



Multi-user rate and power analysis in a cognitive radio network with massive multi-input multi-output*

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Abstract: This paper discusses transmission performance and power allocation strategies in an underlay cognitive radio (CR) network that contains relay and massive multi-input multi-output (MIMO). The downlink transmission performance of a relay-aided massive MIMO network without CR is derived. By using the power distribution criteria, the k^{th} user's asymptotic signal to interference and noise ratio (SINR) is independent of fast fading. When the ratio between the base station (BS) antennas and the relay antennas becomes large enough, the transmission performance of the whole system is independent of BS-to-relay channel parameters and relates only to the relay-to-users stage. Then cognitive transmission performances of primary users (PUs) and secondary users (SUs) in an underlay CR network with massive MIMO are derived under perfect and imperfect channel state information (CSI), including the end-to-end SINR and achievable sum rate. When the numbers of primary base station (PBS) antennas, secondary base station (SBS) antennas, and relay antennas become infinite, the asymptotic SINR of the k^{th} PU and SU is independent of fast fading. The interference between the primary network and secondary network can be canceled asymptotically. Transmission performance does not include the interference temperature. The secondary network can use its peak power to transmit signals without causing any interference to the primary network. Interestingly, when the antenna ratio becomes large enough, the asymptotic sum rate equals half of the rate of a single-hop single-antenna K -user system without fast fading. Next, the PUs' utility function is defined. The optimal relay power is derived to maximize the utility function. The numerical results verify our analysis. The relationships between the transmission rate and the antenna number, relay power, and antenna ratio are simulated. We show that the massive MIMO with linear pre-coding can mitigate asymptotically the interference in a multi-user underlay CR network. The primary and secondary networks can operate independently.

Key words: Massive multi-input multi-output; Cognitive radio; Relay network; Transmission rate; Power analysis
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1 Introduction

In recent years, the spectrum scarcity generated by the evolution of wireless technologies has led us to

rethink the traditional transmission strategies. Cognitive radio (CR) proposed by Mitola and Maguire (1999) is considered as a promising solution to this dilemma. Secondary user (SU) networks can coexist with primary user (PU) networks through opportunistic spectrum access, illustrating that the SU does not affect adversely the PUs' performance. To allow for this spectrum sharing, three models have been considered in the literature: interweave, underlay, and overlay models. In the interweave model,

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SUs sense the availability of spectrum bands that are not occupied by PUs. This model is highly sensitive to sensing errors and PU traffic patterns. In the underlay model, SUs transmit signals simultaneously with the PUs over the same spectrum, illustrating that the SU received signal power levels at all PU receivers remain below a predefined threshold (Haykin, 2005; Goldsmith et al., 2009). In the overlay model, SUs transmit signals simultaneously with the PUs over the same spectrum, illustrating that the SUs aid the PUs transmission by cooperative communication techniques, such as advanced coding or cooperative relaying techniques (Goldsmith et al., 2009).

Today, massive multi-input multi-output (MIMO) (Marzetta, 2010) is regarded as the promising technology for meeting the huge capacity needed for 5th generation (5G) cellular networks (Boccardi et al., 2014) due to various advantages over single-antenna systems, including transmit diversity, higher data rates, and reliability. The massive MIMO system contains a large antenna array at the macrocell base station and can provide preeminent precoding/beamforming capabilities (Hosseini et al., 2013; Rusek et al., 2013). Linear receivers and precoders, such as those based on the zero forcing (ZF) criteria, have often been considered, because they offer significantly lower complexity with tolerable performance (Chen and Wang, 2007).

The combination of these new technologies has attracted a lot of researchers' interest (Manna et al., 2011; Li et al., 2014; Zhang et al., 2015; Wang et al., 2017). Wang et al. (2017) explored the potential benefits of massive MIMO in a spectrum-sharing network. Zhang et al. (2015) investigated the sum rate gains offered by power allocation strategies in multicell massive MIMO systems, assuming time-division duplex transmission. Li et al. (2014) proposed a novel scheme for user scheduling in two-tier networks with massive MIMO and cognitive femtocell technology. Manna et al. (2011) proposed and investigated a solution for spectrum sharing based on the idea that SUs could earn spectrum access in exchange for cooperation with the PUs. Considering that there was no direct path to the user in the cell edge, the relays were used to connect remote users. There has been much related research on dual-hop massive MIMO systems. The performance of multi-pair one-way relay networks with very large relay antenna arrays was

investigated by deriving the asymptotic signal to interference and noise ratio (SINR) and the spectral efficiency in Suraweera et al. (2013). Amarasuriya et al. (2015) studied the asymptotic performance of multi-pair two-way relay networks (TWRNs) with massive MIMO-enabled relays by employing linear precoders/detectors.

The massive MIMO technology will be used widely in 5G. The frequency may be 3.5 GHz, 5 GHz, or some higher frequency band, but the spectrum may not be used fully. The frequency can be reused by other communication systems, such as wireless fidelity (Wi-Fi) or industrial Internet. Therefore, it is essential to discuss the CR network with massive MIMO. Amarasuriya et al. (2015) studied the multi-user relay networks with massive MIMO, but they did not consider the transmission performance in CR networks. In this study, the downlink transmission of the underlay CR network with massive MIMO operating at 3.5 GHz is analyzed.

In this study, two different massive MIMO networks are analyzed: a relay-aided massive MIMO network and an underlay CR network with massive MIMO. The downlink transmission performance of a relay-aided massive MIMO network is discussed first. Then the downlink transmission performance and power allocation strategy in an underlay CR network with massive MIMO are analyzed. In the underlay CR network with massive MIMO, the primary network is a relay-aided massive MIMO network and the secondary network reuses opportunistically the frequency. The asymptotic transmission sum rates of the PUs and SUs are derived. Then the PUs' utility function that relates to relay power and users' SINR is maximized by optimizing the relay power.

The notations used in this paper are as follows: \mathbf{A}^T , \mathbf{A}^H , and $[\mathbf{A}]_{k,l}$ denote the transpose, Hermitian transpose, and (k, l) th element of matrix \mathbf{A} , respectively. $\mathcal{CN}(\mathbf{0}, \mathbf{Y})$ denotes the complex Gaussian distribution with zero mean and covariance matrix \mathbf{Y} . $\|\cdot\|$ denotes the Euclidean norm, and $\text{Tr}(\cdot)$ denotes the trace.

2 System model

In this study, two different relay-aided massive MIMO networks are analyzed. Fig. 1 shows a brief description of the discussed relay-aided massive MIMO network and underlay CR network with

massive MIMO. In the CR network, the macrocell is the primary network and contains the relays that transform continuously the information. The secondary network reuses the frequency. Car networking can be used as a practical example.

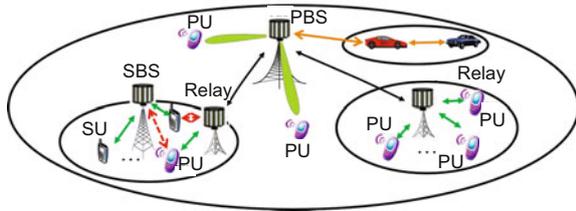


Fig. 1 Simplified diagram of the relay-aided massive multi-input multi-output cognitive radio networks

The relay-aided massive MIMO network in Fig. 2 is discussed. The relay-aided massive MIMO network consists of an N_B -antenna BS, an N_R -antenna relay station, and K single-antenna users. The BS performs the ZF method to precode the signal s . The relay is an amplify-and-forward half duplex relay. N_B and N_R are assumed to be significantly larger than K . The numbers of BS antennas and relay antennas become infinite while keeping a fixed ratio. The direct path from the BS to the K users is not available because of the heavy path loss and shadowing. The additive noise at all the receivers is modeled as complex zero mean Gaussian noise.

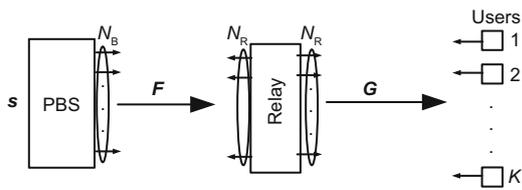


Fig. 2 Downlink transmission in the relay-aided massive multi-input multi-output network

Downlink transmission in the underlay CR network with massive MIMO is analyzed in Section 4. The primary network is the same as that in the relay-aided massive MIMO network. As shown in Fig. 3, the multi-user MIMO primary network consists of a PBS equipped with N_p antennas, a relay equipped with N_r antennas, and K PUs. The secondary network has a secondary base station (SBS) equipped with N_s antennas and an SU. All users are distributed spatially and have only one antenna. N_p and N_s are assumed to be significantly larger than

K . The direct channel between the PBS and PU is negligible due to the severe transmission impairments, such as heavy path loss and shadowing, but the channel between the SBS and PU cannot be ignored because the distance is much closer. The relay just amplifies and transfers the PBS signal. The network is proposed to have two time slots. In the first time slot, the PBS transmits the signal s to the half-duplex relay. In the second time slot, the relay transmits the signal to the PUs and the SBS transmits the signal to the SU. This is because the secondary network is deployed in the edge of the cell and the PBS cannot reach the SBS in the first time slot. The transmission scenario is shown in Fig. 3.

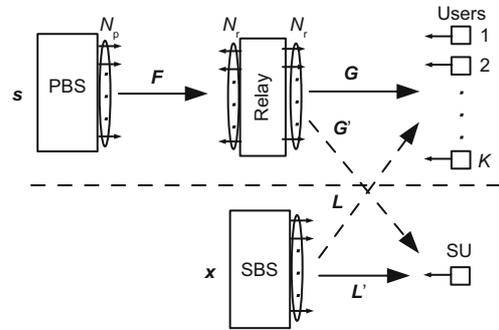


Fig. 3 Downlink transmission in the underlay massive multi-input multi-output cognitive radio network

3 Downlink transmission in the relay-aided massive multi-input multi-output network

In this section, the downlink transmission in the relay-aided massive MIMO network is discussed. The main contribution is to derive the downlink asymptotic SINR and sum rate when the BS and relay antenna numbers grow without bound while keeping a fixed ratio. The relay power assigned to the k^{th} user can be scaled inversely down proportional to N_R . By using power distribution criteria, asymptotic SINR and sum rate expressions are derived. These results are independent of fast fading, and hence the latency in the air interface can be reduced significantly. When the ratio between the BS antenna number and relay antenna number becomes large enough, the asymptotic sum rate equals half of the rate of a single-hop, single-antenna, and K -users system without fast fading. The transmission performance of the relay-aided massive MIMO network is

independent of the BS-to-relay channel parameters and is related only to the relay-to-users stage.

3.1 System, channel, and signal model

The channel matrix from the BS to the relay can be denoted by \mathbf{F} and be derived as

$$\mathbf{F} = \tilde{\mathbf{F}}\mathbf{D}_{\mathbf{F}}^{\frac{1}{2}}, \tag{1}$$

where $\tilde{\mathbf{F}} \sim \mathcal{CN}_{N_{\mathbf{R}} \times N_{\mathbf{B}}}(\mathbf{0}_{N_{\mathbf{R}} \times N_{\mathbf{B}}}, \mathbf{I}_{N_{\mathbf{R}}} \otimes \mathbf{I}_{N_{\mathbf{B}}})$ accounts for the independent and fast Rayleigh fading, and $\mathbf{D}_{\mathbf{F}}$ captures the pathloss and can be defined as $\mathbf{D}_{\mathbf{F}} = \alpha \mathbf{I}_{N_{\mathbf{B}}}$ because antenna arrays at the BS and relay are collocated. Then the channel coefficient between the n^{th} relay antenna and m^{th} BS antenna can be expressed as

$$[\mathbf{F}]_{n,m} = \sqrt{\alpha} [\tilde{\mathbf{F}}]_{n,m}, \tag{2}$$

where $n \in \{1, 2, \dots, N_{\mathbf{R}}\}$ and $m \in \{1, 2, \dots, N_{\mathbf{B}}\}$.

The channel matrix from the relay to the users is denoted by \mathbf{G} and can be derived as

$$\mathbf{G} = \mathbf{D}_{\mathbf{G}}^{\frac{1}{2}} \tilde{\mathbf{G}}, \tag{3}$$

where $\tilde{\mathbf{G}} \sim \mathcal{CN}_{K \times N_{\mathbf{R}}}(\mathbf{0}_{K \times N_{\mathbf{R}}}, \mathbf{I}_K \otimes \mathbf{I}_{N_{\mathbf{R}}})$ and $\mathbf{D}_{\mathbf{G}}$ is a $K \times K$ diagonal matrix that captures the pathloss. Then the channel coefficient between the k^{th} user and n^{th} relay antenna is expressed as

$$[\mathbf{G}]_{k,n} = \sqrt{\beta_k} [\tilde{\mathbf{G}}]_{k,n}, \tag{4}$$

where $n \in \{1, 2, \dots, N_{\mathbf{R}}\}$. Hence, during the first time slot, the BS transmits the signal $\mathbf{s} = [s_1, s_2, \dots, s_k, \dots, s_K]^T$ to the relay. s_k represents the signal to the k^{th} PU which assumes $\varepsilon(|s_k|^2) = 1$, with s_i and s_j ($i \neq j$) being independent of each other. The received signal vector at the relay can be written as

$$\mathbf{y}_{\mathbf{r}} = \sqrt{P_{\mathbf{b}}}\mathbf{F} \frac{\mathbf{W}}{\|\mathbf{W}\|_{\mathbf{F}}} \mathbf{s} + \mathbf{n}_{\mathbf{r}} = \sqrt{P_{\mathbf{b}}}\mathbf{F} \frac{\sum_{k=1}^K \mathbf{w}_k s_k}{\|\mathbf{W}\|_{\mathbf{F}}} + \mathbf{n}_{\mathbf{r}}, \tag{5}$$

where $P_{\mathbf{b}}$ is the BS transmit power. $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_K]$ is the pre-coding matrix which will be given later. $\mathbf{n}_{\mathbf{r}}$ is the Gaussian noise vector at the relay that satisfies $\varepsilon[\mathbf{n}_{\mathbf{r}}\mathbf{n}_{\mathbf{r}}^H] = \sigma_{\mathbf{r}}^2 \mathbf{I}_{N_{\mathbf{R}}}$. Next, the relay employs the amplification factor which is designed to constrain the instantaneous transmit power

on its received signal vector. Then the amplification factor is derived as

$$G_{\mathbf{r}} = \sqrt{\frac{P_{\mathbf{R}}}{P_{\mathbf{b}} \frac{\|\mathbf{F}\mathbf{W}\|_{\mathbf{F}}^2}{\|\mathbf{W}\|_{\mathbf{F}}^2} + N_{\mathbf{R}}\sigma_{\mathbf{r}}^2}}, \tag{6}$$

where $P_{\mathbf{R}}$ is the k^{th} user's transmit power at the relay, which is equal among the users. $G_{\mathbf{r}}$ must have a low bound $G_{\mathbf{r},\text{min}}$. $A = \frac{\|\mathbf{F}\mathbf{W}\|_{\mathbf{F}}^2}{\|\mathbf{W}\|_{\mathbf{F}}^2} = \frac{\text{Tr}(\mathbf{F}\mathbf{W}\mathbf{W}^H\mathbf{F}^H)}{\text{Tr}(\mathbf{W}\mathbf{W}^H)}$ and Eq. (6) can be rewritten as $G_{\mathbf{r}} = \sqrt{\frac{P_{\mathbf{R}}}{P_{\mathbf{b}}A + N_{\mathbf{R}}\sigma_{\mathbf{r}}^2}}$. The K users received signals can form a vector. Therefore, the received users' vector can be written as

$$\mathbf{y} = \mathbf{G}(G_{\mathbf{r}}\mathbf{y}_{\mathbf{r}}) + \mathbf{n}_{\mathbf{u}}, \tag{7}$$

where $\mathbf{n}_{\mathbf{u}} = [n_1, n_2, \dots, n_K]^T$ is the noise matrix at different users' destination and $\varepsilon[\mathbf{n}_k\mathbf{n}_k^H] = \sigma_k^2$ ($k=1, 2, \dots, K$). In the system, the ZF pre-coding method is used for the downlink transmission. Therefore, the pre-coding matrix is given as

$$\mathbf{W} = (\mathbf{G}\mathbf{F})^H [\mathbf{G}\mathbf{F}(\mathbf{G}\mathbf{F})^H]^{-1}. \tag{8}$$

By using Eq. (8), the users' received signal from the BS can be written in an alternative form as

$$\mathbf{y} = \frac{\sqrt{P_{\mathbf{b}}}G_{\mathbf{r}}}{\|\mathbf{W}\|_{\mathbf{F}}} \mathbf{s} + G_{\mathbf{r}}\mathbf{G}\mathbf{n}_{\mathbf{r}} + \mathbf{n}_{\mathbf{u}}. \tag{9}$$

The k^{th} user's received signal can be written as

$$\begin{aligned} \mathbf{y}_k &= G_{\mathbf{r}}\sqrt{P_{\mathbf{b}}}\mathbf{G}_k\mathbf{F} \frac{1}{\|\mathbf{W}\|_{\mathbf{F}}} \mathbf{W}_k s_k \\ &+ G_{\mathbf{r}}\sqrt{P_{\mathbf{b}}}\mathbf{G}_k\mathbf{F} \frac{1}{\|\mathbf{W}\|_{\mathbf{F}}} \sum_{l=1, l \neq k}^K \mathbf{W}_l s_l \\ &+ G_{\mathbf{r}}\mathbf{G}_k\mathbf{n}_{\mathbf{r}} + n_k. \end{aligned} \tag{10}$$

The simplified formula is

$$\mathbf{y}_k = \frac{\sqrt{P_{\mathbf{b}}}G_{\mathbf{r}}}{\|\mathbf{W}\|_{\mathbf{F}}} s_k + G_{\mathbf{r}}\mathbf{G}_k\mathbf{n}_{\mathbf{r}} + n_k. \tag{11}$$

By using Eq. (11), the k^{th} user's SINR of the received signal can be derived as (Amarasuriya et al., 2015)

$$\gamma_k = \frac{G_{\mathbf{r}}^2 P_{\mathbf{b}}}{\|\mathbf{W}\|_{\mathbf{F}}^2 (G_{\mathbf{r}}^2 \|\mathbf{G}_k\|^2 \sigma_{\mathbf{r}}^2 + \sigma_k^2)}. \tag{12}$$

By substituting Eqs. (6) and (8) into Eq. (12) and performing several mathematical manipulations,

the SINR of the k^{th} user can be further expanded as

$$\begin{aligned} \gamma_k &= P_B P_R \cdot \{ P_R \sigma_r^2 (\mathbf{G}\mathbf{G}^H)_{k,k} \text{Tr} [(\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H)^{-1}] \\ &+ P_B \sigma_k^2 [\mathbf{F} (\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H)^{-1} \mathbf{F}^H]_{k,k} \\ &+ N_R \sigma_k^2 \sigma_r^2 [(\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H)^{-1}]_{k,k} \}^{-1}. \end{aligned} \quad (13)$$

3.2 Asymptotic sum rate analysis

In this section, the sum rate of the relay-aided massive MIMO network is defined. Then, using the power distribution criteria at the relay station, the asymptotic sum rate is derived.

3.2.1 Sum rate definitions

The numbers of BS and relay antennas are assumed to become infinite while keeping a fixed ratio. The instantaneous and average achievable sum rate of the relay-aided massive MIMO network can be defined as

$$R = \frac{1}{2} \sum_{k=1}^K \log(1 + \gamma_k) \text{ and } \bar{R} = \frac{1}{2} \sum_{k=1}^K \varepsilon \{ \log(1 + \gamma_k) \}, \quad (14)$$

where γ_k is the k^{th} user's received SINR.

3.2.2 Asymptotic SINR analysis

Asymptotic SINR expression is derived in closed-form in this section. The relay power assigned to the k^{th} user can be scaled inversely down proportional to N_R as $P_R = E_R/N_R$, where E_R is fixed. When N_R and N_B grow without boundary, a fixed ratio $\eta = N_B/N_R$ is maintained. Through some mathematical calculations (including the law of large numbers and statistical analysis), the asymptotic SINR can be derived as

$$\gamma_k^\infty = \frac{E_R \beta_k}{\sigma_k^2 (1 + \sigma_r^2 / (\alpha \eta P_b))}. \quad (15)$$

Through the above mathematical analysis, the relay power assigned to the k^{th} user can be scaled inversely down proportional to N_R . The k^{th} user's asymptotic SINR is independent of fast fading.

3.2.3 Asymptotic sum rate analysis

When the numbers of BSs and relay antennas grow without boundary, the asymptotic sum rate

expressions can be derived by substituting Eq. (15) into Eq. (14), and we can obtain

$$R^\infty = \frac{1}{2} \sum_{k=1}^K \log \left(1 + \frac{E_R \beta_k}{\sigma_k^2 (1 + \sigma_r^2 / (\alpha \eta P_b))} \right). \quad (16)$$

When the ratio between the numbers of BS antennas and relay antennas η becomes large enough, the asymptotic sum rate reduces to $R^\infty = \frac{1}{2} \sum_{k=1}^K \log \left(1 + E_R \beta_k / \sigma_k^2 \right)$, which equals half of the rate of a single-hop, single-antenna, and K -user system without fast fading. The transmission performance of the whole system is independent of the BS-to-relay channel parameters and relates only to the relay-to-users process.

4 Downlink transmission in the underlay cognitive radio network with massive multi-input multi-output

Because time-division duplex is employed, the CSI can be reused in the uplink and downlink transmission as channel reciprocity. The perfect and imperfect CSIs are investigated separately. In the underlay mode, the interference from the secondary network to the PUs cannot exceed a threshold. The main contribution in this section is to investigate the asymptotic user's SINR and sum rate when the numbers of PBSs, SBSs, and relay antennas grow without boundary while keeping a fixed ratio. The results are independent of fast fading, and hence the latency in the air interface can be reduced significantly. The interference between the primary network and secondary network is mitigated asymptotically. The transmission performances of the primary and secondary networks do not include interference temperature. The secondary network can use its peak power to transmit signals.

4.1 Cognitive transmission with perfect channel state information

The PUs' received SINR with perfect CSI is explored in this section. The transmission process in the primary network is the same as that in the previous section. A relay station is added in the primary network to assist the PUs' transmission. The channel matrix from the PBS to the relay is denoted by $\mathbf{F} = \tilde{\mathbf{F}} \mathbf{D}_F^{1/2}$. $\tilde{\mathbf{F}} \sim \mathcal{CN}_{N_r \times N_p}(\mathbf{0}_{N_r \times N_p}, \mathbf{I}_{N_r} \otimes \mathbf{I}_{N_p})$

accounts for the independent and fast Rayleigh fading. \mathbf{D}_F captures the path loss and can be defined as $\mathbf{D}_F = \alpha \mathbf{I}_{N_p}$ because antenna arrays at the PBS and relay are collocated.

The channel matrix from the relay to the PUs is given by $\mathbf{G} = \mathbf{D}_G^{1/2} \tilde{\mathbf{G}}$. $\tilde{\mathbf{G}} \sim \mathcal{CN}_{K \times N_r}(\mathbf{0}_{K \times N_r}, \mathbf{I}_K \otimes \mathbf{I}_{N_r})$ and \mathbf{D}_G is a $K \times K$ diagonal matrix that captures the path loss. The k^{th} element of the diagonal is β_k . Hence, during the first time slot, the PBS transmits the signal $\mathbf{s} = [s_1, s_2, \dots, s_K]^T$ to the relay. s_k ($k=1, 2, \dots, K$) represents the signal to the k^{th} PU which satisfies $\varepsilon(|s_k|^2) = 1$, with s_i and s_j ($i \neq j$) being independent of each other.

The received signal vector at the relay can be written as

$$\mathbf{y}_r = \sqrt{P_p} \mathbf{F} \frac{\mathbf{W}_p}{\|\mathbf{W}_p\|_F} \mathbf{s} + \mathbf{n}_r, \quad (17)$$

where P_p is the PBS transmit power. \mathbf{W}_p is the pre-coding matrix which will be given later. \mathbf{n}_r is the Gaussian noise vector at the relay satisfying $\varepsilon[\mathbf{n}_r \mathbf{n}_r^H] = \sigma_r^2 \mathbf{I}_{N_r}$. The relay employs an amplification factor, which is designed to constrain the instantaneous transmit power on its received signal vector. The amplification factor is derived as

$$G_r = \sqrt{\frac{P_r}{P_p \frac{\|\mathbf{F} \mathbf{W}_p\|_F^2}{\|\mathbf{W}_p\|_F^2} + N_r \sigma_r^2}}, \quad (18)$$

where P_r is the relay transmit power that is assigned to each antenna. G_r must have a low boundary $G_{r,\min}$. The received PUs vector from the PBS can be written as

$$\mathbf{y}_{p1} = \mathbf{G} G_r \mathbf{y}_r + \mathbf{n}_p, \quad (19)$$

where $\mathbf{n}_p = [n_1, n_2, \dots, n_K]^T$ is the noise matrix at different PUs and $\varepsilon[\mathbf{n}_k \mathbf{n}_k^H] = \sigma_k^2$ ($k=1, 2, \dots, K$). The ZF pre-coding method is used for PU transmission; hence, the pre-coding matrix is given as

$$\mathbf{W}_p = (\mathbf{G} \mathbf{F})^H \left[\mathbf{G} \mathbf{F} (\mathbf{G} \mathbf{F})^H \right]^{-1}. \quad (20)$$

Substituting Eq. (17) into Eq. (19), the PUs' received signal from the PBS can be written in an alternative form as

$$\mathbf{y}_{p1} = \frac{\sqrt{P_p} G_r}{\|\mathbf{W}_p\|_F} \mathbf{s} + G_r \mathbf{G} \mathbf{n}_r + \mathbf{n}_p. \quad (21)$$

During the second time slot, the SBS transmits signal $\mathbf{x} = [x_1, x_2, \dots, x_{N_s}]^T$ which satisfies $\varepsilon\{\|\mathbf{x}\|_2^2\} = 1$, but the transmission signal will interfere with the primary network. The channel matrix from the SBS to the PUs can be denoted by \mathbf{L} and is given as

$$\mathbf{L} = \tilde{\mathbf{L}} \mathbf{D}_L^{1/2}, \quad (22)$$

where $\tilde{\mathbf{L}} \sim \mathcal{CN}_{K \times N_s}(\mathbf{0}_{K \times N_s}, \mathbf{I}_K \otimes \mathbf{I}_{N_s})$ and \mathbf{D}_L is an $N_s \times N_s$ diagonal matrix. Then the channel coefficient between the k^{th} PU and n^{th} SBS antenna is denoted as

$$[\mathbf{L}]_{k,n} = \sqrt{t_k} [\tilde{\mathbf{L}}]_{k,n}, \quad (23)$$

where $n \in \{1, 2, \dots, N_s\}$. Therefore, the PUs' received signal vector from the PBS and SBS can be written as

$$\mathbf{y}_p = \frac{\sqrt{P_p} G_r}{\|\mathbf{W}_p\|_F} \mathbf{s} + G_r \mathbf{G} \mathbf{n}_r + \sqrt{P_s} \mathbf{L} \mathbf{W}_s \mathbf{x} + \mathbf{n}_p, \quad (24)$$

where P_s is the SBS transmit power. \mathbf{W}_s is the pre-coding matrix in the secondary network which will be discussed later. Considering the k^{th} PU's received signal, the expression can be written as

$$y_{p,k} = \frac{\sqrt{P_p} G_r}{\|\mathbf{W}_p\|_F s_k} + G_r \mathbf{G}_k \mathbf{n}_r + \sqrt{P_s} \mathbf{L}_k \mathbf{W}_s \mathbf{x} + n_k, \quad (25)$$

where \mathbf{G}_k is the k^{th} row of \mathbf{G} and represents the channel vector from the relay to the k^{th} PU. \mathbf{L}_k is the k^{th} row of \mathbf{L} and represents the channel vector from the SBS to the k^{th} PU. n_k is the noise at the k^{th} PU's.

The channel matrix from the SBS to SU is given by

$$\mathbf{L}' = \tilde{\mathbf{L}}' \mathbf{D}'_L{}^{1/2}, \quad (26)$$

where $\tilde{\mathbf{L}}' \sim \mathcal{CN}_{1 \times N_s}([0, \dots, 0]_{1 \times N_s}, [1, \dots, 1]_{1 \times N_s})$ represents independent and fast Rayleigh fading. \mathbf{D}'_L captures the path loss and can be defined as $\mathbf{D}'_L = \mu \mathbf{I}_{N_s}$. Then the channel coefficient between the n^{th} SBS antenna and the SU is written as

$$[\mathbf{L}']_n = \sqrt{\mu} [\tilde{\mathbf{L}}']_n. \quad (27)$$

The channel matrix from the relay to the SU is modeled as

$$\mathbf{G}' = \tilde{\mathbf{G}}' \mathbf{D}'_G{}^{1/2}, \quad (28)$$

where $\tilde{\mathbf{G}}' \sim \mathcal{CN}_{1 \times N_r}([0, \dots, 0]_{1 \times N_r}, [1, \dots, 1]_{1 \times N_r})$ accounts for the independent and fast Rayleigh fading. \mathbf{D}'_G captures the path loss and can be defined as

$\mathbf{D}'_{\mathbf{G}} = \nu \mathbf{I}_{N_r}$. Then the channel coefficient between the n^{th} relay antenna and the SU is denoted as

$$[\mathbf{G}']_n = \sqrt{\nu} [\tilde{\mathbf{G}}']_n. \quad (29)$$

The ZF pre-coding method is also used by secondary network transmission, and the pre-coding matrix is given by

$$\mathbf{W}_s = (\mathbf{L}')^H [\mathbf{L}'(\mathbf{L}')^H]^{-1}. \quad (30)$$

The SU's received signal from the SBS and PBS can be modeled as

$$y_s = \frac{\sqrt{P_s}}{\|\mathbf{W}_s\|_2} \mathbf{x} + G_r \mathbf{G}' \mathbf{y}_r + n_s, \quad (31)$$

where n_s is the Gaussian noise at the SU.

By substituting Eq. (17) into Eq. (31), the SU's received signal can be derived as

$$y_s = \frac{\sqrt{P_s}}{\|\mathbf{W}_s\|_2} \mathbf{x} + \sqrt{P_p} G_r \mathbf{G}' \mathbf{F} \frac{\mathbf{W}_p}{\|\mathbf{W}_p\|_{\mathbf{F}}} \mathbf{s} + G_r \mathbf{G}' \mathbf{n}_r + n_s. \quad (32)$$

The interference power at all PUs inflicted by the SBS must not exceed the maximum peak interference temperature I_p . To prevent the primary network transmission from harmful interference, the SBS transmit power is given as

$$P_s = \min\left\{\frac{I_p}{Z_1}, P_{\max}\right\}, \quad (33)$$

where $Z_1 = \max_k \left\{ |\mathbf{L}_k (\mathbf{W}_s / \|\mathbf{W}_s\|_2)|^2 \right\}$ and P_{\max} is the SBS's peak transmit power.

By using Eqs. (25), (32), and (33) and several mathematical manipulations, the SINR of the k^{th} PU can be derived as

$$\gamma_k = \frac{P_p G_r^2}{\|\mathbf{W}_p\|_{\mathbf{F}}^2 \left(G_r^2 \sigma_r^2 \|\mathbf{G}_k\|_2^2 + P_s |\mathbf{L}_k \mathbf{W}_s|^2 + \sigma_k^2 \right)}. \quad (34)$$

The SINR of the SU can be written as

$$\gamma_s = \frac{P_s}{\|\mathbf{W}_s\|_2^2 \left(G_r^2 P_p \frac{\|\mathbf{G}' \mathbf{F} \mathbf{W}_p\|_2^2}{\|\mathbf{W}_p\|_{\mathbf{F}}^2} + G_r^2 \sigma_r^2 \|\mathbf{G}'\|_2^2 + \sigma_s^2 \right)}. \quad (35)$$

4.2 Cognitive transmission with imperfect channel state information

In practice, the channel estimation error will reduce significantly system transmission performance

in a massive MIMO network. The channel matrix has to be estimated by receiving pilot sequences at the BS. In the uplink pilot transmission of the primary network, τ_1 symbols are used to estimate \mathbf{G} and τ_2 symbols are used to estimate \mathbf{F} . The corresponding transmit powers of pilot sequences are defined as $\tau_1 P_u$ and $\tau_2 P_u$, respectively, where P_u is the users' uplink transmit power. The minimum mean-square error (MMSE) estimate of channel matrices in the primary network is denoted by (Ngo et al., 2013)

$$\hat{\mathbf{F}} = \left(\mathbf{F} + \frac{1}{\sqrt{\tau_2 P_u}} \mathbf{N}_{\mathbf{F}} \right) \left(\frac{1}{\tau_2 P_u} \mathbf{D}_{\mathbf{F}}^{-1} + \mathbf{I}_{N_r} \right)^{-1} \quad (36)$$

and

$$\hat{\mathbf{G}} = \left(\mathbf{G} + \frac{1}{\sqrt{\tau_1 P_u}} \mathbf{N}_{\mathbf{G}} \right) \left(\frac{1}{\tau_1 P_u} \mathbf{D}_{\mathbf{G}}^{-1} + \mathbf{I}_K \right)^{-1}. \quad (37)$$

In the channel estimation of the secondary network, v symbols are used to estimate \mathbf{L}' . The corresponding transmit power of pilot sequences is defined as $v P_u$. The channel matrix by MMSE estimation is derived as

$$\hat{\mathbf{L}}' = \left(\mathbf{L}' + \frac{1}{\sqrt{v P_u}} \mathbf{N}_{\mathbf{L}'} \right) \left(\frac{1}{v P_u} \mathbf{D}_{\mathbf{L}'}^{-1} + \mathbf{I}_{N_s} \right)^{-1}. \quad (38)$$

Therefore, the pre-coding matrices of the primary network and secondary network are rewritten as

$$\hat{\mathbf{W}}_p = \left(\hat{\mathbf{G}} \hat{\mathbf{F}} \right)^H \left[\hat{\mathbf{G}} \hat{\mathbf{F}} \left(\hat{\mathbf{G}} \hat{\mathbf{F}} \right)^H \right]^{-1} \quad (39)$$

and

$$\hat{\mathbf{W}}_s = \left(\hat{\mathbf{L}}' \right)^H \left[\hat{\mathbf{L}}' \left(\hat{\mathbf{L}}' \right)^H \right]^{-1}. \quad (40)$$

The transmission SINR of PUs and the SU under imperfect CSI can be derived by substituting Eqs. (36)–(40) into Eqs. (34) and (35). This study is concerned mainly about the downlink transmission in the underlay CR network with massive MIMO, but the channel estimation occurs in the uplink. Only the asymptotic downlink users' rate under perfect CSI is discussed.

5 Achievable rate and utility function analysis in the underlay cognitive radio network with massive multi-input multi-output

In this section, the asymptotic transmission performance and utility function are analyzed.

5.1 Asymptotic sum rate analysis

Asymptotic sum rates of PUs and the SU under the perfect CSI are analyzed here. N_p , N_r , and N_s become unlimited while maintaining fixed ratios $\eta = N_p/N_r$ and $\varepsilon = N_s/N_r$. The relay power assigned to the k^{th} user can be scaled inversely down proportional to N_r as $P_r = E_r/N_r$, where E_r is fixed. The SBS power can be scaled down proportional to N_s because $P_s = E_s/N_s$. The asymptotic SBS transmit power is considered: $\lim_{N_s \rightarrow \infty} |\mathbf{L}_k \mathbf{W}_s|^2 = \lim_{N_s \rightarrow \infty} \text{Tr} \left[\left(\frac{\mathbf{L}' \mathbf{L}'^H}{N_s} \right)^{-1} \left(\frac{\mathbf{L}' \mathbf{L}'^H}{N_s} \right) \left(\frac{\mathbf{L}_k \mathbf{L}'^H}{N_s} \right) \left(\frac{\mathbf{L}' \mathbf{L}'^H}{N_s} \right)^{-1} \right] = 0$. Therefore, $Z_1 \rightarrow 0$ and we can obtain

$$\lim_{N_s \rightarrow \infty} P_s = P_{\max}. \quad (41)$$

When the number of antennas becomes infinite, the secondary network can use peak power to transform information without causing any interference to the primary network. The asymptotic SINR of PUs and the SU can be derived as

$$\gamma_k^\infty = \frac{E_r \beta_k}{\sigma_k^2 (1 + \sigma_r^2 / (\alpha \eta P_p))} \quad (42)$$

and

$$\gamma_s^\infty = \frac{E_s \mu}{\sigma_s^2}. \quad (43)$$

The proof is the same as that in Section 3. The results show that the asymptotic transmission performances of the primary and secondary networks are not related to fast fading. Therefore, the latency in the air interface can be reduced significantly. The interference between the primary network and secondary network is canceled asymptotically. The transmission performances of the primary network and secondary network do not contain the interference temperature.

The instantaneous achievable sum rate of the underlay CR network with massive MIMO can be defined as

$$R_p = \frac{1}{2} \sum_{k=1}^K \log(1 + \gamma_k) \quad (44)$$

and

$$R_s = \log(1 + \gamma_s). \quad (45)$$

Then, the asymptotic instantaneous achievable rate of the PUs and the SU can be rewritten as

$$R_p^\infty = \frac{1}{2} \sum_{k=1}^K \log \left[1 + \frac{E_r \beta_k}{\sigma_k^2 (1 + \sigma_r^2 / (\alpha \eta P_p))} \right] \quad (46)$$

and

$$R_s^\infty = \log \left(1 + \frac{E_s \mu}{\sigma_s^2} \right). \quad (47)$$

For large η , Eq. (46) can change to $R_p^\infty = \frac{1}{2} \sum_{k=1}^K \log(1 + E_r \beta_k / \sigma_k^2)$. The asymptotic sum rate of the primary network equals half of the rate of a single-hop, single-antenna, and K -user system without fast fading or secondary network interference. The transmission performance of the primary network is independent of the PBS-to-relay channel parameters and is related only to the relay-to-users stage.

5.2 Utility function object

In the CR network, the SU reuses opportunistically the authorized spectrum resource of the PUs. Li et al. (2017) and Tao et al. (2018) have considered the SU's transmission performance but neglected the PUs' performance. As the previous analysis shows, we need to achieve a tradeoff between the relay power and sum rate in the primary network. To measure the primary network benefits, the utility function related to the sum rate and relay power is defined. Then the PUs' utility function is maximized by optimizing the relay power. ε is assumed to be fixed and η to be large enough. The utility function can be defined as

$$J_k(E_r) = a_k(\gamma_k^\infty - \gamma_{k,\min}^\infty) - b(E_r - E_{\text{th}}), \quad (48)$$

where $\gamma_{k,\min}^\infty$ is the k^{th} PU SINR threshold and E_{th} is the minimum power in the relay station. Therefore, the optimization problem with respect to the primary network can be expressed as

$$\begin{aligned} \max \quad & H = \left[\sum_{k=1}^K a_k(\gamma_k^\infty - \gamma_{k,\min}^\infty) \right] - b(E_r - E_{\text{th}}) \\ \text{s.t.} \quad & \gamma_k^\infty \geq \gamma_{k,\min}^\infty, \quad \forall 1 \leq k \leq K, \\ & E_r \geq E_{\text{th}}. \end{aligned} \quad (49)$$

The closed-form expression cannot be derived immediately. Based on the above hypothesis, we can obtain

$$\frac{\partial H}{\partial E_r} = \left\{ \sum_{k=1}^K \left[a_k(\gamma_k^\infty - \gamma_{k,\min}^\infty) \frac{\partial \gamma_k^\infty}{\partial E_r} \right] \right\} - b = 0. \quad (50)$$

Eq. (50) is a polynomial problem. By substituting Eq. (42) into Eq. (50), the optimal relay transmit power E_r can be derived easily to maximize the PUs' utility function.

6 Numerical results

The simple scenario enables us to validate the expression of the SINR and the transmission rate of the PUs and the SU. Then the utility function of the PUs is discussed based on the same scenario. The central frequency is 3.45 GHz and the bandwidth is 40 MHz. The large-scale fading coefficients are set as $\alpha = \beta_k = t_k = \mu = \nu = 1, \forall 1 \leq k \leq K$. The noises at different places are set as $\sigma_r^2 = \sigma_k^2 = \sigma_s^2 = 1, \forall 1 \leq k \leq K$.

The relay-aided massive MIMO network without CR is simulated first. The practical transmission path loss model is considered in the simulation. The transmission distances $d_F = 10$ m and $d_G = 10$ m are assumed. Fig. 4 shows the relationship between the users' asymptotic sum rate and the relay power. As the relay power E_r grows, the users' asymptotic sum rate increases. More BS power will lead to a higher asymptotic rate, but there is a limit (Fig. 4). Compared to increased relay power, an increase in the BS power is not that useful. The relationship between the asymptotic sum rate and antenna ratio η is shown in Fig. 5. As the ratio grows, the asymptotic sum rate increases. When the ratio between the BS antenna number and relay antenna number η becomes large enough, there is a limit (Fig. 5).

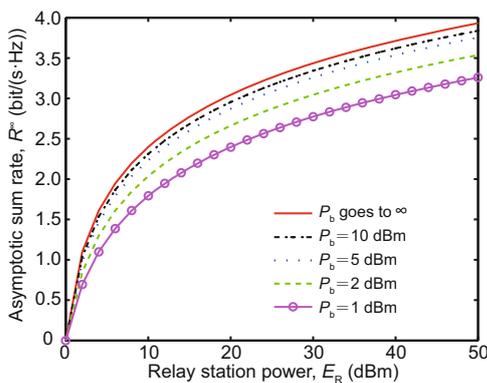


Fig. 4 Asymptotic sum rate versus relay station power for $K=2$ and $\eta=1$ in the massive multi-input multi-output relay network

Then the underlay CR network with massive MIMO is discussed. In the simulation, the practical transmission pathloss model is considered. The powers of the PBS, relay, and SBS are set as $P_p = 23$ dBm, $E_r = 27$ dBm, and $E_s = 20$ dBm, respectively. The ratios between the antenna numbers are assumed as $\eta = 1$ and $\varepsilon = 2$. The transmission

distances $d_F = 10$ m, $d_G = 10$ m, $d_{L'} = 10$ m, $d_L = 20$ m, and $d_{G'} = 20$ m are supposed. The numbers of PUs are set as $K = 2$ and $K = 3$. Fig. 6 shows the sum rate differences between the asymptotic theoretical analysis and the practical transmission pathloss model. As the number of PBS antennas grows, the sum rates of the PUs and SU in this practical scenario tend to the theoretical analysis. The SU asymptotic theoretical analysis has no relationship with the primary network parameters, such as K and interference temperature I_p .

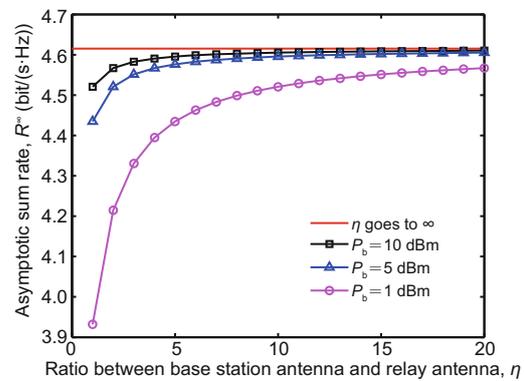


Fig. 5 Asymptotic sum rate versus the antenna ratio for $K=2$ and $E_R=100$ dBm in the massive multi-input multi-output relay network

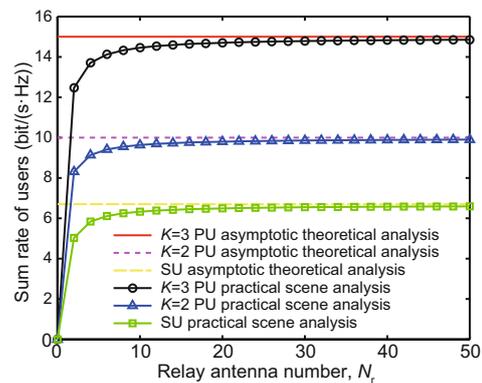


Fig. 6 Sum rate of users versus the relay antenna number in the relay underlay cognitive radio network with massive multi-input multi-output

The relationship between the PUs' asymptotic sum rate and relay station power is shown in Fig. 7. The power of the SBS is set as $E_s = 20$ dBm. The antenna number ratios and transmission distance are assumed as $\eta = 1, \varepsilon = 2, d_F = 10$ m, $d_G = 10$ m, $d_{L'} = 10$ m, $d_L = 20$ m, and $d_{G'} = 20$ m. The number of PUs is set as $K = 2$. The same holds for

the relay-aided massive MIMO network without CR; the asymptotic sum rate increases as the relay power grows. When the PBS power becomes infinite, the asymptotic sum rate tends to a limit (Fig. 7).

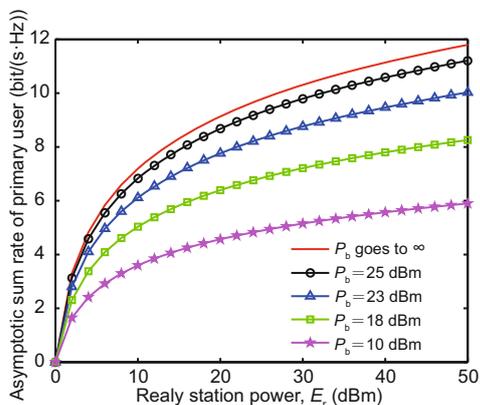


Fig. 7 Asymptotic sum rate of primary users versus relay station power in the relay underlay cognitive radio network with massive multi-input multi-output

The secondary network transmission rate versus the SBS antenna number is simulated. The practical transmission path loss model is employed. The PBS and relay powers are set as $P_p = 23$ dBm and $E_r = 27$ dBm, respectively. The antenna number ratios, transmission distance, and number of PUs are assumed as $\eta = 1$, $\varepsilon = 2$, $d_F = 10$ m, $d_G = 10$ m, $d_{L'} = 10$ m, $d_L = 20$ m, $d_{G'} = 20$ m, and $K = 2$. Fig. 8 shows the relationship between the SU transmission rate and SBS antenna number. As the SBS antenna number grows, the sum rate of the SU in the practical scenario tends toward the theoretical analysis.

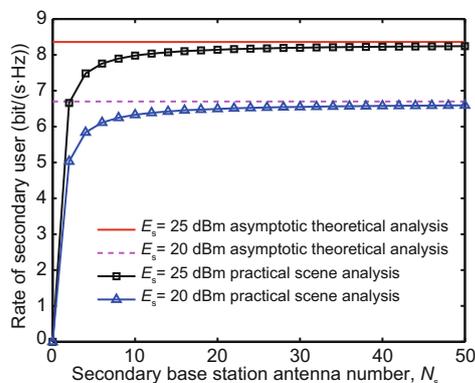


Fig. 8 Secondary user transmission rate versus the number of secondary base station antennas in the relay underlay cognitive radio network with massive multi-input multi-output

Finally, the utility function in the primary network is simulated. Transmission parameters are set as $K = 2$, $a_1 = a_2 = \frac{1}{2}$, $b = 1$, $\gamma_{1,\min} = \gamma_{2,\min} = 2$, $P_{th} = 5$ dBm. Fig. 9 shows the optimal relay power (21.6 dBm) to maximize the utility function. As the relay power grows, the utility function decreases after an initial increase, because as the relay power increases, the sum rate causes more utility increase in the first stage, but when the relay power exceeds an optimal value, the relay power's increase causes a greater decrease in utility. Therefore, as the relay power grows, the utility function increases first and then decreases.

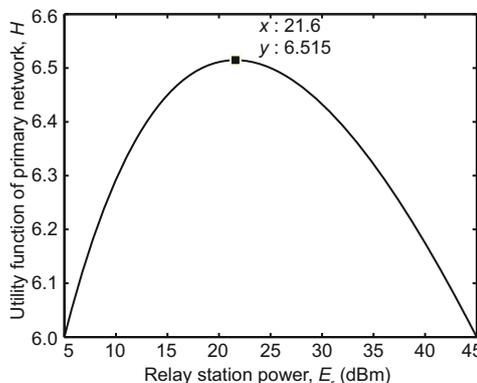


Fig. 9 Primary users' utility function versus relay power in the relay underlay cognitive radio network with massive multi-input multi-output

7 Conclusions

In this paper, transmission performance in a relay-aided massive MIMO network without CR was discussed. Then the transmission performance and power allocation strategy in an underlay CR network with massive MIMO were analyzed. The cognitive transmission performances of PUs, an SU under perfect CSI, and imperfect CSI were derived, including the end-to-end SINR and achievable sum rate. When the numbers of PBS antennas, SBS antennas, and relay antennas became infinite, the asymptotic SINR was independent of fast fading. The interference between the primary network and secondary network was canceled asymptotically. The transmission performance did not include the interference temperature. The secondary network could use its peak power to transmit signals without causing any interference to the primary network. Interestingly, when

the antenna ratio became large enough, the asymptotic sum rate of the primary network equaled half of the rate of a single-hop, single-antenna, and K -user network without fast fading. The PUs' utility function was defined based on relay power and the asymptotic SINR of PUs. The utility function was maximized by optimizing the relay power. Finally, the relationships between the asymptotic sum rate and relay power, the number of relay antennas, and the antenna ratio were simulated. The maximization of the utility function was derived with optimal relay power. It was shown that the massive MIMO with linear pre-coding could cancel asymptotically the interference between the primary network and secondary network in the multi-user underlay relay-aided CR system.

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