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Wheat ear growth modeling based on a polygon*

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Abstract: Visual inspection of wheat growth has been a useful tool for understanding and implementing agricultural techniques and a way to accurately predict the growth status of wheat yields for economists and policy decision makers. In this paper, we present a polygonal approach for modeling the growth process of wheat ears. The grain, lemma, and palea of wheat ears are represented as editable polygonal models, which can be re-polygonized to detect collision during the growth process. We then rotate and move the colliding grain to resolve the collision problem. A linear interpolation and a spherical interpolation are developed to simulate the growth of wheat grain, performed in the process of heading and growth of wheat grain. Experimental results show that the method has a good modeling effect and can realize the modeling of wheat ears at different growth stages.

Key words: Visual inspection; Virtual crop; Three-dimensional modeling
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1 Introduction

There are many kinds of plants in nature that have strange growth processes, strong structural characteristics, and complex three-dimensional (3D) structures. The growth and development processes are very complicated, and there are many control variables and influencing factors. The growth process is random, nonlinear, variable, and mutable, and the interaction between plants and environment is quite complicated. To visualize the plant morphology and natural growth, on one hand, it is of great significance to explore the laws of plant life and growth and to deepen the study of agronomy and botany, which can be used, for example, in plant science research, production process management, production optimization simulation, and urban landscape design. On the other hand, such study shows huge potential markets in the entertainment, business, education, and industries, and new opportunities to improve the quality of human living environments.

In recent years, with the integration of research on digital crop technology in areas such as big data (Wen, 2015), virtual reality (Zhao, 2009), and artificial intelligence (LeCun et al., 2015), an important branch of the technology, crop modeling, is gradually becoming an effective technical tool for intuitive analysis and in-depth understanding of the complex growth and development processes of crops. Therefore, using this crop modeling technology to construct a wheat growth scenario on an agricultural digital platform and to quantitatively analyze the growth data of wheat population's geometric shape, such as plant height, leaf area index, and canopy variables, can realize visual monitoring and scientific

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analysis of wheat growth.

At present, virtual corn and rice plant models have been extensively studied, while wheat models are less studied. Virtual wheat is a research direction of virtual crops. Research on wheat models focuses mostly on growth machine models. Some researchers have studied the morphological growth characteristics of wheat leaves, stems, and leaf sheaths. Simulation of the wheat growing process is an important part of crop morphological simulation, but there are few studies on wheat ear growth simulation.

2 Related work

Ecological characteristics, physiological characteristics, color, texture, geometry, and other 3D morphological characteristics are main scientific bases for wheat growth dynamics modeling. The 3D modeling techniques of wheat are based on a 3D reconstruction of the ecological physiological growth machine model and the 3D morphological model.

2.1 Ecological physiological growth machine model of wheat

Wheat eco-physiological growth models are commonly shown in mathematical models that can dynamically reflect the development of wheat at various stages of growth. The model is composed of many specific growth mechanisms, such as plant soil, water and fertilizer, photosynthesis, nutrient production and distribution, and respiration. In early virtual wheat research, people focused on the study of wheat ecological physiology, which is the starting point of the research on digital modeling of wheat.

Lindemayer (1968) presented the first realistic plant growth modeling method, the L-system, by introducing a mathematical model. The L-system generates a fractal graph by generating empirical expressions and abstracts of the plant growth process. This generates the character development sequence that expresses the plant topological structure, and provides a geometric interpretation of the resulting string.

Prusinkiewicz and Lindenmayer have studied the L-system for a long time, constantly improving the L-system (Prusinkiewicz and Lindenmayer, 2012). The L-system gradually evolved from the original deterministic context-free Lindenmayer (DOL) system (which could express only a strict iteration process) to the stochastic L-system (which could construct a stochastic topological structure) and the context-sensitive L-system (which could simulate the correlation of plant organs). The parametric L-system involves a flexible 3D model of plants by controlling and changing parameters, whereas the open L-system can simulate the information interaction between plants and the external environment.

Peyrat et al. (2008) developed a 2Gmap L-based system, which is based on the L-system and a two-dimensional (2D) generalized graph. The system extends the basic syntax of the L-system, automatically generates a representation of triangular mesh and texture mapping, and processes parameters, so that it can control the model through writing scripts and specific rules.

Aono and Kunii (1984) simulated the wind, light, gravity, and other external conditions in different plant simulations by introducing an attractor algorithm. Barnsley (1988) proposed a fractal interpolation function that provides new thinking for a continuous, but not smooth, natural fractal. At the end of the 20th century, more and more plant growth models appeared. Rauscher et al. (1990) established a plant growth process model based on photosynthesis, called "ECOPHYS." Perttunen et al. (1998) established a plant carbon transport distribution model, called "LIGNUM."

de Reffye et al. (1988) proposed an automaton model, known as "reference axis technology." This method regards the growth of plants as a stochastic process with a certain probability distribution, using the Markov chain theory and the state transition diagram to analyze the topological structure evolution of plants through pattern recognition, to extract a growth rule and simulate the plant growth and development processes. Based on the automaton model, Godin and Caraglio (1998) proposed a plant topology model in multi-scale sense of plant topology with different lengths of time. Based on the automaton model, de Reffye et al. (1997) developed the advanced modeling of architecture of plant (AMAP) system software to simulate plant growth. In the dynamic simulation of plants, the AMAP software needs to use parameter variables input from outside.

According to the topological, physiological, and ecological characteristics of plant growth and the computer graphics method, Zhao et al. (2001) proposed a branch bending simulation algorithm

based on the dual-scale automaton model. On this basis, Chinese and French researchers collaborated to develop a Greenlab system. Yan et al. (2003) described a model in a mathematical formula to compensate for the lack of physiological and ecological functions of the AMAP system in simulated plants, and to overcome the deficiency that the AMAP system can simulate only a simple morphological structure. The system has an advantage of short calculation time. Qu et al. (2009) used the hidden Markov tree to analyze the transition relationship between the structural characteristics of growth units and the growth elements and signals, and proposed a 3D reconstruction based on multi-scale analysis of the branch structure.

2.2 Three-dimensional morphological model of wheat

The 3D morphological model of wheat is a digital model of wheat in a 3D space using computer graphics and computer vision, and it presents its color, texture, and geometric structure in a visual way. The model can help study wheat from an intuitive perspective, to reveal the complex morphological structure and life characteristics of wheat.

Quan et al. (2006) proposed a reconstruction method based on several images to simulate plant stalks, and designed an automatic 3D model system with interactive and manual input parameters. To reduce the effect of jitter and reflection in a single image, Zheng et al. (2011) simulated the fine form of the natural root system using the simulated background as the harmonic function to improve the accuracy of plant root fragments in an image. Based on the study of a single image, Wang et al. (2011) formed a projection matrix with the intrinsic and extrinsic parameters of a camera, matched the image pixel-by-pixel domain sum of squared differences (SSD) values to obtain the parallax map to reconstruct the plot landscape, and then optimized the 3D reconstruction results by clustered adjustment. Bradley et al. (2013) used a small number of images and multiple iterative optimizations through the blade grid to produce an initial model of the plant dense leaves, and optimized the 3D blade model using the blade shape data statistical model during the reconstruction process. Yang et al. (2009) used machine vision to obtain the 3D feature points of the blade from the image, and used B-spline curve fitting to construct the leaf edge and the main vein; the triangulated mesh model of the blade was established by the Delaunay triangulation. Koc-San et al. (2018) proposed a novel approach for the automatic extraction of citrus trees using multi-spectral images from an unmanned aerial vehicle (UAV) and digital surface models.

With the application of sensing and scanning equipment (such as a laser scanner and a 3D digitizer) in the field of agriculture, the plant point cloud data can be obtained quickly without loss, to realize 3D reconstruction of the plant 3D morphological structure. Loch et al. (2005) used a 3D laser scanner to obtain a large set of data points on the leaf surface, and used linear triangulation to precisely model plant leaves. Oqielat et al. (2007) established a highly authentic 3D model of a blade using a radial basis function (RBF) based on a large number of 3D data points obtained by a 3D laser scanner. Oqielat et al. (2009) proposed a hybrid technology for blade surface modeling based on Clough-Tocher and RBF, which could generate an accurate and realistic blade surface representation. Oqielat et al. (2011) proposed a simple mathematical model to generate the natural appearance trajectory of water droplets passing through the virtual leaf surface. Sun et al. (2012) proposed a method of reconstructing the plant leaf surface based on point cloud data. The method uses the Delaunay triangulation to generate the initial mesh surface and optimizes it to generate the mesh surface of plant leaves. Guo et al. (2011) obtained metamorphic root point cloud data by the method of body coloring 3D reconstruction, and the original point cloud data were simulated by noise, outlier removal, and cavitation repair. Then the morphological parameters of the metamorphosis root were extracted for surface modeling. Hu (2013) developed a reference body that could improve the accuracy and angle precision of 3D plant length information, and combined the visualization tool kit (VTK) visual class library with 3D plant visualization of the point cloud, to simulate the 3D morphology of plant leaves based on the plant morphologic characteristics of the point cloud. Wang and Fu (2017) proposed a modeling approach that combines visual basic for applications (VBA) and computer aided design (CAD) technology to design 3D virtual plant models. Tree models were used for numerical simulations of wind flow because trees are more lifelike. In addition, this

method could be applied to create a 3D virtual vegetation library. It was convenient for recording diversified biological indicators by digital plants in a computer. Xue et al. (2018) used a Bezier surface to model the growth of wheat ear, which could simulate the wheat ear growth process. Artru et al. (2018) used the adjusted parameter set, and the simulateur multidisciplinaire pour les cultures standard (STICS) model gave a good prediction of the grain number under all treatments. The model accurately predicted the timing of the grain maturity stage under periodic shade (PS) treatment by reducing only the daily global radiation. Demir (2018) used UAVs for detection of trees in digital surface models, and the quality of the results was assessed by comparison with reference data for correctness of the estimated number of trees. The tree heights were calculated and evaluated with a ground-truth data set. The results showed 80% correctness and 90% completeness. Krishnan et al. (2016) proposed an innovative and efficient approach to evolving crop simulation model, which was used as a decision support tool in the agricultural production system.

Although there has been much research on wheat modeling, the current research work usually involves a static model or a wheat morphological model under specific conditions due to the variability and complexity of wheat morphological characteristics, and changes in dynamic morphological data are difficult to obtain. There are few generalized dynamic models to simulate the growth process. This situation presents an opportunity and a challenge to study the wheat growth model by studying the growth law of wheat and effectively combining the wheat growth machine model and the visualization model. There are few simulation algorithms for wheat growth. Our method focuses on the simulation of wheat growth; however, most of the current simulation algorithms focus on static simulation. We cannot find a suitable method for effective comparison before we completed this study.

3 Model description

Polygonal modeling is a common modeling method. First, an object is converted into an editable polygonal object, and then the modeling process is performed by editing and modifying various sub-objects of the polygonal object. Editable polygonal objects contain five sub-object patterns: vertex, edge, border, polygon, and element. Compared with editable meshes, editable polygons show greater advantages, in which the faces of a polygonal object can be not only triangular and quadrilateral faces, but also polygonal faces with any number of nodes.

In this study, we first establish various organs of wheat, adjust initial and mature states of the wheat model, and perform growth interpolation between the initial and final states. Each time a growth interpolation is performed, a collision is detected. When a collision is detected, adjustment is made to resolve the collision, and then the next growth interpolation is performed and repeated until the final state (Fig. 1).

3.1 Wheat leaf modeling

Through observing and understanding the actual wheat leaves, the wheat leaves have the characteristics of being soft and easily bent. After understanding the physiological and geometric modeling of two commonly used modeling methods, we model the leaves using a geometric and polygonal model based on our goal of speeding up the process. Polygon modeling is flexible and diverse, and we distinguish them based on the degree of bending and the size of the actual wheat leaves, which corresponds to the number of polygons constituting different wheat leaves ranging from 176 to 477. This not only is convenient and quick for establishing models, but also can realistically represent the characteristics of soft and easily bent wheat leaves. The modeling effect using polygons on wheat leaves is shown in Fig. 2.

3.2 Wheat grain modeling

The wheat ear is divided into grain, lemma, palea, and awn. As a key part of the wheat ear, the establishment of a wheat grain model has a great influence on the accuracy of wheat. Because of the large number of wheat grains in the wheat ear, it can have a significant impact on the speed of calculation in wheat grain modeling after wheat ear modeling. By considering these two issues, we use a polygonal model of the grain, lemma, palea, and awn. The shape of the grain is close to that of the spindle. We have made this spindle consist of eight triangular faces. There are three advantages: (1) It can increase

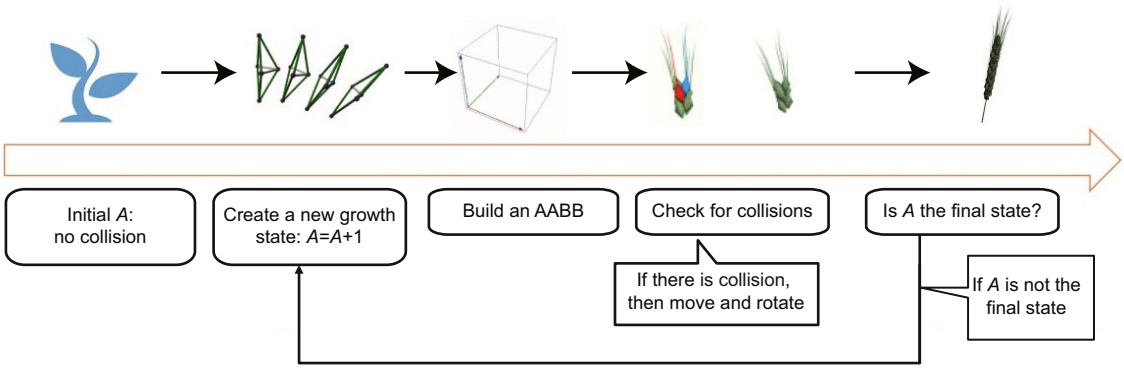

Fig. 1 Overall processing flow (AABB: axis-aligned bounding box)

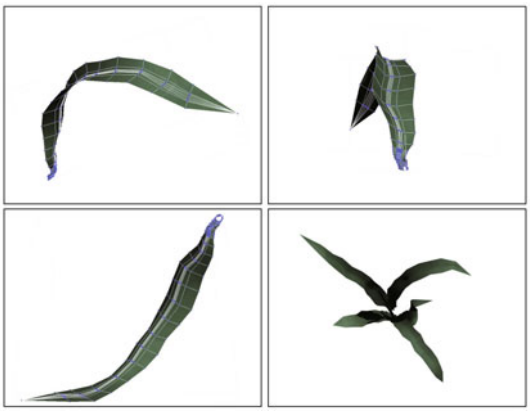

Fig. 2 Simulation diagram of the barley based on a polygon

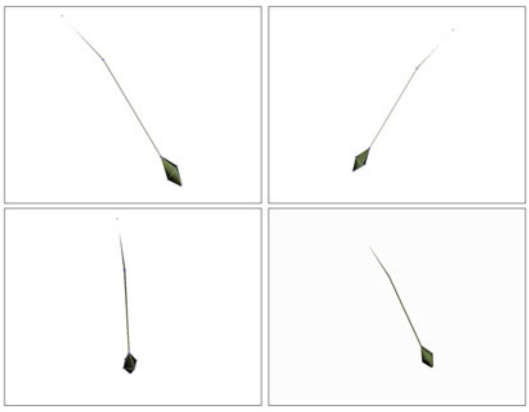

Fig. 3 Simulation diagram of the lemma and palea as one part

the calculation speed for the model; (2) It can reduce the modeling time without greatly affecting the degree of fidelity; (3) Each face is more likely to approach the triangular, which is convenient for re-triangulation. The modeling effect of wheat is shown in Fig. 3.

3.3 Wheat ear modeling

As the top fruit part of wheat stems, wheat ears generally consist of 30–40 grains and wheatgrass. The modeling of wheat ears involves numerous combinations of wheat kernels. By observing the wheat ears, we find that the arrangement of wheat ears follows a certain rule under normal circumstances; that is, wheat ears can be roughly regarded as four grains per layer, with a total of 8–12 layers of wheat. We assemble wheat grains according to this rule to achieve the modeling of wheat ears. After the initial model has been built, the wheat grain models in different parts of wheat ears are manually fine-tuned to make their display realistic. The wheat ear assembly modeling steps are shown in Fig. 4. The overall display is shown in Fig. 5.

3.4 Re-polygonization

After polygonal modeling, the lemma and palea models are delineated using the Delaunay triangulation, so that the lattice and lemma grid topologies will not change in the model's subsequent steps due to the development. The specific division method is described in this subsection.

There are many wheat kernels in the wheat ears. It is difficult to ensure that there is no collision or overlap between wheat grains during the process of simulating wheat ear development. We use these two graphically defined functions to re-grow the grain to polygonize the grain's surface to a non-degenerate triangle. There are some advantages to ensure that: (1) The mesh topology of the grain (outer and inner) does not change because of the development; (2) There are few points on the surface that make up the grain, which can speed up computer calculations;

Fig. 4 Assembly modeling steps of wheat ears

Fig. 5 Overall display of wheat

(3) It can make subsequent collision-detection steps fast.

To re-polygonize the surface of grain, we use a semi-interactive method. We use these two functions defined in Fig. 6 to fine-tune polygons. The first function $\lambda(v)$ defines the vertex sequences $v_0, v_1, \ldots, v_n$ along the v axis. The distance between vertices is calculated by

$$\int_{v_i}^{v_{i+1}} \frac{\mathrm{d}v}{\lambda(v)} = 1, \quad i = 0, 1, \ldots, n-1. \tag{1}$$

In each interval, it distributes points (v_i, v_{i+1}) according to the mean value of the function $\lambda(v)$ along the v axis and ensures that the distribution is robust (not sensitive to small perturbations of λ). The second function $\eta(v)$ defines the vertex numbers along the parameter space of each of the isoparametric lines $v = v_i$. The final polygonal mesh is used as the Delaunay triangulation of the resulting rendezvous (u, v). Triangulation is performed in the parameter space, so that the mesh topology does not change with the growth of grain.

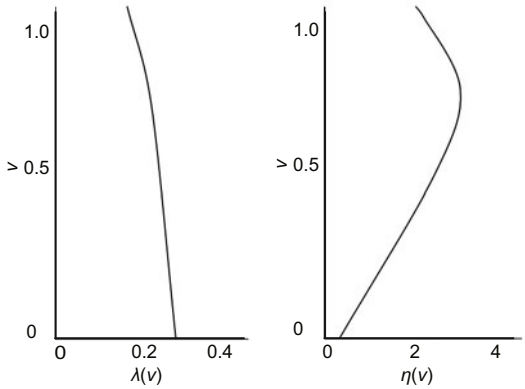

Fig. 6 Functions of trim polygon

The default polygon of the isoparametric series u = const and v = const is used in the surface. These are the graphic definition functions used to create the renavers in the panel. Attentionally, the axis representing the independent variable v is vertical

3.5 Growth interpolation

To simulate the growth of wheat grain, a linear interpolation and a spherical interpolation are performed in the process of heading and growth of wheat grain. The reasons why we use linear interpolation of wheat are as follows: (1) The size of wheat is different from that of earing to maturity; (2) Spherical interpolation of wheat fruits including the head from heading to maturity is appropriate; (3) A direction of transfer may occur in the process of growth. Interpolation occurs in the key positions between heading and maturity.

The rotation minimization frame of each (opening) control polyline running from the v axis is calculated from the bottom of the grain (lemmas and palea) to its tip (Fig. 7). For each vertex between successive segments, the frame revolves around the axis perpendicular to the two segments, so that the next line segment is aligned with the previous one (0 for the collinear segment). Then we linearly interpolate the length of the corresponding segment. Spherical interpolation is performed between the segments to mix the corresponding control polygons in the initial and final postures, which can be used to simulate the development of wheat and wheat grains with a minimum number of critical poses (usually only two or three critical poses).

3.6 Collision detection

It is not enough to model the wheat or the wheat grain, because the wheat is an organ with a lot of

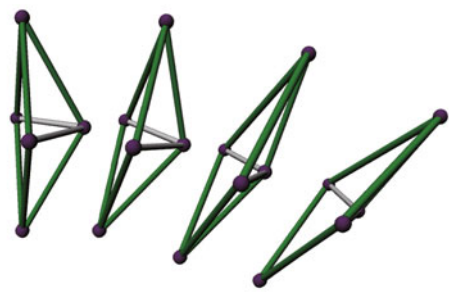

Fig. 7 Grid interpolation control samples

grains. The positional distribution between grains needs to be constrained, which cannot cross each other especially in close-ups of wheat. So, collision detection of grains of wheat is necessary. To solve this problem, we first assume that there is no collision in the initial distribution of grains of wheat, and solve the collision detection problem in grain development. Therefore, we adopt a continuous collision detection model. We use the triangular mesh in the wheat grain model after Delaunay triangulation, because the triangular mesh that expresses the lemma and the glumellule of wheat grains change as time goes on. The intersections and movements of edges and sides of a triangle represent the movement of the triangular mesh, and the intersections may be a narrow or wide part of the wheat grain. Therefore, we need to first distinguish the intersections and vertices of edges and sides between triangles, consider them separately, and divide the situation of each point into a wide or narrow phase. For the width phase, we divide it into a regular grid in space to accelerate it. For the vertices, we use voxels to facilitate statistics and detect each voxel to store the IDs of all triangles that intersect with voxels for a certain time step. First, we construct an axis-aligned bounding box (AABB), and then complete it with what we have constructed. The bounding box contains the space edges of triangles when it passes through its space and intersects with voxels. When detecting the intersections, we can enlarge the bounding box a little bit to reduce the potentially inaccurate value. The information in each voxel is time-stamped, so that it is convenient to remove obsolete voxels if we access it in the next step. The AABB is created around the lines representing the vertex motion to facilitate detection of whether the moving vertices intersect with triangles. All the triangles in the voxels that intersect with this AABB are in a narrow phase. In the wide phase a similar method is used to detect the potential edge-to-edge collisions; the information contained in voxels is the information of edges that intersect with them during movement through the space, rather than the information of the whole triangles. In the collision detection of the narrow phase the Provot method is used. In a given time interval, we detect the four points that define a moving point and a triangle, or the endpoints of two lines, to see if they are coplanar. If the point belongs to one of the cases, we will test them to determine if the moving points are within the triangle or the intersection is within the two lines. If we detect a collision, we will adjust the direction of wheat grain and rotate the wheat grain to resolve the collision problem (Fig. 8).

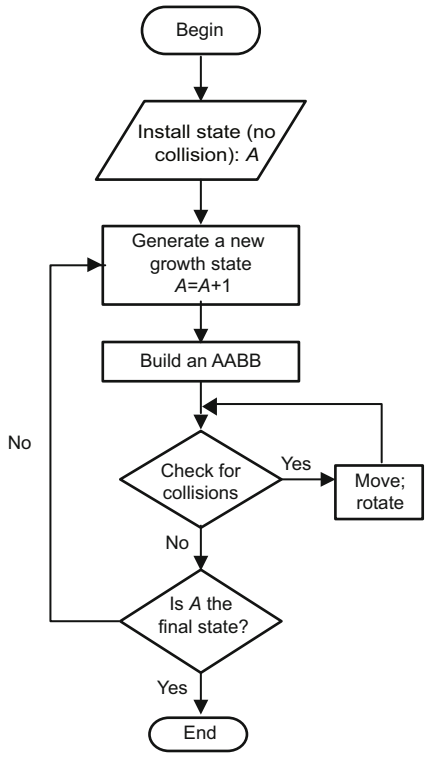

Fig. 8 Collision detection flowchart (AABB: axis-aligned bounding box)

4 Experimental results and discussion

We used Visual Studio and 3D Studio Max to evaluate our approach for wheat growth modeling.

The inputs of the algorithm included key postures for wheat kernels (outside and inside), key postures for wheat ears, wheat leaf models, and graphic

functions that define grain polygons. The model editor interactively specified the key postures and leaf models of the wheat grain. The model was finally assembled by a C++ program, which implements a model that can solve the colliding growth problem of wheat ears. We used 3D Studio Max to do the final rendering.

The experiments modeled the wheat grain (lemma and palea), and the polygons were modeled in the editing window provided by the 3D Studio Max. Based on the characteristics of wheat kernels and the nature of polygons, we used polygons to model the wheat grain. The initial model of the wheat kernels (outer and inner malleolus) was wheat kernels at the time of ripening, in which there were 17 polygonal control points of the mature wheat kernels (outer, inner, and indica), as shown in Table 1.

According to observations of the leaf blade, the blade was modeled with polygons according to the characteristics of the blade (Fig. 9).

3D Studio Max supports MAXScript. We used the MAXScript and C language to control the growth of wheat ears, and used the material editor in the system to map the wheat model.

After that, we re-polygonized the wheat grain using a graph definition function that defines the grain polygons, making it a topologically stable Delaunay triangulation. On the one hand, this kind of triangulation re-polygonization can speed up the simulation. On the other hand, it maintains the stability of the topological structure of the grain model.

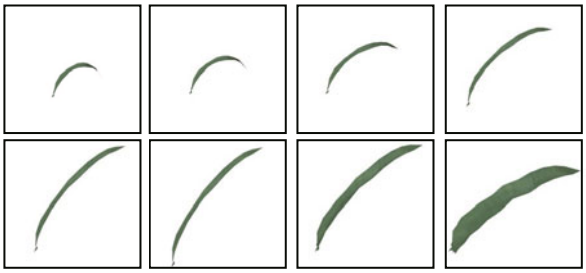

Fig. 9 Leaf growth process

After polygonization of the wheat grain, we used the model editor to interactively specify the critical poses of wheat grain, so that it could further simulate the growth wheat ears, highlighting the process of wheat grain in ripening of wheat ears. The editor used wheat growth characteristics to start internal interpolation and spherical interpolation. The internal interpolation was based on different wheat grain sizes in the early and late stages. The spherical interpolation was based on the fact that the growth of wheat grains will change the angle during the growth of wheat ears. Therefore, the wheat grain was interpolated in a sphere, which makes it realistic.

Finally, we assembled the model through a C++ program and rendered the model using 3D Studio Max. The experimental results in Figs. 9–11 show that our method is effective in modeling wheat ear growth and can reproduce the wheat ear growth process from heading to maturity.

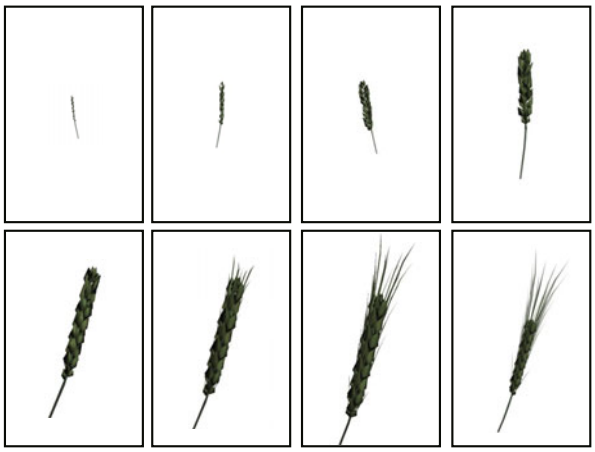

Fig. 10 Wheat ear growth process

Table 1 Polygon control points of grain (lemma and palea)

No.	Polygon control points		
	Vertex sequence 1	Vertex sequence 2	Vertex sequence 3
1	(1.90, 59.91, 1.36)	(2.31, 60.51, 1.56)	(1.85, 60.69, 1.18)
2	(1.83, 59.94, 1.30)	(1.93, 59.86, 1.31)	(2.48, 60.18, 1.20)
3	(1.86, 59.89, 1.25)	(2.03, 60.36, 0.83)	(4.41, 65.22, 0.67)
4	(4.37, 65.24, 0.63)	(2.50, 61.14, 1.04)	(2.55, 61.12, 1.08)
5	(4.38, 65.22, 0.61)	(2.51, 61.12, 1.01)	(4.42, 65.20, 0.64)
6	(2.56, 61.10, 1.05)	(5.65, 67.29, 0.14)	

Fig. 11 Wheat growth process

5 Conclusions

The morphological structure and the growth process of wheat ears are very complex, so it is difficult to construct a 3D visualization model of wheat ear growth. In this study, we have used morphological data to extract the morphologic and geometric information, and then characteristic parameters to establish an algorithm for modeling the main wheat ear structures. A polygon-based geometric model has been used to construct 3D models of wheat ears in different growth stages. The models constructed in the study were accurate and close to the measured model. Compared with traditional methods such as the 3D point-cloud method and the stereopsis method, the presented method is simple and cost-effective, and can simulate the dynamic process of wheat ear growth.

Compliance with ethics guidelines

Jun-xiao XUE, Chen-yang SUN, Jun-jin CHENG, Ming-liang XU, Ya-fei LI, and Shui YU declare that they have no conflict of interest.

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