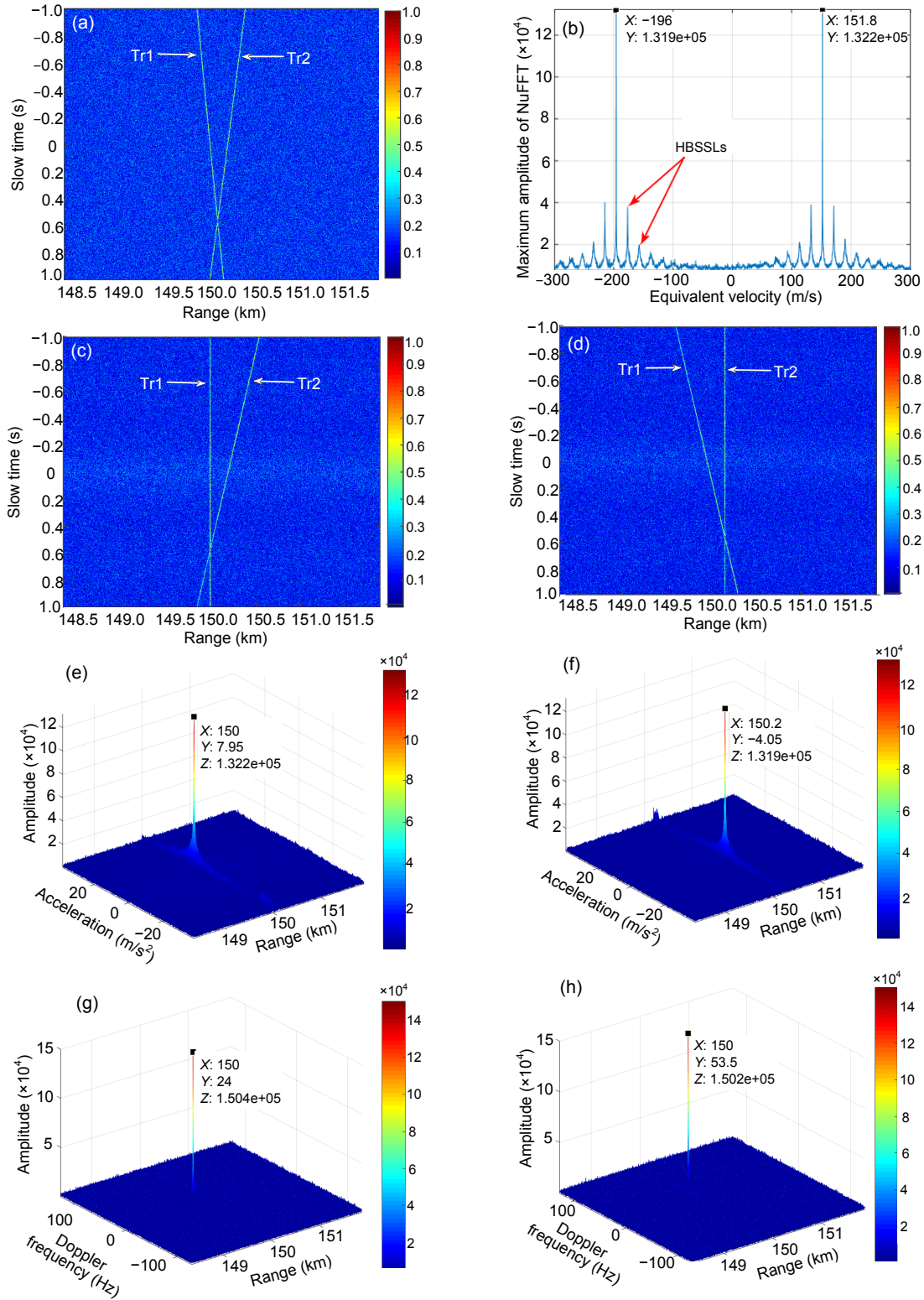


Fig. 4 Coherent integration simulation results of LRT-NuFFT for multiple targets

(a) results of pulse compression; (b) equivalent velocity search results; (c) range migration correction results of Tr1; (d) range migration correction results of Tr2; (e) acceleration estimation results of Tr1; (f) acceleration estimation results of Tr2; (g) integration results of Tr1; (h) integration results of Tr2

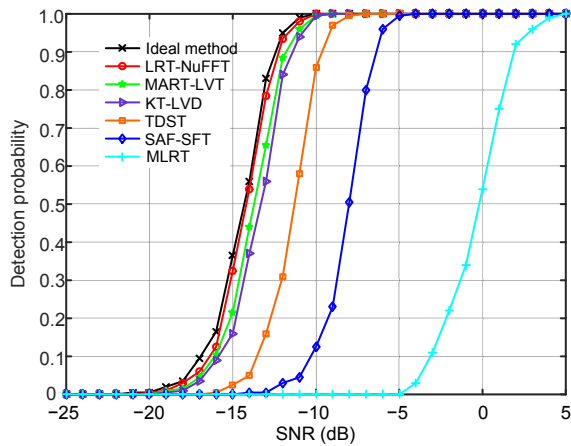


Fig. 5 Detection probability against the signal-to-noise ratio (SNR)

estimate the equivalent velocity and true velocity of the target to be $\hat{V}_e = 1$ m/s and $\hat{v} = 1.2105$ m/s, respectively. After phase compensation, LRM is corrected (Fig. 6e) and the acceleration of the UAV is obtained via NuFFT (Fig. 6f), i.e., $\hat{a} = 0.2231$ m/s².

Table 4 Frequency modulated continuous wave radar parameters

Parameter	Value	Parameter	Value
Carrier frequency (GHz)	9.5	PRF (Hz)	50
Bandwidth (GHz)	1	Sampling frequency (MHz)	1
Pulse width (s)	0.0102	Coherent time (s)	0.92

PRF: pulse repetition frequency

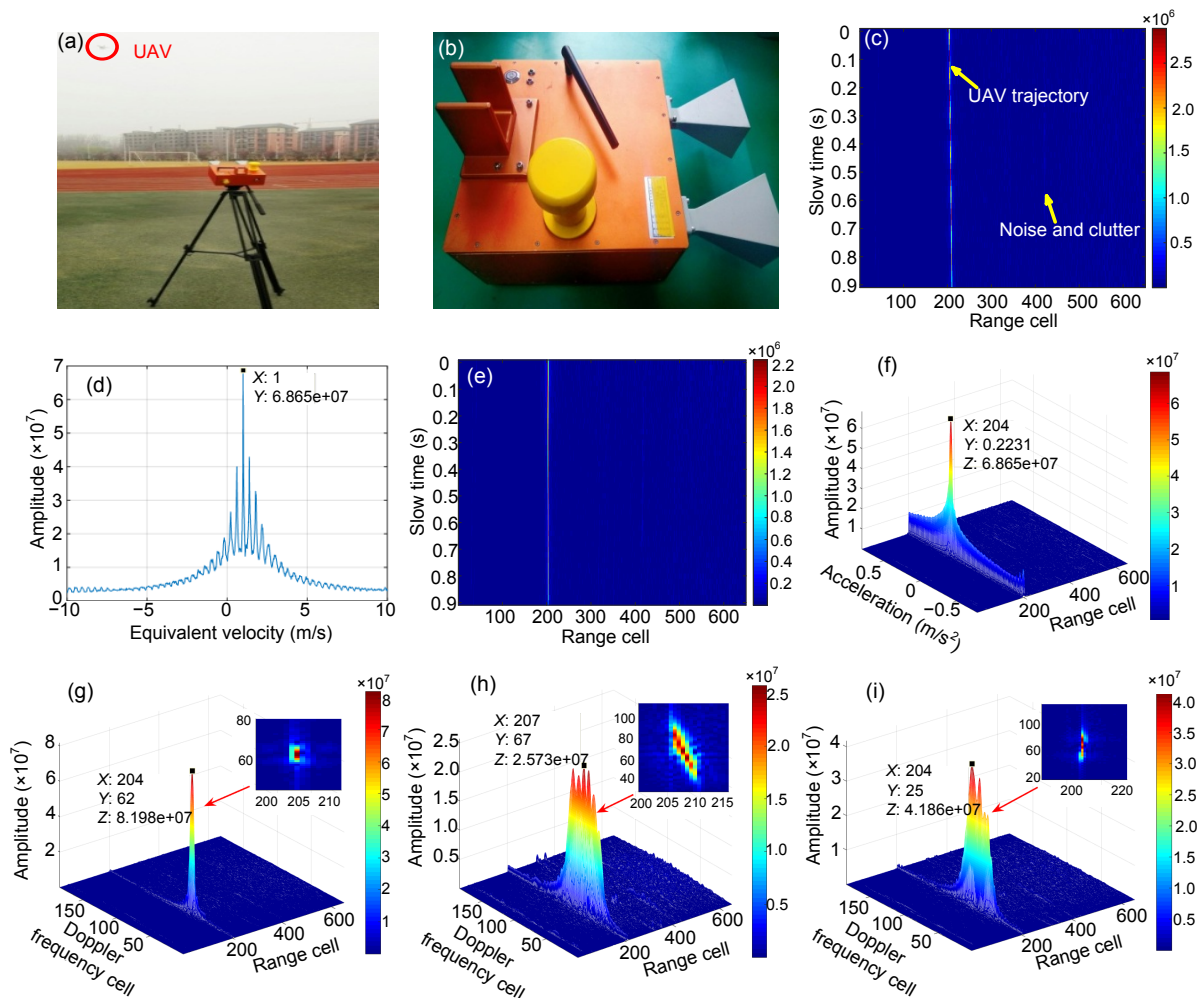


Fig. 6 Processing results with real data

(a) experimental scene; (b) frequency modulated continuous wave radar system; (c) results of pulse compression; (d) equivalent velocity estimation results; (e) results of LRM correction; (f) acceleration estimation results of NuFFT; (g) integration results of LRT-NuFFT; (h) integration results of MTD; (i) integration results of MLRT

Fig. 6g depicts the coherent integration results of LRT-NuFFT, where a well-focused peak is observed in the range–Doppler frequency domain. At the same time, the integration results of MTD and MLRT are presented for comparison in Figs. 6h and 6i, respectively. However, the target energy is dispersed over multiple range cells and velocity cells, which creates difficulties for target detection.

6 Conclusions

In this study, we have proposed a novel fast method based on LRT and NuFFT for maneuvering target detection. The effects of LRM, QRM, and DFM have been eliminated via LRT, SoKT, and NuFFT, respectively. At the same time, an equivalent velocity search method has been derived to avoid interpolation. Simulation results showed that the proposed LRT-NuFFT method could achieve nearly ideal detection performance at low computational cost. Experimental results from UAV echoes demonstrated the effectiveness of the proposed method.

Contributors

Ke JIN designed the research and drafted the manuscript. Tao LAI and Yan-li QI processed the data. Jie HUANG and Yong-jun ZHAO helped organize the manuscript. Ke JIN and Tao LAI revised and finalized the paper.

Compliance with ethics guidelines

Ke JIN, Tao LAI, Yan-li QI, Jie HUANG, and Yong-jun ZHAO declare that they have no conflict of interest.

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Appendix: Derivation of HBSSLs

Because of the discrete pulse sampling, finite range resolution, and limited integration time, HBSSLs will inevitably appear. When deriving the HBSSLs of LRT-NuFFT, we may refer to the blind speed sidelobes (BSSLs) of RFT (Xu et al., 2011b).

Assume that VS-NuFFT is used to achieve the LRT operation. Consider the discrete form of Eq. (35) as

$$\begin{aligned}
 & s_{\text{rot}}(n', m'; V_s) \\
 &= A_c \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_0}\right) \exp\left(-j\frac{4\pi}{\lambda} q_b \Delta v m' T\right) \\
 & \cdot \exp\left(-j\frac{2\pi}{\lambda} a(m'T)^2\right) \text{sinc}\left[\frac{\rho_r}{\Delta r}(n' - n_{R_0})\right. \\
 & \left. + \frac{m'T}{\Delta r}(s - N_b)v_b + \frac{m'T}{\Delta r}\left(q_s - \frac{q_b}{2}\right)\Delta v\right], \tag{A1}
 \end{aligned}$$

where $V_s = sv_b + q_s \Delta v$, $V_e = N_b v_b + q_b \Delta v / 2$, $v_0 = q_b \Delta v$, and $s = \text{round}(V_s / v_b)$ ($\text{round}(\cdot)$ denotes the round up operation).

The phase compensation function in Eq. (22) can be written as

$$H_1(m') = \exp\left(j\frac{4\pi}{\lambda} 2q_s \Delta v m' T\right). \tag{A2}$$

Multiplying Eq. (A2) with Eq. (A1) and performing NuFFT to accumulate target energy, we have Eq. (A3), which is expressed at the bottom of this page.

To simplify the derivation, only $f_{(m'T)^2} = -a/\lambda$ is considered. When $f_{(m'T)^2} \neq -a/\lambda$, the second-order

terms of $m'T$ defocus the target energy, and thus cannot form a sharp peak. Consequently, Eq. (A3) is simplified as

$$\begin{aligned}
 s_{\text{NuFFT}}(n', s) &= \sum_{m'=-N_a/2}^{N_a/2-1} A_c \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_0}\right) \\
 & \cdot \exp\left[j\frac{4\pi}{\lambda}(2q_s - q_b)\Delta v m' T\right] \\
 & \cdot \text{sinc}\left[\frac{\rho_r}{\Delta r}(n' - n_{R_0}) + \frac{m'T}{\Delta r}(s - N_b)v_b\right. \\
 & \left. + \frac{m'T}{\Delta r}\left(q_s - \frac{q_b}{2}\right)\Delta v\right]. \tag{A4}
 \end{aligned}$$

It is easy to see that Eq. (A4) is the derivation of the BSSLs in RFT. We may refer to the results of RFT (Xu et al., 2011b) and directly give the result, i.e.,

$$s_{\text{NuFFT}}(n', s) \approx I_0 + \sum_{p=1, p \neq s} I_p, \tag{A5}$$

where

$$I_0 = A_c N_a \text{sinc}\left[\frac{\rho_r}{\Delta r}(n' - n_{R_0})\right] \delta\left(q_s - \frac{q_b}{2}\right), \tag{A6}$$

$$\begin{aligned}
 I_p &= \frac{A_c f_p \Delta r}{(s - N_b)v_b} \text{rect}\left[\frac{(n' - n_{R_0})}{(s - N_b)v_b T} \Delta r\right] \\
 & \cdot \text{sinc}\left[\frac{\Delta r(2q_s - q_b)}{v_b T(s - N_b)}\right], \tag{A7}
 \end{aligned}$$

where $f_p = 1/T$ denotes the pulse repetition frequency.

It is evident that the response of a single target in the NuFFT domain is composed of one main peak I_0 , which indicates the initial range and equivalent

$$\begin{aligned}
 s_{\text{NuFFT}}(n', s, f_{(m'T)^2}) &= \text{NuFFT}[s_{\text{rot}}(n', m'; V_s)H_1(m')] \\
 &= \sum_{m'=-N_a/2}^{N_a/2-1} s_{\text{rot}}(n', m'; V_s)H_1(m') \exp\left[-j2\pi f_{(m'T)^2}(m'T)^2\right] \\
 &= \sum_{m'=-N_a/2}^{N_a/2-1} A_c \exp\left(-j\frac{2\pi c}{\lambda KB} n_{R_0}\right) \exp\left[j\frac{4\pi}{\lambda}(2q_s - q_b)\Delta v m' T\right] \exp\left[-j\frac{2\pi}{\lambda}\left(f_{(m'T)^2} + \frac{a}{\lambda}\right)(m'T)^2\right] \\
 & \cdot \text{sinc}\left[\frac{\rho_r}{\Delta r}(n' - n_{R_0}) + \frac{m'T}{\Delta r}(s - N_b)v_b + \frac{m'T}{\Delta r}\left(q_s - \frac{q_b}{2}\right)\Delta v\right]. \tag{A3}
 \end{aligned}$$

velocity of the target. The interval between the two peaks of I_p is half the blind speed. By Eq. (A7), the velocity slice at $n' = n_{R_0}$ is

$$s_{\text{NuFFT}}(n_{R_0}, s) \approx A_c N_a \delta\left(q_s - \frac{q_b}{2}\right) + \frac{A_c f_p \Delta r}{(s - N_b) v_b} \sum_{p=1, p \neq s} \text{sinc}\left[\frac{\Delta r(2q_s - q_b)}{v_b T(s - N_b)}\right]. \quad (\text{A8})$$

Eq. (A8) indicates the HBSSLs obtained by the proposed method. When QRM and SoKT are neglected, HBSSLs will become BSSLs.