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Consensus-reaching methods for hesitant fuzzy multiple criteria group decision making with hesitant fuzzy decision making matrices*

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Abstract: Group decision making plays an important role in various fields of management decision and economics. In this paper, we develop two methods for hesitant fuzzy multiple criteria group decision making with group consensus in which all the experts use hesitant fuzzy decision matrices (HFDMs) to express their preferences. The aim of this paper is to present two novel consensus models applied in different group decision making situations, which are composed of consensus checking processes, consensus-reaching processes, and selection processes. All the experts make their own judgments on each alternative over multiple criteria by hesitant fuzzy sets, and then the aggregation of each hesitant fuzzy set under each criterion is calculated by the aggregation operators. Furthermore, we can calculate the distance between any two aggregations of hesitant fuzzy sets, based on which the deviation between any two experts is yielded. After introducing the consensus measure, we develop two kinds of consensus-reaching procedures and then propose two step-by-step algorithms for hesitant fuzzy multiple criteria group decision making. A numerical example concerning the selection of selling ways about 'Trade-Ins' for Apple Inc. is provided to illustrate and verify the developed approaches. In this example, the methods which aim to reach a high consensus of all the experts before the selection process can avoid some experts' preference values being too high or too low. After modifying the previous preference information by using our consensus measures, the result of the selection process is much more reasonable.

Keywords: Multiple criteria group decision making; Group consensus; Consensus-reaching process; Hesitant fuzzy decision making matrices; Aggregation operators

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1 Introduction

Group decision making (GDM), which consists of finding the optimal alternative(s) from a set of feasible alternatives according to the preferences provided by a group of experts, takes place widely in various fields of management decision and economics. Let $A=\{a_1, a_2, \ldots, a_n\}$ be the set of alternatives, $C=\{c_1, c_2, \ldots, c_m\}$ the set of different criteria, and $E=\{e_1, e_2, \ldots, e_s\}$ the set of experts. Decision matrices, which are constructed by the mutual relationship of the alternatives and the criteria, can express the preferences of experts intuitively. Generally, the values of decision matrices can be represented in many forms such as fuzzy numbers (Zadeh, 1965; Gong and Feng, 2016), intuitionistic fuzzy numbers (Atanassov, 2012; Zhou, 2016), and interval-valued intuitionistic fuzzy numbers (Atanassov and Gargov, 1989; Azarnivand and Malekian, 2016). However, in some cases the experts cannot provide their preferences with a certain value or some interval values because of the experts' different and uncertain ideas about one alternative from the criteria. Compared with the above extended fuzzy sets, the hesitant fuzzy sets (Torra, 2010), characterized as a set of several possible

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values, can describe human's various opinions of uncertainty and diversity more reasonably and conveniently. The hesitant fuzzy decision matrix (HFDM), whose elements are depicted by hesitant fuzzy elements (HFEs), can express the information more accurately and roundly.

There is much successful work that has a great influence on decision making with hesitant fuzzy information. Xia and Xu (2011) and Xia *et al.* (2013) conducted an intensive study on hesitant fuzzy aggregation operators, and proposed two methods to determine the weight vectors and aggregation operators, which aim to reflect the correlations of the aggregation arguments. Liao and Xu (2014; 2015) proposed a family of hesitant fuzzy hybrid weighted aggregation operators to aggregate hesitant fuzzy information, and the properties of these aggregation operators were investigated. Based on this, a study on their application in decision making was undertaken. Considering that the criteria have different priority levels in practical decision making problems, Wei (2012) developed some prioritized aggregation information and applied it to develop some models for hesitant fuzzy multiple criteria group decision making (HFMCGDM). To arrange the values of any two HFEs, Xu and Xia (2011a; 2011b) proposed a variety of distance and similarity measures for hesitant fuzzy sets. Many researchers have studied the correlation coefficient since it has been widely used in data analysis. Liao *et al*. (2015b) introduced some novel correlation measures between HFSs which are better than other correlation measures previously proposed. Zhang *et al*. (2013) defined an improved distance measure for a hesitant fuzzy set considering optimistic and pessimistic preference information simultaneously, and used the TOPSIS method to handle the HFMCGDM problems, which can avoid dealing with complex hesitant fuzzy information. To handle more complex HFMCGDM problems, Zeng *et al*. (2013) proposed the MULTIMOORA-HF method, providing a way related to uncertain and complex assessments for HFMCGDM problems to reduce bias and subjectivity.

The experts who are involved in the HFMCGDM problems come from a variety of research fields, so they may have unique perspectives on an issue because of different knowledge, experience, skills, and personality. Absolutely, it is important and necessary to let all experts achieve a high level of consensus before the selection processes. Thus, it is necessary to develop a consensus-reaching process to obtain a more reasonable solution that can be accepted by all decision makers. Although consensus has received much attention in some research, there are also several issues to be dealt with, such as those proposed by Cabrerizo *et al*. (2015). Many scholars have proposed a series of methods for consensus measures, a consensus-reaching process with fuzzy preference relations (Herrera-Viedma *et al*., 2007; Cabrerizo *et al.*, 2010) and intuitionistic fuzzy preference relations (Zhang *et al*., 2013; Liao *et al.*, 2015a; 2016; Xu *et al*., 2016). Herrera-Viedma *et al*. (2007) presented a consensus model that uses two different kinds of measures to guide the consensus-reaching process for GDM problems with incomplete fuzzy preference relations. Cabrerizo *et al*. (2010) analyzed the advantages and drawbacks of the consensus approach used in fuzzy GDM. Zhang *et al*. (2013) developed a different methodology for intuitionistic fuzzy GDM with group consensus. To make the result more reasonable, Xu *et al*. (2016) developed a method to check the consistency and consensus of intuitionistic fuzzy preference relations. Furthermore, to better understand consensus among the experts with intuitionistic fuzzy preference relations in GDM, Liao *et al*. (2015a) used a different consistency checking method and a consensus-reaching method to deal with the intuitionistic fuzzy GDM problem, in which all experts use intuitionistic fuzzy preference relations to express their preferences. Recently, Liao *et al.* (2016) enhanced their consensus-reaching process for GDM with intuitionistic fuzzy preference relations by removing only some information of the expert(s) as alternative(s) to remove the expert from the decision group. As the above-mentioned methods include the consensus-reaching process before selection, the decision making results are much better than those derived by only the selection process.

However, up to now, there is little research on the consensus-reaching process for multiple criteria group decision making (MCGDM) problems with hesitant fuzzy information. Zhang *et al*. (2014; 2015a; 2015b) presented a consensus support model and a decision making model, composed of a consensus-reaching process and a selection process for MCGDM with hesitant fuzzy information. Their method has some drawbacks as it focuses only on how to measure the consensus of a group and uses a feedback mechanism to interact with the experts, which wastes much time and is very complicated in practical applications. In addition, the method cannot help the group check what are the real factors resulting in the low level of consensus. To avoid these drawbacks, in this research we develop two methods for HFMCGDM with group consensus, in which all experts use HFDMs to express their preferences. We introduce two novel methods to measure, check, and reach the consensus of a group and give two complete algorithms for GDM with HFDMs, which are quite flexible and reasonable, and thus can match the practical GDM situations well.

2 Preliminaries

In this section, some basic concepts related to HFSs and HFDMs are introduced which will be used in the following analyses.

2.1 Hesitant fuzzy set

Let X be a fixed set. A hesitant fuzzy set (HFS) (Torra, 2010) on X permits the membership of an element to a set of several possible values between 0 and 1. To be easily understood, Xia and Xu (2011) expressed an HFS mathematically as

$$H=\{\langle x,h_A(x)\rangle | x\in X\},\tag{1}$$

where $h_A(x)$ is a set of values in [0, 1], denoting the possible membership degrees of the element $x\in X$ to the set H. $h=h_A(x)$ is a hesitant fuzzy element (HFE) and H is the set of all HFEs.

Xu and Xia (2011a) defined a simplified hesitant normalized Hamming distance to reflect relationship and difference between two variables as follows:

$$d(A_1,A_2)=\frac{1}{m}\sum_{j=1}^{m}\left[\frac{1}{l_{x_j}}\sum_{q=1}^{l_{x_j}}\left|h_{A_1}^{\sigma(q)}(x_j)-h_{A_2}^{\sigma(q)}(x_j)\right|\right],\tag{2}$$

where $h_{A_1}^{\sigma(q)}(x_j)$ and $h_{A_2}^{\sigma(q)}(x_j)$ are the qth largest values in $h_{A_1}(x_j)$ and $h_{A_2}(x_j)$, respectively. The Hamming distance satisfies $0\le d(A_1, A_2)\le 1$. There are many other different forms of distance measures for HFEs (refer to Torra and Narukawa (2009) and Xu (2014) for details). In this study, we use the hesitant normalized Hamming distance as a representation.

For an HFE h,

$$s(h)=\frac{1}{l_h}\sum_{\gamma\in h}\gamma\tag{3}$$

denotes the score of h, where l_h is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1)>s(h_2)$, then h_1 is superior to h_2, denoted as $h_1>h_2$; if $s(h_1)=s(h_2)$, then h_1 is the same as h_2, denoted as $h_1\sim h_2$. Chen *et al.* (2013) defined the concept of deviation degree to better compare the superiority between two HFEs when h_1 and h_2 have the same scores. Here we do not review it. Interested readers may refer to Chen *et al.* (2013).

2.2 MCGDM problem with hesitant fuzzy information

We consider the case where a group of experts provide their preference information in the form of matrices and express their possible preference values by HFEs when evaluating several existing alternatives. For an HFMCGDM problem with a discrete set of n alternatives $A=\{a_1, a_2, \ldots, a_n\}$, a finite set of m criteria $C=\{c_1, c_2, \ldots, c_m\}$, and a set of experts e_k (k=1, 2, …, v, r, …, s), where s is the number of experts, and v and r are two positive integers between 1 and s, the weight vector of all criteria, determined by the experts according to the importance of each criterion, is denoted as $\boldsymbol{\omega}=[\omega_1, \omega_2, \ldots, \omega_m]^{\mathrm{T}}$, where $0\le\omega_j\le 1$, j=1, 2, …, m, and $\sum_{j=1}^{m}\omega_j=1$. The weight vector of experts e_k (k=1, 2, …, v, r, …, s) is $\boldsymbol{\lambda}=[\lambda_1, \lambda_2, \ldots, \lambda_v, \lambda_r, \ldots, \lambda_s]^{\mathrm{T}}$, where $\lambda_k>0$ (k=1, 2, …, s) and $\sum_{k=1}^{s}\lambda_k=1$, which can be determined according to how deep and important the experts' professional knowledge, experience, status, and impact are. In this study, we assume that the weights of the experts are the same and focus on the different weights of the criteria. If all the experts use HFEs to express their assessments, we can construct an HFDM $\boldsymbol{H}^k$ as follows:

$$\boldsymbol{H}^k=(h_{ij}^k)_{n\times m}=\begin{bmatrix} h_{11}^k & h_{12}^k & \cdots & h_{1m}^k \\ h_{21}^k & h_{22}^k & \cdots & h_{2m}^k \\ \vdots & \vdots & & \vdots \\ h_{n1}^k & h_{n2}^k & \cdots & h_{nm}^k \end{bmatrix}. \tag{4}$$

2.3 Aggregation operators for HFEs

Xia and Xu (2011) developed many kinds of aggregation operators for HFEs. Then, Liao *et al.* (2014) proposed some adjusted aggregation operators for HFEs, which simplify the computation process due to the decreasing dimension of HFEs. According to the constructed HFDM $\boldsymbol{H}^k$, we can obtain the form of presentation for the adjusted hesitant fuzzy weighted averaging (AHFWA) operator as follows:

Definition 1 (Liao *et al.*, 2014) Let h_j (j=1, 2, …, z) be a collection of HFEs. An AHFWA operator is a mapping $P^n\rightarrow P$ such that

$$\begin{aligned}&\text{AHFWA}(h_1,\ h_2,\ ...,\ h_z)=\mathop{\oplus}_{j=1}^{z}(\omega_j h_j)\\&=\left\{1-\prod_{j=1}^{z}(1-h_j^{\sigma(q)})^{\omega_j}\,|q=1,\ 2,\ ...,\ l\right\},\end{aligned} \tag{5}$$

where $h_j^{\sigma(q)}$ is the qth largest value in h_j, l is the number of elements in the HFE, and $\boldsymbol{\omega}=[\omega_1, \omega_2, …, \omega_z]^{\mathrm{T}}$ is the weight vector of h_j (j=1, 2, …, z) with $\omega_j\in[0, 1]$ and $\sum_{j=1}^{z}\omega_j=1$.

Remark 1 There are many different forms of aggregation operators for HFEs (Xia and Xu, 2011; Xia *et al.*, 2013; Liao *et al.*, 2014; Liao and Xu, 2014; 2015). Generally, we should select the appropriate aggregation operator according to different practical issues. Here we just use the aggregation operator in Eq. (5) as a representation. The numbers of values in the HFEs are always different so that we should extend the shorter one until they have the same length when we aggregate them. We can extend the shorter HFE by adding the minimum value in it if the experts are considered pessimistic as in an example here.

3 Consensus-reaching process for HFMCGDM

In MCGDM situations, most researchers always rank the alternatives according to the overall scores of these alternatives, which are calculated by some aggregation operators. However, if the aggregation process does not involve the consensus or reach the expected consensus degree δ among the experts, the decision result may be unreasonable. Consensus is a basic idea in GDM, expected to occur after the experts exchange opinions (Ben-Arieh and Chen, 2006). In this section, we give a detailed algorithm for HFMCGDM with a consensus-reaching process. An extensional consensus-reaching process will be presented in Section 4.

3.1 Consensus-reaching process

Let $h(a_i)$ (i=1, 2, …, n) be the aggregated values of alternatives a_i (i=1, 2, …, n) over all criteria c_j (j=1, 2, …, m) by the AHFWA operator. Then we can give a distance measure between experts e_v and e_r as

$$d(e_v,e_r)=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{l}\sum_{q=1}^{l}\left|h_{e_v}^{\sigma(q)}(a_i)-h_{e_r}^{\sigma(q)}(a_i)\right|\right], \tag{6}$$

where $h_{e_v}^{\sigma(q)}(a_i)$ is the qth largest value in $h(a_i)$ of expert e_v, $h_{e_r}^{\sigma(q)}(a_i)$ is the qth largest value in $h(a_i)$ of expert e_r, and l denotes the number of all the different membership degrees.

Note that the larger the distance measured between two experts, the smaller the consensus degree between the two experts. Based on the distance measure between any two experts shown in Eq. (6), a consensus degree between any two experts can be defined mathematically as

$$\Delta(e_v,e_r)=1-d(e_v,e_r),\ \ v,r=1,\ 2,\ \ldots,\ s. \tag{7}$$

It is important to maximize the group consensus in GDM, which can ensure that the final result is more reliable and more reasonable. We can find the minimum consensus degree Δ by Eq. (7). Since $0\le d(e_v, e_r)\le 1$, the consensus degree $\Delta\in[0, 1]$. We know the expected consensus degree δ that the decision maker expects to reach was given previously. Thus, if $\Delta>\delta$, we can say the group reaches consensus; if $\Delta<\delta$, then there is at least one expert who should be advised to modify his/her preference values to reach consensus.

When $\Delta<\delta$, we should provide a procedure to reach a higher consensus degree which is at least equal to δ in the end. In this situation, the experts should usually have some discussions and the expert who does not want to change his/her own preference information may persuade others to adopt his/her opinion. If no preference information is changed or if the final consensus degree which we obtain from the modification preference values changed by some experts is still $\Delta<\delta$ after discussions, then we should develop a consensus-reaching procedure to reach a higher consensus effectively. The method is described as follows:

1. Through a series of effective computing methods, we first should find and pick out the expert who should change his/her preference information to reach a higher consensus. This is the key and the most difficult step of this consensus-reaching process.

We define that the two experts who have the largest distance between each other should be picked out, which can be depicted as

$$e_k \in \forall\{e_v, e_r\} = \left\{ d(e_v, e_r) = \max_{v,r=1,2,\ldots,s} \{d(e_v, e_r)\} \right\}. \quad (8)$$

Suppose that the two experts e_v and e_r who have a distance $d(e_v, e_r)$ from each other satisfy $e_k \in \forall\{e_v, e_r\}$, where $\forall\{e_v, e_r\}$ expresses any two experts. The next important job is to judge whether e_v or e_r should be selected to change his/her assessments. We should pick out the expert e_{k^*} according to

$$e_{k^*} = e_k = \min d\left(e_v \in \forall\{e_v, e_r\} \setminus e_k, e_r \in E \setminus \forall\{e_v, e_r\}\right). \quad (9)$$

If there are several experts satisfying Eqs. (8) and (9) simultaneously, it is permissible for anyone to adjust his/her preference values. However, this hardly ever occurs because it is so special and unusual. Generally, there is always only one expert who satisfies Eqs. (8) and (9) simultaneously.

2. Then we ask the expert e_{k^*} who is selected whether he/she agrees to change his/her preference values. If the expert agrees, then we calculate a new consensus degree and compare the size between the calculation consensus degree and the expected consensus degree. If $\Delta\geq\delta$, then we can say that the group reaches consensus. If $\Delta<\delta$, then we repeat the process from the beginning.

3. If the expert does not agree to change his/her preference values, we will exclude him/her from the group because his/her preference information is different from that of other experts. Then, we calculate a new consensus degree and compare the size between the calculation consensus degree and the expected consensus degree. If $\Delta\geq\delta$, then we can say the group reaches consensus. If $\Delta<\delta$, then we repeat the process from the beginning.

Here, we also introduce a variable τ representing the expected majority degree which should be determined according to the demand of decision in advance. Suppose that there are s^* experts e_k (k=1, 2, …, s^*) who reach the final consensus $\Delta^*\geq\delta$. Then, the majority degree of the final group can be defined as $T^*=s^*/s$. If some experts are not willing to change their minds or are excluded from the group which results in a final $T^*<\tau$, then this GDM is not meaningful.

4. Finally, we can calculate the overall scores of the alternatives a_i (i=1, 2, …, n) and make a ranking of them. The overall score $\tilde{s}(h_i)$ is described as

$$\tilde{s}(h_i) = \frac{1}{sl}\sum_{k=1}^{s}\sum_{q=1}^{l} h_{ik}^{\sigma(q)}. \quad (10)$$

Furthermore, we can obtain $\tilde{s}(h_i)$ by Eqs. (11a) and (11b) if we assume that each expert has the same weight:

$$\begin{cases} \tilde{s}(h_i) = \dfrac{1}{s}\displaystyle\sum_{k=1}^{s} s(h_{ik}), & (11a) \\ s(h_{ik}) = \dfrac{1}{l}\displaystyle\sum_{q=1}^{l} h_{ik}^{\sigma(q)}, & (11b) \end{cases}$$

$$\Rightarrow \tilde{s}(h_i) = \frac{1}{sl}\sum_{k=1}^{s}\sum_{q=1}^{l} h_{ik}^{\sigma(q)}, \quad (11c)$$

where $s(h_{ik})$ is the score of alternative a_i about expert e_k, and $h_{ik}^{\sigma(q)}$ is the qth smallest value in h_{ik}.

Also, we introduce the deviation degree $\bar{\sigma}(h_i)$ as follows:

$$\bar{\sigma}(h_i) = \left[\frac{1}{s}\sum_{k=1}^{s}\left(s(h_{ik}) - \tilde{s}(h_i)\right)^2\right]^{\frac{1}{2}}. \quad (12)$$

Usually, the overall scores $\tilde{s}(h_i)$ are different from each other, and we can just compare the size through the overall scores. If there are several overall scores which are equal, we should compare alternative a_i further through the deviation degree $\bar{\sigma}(h_i)$. In the end, we can choose the best alternative a^*.

The consensus-reaching process introduced above has the following three advantages:

1. It is much closer to a practical decision making process as it takes the experts' feedbacks into account.

2. It not only measures the consensus of a group, but also introduces some procedures to improve the consensus of a group.

3. It makes the result of GDM more reasonable as each expert's preference information is accepted by other experts even when they have different viewpoints due to different knowledge and experience.

3.2 Algorithm for HFMCGDM with consensus-reaching process

Based on the above analysis, we assume that there are s^* experts e_k (k=1, 2, …, s) who reach the final consensus $\Delta^*\geq\delta$ and also the majority degree is finally $T^*=s^*/s\geq\tau$ after the consensus-reaching process. A step-by-step process for HFMCGDM can be given as Algorithm 1.

Algorithm 1 Step-by-step process for HFMCGDM

1. A group of experts e_k (k=1, 2, …, v, r, …, s) is invited to evaluate the alternatives a_i (i=1, 2, …, n), and each expert e_k expresses his or her opinions by each HFDM $\boldsymbol{H}^k=(h_{ij}^k)_{n\times m}$ in the form of Eq. (4). In addition, the weights ω_j (j=1, 2, …, m) of the criteria c_j (j=1, 2, …, m) are completely known. To find a consensus solution, the two parameters on the expected consensus degree of group δ and the expected majority degree τ are established in advance by the group of experts. Then go to the next step.

2. Use Eq. (5) to obtain the aggregation values of each alternative a_i over all criteria c_j (j=1, 2, …, m) with weights ω_j (j=1, 2, …, m), and calculate the distance $d(e_v, e_r)$ between any pair of experts (e_v, e_r), v, r=1, 2, …, s according to Eq. (6). We obtain the consensus degree Δ of the group through Eq. (7). Then go to the next step.

3. Compare the values between the consensus degree of the group Δ and the expected consensus degree δ. If $\Delta\geq\delta$, then go to step 5; otherwise, go to the next step.

4. Find e_{k^*} who should be required to adjust his or her preference in order to reach a higher group consensus according to Eqs. (8) and (9). If expert e_{k^*} agrees to change his/her opinions and provides new information in the HFDM $\boldsymbol{H}^{k^*}=(h_{ij}^{k^*})_{n\times m}$, then we let $\boldsymbol{H}^k=\boldsymbol{H}^{k^*}=(h_{ij}^{k^*})_{n\times m}$ and go back to step 2. If expert e_{k^*} refuses to change his/her opinions, then we exclude him/her from the group and let $s^*=s^*-1$ ($s^*=s$ represents the number of experts which are at first considered in the decision making). After that, we calculate the majority degree of the group $T=s^*/s$ (s^* is the number of experts after the final iteration). If $T<\tau$, then the algorithm ends and there is not a consensus solution for this group decision making problem; otherwise, go back to step 3.

5. Calculate the overall scores and deviation degrees for the alternatives according to Eqs. (10) and (12), and compare all the scores and find the best alternative a^*.

6. End.

According to the Algorithm 1 and the consesus-reaching procedure introduced before, we can solve the GDM problem with consensus easily.

4 Extensional consensus-reaching method for HFMCGDM

In this section, an extensional consensus-reaching method using the removal of inappropriate alternatives for HFMCGDM is proposed which is a little different from the method in Section 3.

4.1 Extensional consensus-reaching process

It is recognized that removing the expert from the group may result in the loss of useful information from this expert. To avoid losing any useful information, we do more detailed research on the consensus-reaching process. Liao *et al*. (2015a) proposed the idea for removing the alternative(s) where the opinion of expert e_{k^*} is far from the other experts from the group in intuitionistic fuzzy group decision making. In our study, we will introduce this idea and apply it to HFMCGDM.

After finding the expert e_{k^*} who should modify his/her preference in Section 3, we try to find the alternative(s) where expert e_{k^*}'s opinion is far away from those of the other experts who reach the lowest degree of consensus. Generally, there may be not only one alternative that leads to the furthest distance between e_{k^*} and the other experts who reach the lowest consensus degree. A set of such alternatives A={a_1, a_2, …, a_n} can be selected by computing

$$a_i \in A = \{ \max_{i=1,2,\ldots,n} \max_{e_r \in \forall \{e_v, e_r\}} d(e_{k^*}, e_r) \}, \tag{13}$$

where $d(e_{k^*}, e_r)$ is the distance of HFEs between experts e_{k^*} and e_r according to Eq. (5).

After picking out alternative a_i in set A, we ask expert e_{k^*} whether he/she is willing to change his/her assessment about any alternative in A. If yes, the new assessment(s) will replace the old ones and we will obtain a new decision matrix from expert e_{k^*} and make a decision again about new information; if not, we will remove the alternative a_{i^*} $(a_{i^*} \in A)$ from expert e_{k^*} as follows:

$$a_{i^*} = \min_{a_i \in A} \max_{r=1,2,\ldots,s; r \neq k^*} d_{\bar{a}_{i^*}}(e_{k^*}, e_r), \tag{14}$$

where $d_{\bar{a}_{i^*}}(e_{k^*}, e_r)$ is the revised distance measure between experts e_{k^*} and e_r by removing alternative a_{i^*} $(a_{i^*} \in A)$ from expert e_{k^*}, that is,

$$d_{\bar{a}_{i^*}}(e_{k^*}, e_r) = \frac{1}{n-1} \sum_{i=1, i \neq i^*}^{n} \left[\frac{1}{l} \sum_{t=1}^{l} \left| h_{e_{k^*}}^{\sigma(t)}(a_i) - h_{e_r}^{\sigma(t)}(a_i) \right| \right]. \tag{15}$$

Here we just remove alternative a_{i^*} from expert e_{k^*}. That is to say, we delete HFEs h_{i^*j} from expert e_{k^*}. Then, we should know how to calculate the distance measure and the overall scores after deleting HFE h_{i^*j} from expert e_{k^*}. Here we just ignore HFE h_{i^*j}, which means the distance measure between HFEs h_{i^*j} and $h_{ij} (i \neq \overline{i}^*)$ is not computed in the following procedure, and then calculate the distance measures and the overall scores for the rest of the HFEs.

4.2 New algorithm for HFMCGDM

Based on the above analysis, we remove an expert's assessment on just one or several alternatives instead of all of his/her assessments to revise the consensus degree. Thus, a new step-by-step process for HFMCGDM is as given in Algorithm 2.

Algorithm 2 New step-by-step process for HFMCGDM

1. Same as step 1 in Algorithm 1.
2. Same as step 2 in Algorithm 1.
3. Compare the values between the consensus degree of group Δ and the expected consensus degree δ. If $\Delta \geq \delta$, then go to step 8; otherwise, go to the next step.
4. If expert e_k is persuaded by other experts through communication to modify his/her HFMADM matrix $\boldsymbol{H}^k$ into a new one $\boldsymbol{H}^{k'}$, then let $\boldsymbol{H}^k = \boldsymbol{H}^{k'}$ and go back to step 2; if no expert agrees to change the preferences at this time, then go to the next step.
5. Find the e_{k^*} who should be required to adjust his/her preference to reach a higher group consensus according to Eqs. (8) and (9) as in Algorithm 1. Go to the next step.
6. Find the alternative(s) where expert e_{k^*}'s opinions are the most far away from those of the other experts who reach the lowest consensus degree by Eq. (13). If expert e_{k^*} is willing to change his/her assessment on any alternatives in A, then the old assessment(s) from expert e_{k^*} is (are) replaced by the new ones, and go back to step 1; otherwise, go to the next step.
7. Exclude a_{i^*} $(a_{i^*} \in A)$ from the expert e_{k^*} that satisfies Eqs. (14) and (15) simultaneously, and go to step 2.
8. Same as step 5 in Algorithm 1.
9. End.

The idea of Algorithm 2 is a little different from that of Algorithm 1. Algorithm 2 aims at removing alternative(s) to reach consensus. However, Algorithm 1 aims at removing the expert(s) to reach consensus. These two ideas are applied to different situations according to the practical issue.

Comparing Algorithm 1 with Algorithm 2, we can see that the consensus measures are a little different. These two methods can be used in different decision making situations. For example, if we want to find the experts who result in the low level of consensus in a decision making problem, we can use Algorithm 1 to address this issue which is relatively easy and straightforward. However, if we want to find the alternatives that result in the low level of consensus in a decision making problem, we can use Algorithm 2 to address this issue because it retains the partial information of the experts, and the parameter τ (expected majority degree) does not need to be considered. Thus, how we apply the two methods is usually related to the practical decision making situations.

5 Numerical example and discussion

5.1 Numerical example

In what follows, we present a numerical example on selecting selling ways for 'Trade-ins' for Apple Inc. to illustrate the proposed decision analysis process and validate the proposed algorithms in aiding group decision making.

Example 1 The 'Trade-Ins' policy is researched and pushed by government and many enterprises. In May 2009, the CPC Central Committee and the State Council introduced a 'Trade-Ins' policy for cars and appliances, which is aimed to promote consumption through fiscal subsidies. 'Trade-Ins' refers to the position that if you can give the same old goods to the store when you are buying new goods, you can obtain discount coupons. Kong (2010) proposed the Trade-Ins of appliances, which can not only promote the economic effect of consumption, but also promote the environmental effect of electronic waste management. Wu *et al*. (2014) indicated that the influence of Trade-Ins on sales and profits depends on the enterprise's cost structure, the difference values between old and new products, and the proportion of different consumers in the market.

Apple Inc. has carried out a 'Trade-Ins' plan for the iPhone in the United States market since 2013. In 2015, the company officially launched the reusing and recycling program in China. 'Trade-Ins' activity has a variety of formats, including:

a_1: no Trade-Ins;

a_2: Trade-Ins on only Apple products;

a_3: Trade-Ins on the same products like Apple;

a_4: Trade-Ins on both Apple products and competing products.

There are four experts who are asked to choose which alternative is the best to be carried out. Considering two sides, enterprise benefit and social responsibility, several criteria are established according to the two sides in Fig. 1, involving:

c_1: unit benefit (unit selling price–unit cost+value of recycling product–cost of recycling);

c_2: number of customers (new customers+old customers);

c_3: benefit of customers (comparing the demand of changing old products and the use of discount coupons with the value of old products);

c_4: environmental protection (reducing environmental pollution by uniform recycling);

c_5: fewer security risks (old products may not be good for health because of higher radiation and being more highly combustible).

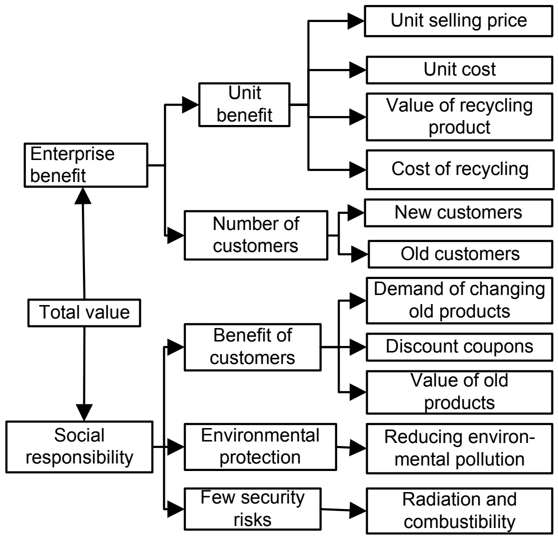

Fig. 1 The relative factors about the total value of Apple Inc.

For an enterprise, each criterion has a different weight, and the enterprise benefit is the most important. So, the weight vector $\boldsymbol{\omega}_j=[\omega_1, \omega_2, \omega_3, \omega_4, \omega_5]^{\mathrm{T}}$ is considered as $\boldsymbol{\omega}_j=[0.3, 0.3, 0.2, 0.1, 0.1]^{\mathrm{T}}$. Since there is no significant difference among the experts, the weights of them can be set to be equal. The expected majority degree τ is assumed to be 3/4, which means at least three out of four experts in the committee are needed to make a decision. Since the alternatives are important for the development of Apple Inc., the group of experts wants to be unanimous in choosing the selling ways and requires the expected consensus degree to be δ=0.9.

We assume that senior managers, acting as a decision organization, provide preferences over the alternatives a_i (i=1, 2, 3, 4) under each criterion $C=\{c_1, c_2, \ldots, c_m\}$ when they make decisions about selling ways about 'Trade-Ins' for Apple Inc. These senior managers may provide different preferences over the alternatives under each criterion. For example, when the members of the decision organization discuss the degree which should be over a_1 under c_1, some members may provide several values like 0.9, 0.8, and 0.7. The preference information can be contained in an HFE, which is expressed as h_{11}={0.9, 0.8, 0.7}. So, the results evaluated by the experts are contained in an HFDM (Tables 1–4).

We could use Algorithm 1 to solve this problem. Step 1 has been performed above, so we begin the calculation process from step 2.

Step 2: We consider that the experts are pessimistic, and change the hesitant fuzzy data by adding the minimum values, and then use Eq. (5) to obtain the aggregation operators (Tables 5–8). Then, calculate the distance $d(e_v, e_r)$ between any pair of experts (e_v, e_r), v, r=1, 2, 3, 4 according to Eq. (6). We obtain the consensus degree Δ of the group through Eq. (7). Then go to the next step.

We can calculate the distance between any pair of experts and obtain the consensus degree Δ of the group as follows:

$$\begin{aligned}&d(e_1,e_2)=0.0279,\ d(e_1,e_3)=0.0615,\\&d(e_1,e_4)=0.0754,\ d(e_2,e_3)=0.0649,\\&d(e_2,e_4)=0.0816,\ d(e_3,e_4)=0.1332,\\&\Delta=0.8668.\end{aligned}$$

Table 1 Hesitant fuzzy decision matrix e_1

	c_1	c_2	c_3	c_4	c_5
a_1	{0.9, 0.8}	{0.8}	{0.7, 0.5}	{0.7, 0.6}	{0.5}
a_2	{0.8, 0.7}	{0.8, 0.6}	{0.8, 0.6}	{0.6}	{0.7}
a_3	{0.6}	{0.9, 0.8}	{0.8}	{0.8, 0.7}	{0.9, 0.7}
a_4	{0.5}	{0.9}	{0.7, 0.6}	{0.8}	{0.9, 0.8}

Table 2 Hesitant fuzzy decision matrix e_2

	c_1	c_2	c_3	c_4	c_5
a_1	{0.9, 0.8}	{0.7}	{0.8, 0.6}	{0.6}	{0.7, 0.5}
a_2	{0.8}	{0.8, 0.6}	{0.7}	{0.6}	{0.7, 0.6}
a_3	{0.9}	{0.9}	{0.6, 0.5}	{0.7, 0.5}	{0.4}
a_4	{0.6}	{0.9}	{0.7, 0.6}	{0.8}	{0.6, 0.5}

Table 3 Hesitant fuzzy decision matrix e_3

	c_1	c_2	c_3	c_4	c_5
a_1	{0.7, 0.5}	{0.8}	{0.7}	{0.7, 0.6}	{0.5}
a_2	{0.5}	{0.6}	{0.8, 0.6}	{0.8, 0.7}	{0.7}
a_3	{0.7}	{0.9, 0.8}	{0.8}	{0.7}	{0.9, 0.7}
a_4	{0.8}	{0.9}	{0.8, 0.7}	{0.8}	{0.9}

Table 4 Hesitant fuzzy decision matrix e_4

	c_1	c_2	c_3	c_4	c_5
a_1	{0.9}	{0.8}	{0.7, 0.5}	{0.7}	{0.6, 0.5}
a_2	{0.8, 0.7}	{0.9}	{0.8, 0.6}	{0.7}	{0.5}
a_3	{0.6}	{0.6}	{0.9}	{0.8, 0.7}	{0.9, 0.8}
a_4	{0.5}	{0.6}	{0.7, 0.6}	{0.7}	{0.5}

Step 3: Since $\Delta<\delta$=0.9, go to step 4.

Step 4: We suppose that no expert agrees to change his/her mind without any tip. Since $d(e_3, e_4)$ is the furthest distance, then one of them should be asked to change his/her preferences. Removing e_3 and e_4 from the group respectively, we obtain the corresponding maximum distance of the remaining group as $d(e_2, e_4)$=0.0816 and $d(e_2, e_3)$=0.0649. Since $d(e_2, e_3)<d(e_2, e_4)$, the fourth expert should be asked to change his/her preferences. If expert e_4 refuses to change his/her opinions, then we should exclude him/her from the group and let s^*=3, and calculate the majority degree of the group T=3/4≥τ. In this case, $d(e_2, e_3)$=0.0649 turns out to be the furthest distance and Δ=0.9351>δ, which implies that the new group reaches an acceptable consensus. Then, go to the next step.

Step 5: Calculate the overall scores as follows:

$$\begin{aligned}&\tilde{s}(h_1)=0.7320,\ \tilde{s}(h_2)=0.6884,\\&\tilde{s}(h_3)=0.7884,\ \tilde{s}(h_4)=0.7896.\end{aligned}$$

So, the best alternative is a_4. That is to say, the best way of selling is Trade-Ins on both Apple products and competing products.

We will use Algorithm 2 to solve this problem. Steps 1–5 have been carried out above, so we begin the calculation process from step 6.

Step 6: Find the alternative(s) where expert e_4's opinions are the most far away from those of the other experts. Then, we calculate the distances:

$$\begin{aligned}&d_{a_1}(e_3,e_4)=0.0979,\ d_{a_2}(e_3,e_4)=0.1577,\\&d_{a_3}(e_3,e_4)=0.0489,\ d_{a_4}(e_3,e_4)=0.2283.\end{aligned}$$

Thus, $a_{i^*}=\{4\}$. Then, we ask expert e_4 whether he/she is willing to change his/her assessment for the fourth alternative. As it is assumed that expert e_4 does not want to change his/her mind, go to the next step.

Table 5 Hesitant fuzzy decision matrix e_1

	c_1	c_2	c_3	c_4	c_5	AHFWA
a_1	{0.9, 0.8}	{0.8, 0.8}	{0.7, 0.5}	{0.7, 0.6}	{0.5, 0.5}	{0.7989, 0.7178}
a_2	{0.8, 0.7}	{0.8, 0.6}	{0.8, 0.6}	{0.6, 0.6}	{0.7, 0.7}	{0.7768, 0.6435}
a_3	{0.6, 0.6}	{0.9, 0.8}	{0.8, 0.7}	{0.8, 0.7}	{0.9, 0.7}	{0.8134, 0.7104}
a_4	{0.5, 0.5}	{0.9, 0.9}	{0.7, 0.6}	{0.8, 0.8}	{0.9, 0.8}	{0.7836, 0.7544}

Table 6 Hesitant fuzzy decision matrix e_2

	c_1	c_2	c_3	c_4	c_5	AHFWA
a_1	{0.9, 0.8}	{0.7, 0.7}	{0.8, 0.6}	{0.6, 0.6}	{0.7, 0.5}	{0.7952, 0.6952}
a_2	{0.8, 0.8}	{0.8, 0.6}	{0.7, 0.7}	{0.6, 0.6}	{0.7, 0.6}	{0.7579, 0.6933}
a_3	{0.9, 0.9}	{0.9, 0.9}	{0.6, 0.5}	{0.7, 0.5}	{0.4, 0.4}	{0.8238, 0.8061}
a_4	{0.6, 0.6}	{0.9, 0.9}	{0.7, 0.6}	{0.8, 0.8}	{0.6, 0.5}	{0.7675, 0.7482}

Table 7 Hesitant fuzzy decision matrix e_3

	c_1	c_2	c_3	c_4	c_5	AHFWA
a_1	{0.7, 0.5}	{0.8, 0.8}	{0.7, 0.7}	{0.8, 0.8}	{0.5, 0.5}	{0.7204, 0.6646}
a_2	{0.5, 0.7}	{0.6, 0.6}	{0.8, 0.6}	{0.8, 0.7}	{0.7, 0.7}	{0.6625, 0.5962}
a_3	{0.7, 0.7}	{0.9, 0.8}	{0.8, 0.8}	{0.7, 0.7}	{0.9, 0.7}	{0.8217, 0.7551}
a_4	{0.8, 0.8}	{0.9, 0.9}	{0.8, 0.7}	{0.8, 0.8}	{0.9, 0.9}	{0.8484, 0.8356}

Table 8 Hesitant fuzzy decision matrix e_4

	c_1	c_2	c_3	c_4	c_5	AHFWA
a_1	{0.9, 0.9}	{0.8, 0.8}	{0.7, 0.5}	{0.7, 0.7}	{0.6, 0.5}	{0.8034, 0.7773}
a_2	{0.8, 0.7}	{0.9, 0.9}	{0.8, 0.6}	{0.7, 0.7}	{0.5, 0.5}	{0.8146, 0.7595}
a_3	{0.6, 0.6}	{0.6, 0.6}	{0.9, 0.9}	{0.8, 0.7}	{0.9, 0.8}	{0.7538, 0.7252}
a_4	{0.6, 0.6}	{0.6, 0.6}	{0.7, 0.6}	{0.7, 0.7}	{0.5, 0.5}	{0.6248, 0.6026}

Step 7: Since there is only one alternative in A, expert e_4 on the fourth alternative is unreliable compared to the other experts' evaluations, and thus it needs to be removed. Then go back to step 2. We can calculate the distance between any pair of the experts and obtain the consensus degree $\varDelta$ of the group as follows:

$$d(e_1,e_2)=0.0279,\ d(e_1,e_3)=0.0615,$$
$$d(e_1,e_4)=0.0487,\ d(e_2,e_3)=0.0649,$$
$$d(e_2,e_4)=0.0607,\ d(e_3,e_4)=0.1015,$$
$$\varDelta=0.8985.$$

Because $\varDelta<\delta=0.9$ and $d(e_2, e_4)<d(e_2, e_3)$, the third expert should be asked to change his/her preferences. Finding the alternative(s) where expert e_3's opinions are the most far away from expert e_4's, we can calculate the distances:

$$d_{a_1}(e_3,e_4)=0.0979,\ d_{a_2}(e_3,e_4)=0.1577,$$
$$d_{a_3}(e_3,e_4)=0.0489.$$

Thus, $a_{i^*}=\{2\}$. Then, we ask expert e_3 whether he/she is willing to change his/her assessment for the second alternative. As it is assumed that expert e_3 does not want to change his/her mind, expert e_3 on the second alternative is unreliable compared to the other experts' evaluations and needs to be removed. Then go back to step 2.

We can calculate the distance between any pair of experts and obtain the consensus degree $\varDelta$ of the group as follows:

$$d(e_1,e_2)=0.0279,\ d(e_1,e_3)=0.0551,$$
$$d(e_1,e_4)=0.0487,\ d(e_2,e_3)=0.0545,$$
$$d(e_2,e_4)=0.0607,\ d(e_3,e_4)=0.0734,$$
$$\varDelta=0.9266>\delta.$$

The results aggregated using the AHFWA operators for each alternative under four experts are shown in Table 9. Then, go to the next step.

Step 8: Calculate the overall scores as follows:

$$\tilde{s}(h_1)=0.7461,\ \tilde{s}(h_2)=0.7409,$$
$$\tilde{s}(h_3)=0.7762,\ \tilde{s}(h_4)=0.7896.$$

So, the best alternative is a_4. That is to say, the best way of selling is Trade-Ins on both Apple products and competing products.

In this example, we obtain the same results from Algorithms 1 and 2. However, sometimes we may obtain different results through the two methods of excluding experts and removing the alternative from the expert.

5.2 Comparison analysis

In this part, we give some in-depth comparison analyses between our new algorithms and the existing algorithm for hesitant fuzzy group decision making with the consensus method.

The method of Zhang *et al*. (2015a) is as follows:

Step 1: Use the additive aggregation operator to obtain the group decision matrix $\tilde{\boldsymbol{H}}=(\tilde{h}_{ij})_{n\times m}$:

$$\tilde{h}_{ij}=\frac{1}{s}\sum_{1}^{s}h_{ij}^{k},\ i=1,\ 2,\ ...,\ n,\ j=1,\ 2,\ ...,\ m. \quad (16)$$

Step 2: Compute the consensus measure by the distance measure:

$$d(H^k,\tilde{H})=\frac{1}{nms}\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{s}\left|h_{ij}^{k}-\tilde{h}_{ij}\right|. \quad (17)$$

Step 3: Control the consensus degree by the following formula:

$$d(\boldsymbol{H}^k,\tilde{\boldsymbol{H}})\le 1-\delta. \quad (18)$$

Step 4: Identify the preference values which are calculated as follows:

$$\begin{aligned}\mathrm{IJ}&=\left\{(i,j)\left|d(h_{ij},\tilde{h}_{ij})>1-\delta\right.\right\}\\&=\left\{(i,j)\left|\frac{1}{s}\sum_{k=1}^{s}\left|h_{ij}^{k}-\tilde{h}_{ij}\right|>1-\delta\right.\right\}.\end{aligned} \quad (19)$$

The preference values $h_{ij}^{k}\in\tilde{h}_{ij}$ that should be changed are calculated by

$$\mathrm{IJK}=\left\{(i,j,k)\left|(i,j)\in\mathrm{IJ}\wedge d(h_{ij}^{k},\tilde{h}_{ij})>1-\delta\right.\right\}. \quad (20)$$

Step 5: Experts receive the recommendation. Then we obtain the new group decision matrix and rank the alternatives by

$$a_i=\sum_{j=1}^{m}\omega_j\tilde{h}_{ij}. \quad (21)$$

Here we use the additive aggregation operator to obtain the group decision matrix $\tilde{\boldsymbol{H}}=(\tilde{h}_{ij})_{n\times m}$ (Table 10).

Table 10 The group decision matrix $\tilde{\boldsymbol{H}}$

	c_1	c_2	c_3	c_4	c_5
a_1	{0.85}	{0.78}	{0.73}	{0.7}	{0.58}
a_2	{0.73}	{0.78}	{0.78}	{0.68}	{0.65}
a_3	{0.7}	{0.83}	{0.78}	{0.75}	{0.78}
a_4	{0.63}	{0.83}	{0.73}	{0.78}	{0.73}

We obtain the following results from the method of Zhang *et al*. (2015a) according to the above algorithm:

$$\tilde{s}(h_1)=0.763,\ \tilde{s}(h_2)=0.742,$$
$$\tilde{s}(h_3)=0.768,\ \tilde{s}(h_4)=0.735.$$

Table 9 The AHFWA operators for each alternative under five experts

	e_1	e_2	e_3	e_4
a_1	{0.7989, 0.7178}	{0.7952, 0.6952}	{0.7204, 0.6646}	{0.8034, 0.7773}
a_2	{0.7768, 0.6435}	{0.7579, 0.6933}	–	{0.8146, 0.7595}
a_3	{0.8134, 0.7014}	{0.8238, 0.8061}	{0.8217, 0.7551}	{0.7538, 0.7252}
a_4	{0.7836, 0.7544}	{0.7675, 0.7482}	{0.8484, 0.8356}	–

So, the best alternative is a_3. That is to say, the best way of selling is Trade-Ins on the same products like Apple.

All the results derived by different methods are listed in Table 11.

Table 11 The results derived by different methods

Method	Ranking
Our method 1	$A_4>A_3>A_1>A_2$
Our method 2	$A_4>A_3>A_1>A_2$
Zhang *et al.* (2015a)	$A_3>A_1>A_2>A_4$

From Table 11, we know that the three methods have derived different optimal alternatives and there are slight differences in the ranking order among the alternatives. In a method proposed by Zhang *et al.* (2015a), we can see that there are some drawbacks. For example, the step which generates suggestions to help the experts change their preferences wastes much time and sometimes the preferences are ineffective. In addition, the authors did not consider that the experts are unwilling to modify their preferences. In contrast, our two methods consider all the factors above, and thus are more comprehensive, reasonable, and practical. Based on the analyses, our methods have many advantages compared with the method proposed by Zhang *et al.* (2015a), which can be further summarized as follows:

1. More preference information is considered. We consider the situation in which each expert's preference information of alternatives a_i (i=1, 2, 3, 4) under attributes c_j (j=1, 2, …, 5) may be expressed as several possible values, which can be widely applied in many GDM problems without losing any expert's information. For example, expert e_1 provided two values 0.9 and 0.8 of alternative a_1 under attribute c_1 because of uncertainty and hesitancy. However, in this case, the values provided by expert e_1 are not taken into account in the method of Zhang *et al.* (2015a).

2. The consensus checking and reaching processes are more specific and flexible because of the following steps: We first ask the experts to discuss and adjust their preferences. If no one changes his/her preference information, then we use our consensus approaches whose computing processes are simpler and more accurate to find the experts who should modify their judgments or find the alternatives where the experts' opinions should be modified. If they are still not willing to do this, then we exclude their preference information, which is done over the experts one by one. The two methods consider all the possible situations whether the experts are willing to change their opinions or not, and we have a series of corresponding methods for these situations, which are reliable in a practical decision making process.

3. The preference information that needs to be modified is picked up reasonably. We ask the expert to change his/her preference information according to his/her own opinions, rather than following the fixed recommendations, which is more flexible and reasonable in a decision making process.

6 Conclusions

We have developed two consensus methods to solve the HFMCGDM problems with aggregation information. After giving the consensus measure, two interesting consensus-reaching processes have been established with step-by-step algorithms for HFMCGDM and have been given for applications. A numerical example concerning selecting selling ways about 'Trade-Ins' for Apple Inc. has been provided to illustrate and validate the developed approaches. Finally, we have made a comparison analysis to show the advantages of our two methods. Based on a numerical analysis between our methods and the existing hesitant fuzzy group decision making methodologies with a consensus method in the literature, we find that our methods are more comprehensive and convincing because we have considered the situation where each expert's preference information of alternatives a_i (i=1, 2, 3, 4) under criteria c_j (j=1, 2, …, 5) may be expressed as several possible values. In addition, we have proposed the consensus checking and reaching processes to find the experts who should modify their judgments. These processes can be done over the experts one by one. Thus, our methods allow to achieve consensus solutions before the selection process and can avoid some experts' preference values being too high or too low. In the future, we will apply our proposed consensus approaches to GDM using R language, linguistic term sets, etc.

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