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REVIEW ARTICLE

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Review of stochastic optimization methods for smart grid

Abstract This paper presents various approaches used by researchers for handling the uncertainties involved in renewable energy sources, load demands, etc. It gives an idea about stochastic programming (SP) and discusses the formulations given by different researchers for objective functions such as cost, loss, generation expansion, and voltage/V control with various conventional and advanced methods. Besides, it gives a brief idea about SP and its applications and discusses different variants of SP such as recourse model, chance constrained programming, sample average approximation, and risk aversion. Moreover, it includes the application of these variants in various power systems. Furthermore, it also includes the general mathematical form of expression for these variants and discusses the mathematical description of the problem and modeling of the system. This review of different optimization techniques will be helpful for smart grid development including renewable energy resources (RERs).

Keywords renewable energy sources, stochastic optimization, smart grid, uncertainty, optimal power flow (OPF)

1 Introduction

World population is increasing drastically and these huge mass of human also has a great consumption of energy. Therefore, to meet this requirement of increasing demand, renewable energy can be used along with fossil fuel. Fossil fuels are not enough to meet the increasing demand for a long time and their use is not advisable in environmental terms. Renewable energy resources (RERs) are the only solution to global warming and higher cost of fossil fuels issues [1]. They produce almost no waste products such as carbon dioxide or any other harmful chemicals. Since renewable energies have some technical and economic challenges such as reliability, as their generation depends on the availability of different RERs such as wind and solar, they involve unpredictable and random behavior. This makes the whole power system unpredictable. In this paper, different OPF methods, i.e., linear, nonlinear and advanced technologies are evaluated and compared.

RERs can also be used as optimized source of distributed generation [2]. As it is a decentralized generation, it can be generated near the customer point and the losses can be extensively minimized. The investment in transmission and distribution equipment will also be reduced due to the distributed generation approach. Moreover, the control action will also be fast and more reliable compared to the conventional generation approach. The distributed generation approach has benefited the deregulation of energy. Consumers are also getting benefits from the low cost and better quality of supply [3]. The distributed generation also helps in peak shaving. It has some major issues with safety and security, protection and control, efficiency and connection, etc. It can be used as standalone or grid connected source of energy. Renewable energy suffers from the reliability problem. Therefore, with storage into the system, the extra power generated can be stored and can be utilized when there is wind or solar power available. Researchers have tried to perform optimal power flow (OPF) operation with renewable energy and storage integrated into the system [4]. Even though renewable energy has been discussed by many researchers for various OPF algorithms [5,6], a study which reviews the stochastic programming (SP) with RERs in the smart grid context is required.

Stochastic nonlinear programming (S-NLP) is proposed by many researchers as it uses randomness as part of optimization processes [7–9]. The basic idea for solving the stochastic problem is to first convert the problem into

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an equivalent deterministic form so that the conventional optimization techniques can be applied to it [10]. Reference [10] presented the hybrid of two variants-recourse model and chance constrained programming (CCP) to deal with the complex problem of available transfer capability (ATC) evaluation. Reference [11] discussed the stochastic OPF using constraint relaxation, and the challenges attached to it. The algorithm presented solves this problem through potential benefits such as restricted attention to the small set of violated constraints through constraint relaxation. Besides, for each different contingency, it tests the newly violated line flow constraints without exclusively computing the distribution matrix for each contingency by using rank-one updates to calculate post contingency power flows. Stochastic OPF was first developed by Yong and Lasseter as described in Ref. [12]. The optimization was developed as a two stage SP problem. Reference [13] presents both linear and nonlinear formulations for the stochastic OPF. Reference [14] uses the SP approach for the vulnerability constrained transmission planning problem. The equivalent deterministic model using mixed integer nonlinear programming is utilized to find the optimal value.

The SP problem with deterministic constraints can be formulated as an expected value function subject to deterministic constraints [15]. Reference [15] discusses stopping rules and a validation analysis for sample average approximation (SAA) optimization procedures using some illustration examples. The recourse model is constructed through actions followed by observations and reaction. Another model, named as the chance constrained method, is developed by Cooper and Charnes using the statistical decision theory [16]. The size of stochastic program depends upon the number of realizations of random parameters presented in the problem. According to theory of “curse of dimensionality”, as the number of possible realizations increases, the time periods also increases exponentially [4].

Reference [17] formulates a stochastic problem for microgrid energy scheduling. It minimizes the expected operational cost of the microgrid and power losses while accommodating the intermittent nature of RERs. A hybrid stochastic/robust optimization model is proposed in Ref. [18] to minimize the expected net cost, i.e., expected total cost of operation minus total benefit of demand. This formulation can be solved by mixed-integer linear programming. A probabilistic model for small scale energy resources and load demand which is used to determine the optimal scheduling of microgrids with minimum operating cost is presented in Ref. [19]. In Ref. [20], the key features of microgrids and a comprehensive literature survey on the stochastic modeling and optimization tools for a microgrid are presented. Reference [21] proposes a stochastic model for optimal energy management with the goal of cost and emission minimization. In this model, the uncertainties related to the forecasted values for load demand, available output power of wind and photovoltaic units and market price are modeled by scenario-based SP. Reference [22] formulates a two-stage SP, where the first-stage is associated with the electricity market and its rules and constraints, and the second-stage is related to the actual operation of the power system and its physical limitations in each scenario. In Ref. [23], a novel stochastic probabilistic energy and a reserve market clearing scheme are proposed in the presence of plug-in vehicles (PEV) and wind power introducing a new model for PEV aggregators. In Ref. [24], the problem of modeling and stochastic optimization for home energy management is considered.

Reference [25] proposes a probabilistic model using the point estimate method to reduce the probability of congestions and voltage violations in a smart grid located in a radial distribution system. A detailed review of literature using agent based modeling and simulation techniques for analyzing smart grids from the perspective of the system is presented in Ref. [26]. A comprehensive review on mathematical modeling methods of photovoltaic (PV) solar cell/module/array which can be used for power system dynamic modeling purpose is given in Ref. [27]. Reference [28] proposes a robust optimization model for optimal self scheduling of a hydro-thermal generating company. A stochastic framework based on the cloud theory to handle the uncertainty effects in the optimal operation of microgrids is proposed in Ref. [29]. A review of various mathematical models proposed by different researchers is conducted in Ref. [30]. These models are developed based on objective functions, economics and reliability studies involving design parameters. Reference [31] proposes a unit commitment formulation for the microgrid using the two stage scenario based SP method. The day-ahead scenarios and hour-ahead scenarios providing the information about the location of electric vehicles, and the historical data are presented in Ref. [32]. In Ref. [33], a co-optimization based OPF solver is developed to solve over contingencies and renewable uncertainties. A stochastic model with high number of scenarios and the modeling of different strategies of EV integration are proposed in Ref. [34]. Reference [35] proposes the effects of uncertain renewable energy and loads on optimizing profit and cost in a microgrid power market. A cloud computing framework in a smart grid environment by creating small integrated energy hub supporting real time computing for handling huge storage of data is proposed in Ref. [36].

In Ref. [37], an optimal operation planning method taking into consideration the uncertainties of renewable power generations and load demand is proposed. In Ref. [38], the performance of four different scheduling algorithms is compared to integrate the electric vehicles into smart grid architectures by optimizing their charging and feeding periods. A dynamic optimization framework to formulate the optimal charging problem is proposed in Ref. [39]. A methodology for modeling and controlling of the load demand in a residential distribution grid due to plug-in hybrid electric vehicle (PHEV) battery charging and discharging is proposed in Ref. [40]. A statistical model based optimization for developing dynamic, stochastic, computationally efficient, and scalable platforms is proposed in Ref. [41]. Reference [42] proposes an innovative technology to handle the growing complexity of the smart grid and stochastic bidirectional OPFs to maximize the penetration of renewable energy and to provide maximum utilization of available energy storage, especially plug-in electric vehicles. Reference [43] proposes a stochastic energy scheduling model for a local area smart grid system with a single energy source and multiple energy consumers.

In this paper, various OPF methods i.e., linear, nonlinear and advance techniques are evaluated and compared for different objective functions such as cost, loss, generation expansion, voltage/V, etc. aiming at developing an OPF for RERs using the best technology available considering the uncertainty and randomness added to the system due to RERs. The SP scheme and its variants are studied in detail to deal with the uncertainty and randomness involved in the system. The objective of this paper is to provide an initial framework in the direction of smart grid with the help of SP to optimize the system with renewable energy sources.

2 Evaluation of stochastic optimization methods for smart grid

This section describes various SP techniques.

2.1 Recourse method

The recourse method is composed of more than one stage. By solving the first stage, some uncertainties will be resolved and other decision can be made based on the values obtained from the first stage and this will be continued till the last stage is solved. The most difficult part of this recourse problem is the evaluation of the expected value at each stage except the first stage [44]. In the recourse problem, one decision can be made now and the expected costs of consequences of the decision made in the first stage can be minimized. This method uses randomly generated observations from random variables to create statistical estimates. Stochastic decomposition methods can be further subdivided into the decomposition based method and the stochastic approximations based method.

Let x be a vector of decisions that must be taken, and $y(w)$ be a vector of decisions that describes the consequences of x. Two different set of y's can be chosen for each possible outcome w. The recourse problem formulation is expressed as [44]

$$\text{Minimize}, f_1(x) + E[f_2(\mathrm{y}(w)),w], \tag{1}$$

$$\text{s.t. } g_1(x) \leqslant 0, g_{\mathrm{m}}(x) \leqslant 0, \tag{2}$$

$$h_1(x,y(w)) \leqslant 0, \text{for all } w \in W, \tag{3}$$

$$h_K(x,y(w)) \leqslant 0, \text{for all } w \in W, \tag{4}$$

where $x \in X$, $y(w) \in Y$. The set of constraints h_1, h_2,..., h_k represent the links between the first stage decisions x, and the second stage decisions $y(w)$. Recourse models can be extended in a number of ways. The most common way is to include more stages. In the multi-stage problem, the decision is made now, waiting for some uncertainty to be resolved, and then another decision is made based on what has happened. The objective is to minimize the expected costs of all decisions made.

Solving a recourse problem is difficult. Especially, the hardest part is to solve the expected value expect the first stage [45]. In this model, the random constraints are modeled as 'soft constraint'. The possible violations are accepted but the cost of these violations should be added. If the recourse model does not change with the scenario, it is a fixed recourse problem [44].

2.2 CCP

CCP is an approach in which the probability distribution of random parameters are fixed and already known. This method is used where high uncertainty is involved and reliability has a higher priority than other issues. In such application, one would like to settle up with an optimal value that guarantees feasibility 'as much as possible' [46]. The CCP is expressed as [45]

$$P\ \{A^i(\omega)x \geqslant h^i(\omega)\} \geqslant \xi^i, \tag{5}$$

where $0 < \xi^i < 1$, $i = 1,2,...,$ which is an index of constraints. These constraints can be modeled in a general expectation form as $E_\omega\ (f^i(\omega,\ x(\omega))) \geqslant \xi^i$ where f^i is an indicator of $\{\omega / A^i(\omega)x \geqslant h^i(\omega)\}$. The objectives for CCP are in one of the following forms: (a) It can be an expectation function (E-model); (b) It can be the variance of some value (V-model); (c) It can be the probability of some occurrence (P-model); (d) It can be the quintile of a random function. In CCP, the generalized concavity theory plays a vital role, as it is inclusive of powerful tools for convex analysis [4].

The following is a step-by-step procedure to solve stochastic CCP.

Step 1: Create stochastic model

$$\text{maximize } z=\sum_{j=1}^{n}c_jx_j, \tag{6}$$

such that $p\left\{\sum_{i=1}^{n}a_{ij}x_j\leqslant b_i\right\}\geqslant 1-\alpha_i$, $i=1,2,\ldots,m$, $0<\alpha_i<1$. a_{ij} is normally distributed with a mean of $E\{a_{ij}\}$, a variance of $\text{var}\{a_{ij}\}$, and a covariance of $\text{cov}\{a_{ij},a_{i'j'}\}$.

Step 2: Define $h_i=\sum_{j=1}^{n}a_{ij}x_j$, and h_i is normally distributed with $E\{h_i\}=\sum_{j=1}^{n}E\{a_{ij}\}x_j$.

$$\text{var}\{h_i\}=X^{\text{T}}D_iX, \tag{7}$$

$$X=\{x_1,x_2,x_3,\ldots x_n\}^{\text{T}}, \tag{8}$$

Covariance matrix

$$D_i=\begin{bmatrix}\text{var}\{a_{i1}\} & \ldots & \text{cov}\{a_{i1},a_{in}\}\\ \cdots & \cdots & \cdots\\ \text{cov}\{a_{in},a_{i1}\} & \ldots & \text{var}\{a_{in}\}\end{bmatrix}, \tag{9}$$

$$\{h_i\leqslant b_i\}=P\left\{\frac{h_i-E\{h_i\}}{\sqrt{\text{var}\{h_i\}}}\leqslant\frac{b_i-E\{h_i\}}{\sqrt{\text{var}\{h_i\}}}\right\}\geqslant 1-\alpha_i, \tag{10}$$

where $(h_i-E\{h_i\})/\sqrt{\text{var}\{h_i\}}$ is a standard normal distribution with a mean of zero and a variance of one. This means that

$$P\{h_i\leqslant b_i\}=\Phi\left(\frac{h_i-E\{h_i\}}{\sqrt{\text{var}\{h_i\}}}\right), \tag{11}$$

where Φ represents the CDF of the standard normal distribution. Let K_{ai} be the standard normal value such that

$$\Phi(K_{ai})=1-a_i. \tag{12}$$

$\text{P}\{h_i\leqslant b_i\}\geqslant 1-\alpha_i$ is realized if and only if

$$\left(\frac{b_i-E\{h_i\}}{\sqrt{\text{var}\{h_i\}}}\right)\geqslant K_{\alpha i}. \tag{13}$$

This yields the following nonlinear constraint

$$\sum_{j=1}^{n}E\{a_{ij}\}x_j+K_{\alpha i}\sqrt{X^{\text{T}}D_iX}\leqslant b_i, \tag{14}$$

which is equivalent to the original stochastic constraint. For the special case where the normal distributions are independent,

$$\text{cov}\{a_{ij},a_{ij},a_{i'j'}\}=0, \tag{15}$$

and the last constraint reduces to

$$\sum_{j=1}^{n}\{a_{ij}\}x_j+K_{\alpha i}\sqrt{\sum_{j=1}^{n}\text{var}\{a_{ij}\}x_j^2}\leqslant b_i. \tag{16}$$

This constraint can now be put in the separable programming form by using the substitution

$$y_i=\sqrt{\sum_{j=1}^{n}\text{var}\{a_{ij}\}x_j^2}, \tag{17}$$

$$P\left\{b_i\geqslant\sum_{j=1}^{n}a_{ij}x_j\right\}\geqslant\alpha_i, \tag{18}$$

$$P\left\{\frac{b_i-E\{b_i\}}{\sqrt{\text{var}\{b_i\}}}\geqslant\frac{\sum_{j=1}^{n}a_{ij}x_j-E\{b_i\}}{\sqrt{\text{var}\{b_i\}}}\right\}\geqslant\alpha_i. \tag{19}$$

The necessary condition for this is

$$\frac{\sum_{j=1}^{n}a_{ij}x_j-E\{b_i\}}{\sqrt{\text{var}\{b_i\}}}\leqslant K_{\alpha i}. \tag{20}$$

Therefore, the stochastic constraint is now converted into the deterministic linear constraint as

$$\sum_{j=1}^{n}a_{ij}x_j\leqslant E\{b_i\}+K_{ai}\sqrt{\text{var}\{b_i\}}. \tag{21}$$

2.3 Monte Carlo simulation (MCS) / sample average approximation (SAA)

Stochastic programming incorporates deterministic optimization with random variables and probabilistic constraints. Large size of these problems involves a very large number, and sometimes an infinite number of scenarios. Eventually, this may lead to intractable models which require specially designed algorithm to solve them. The Monte Carlo sampling based method is suggested for such kind of problem to limit the number of scenarios and still obtain reasonable solutions [7,47]. The Monte Carlo simulation (MCS) may be the best solution to estimating the expectation function [8]. When there exists a very large number of scenarios, the sample of N replications of a random variable δ, i.e., δ^1, δ^2,..., δ^N can be generated. The sample generated can also be stored in computer memory and alternatively, it can also be generated by using the common random number generation method. This method is explained clearly in Ref. [4]. However, the result obtained using this method does not give the assurance of quality. δ^j (j = 1, 2,..., N) has the identical probability distribution as δ. If δ^j is mutually exclusive, the sample evaluated is IID (independently and identically distributed). For a sample evaluated, the expectation function $p(x) = E[P(x, \delta)]$ can be approximated by the average,

$$\hat{p}_N(x) = N^{-1}\sum_{j=1}^{N} P(x,\delta^j). \tag{22}$$

This method is also known as sample average approximation (SAA). By the law of large numbers (LLN), under some conditions, $\hat{p}_N(x)$ converges point wise with point 1 to $p(x)$ as $N\to\infty$. This is true if the sample is IID according to LLN and the convergence is uniform [4]. SAA is not an algorithm for solving any SP. Therefore, a numerical procedure should be followed. The recourse function of any stochastic problem can be replaced by Monte Carlo estimate to solve it with the SAA method [45]. This approach gives reasonably accurate results if its variability is not too large and it has a relatively complete recourse [4]. The sample average functions $\hat{p}_N(x)$ on a probability space (Ω, $\mathcal{P}$, P) can be defined. The advantage of this method is that it does not require any tuning parameter. However, this method is computationally expensive and it requires nonlinear optimization software.

MCS is a technique that uses random numbers and their PDFs to solve problems. This method is often used when the model is complex, nonlinear, or involves many uncertain parameters. A comprehensive approach to evaluating the system reliability concerning the stochastic modeling of plug-in-hybrid-electric vehicles (PHEVs), renewable resources, availability of devices, and etc. is proposed in Ref. [48]. In addition, a novel risk management method in order to reduce the negative PHEVs effects is introduced. Reference [49] deals with the issues concerning transaction costs and financial risks by introducing a MCS approach to risk analysis based on an entire life-cycle representation of renewable energy technology investment projects. Reference [50] proposes a methodology that uses MCS to estimate the behavior of economic parameters which may help decision-making, considering the risk in project sustainability. The flowchart for handling the uncertainty using the MCS method in the smart grid context is shown in Fig. 1.

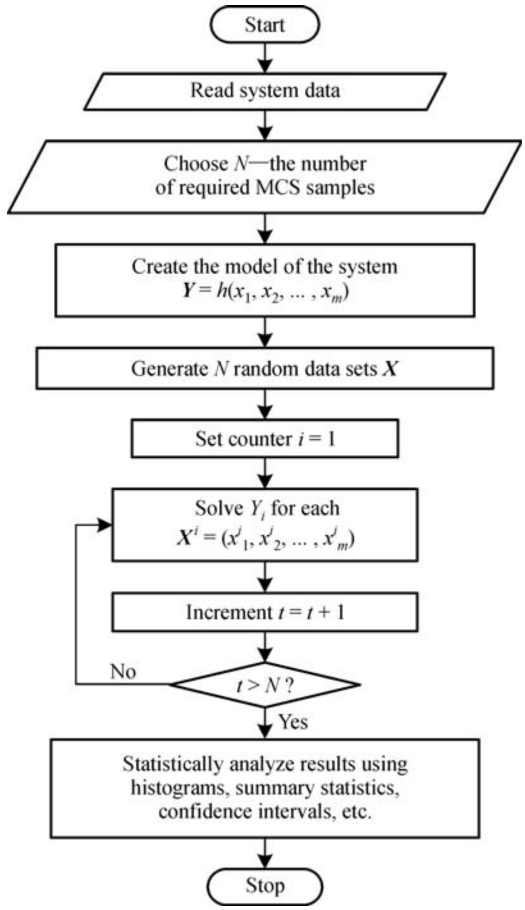

Fig. 1 Flowchart for handling the uncertainty using MCS in the smart grid context

2.4 Risk averse optimization

Risk averse optimization is based on the economics theory of expected utility [4]. The risk averse method does not depend upon the law of large numbers. In this method, two random outcomes are compared by expected values of some scalar transformation f: $R \to R$ of the realizations of these outcomes. A random outcome Y_1 is preferred over Y_2 if

$$E[f(Y_1)]<E[f(Y_2)]. \tag{23}$$

The function f is known as disutility function, and is assumed as non-decreasing and a convex function for the minimization function, and vice versa for the maximization function. Therefore, the objective function can be formulated as

$$\underset{x\in X}{\text{Min}}\, E[f(F(x,w))]. \tag{24}$$

As function f is a convex function, by using Jensen's inequality,

$$f(E[F(x,w)])\leqslant E[f(F(x,w))]. \tag{25}$$

So, the sure outcome of $E[F(x,w)]$ is similar to random outcome $F(x,w)$.

3 Comparison of SP techniques

The use of SP application depends upon the parameters such as application, accuracy, CPU time, robustness, convexity, continuity and stability properties. A comparison of parameters of SP techniques is given in Table 1. The structure of the stochastic problem expands proportionally to the number of possible realizations of uncertain parameters [4], which eventually increases exponentially in the number of time periods and parameters. Besides, some of the methods may be very efficient for small size problems. However, they may not give similar results for large scale problems. The emphasis in this paper is to check the validity of selected methods for various applications from small scale to large scale. Table 2 presents the comparison of various SP variants in terms of initialization, necessary conditions and feasibility.

4 Probabilistic OPF (P-OPF)

OPF is a tool to optimize a power system objective over power network variables under certain constraints. It is a gadget for better reliability and control of power systems. It is used at each phase of power systems for better reliability and control. Renewable energy is a kind of pollution-free, economic and easily available source of energy. The output received from RERs flutters widely, rapidly and randomly which results in an increased operational complexity. Moreover, the availability of RERs is usually in remote locations. Therefore, transmission losses are very high as the power has to be transmitted over a long distance. Researchers have considered the storage technologies as a solution to the above problems [1]. Some work has also been done to use storage in the OPF problem either as a constraint or in objective formulation with the integration of renewable energy and micro grid [2–5].

In recent years, point estimate methods have been widely used to solve P-OPF problems. The aim of any point estimate method is to compute the moments of a random variable Z that is a function of m random input variable p_l, i.e., $Z = F(p_1, p_2, ..., p_m)$. The first point estimate method was developed by Rosenblueth in 1975 [51] for

Table 1 Comparison of SP variants [9–13]

Parameters	Recourse model	Chance constrained/ probability constraint programming	Sample average approximation/ Monte Carlo simulation	Risk aversion
Handling of nonlinearity	It can efficiently handle nonlinearities	It can efficiently handle nonlinearities	Most of the approximation methods involve some discretization of an underlying probability function	It can handle the nonlinearity through the linearization which may approximate nonlinear utilities
Complexity	The fixed recourse model is simple and easily applicable for any stochastic problem	It involves complexity. Specially, it involves more complexity in joint CCPs compared to recourse problems	Choosing approximations and models is difficult and deriving solutions for models based on the discrete approximations are complex	It involves less complexity compared to other methods
Convex problems	It is efficient for convex problems (with discrete random variables, and piecewise linear)	It can handle convex problems	It can handle convex problems	It can handle convex problems
Large scale problems	It can be applied to large scale problems	It can be applied to large scale problems	It can either be applied to large sample approximation, where asymptotic statistical characteristics may apply or be implemented with small sample batches attached to an algorithm that converges asymptotically	It is not very efficient for large scale problems
Iterations	For fixed recourse, the number of iterations is less compared to other methods	It depends upon the number of probabilistic constraints	It depends upon the number of samples selected	After each stage, the size of the problem grows, by applying multi cut strategy, and the number of iterations can be minimized
Speed	It leads to a considerable size expansion of the problem and eventually increases computational burden, if the variables are continuous	It is a very less time consuming method	It is inversely proportional to the number of samples	High
Accuracy	It is a highly accurate method	The accuracy is lower than the recourse model but higher than other models	It depends upon the size of sampling	High
Robustness	It is a robust method. It involves different scenarios and it is suitable for all conditions	Good	Robustness depends upon the sample size. It is very good for a large number of samples	Good

symmetric variables and was later revisited in 1981 [52] to consider asymmetric variables. In Ref. [53], four different Hong's point estimate schemes are presented and tested on the probabilistic power flow problem. Reference [54] presents an application of a two-point estimate method (2PEM) to account for uncertainties in the OPF problem in the context of competitive electricity markets. The 2PEM needs $2n$ runs of the deterministic OPF for n uncertain variables to get the result in terms of the first three moments of the corresponding PDFs. More details about the MCS and the point estimate methods can be found in Refs. [55–57].

OPF problems involve different power system objectives such as cost minimization, environmental dispatch, maximum power transfer, reactive power objectives — minimization of MW and MVA losses, general objectives — minimum deviation from a target schedule, minimum control shifts, least absolute shift approximation of control shift, and constraints such as limits on control variables — generator output in MW, transformer tap limits, shunt capacitor limits, operating limits on line and transformer flows — MVA, amps, MW and MVA, MW and MVA reserve margins, voltage and angle, control parameters — control effectiveness, limit priorities through engineering rules and operating limit enforcement, voltage stability limits, local and non optimized controls — generator voltage, general real power, transformer output voltage, MVA and shunt/SVC controls, equipment ganging and sharing — tap changing, generator MVA sharing, and control ordering [6]. It is a mathematical form of any power system problem, and the optimal solution leads to the ideal operation of the power system. For any typical OPF problem, first, all inputs need to be modeled so that it can fit to the mathematical form. According to the desired form of output, objective functions and constraints should be defined. The problem formulated can either be solved by using classical methods such as linear programming, nonlinear programming, quadratic programming, integer programming, dynamic programming or any of the advanced methods of optimization such as adaptive dynamic programming, evolutionary programming, artificial intelligence methods and heuristic programming, etc. Table 3 is generated for comparison of the above methods for different objective functions. It gives the guidelines for selecting a method for a given optimization problem and its relative application.

One needs to be aware of the optimization formulations relative to each objective function. Table 4 is prepared to satisfy the same need and it can be customized to fit into the system requirement.

OPF formulations should be tested for various contingencies [47,70] in order to evaluate the robustness of solution for different situations. As the power system should be designed to work satisfactorily under contingencies such as line outage, generation outage, transformer problems, and relay malfunctions, etc. For a typical OPF problem, the base case cost is mostly higher when contingencies are imposed as the system behaves conservatively. As a matter of fact, it is not possible to model each possible contingency. Therefore, usually

Table 2 Comparison of SP variants in terms of initialization, necessary and feasible conditions

SP variant	Recourse model	Chance constrained programming	Sample average approximation	Risk averse optimization
Initialization	$Z=X+E_{\xi}\theta Y$ with $\xi^{\mathrm{T}}=(\xi^1,\xi^2,\ldots)$ with probability $p^{\mathrm{r}}=(p^1,p^2,\ldots)$	Min $z^P(x,\ \xi^r,\ \xi^k)=c^{\mathrm{T}}x+p^rq^{\mathrm{T}}y(\xi^r)+(1-p^{\mathrm{r}})q^{\mathrm{T}}y(\xi^k)$	Min $z=c^{\mathrm{T}}x+\theta$ such that constraints with ξ uniformly distributed	Min $c^{\mathrm{T}}x+q^{\mathrm{T}}y$: $Ax+By=z(w)$, $x\in X$, $y\in Y$
Necessary condition	Solve equations $E_{s+1}=\sum_{k=1}^{K}p_k.(\pi_k^v)T_k$ and $e_{s+1}=\sum_{k=1}^{K}p_k.(\pi_k^v)$	$\mathrm{SPEV}\geqslant p^rz_r^*+\sum_{k\neq r}p^kz_k^*=\mathrm{WS}$	Jensen lower bound and Edmundon-Madansky upper bound $£_j^{\mathrm{L}}(x^v)=\int_{\Omega}Q_j^{\mathrm{L}}(x^v,\xi)P_j^{\mathrm{L}}\mathrm{d}\omega$ $£_j^{\mathrm{U}}(x^v)=\int_{\Omega}Q_j^{\mathrm{U}}(x^v,\xi)P_j^{\mathrm{U}}\mathrm{d}\omega$	decide x–observe ω–decide $y=y(x,\ \omega)$
Feasibility check	$\theta^v\geqslant w^v$?	–0 $\leqslant$EVRS–EPEV $\leqslant$VSS $\leqslant$EVRS–SPEV $\leqslant$EVRS–WS	$\theta^v\geqslant £_j^{\mathrm{L}}(x^v)$ and $\theta^v\geqslant £_j^{\mathrm{U}}(x^v)$	Risk measures satisfied?
Update	$s=s+1$, add feasibility cut	ξ^r, ξ^k	$s=s+1$, add feasibility cut	Fix x for non-anticipatively
Optimization	x* is optimal solution	x*	x* is optimal solution	$\hat{x}^*$ is the optimal solution

Table 3 Comparison of OPF methods [15,16], [45,46], [58–61]

	Classical methods—linear programming	Classical methods—nonlinear programming	Intelligent systems—artificial neural network	Intelligent systems—genetic algorithm	Intelligent systems—ant colony	Intelligent systems—article swarm optimization
Objective functions	Generation cost, power generation dispatch, power generation scheduling, power transmission planning, economic dispatch (minimum cost allocation of demand), locational marginal price (LMP)	Generation fuel cost minimization, minimization of power losses, minimization of production cost, minimization of generation cost, LMP, economic dispatch	Reactive power and voltage control, generation scheduling, generation cost, loss minimization, reactive power dispatch, cost minimization	Transmission loss minimization, reactive power control, unit commitment, economic dispatch and load allocation, voltage control, optimal allocation of static VAR compensators	Loss reduction, optimal load shedding, voltage stability, voltage control, economic load dispatch, optimal capacitor placement, VAR planning	Economic emission dispatch, reactive power optimization, generation cost minimization, minimize fuel cost, voltage stability enhancement, minimization of polluted gas emission, reactive loss minimization
Merits	They can efficiently solve local constraints; they are reliable for most engineering applications	They give solutions involving nonlinearity in the objective function as well as constraints; they are better fitted for highly constrained methods	It requires less training compared to other intelligent systems; it can detect nonlinear relationships between dependent and independent variables	It does not require any external rules because it works on internal rules	It has inherent parallelism; it is useful for traveling salesman problem and similar applications; it can efficiently handle dynamic problems	It has very less computational burden; it is a simple method and easy to implement; it is a robust method compared to other methods
Demerits	They are less accurate compared to other methods; they cannot handle the randomness and stochastic nature of the problem; there is no correlation among variables	They have slow convergence near optimal solution; they are not suitable for multi objective optimization problems	It has a "black box" kind of structure, so the results obtained are highly questionable; lots of computations are involved	It does not give exact solution; it is a slow method	Theoretical analysis is difficult; probability distribution changes by iteration	It is less accurate; it is not efficient for local search space
Significant contribution	They make best possible use of time, machines, labor, etc.	The introduction of Krush Kuhn Tucker conditions and penalty function gives the optimal solution	It is used to model complex relationships between input and output and discover patterns in data	It is useful when the search space is large and complex; it is useful when mathematical analysis is not possible	Convergence is guaranteed but the time required for convergence is uncertain	It has flexibility in global and local control exploration of the search space
Nonlinear problems	The objective function and constraints must be linear	They can solve nonlinear problems	It can solve nonlinear problems	It can solve nonlinear problems	It can solve nonlinear problems	It can solve nonlinear problems
Uncertainty and randomness	They cannot handle the uncertainty and the randomness in the problem	They cannot handle the uncertainty and the randomness in the problem	It can work for problems involving uncertainty and randomness	It can work for problems involving uncertainty and randomness	It can work for problems involving uncertainty and randomness	It can work for problems involving uncertainty and randomness
Speed	The solution can be achieved very fast for some of the applications	They have slow convergence compared to other methods	It depends on the method used and nature of the problem; it has slow convergence when the optimal solution is far from local maxima/minima	It is a slow method	It requires very high computational time	It is a fast method as low computation is required and it has a fast convergence speed

(*Continued*)

	Classical methods—linear programming	Classical methods—nonlinear programming	Intelligent systems—artificial neural network	Intelligent systems—genetic algorithm	Intelligent systems—ant colony	Intelligent systems—article swarm optimization
Working principal	They work on mathematical principal; they work for linear objective and linear constraint; the constraints may be an equality or an inequality function. some variables may be restrictive to non-negative	They are mathematical methods that can solve linear and nonlinear objective functions and constraints	It mimics the behavior of biological neurons; it is basically an adaptive network that accepts information internally and externally; it uses weights to change the network parameter	It mimics the Darwinian selection process; it gives solutions to optimization process through simulation of natural evaluation phases such as inheritance, mutation, selection and cross over	It is a meta heuristic optimization method based on ant behavior seeking a path through their colony in search of food	It is a stochastic optimization method mimicking the social behavior of bird flocking and fish schooling
Methods	Graphical method, duality, integer programming, Langrangian, simplex method, etc.	Branch and bound, Langrangian, mixed integer programming etc.	Clustering, back propagation, feed-forward	—	—	—
Applicatio-ns (power systems)	Power system planning, operation and economics problems	Optimal control, power dispatch, unit commitment, and power generation planning problems	Online-load-frequency control, security assessment, voltage stability, minimization of power losses, fault analysis, and etc.	Loss minimization, power system planning, operation, maintenance and control applications	Real time applications along with energy management system, power loss minimization, power system planning and design applications	Economic dispatch, power system operation, and voltage control
Real world large scale power system problems	They are not able to solve these problems as they require complex mathematical work and they are unable to solve multi objective problems	They are not able to solve these problems as they require complex mathematical work and they are unable to solve multi objective problems	The convergence is not guaranteed when optimal solution is very far from local minima; so, it cannot be used for real life applications	It is useful for most of the real life applications	It is useful for real world applications; it is not useful for large size applications	It is capable of solving large problems

researchers account for most probable contingencies only depending upon particular power system operation.

The OPF methods presently under research require the accommodation of load flow control and monitoring devices such as load tap-changing/phase-shifting transformers, the modern flexible AC transmission system (FACTS), the phasor measurement unit (PMU), renewable energy, smart meters and storage and many other advanced communication devices. It is the advanced distribution management system (DMS) and the SCADA technology that support the smart grid. The advance DMS extends its work from transmission to distribution and addresses control functions such as reactive dispatch, voltage regulation, contingency analysis and ranking, capability maximization or line switching. Improved and advanced OPF tools also address the requirements of smart distribution network which can also be attached to the advanced DMS.

5 Conclusions

This paper presented the review of stochastic optimization techniques for use in the smart grid. It aimed to provide the initial work in this direction with the help of SP to optimize a system with renewable energy involvement. It reviewed OPF techniques and formulations given by various researchers, the relative pros and cons, the applications, the adaption of these techniques, and formulations for large and complex systems, etc. Though the review of SP has been presented in literature, a detailed review and formulations in presence of variability and uncertainty of renewable energy sources are not presented. Therefore, this paper made an attempt at bridging this gap. It presented the development of SP variants and their application, algorithm and comparison. It also discussed different variants of SP in detail. It compared these variants stage by stage from the initialization point to the optimization point.

Table 4 Various optimization problem formulations for specific objective using different methods

Objective function	Optimization methods		
	Nonlinear programming [62]	SP-benders decomposition [63]	Genetic algorithm [64]
Cost	$f(\cdot) = -\sum_{i=1}^{n}\sum_{j=1}^{n} G_{ij}\{(f_i - f_j)^2 + (e_i - e_j)^2\},$ $-\sum_{i=1}^{n}\sum_{j=1}^{n} B_{ij}\{(f_i - f_j)^2 + (e_i - e_j)^2\},$ $\sum_{i\in S_G}(a_{2i}P_{Gi}^2 + a_{1i}P_{Gi} + a_{0i}),$ $\sum_{i\in S_G} P_{Gi},$ power flow equations: $P_{Gi} = P_{Di} + \sum_{j=1}^{n}(e_i(e_jG_{ij} - f_jB_{ij}) + f_i(f_jG_{ij} + e_jB_{ij}),$ $Q_{Ri} = Q_{Di} + \sum_{j=1}^{n}(f_i(e_jG_{ij} - f_jB_{ij})e - e_i(f_jG_{ij} + e_jB_{ij}),$ system constraints: $P_{ij}\min \leqslant P_{ij} \leqslant P_{ij}\max,$ $P_{ij} = (e_i^2 + f_i^2 - e_ie_j - f_if_j)G_{ij} + (e_if_j - e_jf_i)B_{ij},$ $V_{i\min}^2 \leqslant (e_i^2 + f_i^2) \leqslant V_{i\max}^2,$ $P_{Gi\min} \leqslant P_{Gi} \leqslant P_{Gi\max},$ $Q_{Gi\min} \leqslant Q_{Gi} \leqslant Q_{Gi\max},$ $Q_{Ri\min} \leqslant Q_{Ri} \leqslant Q_{Ri\max},$	Minimize$\sum_{k\in K}\sum_{j\in J}\{F_jv_j(k) + A_jv_j(k) + E_j(t_j(k))\}$ s.t. $\sum_{j\in A_n} t_j(k) + \sum_{p\in\Omega_n} B_{np}(\delta_p(k) - \delta_n(k)) + \sum_{i\in x} H_iu_i(k) - \sum_{p\in\Omega_n} K_{np}(1 - \cos(\delta_p(k) - \delta_n(k))), = D_n(k) : \phi_n(k),$ spinning reserve constraint per period is $\sum_{j\in J}\overline{T}_jv_j(k) + \sum_{i\in I}\prod_i\overline{U}_i \geqslant \sum_{n\in N} D_n(k) + R(k) \quad \forall k \in \mathcal{K},$ $\underline{T}_jv_j(k) \leqslant l_j(k) \leqslant \overline{T}_jv_j(k) \ \forall j \in J, \forall k \in \mathcal{K},$ $-C_{np} \leqslant B_{np}(\delta_p(k) - \delta_n(k)) \leqslant C_{np} \ \forall n \in N, \forall p \in \Omega_n, \forall k \in \mathcal{K},$ hydraulic continuity equations $x_i(k) = x_i(k-1) - u_i(k) + W_i(k) + \sum_{p\in\Omega_i} u_p \ \forall k \in K, \forall i \in I$ $\underline{X}_i \leqslant x_i(k) \leqslant \overline{X}_i \ \forall k \in K, \forall i \in I$ $y_i(k) \geqslant v_j(k) - v_j(k-1) \ \forall k \in K, \forall j \in J$ $v_j(k), y_j(k) \in \{0,1\} \ \forall k \in K, \forall j \in J$	$f_i(P_i) = a_iP_i^2 + b_iP_i + c_i;$ $\sum_{i\in TPP} P_i = P_{\text{load}} + P_{\text{loss}} - \sum_{k\in HPP} P_k^{\text{Scheduled}};$ $P_i^{\min} \leqslant P_i \leqslant P_i^{\max}$

Objective function	Optimization methods		
	Interior point method [65]	Extended conic quadratic programming [66]	SP-benders decomposition [67]
Loss	Min $f(x) - \mu^k\sum_{j=1}^{p}(\ln s_{1j} + \ln s_{2j})$ $-\mu^k\sum_{j=1}^{p}(\ln s_{3j} + \ln s_{4j})$ such that $g(x)=0$; $-s_1 - s_2 - h^{\min} + h^{\max} = 0;$ $-h(x) - s_2 + h^{\max} = 0;$ $-s_3 - s_4 - \hat{x}^{\min} + \hat{x}^{\max} = 0;$ $-\hat{I}_X - s_4 + \hat{x}^{\max} = 0$	Minimize P_s s.t. $P_i = P_{Gi} - P_{Di},$ $Q_i = Q_{Gi} - Q_{Di} + Q_{Ci},$ $P_{Gi}\min \leqslant P_{Gi} \leqslant P_{Gi}\max,$ $Q_{Gi}\min \leqslant Q_{Gi} \leqslant Q_{Gi}\max$	Min$\underline{1}^T.\varepsilon_k$ s.t.$g_k(x_k^0, u_0 + \varepsilon_k) = 0,$ $h_k^{\min} \leqslant (x_k^0, u_0 + \varepsilon_k) \leqslant h_k^{\max},$ $1^T.\varepsilon_k + \lambda(u_0^* - u_0) \leqslant 0$

(*Continued*)

	Optimization methods		
	Sequential quadratic programming (SQP) [68]	Nonlinear programming [69]	Mixed integer nonlinear programming [70]
Generation capacity/ expansion	$f\ (P_{\mathrm{G}},\ P_{\mathrm{T}}) = \sum_{\mathrm{g}} C_{\mathrm{g}}(P_{\mathrm{g}}) + \sum_{\mathrm{T}} C_{\mathrm{T}}(P_{\mathrm{T}})$, $\sum_{\mathrm{t}}(P_{\mathrm{t}k} + jQ_{\mathrm{t}k}) + \sum_{\mathrm{g}}(P_{\mathrm{g}k} + jQ_{\mathrm{g}k}) + \sum_{\mathrm{d}}(P_{\mathrm{d}k} + jQ_{\mathrm{d}k}) = 0$ $V_{\mathrm{b}}^{\min} < V_{\mathrm{b}} < V_{\mathrm{b}}^{\max}$, $LB_{\mathrm{g}} < P_{\mathrm{g}} < UB_{\mathrm{g}}$, $S_{\mathrm{t}} < S_{\mathrm{t}}^{\max}$, $P_{\mathrm{T}} < P_{\mathrm{T}}^{\max}$, $Q_{\mathrm{T}}^{\min} < Q_{\mathrm{T}} < Q_{\mathrm{T}}^{\max}$	$\min_{P_{\mathrm{G}i}} \left\{ \sum_{i \in \mathrm{G}} C_{\mathrm{G}i}(P_{\mathrm{G}i}) + \sum_{i \in \mathrm{G}+} I\mathrm{G}_{\mathrm{G}i}(P_{\mathrm{G}i}) \right\}$ s.t. $\sum_{i \in \mathrm{G}}(P_{\mathrm{G}i} + Q_{\mathrm{G}i}) = \sum_{s \in \mathrm{D}}(P_{\mathrm{Ds}} + Q_{\mathrm{Ds}})$ $\lvert I_{\mathrm{s}}^{f}(\tilde{Z}_{\mathrm{SFCL}}) \rvert \leqslant I_{\mathrm{s}}^{\max}$, $V_{\mathrm{s}}^{\min} \leqslant V_{\mathrm{s}} \leqslant V_{\mathrm{s}}^{\max}$, $f_{\mathrm{srk}} \leqslant f_{\mathrm{srk}}^{\max}$, $P_{\mathrm{G}i}^{\min} \leqslant P_{\mathrm{G}i} \leqslant P_{\mathrm{G}i}^{\max}$, $Q_{\mathrm{G}i}^{\min} \leqslant Q_{\mathrm{G}i} \leqslant Q_{\mathrm{G}i}^{\max}$	$F_1 = \sum_{i=1}^{N\mathrm{G}} a_i + b_i P_{\mathrm{G}i} + c_i P_{\mathrm{G}i}^2$ (\$), $F_2 = P_{\mathrm{Loss}}(x,u) = \sum_{l=1}^{Nl} P_l$, $F_3 = \rho(x,u)$, $F_4 = \frac{C \times S \times 1000}{8760 \times 5}$ (\$), s.t. $P_{\mathrm{G}i} - \rho P_{\mathrm{D}i} = f_{pi}(x,u)$, $Q_{\mathrm{G}i} - \rho Q_{\mathrm{D}i} = f_{\mathrm{Q}i}(x,u)$, $V_{\mathrm{G}i}^{\min} \leqslant V_{\mathrm{G}i} \leqslant V_{\mathrm{G}i}^{\max}\quad i \in 1,\ldots,NG$, $P_{\mathrm{G}i}^{\min} \leqslant P_{\mathrm{G}i} \leqslant P_{\mathrm{G}i}^{\max}\quad i \in 1,\ldots,NG$, $Q_{\mathrm{G}i}^{\min} \leqslant Q_{\mathrm{G}i} \leqslant Q_{\mathrm{G}i}^{\max}\quad i \in 1,\ldots,NG$, $V_{\mathrm{L}i}^{\min} \leqslant V_{\mathrm{L}i} \leqslant V_{\mathrm{L}i}^{\max}\quad i \in 1,..,Nd$, $S_{\mathrm{L}i} \leqslant S_{\mathrm{L}i}^{\max}\quad i = 1,\ldots,\ Nl$, PST : $\sigma^{\min} \leqslant \sigma \leqslant \sigma^{\max}$, $S_{\mathrm{PST}} \leqslant S_{\mathrm{PST}}^{\max}$, HFC : $k^{\min} \leqslant k \leqslant k^{\max}$, $0 \leqslant K_C \leqslant K_C^{\max}$, $0 \leqslant K_{\mathrm{L}} \leqslant K_{\mathrm{L}}^{\max}$, $0 \leqslant K_{\mathrm{m}} \leqslant \mathrm{K}_m^{\max}$, $S_{\mathrm{HFC}} \leqslant S_{\mathrm{HFC}}^{\max}$, UPFC : $r^{\min} \leqslant r \leqslant r^{\max}$, $\gamma^{\min} \leqslant \gamma \leqslant \gamma^{\max}$, $S_{\mathrm{UPFC}} \leqslant S_{\mathrm{UPFC}}^{\max}$

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