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A novel method for PAPR reduction of the OFDM
signal using nonlinear scaling and FM[*]

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Abstract: Orthogonal frequency division multiplexing (OFDM) has been adopted as standard beginning with the 4th generation mobile communication system because of its high-bit-rate transmission capability under frequency selective fading channel conditions. However, a major disadvantage of OFDM is the non-constant envelope signal with a high peak-to-average power ratio (PAPR). The high peak signal in OFDM is distorted through a nonlinear amplifier, which causes bit error ratio (BER) reduction. Many techniques have been developed for reducing PAPR at the cost of inefficient bandwidth usage or throughput because of the additional information about PAPR reduction. We propose a novel method, in which the high peak signal above the threshold of the nonlinear amplification region is nonlinearly downscaled to lower the PAPR. The time slot location and scaling ratio for where and how the high peak baseband OFDM signal is downscaled are transmitted using frequency modulation (FM) combined with OFDM, which requires less additional bandwidth than the previously proposed methods. Simulation results show that the proposed novel method provides a lower PAPR and elicits a better BER performance compared with other conventional methods, because it reduces the PAPR by nonlinear scaling and restores the pre-distorted signal using FM.

Key words: Orthogonal frequency division multiplexing; Peak-to-average power ratio; Nonlinear scaling; Frequency modulation
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1 Introduction

Orthogonal frequency division multiplexing (OFDM) has been used in high-bit-rate wireless and mobile communication because of its robust transmission under frequency selective fading channel conditions. OFDM signals generated using inverse fast Fourier transform (IFFT) signal processing are equivalent to the summation of multiple subcarriers' signals. Therefore, the OFDM signals are non-constant envelope modulation signals that show dynamic fluctuations in the envelope, some of which are above the linear amplification region and distorted through a nonlinear amplifier.

Therefore, many techniques have been proposed for reducing the peak-to-average power ratio (PAPR) in the OFDM signal. Techniques for reducing PAPR include partial transmit sequence (PTS) and selective mapping (SM), in which one code sequence is selected to reduce the occurrence probability of a high peak signal to control the phases of each OFDM signal subcarrier (Pamungkasari and Sanada, 2015; Joo et al., 2017; Sravanti and Vasantha, 2017). This process requires the transmitter send the code sequence to the receiver, meaning that additional information must be transmitted. The transmission of additional information reduces the throughput. Single

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carrier-frequency division multiple access (SC-FDMA) system has been exploited for PAPR reduction in the 4th generation mobile communication system. Because the additional discrete Fourier transform (DFT) is processed before IFFT in SC-FDMA, a weak wideband signal might be generated in the frequency selective fading channel.

In this study, we present a scaling method, in which the high peak signal is downscaled near the threshold level between linear and nonlinear amplification regions to lower the PAPR close to that of the clipping method. However, different from the clipping method, the downscaled signal can be upscaled because the scaling ratio is shared by both transmitter and receiver, which means that there is no performance degradation through the scaling process. Normally, the PAPR reduction method is evaluated using the complementary cumulative distribution function (CCDF) of the PAPR and the bit error ratio (BER) performance. The nonlinear scaling process shows that its CCDF gets close to that of the clipping method. Because the scaling is done in some of the time slots of the baseband OFDM symbol (the time slots consist of discrete time signals fed in each of N time slots of N-point IFFT), the time slot location where the high peak signal is downscaled should be transmitted using frequency modulation (FM), given that the carrier of FM is an edge subcarrier of the baseband OFDM signal, whereby input data is not assigned.

The scaling ratio information concerning how high the peak signal is above the threshold level must be sent. It is possible to obtain the scaling ratio information, because the scaling ratio is proportional to the deviation of the FM whose frequency band is in the main lobe bandwidth, which corresponds to the inverse of one discrete time slot in the baseband OFDM symbol (Kumar Singh and Dalal, 2017). The characteristics of the CCDF scaling method with FM, which sends scaling ratio and time slot location, are close to those of conventional clipping method. However, a better BER performance can be guaranteed because the pre-distorted signal in the downscaling method can be recovered after frequency demodulation at the receiver.

Simulation results show that the proposed method has better BER performance compared with conventional clipping method and lower PAPR

compared with PTS and SM techniques. Concerning the additional bandwidth due to the insertion of additional information, a comparison table shows that the proposed method consumes less additional bandwidth than other conventional PAPR reduction methods.

2 PAPR of the OFDM signal

OFDM is inherently multi-carrier modulation. The carrier modulated signal can be expressed as

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[k]\exp\left(\frac{\mathrm{j}2\pi kn}{N}\right), \tag{1}$$

where $n=0, 1, \ldots, N-1$, and $x[k]$ is the input data symbol in a complex number, consisting of an in-phase signal and a quadrature signal. Therefore, the OFDM signal has the characteristics of fluctuations in the signal envelope, causing the high peak signal to be distorted through a nonlinear amplifier. The occurrence probability of high peak signal in the OFDM symbol after IFFT is dependent on the combination of the input data sequences, i.e., $x=\{x[0], x[1], \ldots, x[N-1]\}$, which can be analyzed with a statistical approach. Therefore, the ratio of the maximum power to the averaged power of the signal is defined as (Kumar Singh and Dalal, 2018)

$$\mathrm{PAPR}\left\{x[n]\right\} = \frac{\max\left|x[n]\right|^2}{E\left\{\left|x[n]\right|^2\right\}}. \tag{2}$$

The signal with a high PAPR should be backed off for being linearly amplified with a nonlinear amplifier, making the signal's averaged power to be low, i.e., low signal-to-noise ratio (SNR). The signal with a low PAPR can have a higher SNR than the signal with a high PAPR (Ni et al., 2016; Anoh et al., 2018), as shown in Fig. 1.

The above power characteristics can be described in terms of magnitudes by defining the crest factor (CF) as

$$\mathrm{CF} = \sqrt{\mathrm{PAPR}}. \tag{3}$$

Because both the real part and imaginary part of

an OFDM signal have Gaussian distribution, an envelope of the OFDM signal has Rayleigh distribution. $\{Z_n\}$ is defined as the magnitude of complex samples as output after IFFT in OFDM. Assuming that the average power of $x[n]$ is equal to 1, that is, $E\{|x[n]|^2\}=1$, then $\{Z_n\}$ is the set of independent and identically distributed Rayleigh random variables normalized by their own average power.
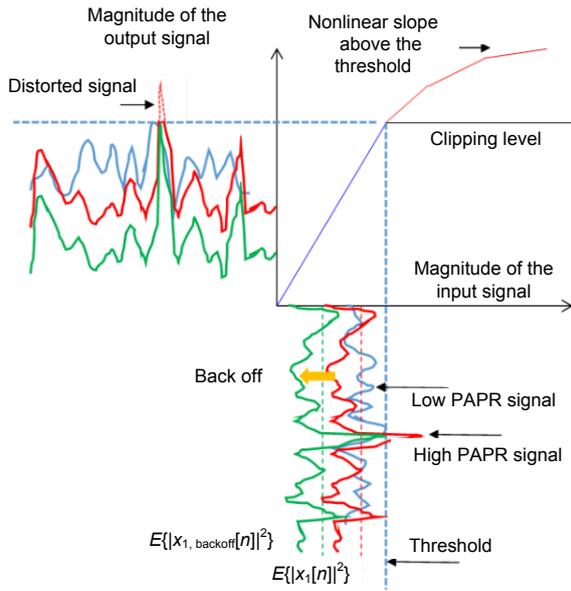

Fig. 1 Amplitude-amplitude (AM/AM) and high peak-to-average power ratio (PAPR) characteristics of the nonlinear amplifier

Because Z_{max} denotes the CF, the cumulative distribution function (CDF) of Z_{max} is given as

$$F_{Z_{max}}(z) = P(Z_{max} \leq z) = \left(1 - \exp(-z^2)\right)^N. \quad (4)$$

To find the probability that the CF exceeds z, consider the CCDF as follows:

$$\overline{F}_{Z_{max}}(z) = P(Z_{max} > z) = 1 - P(Z_{max} \leq z) = 1 - \left(1 - \exp(-z^2)\right)^N. \quad (5)$$

CCDF is an efficient analytical method to evaluate the distribution of the PAPR of a non-constant envelope modulation signal (Ochiai and Imai, 2001; Pandey and Tripathi, 2013; Lee et al., 2017).

The clipping method, in which the high peak signal above the threshold is clipped, is the simplest

PAPR reduction scheme with signal distortion. The PTS and SM methods use the signal scrambling technique. These methods manipulate and sort the input data to the IFFT to achieve a low PAPR.

3 Scaling method for PARP reduction

3.1 Scaling method

The PAPR reduction method with downscaling is a signal distortion technique and different from other methods with scrambling techniques. The range of nonlinear amplifier's operations is divided into a linear region and a nonlinear region. The nonlinear region has nonlinear transfer characteristics of amplitude-amplitude (AM/AM) and amplitude-phase (AM/PM) with distortion. A baseband OFDM signal after IFFT in the time domain is expressed as follows:

$$\begin{aligned} x_{OFDM,b}(t) &= \sum_{n=0}^{N-1} x[nT]\Pi\left(\frac{t-nT-T/2}{T}\right) \\ &= \sum_{n=0}^{N-1} \{x_{Re}[nT] + jx_{Im}[nT]\}\Pi\left(\frac{t-nT-T/2}{T}\right), \end{aligned} \quad (6)$$

where $\Pi(t/T)=1$ when $-T/2 \leq t \leq T/2$; otherwise, $\Pi(t/T)=0$. N is the number of discrete time slots and T is the time duration. The real part and imaginary part of the baseband OFDM signal are multiplied by $\cos(\omega_c t)$ and $\sin(\omega_c t)$, respectively, for passband processing. Therefore, the passband OFDM signal is expressed as follows:

$$\begin{aligned} & x_{OFDM,p}(t) \\ &= \sum_{n=0}^{N-1} x_{Re}[nT]\Pi\left(\frac{t-nT-T/2}{T}\right)\cos(\omega_c t) \\ &\quad - \sum_{n=0}^{N-1} x_{Im}[nT]\Pi\left(\frac{t-nT-T/2}{T}\right)\sin(\omega_c t) \\ &= \sum_{n=0}^{N-1} \sqrt{x_{Re}^2[nT] + x_{Im}^2[nT]}\,\Pi\left(\frac{t-nT-T/2}{T}\right) \\ &\quad \cdot \cos(\omega_c t + \theta_n), \end{aligned} \quad (7)$$

where $\theta_n = \arctan\{x_{Im}[nT]/x_{re}[nT]\}$ is the phase of the passband OFDM signal, and $\sqrt{x_{Re}^2[nT] + x_{Im}^2[nT]}$ presents the envelope of the baseband OFDM signal in each discrete time slot. Pilot signals for channel estimation and zero paddings for guard band are not

considered in focusing on PAPR reduction for the time being. V_{th} is defined as the threshold envelope, which corresponds to the input power near the boundary between the linear and nonlinear regions of the nonlinear amplifier.

If the mth envelope, $|x[mT]| = \sqrt{x_{Re}^2[mT] + x_{Im}^2[mT]}$, is larger than V_{th}, both $x_{Re}[mT]$ and $x_{Im}[mT]$ will be downscaled or clipped.

For reduction of a high peak envelope above V_{th}, both $x_{Re}[mT]$ and $x_{Im}[mT]$ are downscaled with the constant scaling factor α. However, if the highest peak envelope above V_{th} is downscaled near V_{th}, the high peak envelope slightly above V_{th} becomes lower far from V_{th}. Therefore, the CCDF of the linear downscaling method is inferior to that of the clipping method. Now, instead of a constant scaling factor, a nonlinear scaling factor is used to make the peak envelope signal slightly above V_{th} less downscaled than the highest peak envelope signal above V_{th}, and then those signals with a high peak envelope above V_{th} are distributed slightly below V_{th} (Fig. 2). The CCDF performance of the nonlinear scaling method is better than that of the linear scaling method, but still inferior to that of the clipping method.

3.2 FM for sending time slot location information in scaling method

Now, we need to devise a method to advise the receiver of discrete time slot location, where the high peak envelope is downscaled, which should upscale the pre-distorted signal. The edge OFDM subcarrier is responsible for sending time slot location information to the receiver, where downscaling is processed in the baseband OFDM signal $x_{OFDM,b}(t)$. Because the passband OFDM signal $x_{OFDM,p}(t)$ is equivalent to the summation of N subcarriers' signals, which is multiplied by the input data symbol sequence, expressed as

$$x_{OFDM,p}(t) = \sum_{k=0}^{N-1} x[k] \cos\left[2\pi\left(f_c + \frac{N/2-k}{T}\right)t\right]. \quad (8)$$

Considering the main lobe bandwidth of $x[k]$, which is equal to $2/T$ and modulated with each subcarrier, the bandwidth of the passband OFDM signal with subcarriers is expressed as

$$\frac{N+1}{T} = \left[\left(f_c + \frac{N/2}{T}\right) - \left(f_c + \frac{N/2+1}{T}\right)\right]. \quad (9)$$

Considering the specification of the normal OFDM standard, there are some edge subcarriers to which null data is assigned to escape from the adjacent channel interference. The dotted line spectra are vacant spectra corresponding to null data in Fig. 3.

Because the baseband M-ary OFDM signal consists of a real signal and an imaginary signal, these two edge subcarriers can be assigned to send the downscaling location information for diversity. The additional spectrum can be located in both edges, and these edge subcarriers can be assigned to send sides of the spectrum range. The one for the real baseband OFDM signal is between $f_c+(N/2-2)/T$ and $f_c+(N/2)/T$ around subcarrier $f_c+(N/2-1)/T$ and the other one for imaginary baseband OFDM signal is between $f_c-(N/2)/T$ and $f_c-(N/2-2)/T$ around subcarrier $f_c-(N/2-1)/T$ (the bold line spectrum in Fig. 3).

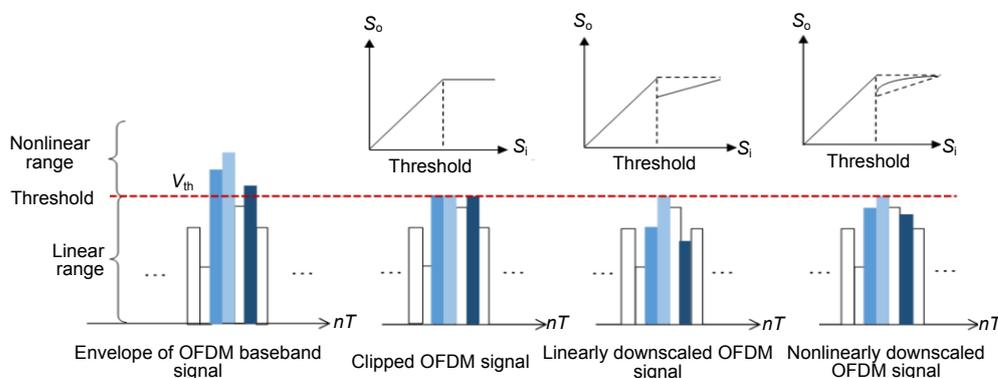The passband OFDM signal with the downscaled signal is expressed as

Fig. 2 Envelope distribution after applying signal pre-distortion in the cases of clipping, linear scaling, and nonlinear scaling

$$x_{\mathrm{OFDM,p,s}}(t) = \sum_{n,m=0}^{N-1} \left(\sqrt{x_{\mathrm{Re}}^2[nT] + x_{\mathrm{Im}}^2[nT]} \right.$$

$$\cdot \Pi\left(\frac{t - nT - T/2}{T}\right) \cos(\omega_{\mathrm{c}}t + \theta_n)$$

$$+ \alpha x_{\mathrm{Re}}[mT]\Pi\left(\frac{t - mT - T/2}{T}\right)\cos(\omega_{\mathrm{c,FM,Re}}t)$$

$$\left. + \alpha x_{\mathrm{Im}}[mT]\Pi\left(\frac{t - mT - T/2}{T}\right)\sin(\omega_{\mathrm{c,FM,Im}}t)\right),$$

$$(10)$$

where α is the scaling factor in Fig. 4, and $\omega_{\mathrm{c,FM,Re}}$ and $\omega_{\mathrm{c,FM,Im}}$ are angular frequencies of the edge subcarrier

for the real signal in the in-phase branch and the imaginary signal in the quadrature branch, respectively, indicating the time slot scaling location.

The downscaled OFDM signal can be upscaled at the receiver at the instant when the edge subcarrier is detected through band pass filter (BPF) (Fig. 5).

To send the scaling ratio in the scaling method for PAPR reduction, the frequency deviation from the edge subcarrier is proportional to the downscaled magnitude of the high peak signal above V_{th} in FM at the in-phase branch and quadrature phase branch (Fig. 4). In Eq. (11), the frequency deviation of the voltage controlled oscillator (VCO) in FM
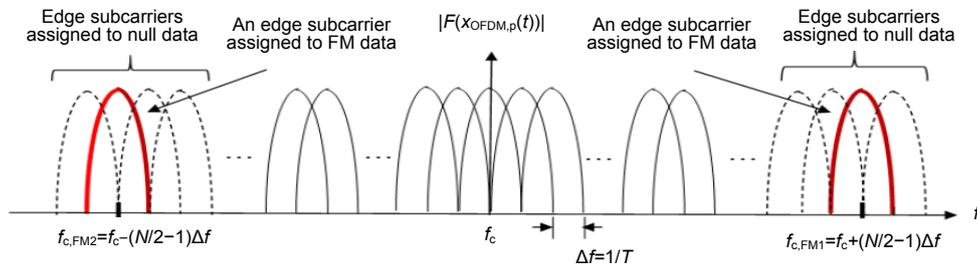

Fig. 3 Spectrum of the OFDM and additionally assigned spectrum of the FM for sending the scaling location information
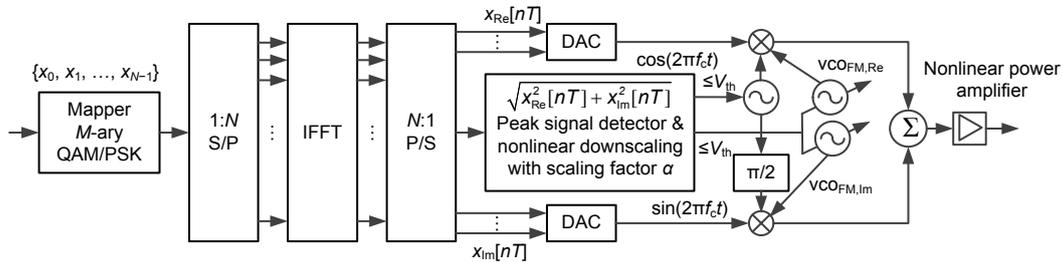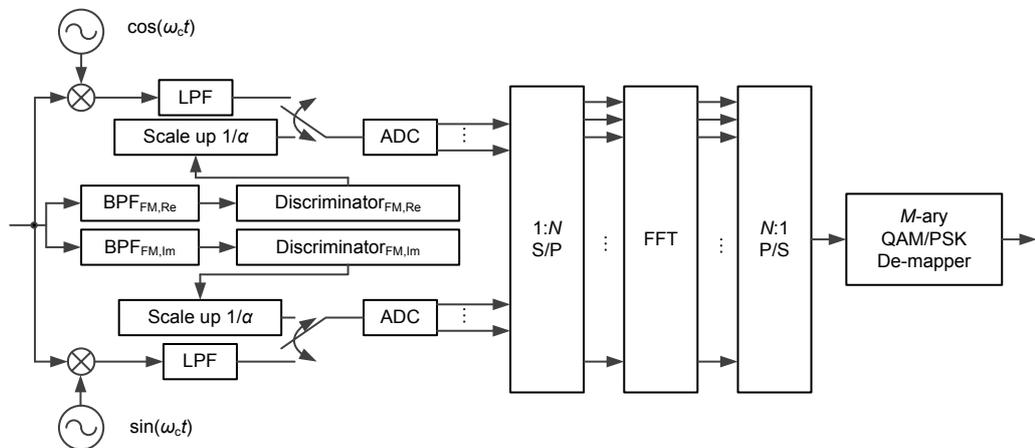

Fig. 4 OFDM transmitter with FM for scaling down

Fig. 5 OFDM receiver with an FM discriminator for scaling up

corresponding to the downscaled magnitude of the high peak signal above V_{th} is in the range of the main lobe bandwidth around the edge subcarrier.

$$
\begin{cases}
s_{FM,Re,VCO}(t) = \cos\Big\{(\omega_c + \omega_{c,FM,Re})t + \\
\qquad k_{FM}\int_{mT}^{(m+1)T}(V_{th} - x_{Re}[\tau])\mathrm{d}\tau\Big\}, \\
s_{FM,Im,VCO}(t) = \sin\Big\{(\omega_c + \omega_{c,FM,Im})t + \\
\qquad k_{FM}\int_{mT}^{(m+1)T}(V_{th} - x_{Im}[\tau])\mathrm{d}\tau\Big\},
\end{cases} \tag{11}
$$

where k_{FM} is the frequency modulation index. Assuming that the magnitude of the downscaled signal above V_{th} in some discrete time slots is constant, the FM signal has a single frequency proportional to the constant magnitude of the downscaled signal, expressed as

$$
\begin{cases}
f_{c,FM,Re} = f_c + \dfrac{N+1}{2T} + k_{FM}\dfrac{V_{th} - x_{Re}[mT]}{2\pi}, \\
f_{c,FM,Im} = f_c - \dfrac{N+1}{2T} + k_{FM}\dfrac{V_{th} - x_{Im}[mT]}{2\pi}.
\end{cases} \tag{12}
$$

At the receiver, as shown in Fig. 5, the downscaled signal can be restored using the frequency discriminator at the instant when BPF detects the signal in the ranges of $f_c+(N/2-2)/T$ to $f_c+(N/2)/T$ and $f_c-(N/2)/T$ to $f_c-(N/2-2)/T$.

3.3 Comparison of additional bandwidths for sending side information in each PAPR reduction method

In the clipping method, because the distorted signal with the clipped portion of the high peak signal is sent and not restored at the receiver, the distorted signal causes errors to be generated. Those errors would be corrected at the receiver with an error correction block. Therefore, the BER in the clipping method might be dependent on the capability of an error correction block embedded in the receiver, which requires a complex integrated circuit block with power consumption.

In PTS and SM methods, the number of additional required subcarriers is proportional to the number of bits that can identify the possible combination of random sequences.

However, in our novel method, only two edge subcarriers are assigned to send time slot location information, where the scaling happens in discrete time slots of the baseband OFDM symbol. The frequency deviation from the edge subcarrier in FM, which is proportional to scaling ratio, is in the range of the main lobe bandwidth, corresponding to the inverse of time duration T of one discrete time slot in the baseband OFDM signal.

As shown in Table 1, the proposed method consumes less additional bandwidth than PTS and SM methods.

Table 1 Comparison of additionally required bandwidth in each PAPR reduction method

Method (QPSK)	Number of bits (side information)	Number of subcarriers	Bandwidth
SM	9 ($\log_2 U$, U=512)	5	$10\Delta f$
PTS	63 ($\log_2 H^{V-1}$, V=64 and H=2)	32	$64\Delta f$
NLSD+FM		2	$4\Delta f$

QPSK: quadrature phase shift keying; SM: selective mapping; PTS: partial transmit sequence; NLSD: nonlinear scaled-down; FM: frequency modulation

4 Simulation results

In this section, we compare the performances of the CCDF and BER for the clipping method, linear scaled-down (LSD) method, nonlinear scaled-down (NLSD) method, PTS method, and SM method. The threshold level for downscaling high peak signal is defined by the clipping ratio (CR), expressed as

$$
CR = \frac{A}{\sigma}, \tag{13}
$$

where A is the threshold level (clipping level) and σ is the standard deviation of signal $x[n]$. In the simulation, we considered a 64-subcarrier system with quadrature phase shift keying (QPSK) modulation.

Fig. 6 shows the comparison of the CCDF performances of several PAPR reduction methods, namely, clipping method, LSD method, NLSD method, PTS method, and SM method. It is clear in Fig. 6 that the clipping method has the narrowest distribution of PAPR considering the OFDM signal's envelope fluctuations according to all the random input data sequences.

Fig. 7 shows the comparison of BER performances of several PAPR reduction methods, namely, clipping method, LSD method, NLSD method, PTS method, and SM method. Additive white Gaussian noise (AWGN) is added to the OFDM signal with pre-distortion of the signal for PAPR reduction. As can be expected, the BER performance of the original clipping method is worse than other methods because of severe signal distortion caused by the simple clipping effect, even though it has the lowest PAPR among the abovementioned methods. However, the NLSD method shows better CCDF and BER performances than other methods.
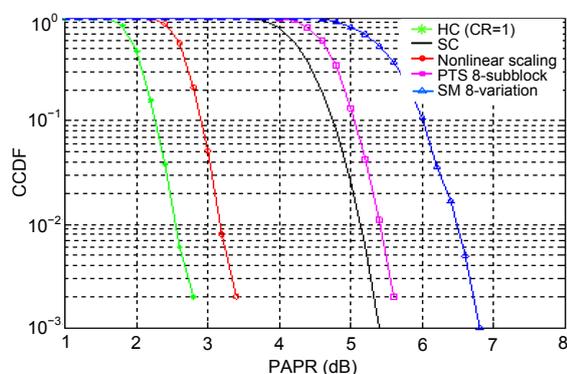

Fig. 6 Comparison of the CCDF performances of several PAPR reduction methods in a 64-subcarrier system
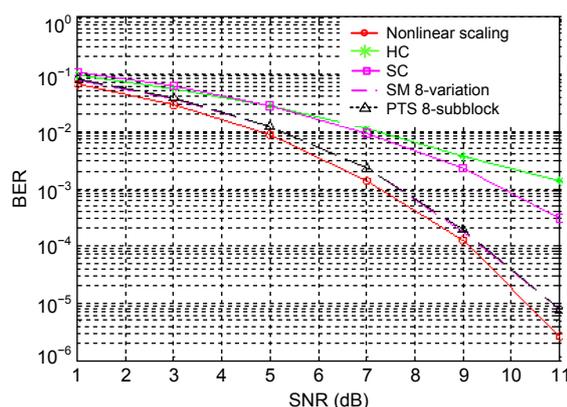

Fig. 7 Comparison of the BER performances of several PAPR reduction methods in a 64-subcarrier system

5 Conclusions

In this study, we have proposed a nonlinear scaled-down method with FM to reduce the PAPR in an OFDM system, and its CCDF and BER performances have been compared with those of other conventional methods. In this nonlinear scaled-down method, the high peak signal was nonlinearly downscaled, enabling the CCDF of the OFDM signal with the downscaled signals to be close to that of the clipping method. Different from the clipping method, in which the clipped signals cannot be restored at the receiver, the pre-distorted signal for PAPR reduction can be restored because scaling ratio and time slot scaling location in the baseband OFDM can be sent to the receiver using FM. Therefore, the proposed method had better CCDF and BER performances than conventional methods. With respect to the spectrum inefficiency due to insertion of additional information related to the scaling location and scaling ratio in the PAPR reduction process, the proposed method showed that less additional bandwidth was assigned than in other conventional methods.

Compliance with ethics guidelines

Yi-hu XU, Jung-Yeol OH, Zhen-hao SUN, and Myoung-Seob LIM declare that they have no conflict of interest.

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