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RESEARCH ARTICLE

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A new density model of *Cryptomeria fortunei* plantation

Abstract According to the volume increase model of an average individual tree in a plant population and the theory of invariable final output, we put forward a new density model of plant population: $V^{-\beta}=AN^{\beta}+B$. Here N means the stand density and V stands for average individual tree volume; A, B and β are parameters that change with growth stage. Using the density variation of standard plots of *Cryptromeria fortunei* plantation to verify the new model, it turns out that this model can well simulate the population density effect law of *C. fortunei* plantation, and it is markedly better and shows higher accuracy than the commonly used reciprocal model of density effect and secondary-effect model. Let β=1, we can obtain the reciprocal model of density effect, which means the reciprocal model of density effect is only a special case of this new model.

Keywords density effect, model, *Cryptromeria fortunei*

1 Introduction

The density effect is the foundation of the stand density control theories. After a great deal of research on this subject, the density effect rule has been understood and some mathematic models (density effect models) that describe the relationship between the average individual tree volume (V), the average diameter, the per hectare growing stock and the stumpage density (N) have been put forward (Yoda, 1963; Yoshitatsu, 1963; An, 1968; Drew and Flewelling, 1977; Yin et al., 1978; Liu, 1980; Yin, 1984; Chen et al., 1992; Meng, 1996). The most typical reciprocal model of density effect $V^{-1}=AN+B$ was put forward by Yoshitatsu (1963), in which A and B are the parameters that change with stand growth stages separately. This model reveals the size of an individual plant in one population and the relationship between the gross harvest yield and the plant population density, which becomes an important theoretical criterion for making stand-density-control maps and is used widely. Then Drew and Flewelling (1977), based on the Japanese density effect theory, demonstrated anew the reciprocal model of density effect according to biological hypothesis and dimensional analysis.

The research indicates that the Logistic equation is the basic hypothesis of the reciprocal model of density effect, which means the growth of average individual follows the common Logistic curve equation. However, the maximum of the increase rate is at the inflexion of the Logistic curve, and the relative increase rate of plant population biomass and the size of plant population follow the linear hypothesis, which sometimes does not agree with the increase model of the plant population. So the reciprocal model of density effect based on the Logistic equation has some limitations in theory, which may be one of the reasons that the stand-density-control map is not accurate enough.

The *Cryptomeria fortunei* is one of the key plants for protection in the Fujian Province, and natural *C. fortunei* forest can only be found in the Tianbaoyan Nature Reserve. At present, there are a lot of *C. fortunei* plantations in Ningde, Fuzhou, Nanping, Sanming, Longyan in the Fujian Province. The *C. fortunei* has become one of the main fast-growing tree species in the Fujian Province, but density optimization of *C. fortunei* plantation is still a pressing problem in the management. This article proposes a new density model according to the increase model of the plant population biomass and the theory of final yield invariableness.

2 Material origin

We set 218 standard plots (0.06–0.08 hm^2 each) in different

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types of stands in the distributing regions of *C. fortunei* plantation in the Fujian Province, and then used the common method to investigate factors such as tree diameter and height in the standard plots. The highest tree for every 100 m^2 was measured to determine the average height of superior trees, and the tree volume was calculated using the duality tree volume diagram.

3 Model deduction

The rate of growth of the average individual tree volume changes with the age and is restricted by density, whose growth curve presents a 'S' type. We can use the Logistic equation to describe this 'S' type curve.

$$\frac{dV}{dt} = r \times V \times [1 - (\frac{V}{K})] \quad (1)$$

where V is the average individual tree volume, t is the time, K is the upper limit value of V and r means increase rate.

This equation has been widely applied to simulate animal or plant population dynamics in the ecology. But in this equation the maximum of the increase speed is at $K/2$, i.e., the inflexion of the curve, which is not in accordance with the fact sometimes, and the linearity hypothesis of the rate of growth and V produces the model deviation. Based on Eq. (1), it is better to introduce a special parameter θ which can make different growth forms compatible to describe the growth curve of the average individual tree volume (Zhang and Zhao, 1985).

$$\frac{dV}{dt} = r \times V \times [1 - (\frac{V}{K})^{\theta}] \quad (2)$$

Eq. (2) suggests that the increase rate of population reaches the peak at the point of $K/(1+\theta)^{1/\theta}$. It means that if $\theta>1$, the maximum is after $K/2$ point, and, otherwise, before $K/2$ point. When $\theta<1$, $\theta=1$ or $\theta>1$, Eq. (2) can simulate down-type growth, Logistic growth, up-type growth, respectively. When θ tends to infinite, the model goes to exponential growth. When θ tends to zero, the model tends to maintain the initial value of growth invariable. Because Eq. (2) can describe different increase models of plant population, it can be considered a better mathematical formula to describe the changes of average individual tree volume over time.

Calculate Eq. (2), we obtain

$$V = \frac{K}{(1 + ce^{-at})^{b}} \quad (3)$$

where c is an integral constant, $a = \theta r$, and $b=1/\theta$. According to the Japanese theory that states that the final harvest yield is constant, which means the final harvest yield per hectare (V_m) is an invariable constant and does not depend on the stumpage density (N), we work out Eq. (4):

$$K = \frac{V_m}{N} \quad (4)$$

According to Eq. (3), the average individual tree volume (V_0) of stand growth initial stage does not depend on the stumpage density, which means when t tends to zero, we will get:

$$V_0 = \frac{K}{(1 + c)^{b}} \quad (5)$$

Place Eq. (4) into Eq. (5), we obtain:

$$V_0 = \frac{V_m}{N(1 + c)^{b}} \quad (6)$$

Calculate Eq. (6) for the answer c:

$$c = (\frac{V_m}{N \times V_0})^{\frac{1}{b}} - 1 \quad (7)$$

According to Eq. (3), we have:

$$V^{-\frac{1}{b}} = \frac{(1 + ce^{-at})}{K^{\frac{1}{b}}} = \frac{e^{at} + c}{K^{\frac{1}{b}} e^{at}} \quad (8)$$

Put k in Eq. (4) and c in Eq. (7) into Eq. (8), and let $\beta=1/b$, we work out Eq. (9):

$$V^{-\beta} = \frac{e^{at} + (\frac{V_m}{V_0 \times N})^{\beta} - 1}{(\frac{V_m}{N})^{\beta} e^{at}} \quad (9)$$

The equivalent of Eq. (9) is:

$$V^{-\beta} = \frac{1 - e^{-at}}{V_m^{\beta}} \times N^{\beta} + \frac{e^{-at}}{V_0^{\beta}} \quad (10)$$

At the same growth stage, which means that t is invariable, we let

$$A = \frac{1 - e^{-at}}{V_m^{\beta}}, B = \frac{e^{-at}}{V_0^{\beta}}$$

then Eq. (10) turns to be:

$$V^{-\beta} = AN^{\beta} + B \quad (11)$$

Equation (11) is the density effect model we put forward in this paper. It reflects the law that the average individual tree volume declines with the increase of stumpage density and tends to zero finally. Considering the different growth states of the plant population under various site conditions, the law of density effect also varies greatly. So, even though the stand density is consistent, the average individual tree volume is still different. In order to better describe the density effect law of the plant population, we should consider the site factors. The superior height H is the synthetical embodiment of stand age and site factors. We introduce the superior height to express the parameter values of stand growth stage under different site conditions:

$$A = a_1 H^{a_2} \qquad B = b_1 H^{b_2} \qquad \beta = c_1 H^{c_2}$$

then we get Eq. (12):

$$V^{-c_1 H^{c_2}} = a_1 H^{a_2} N^{c_1 H^{c_2}} + b_1 H^{b_2} \quad (12)$$

Equation (12) describes the plant population density effect law of different sites and different growth stages.

4 Model verification

A good density effect model requires not only theoretical basis, but also convenience of application. According to the investigation data collected in the 218 standard plots of *C. fortunei* plantation with different site conditions and ages to fit the density effect model Eq. (12), we work out the *C. fortunei* density effect model:

$$V = (0.224,3H^{-1.375,9} N^{0.637,6H^{0.141,7}} + 858.809H^{-2.455,2})^{-\frac{-1}{0.637,6H^{0.141,7}}} \quad (13)$$

The goodness of model regression is T=U/(U+Q)×100%= 93.65%, and multi-correlation coefficient is R=0.987,7, which shows that the model fitness is extremely significant.

In order to explain conveniently, we use the reciprocal model of density effect and secondary-effect model to simulate the population density effect law of the *C. fortunei* plantation, and compare with the density effect model put forward in this article. We carried out the most superior fitting using the modified simplex method and according to the investigation data collected in 218 standard plots of the *C. fortunei* plantation with different site conditions and ages, the result is:

Reciprocal model of density effect is

$$V^{-1} = 0.091,67\ H^{1.143,7}N + 227.377,5\ H^{1.951,6} \quad (14)$$

Secondary-effect model is

$$V = 8.931\times10^{-4}\ H^{2.131,2} - 1.448,3\times10^{-8}\ H^{3.236,3}N \quad (15)$$

The goodness of model regression in Eq. (14) T=88.33%, multi-correlation coefficient R=0.939,8, and those in Eq. (15) is T=86.53%, multi-correlation coefficient R=0.930,2. The new density effect model put forward in this article considers the growth mechanism of the plant population, so the model fitness is better than the other two models. The new density effect model is more superior and accurate, so it can be widely used in the research of plant population density effect and production practice.

5 Conclusion

In Eq. (11), if $\beta=1$ (θ=1), we can obtain the result V^{-1}=AN+B. This means that the density effect model referred in this paper includes the reciprocal model of density effect, which is just a special case of Eq. (11). In fact, it can be inferred from above that the growth process of plants included in the reciprocal model of density effect follows the Logistic curve. But because of the defects of the Logistic curve, it does not fit with the reality when it describes the maximum speed of plant population at the point of inflection of this curve. For, the maximum increase rate may appear before or after the capacity, not always just at the half capacity. So the reciprocal model of density effect is not perfect in theory and it has some limitations when describing the growth process of plants. Accordingly, we make further improvements to it, introduce a specific parameter θ that can integrate different growth forms and have the general non-linear restriction functions, then obtain a new density effect model, which is more strict and reasonable in theory and can well depict the plant the actual growth process. Using the sample materials of *C. fortunei* plantations concerning density change, we carry out the comparative analysis for the three density effect models. It is found that the density effect model proposed in this article is superior and possesses higher accuracy.

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