

## **Supplementary Material**

for

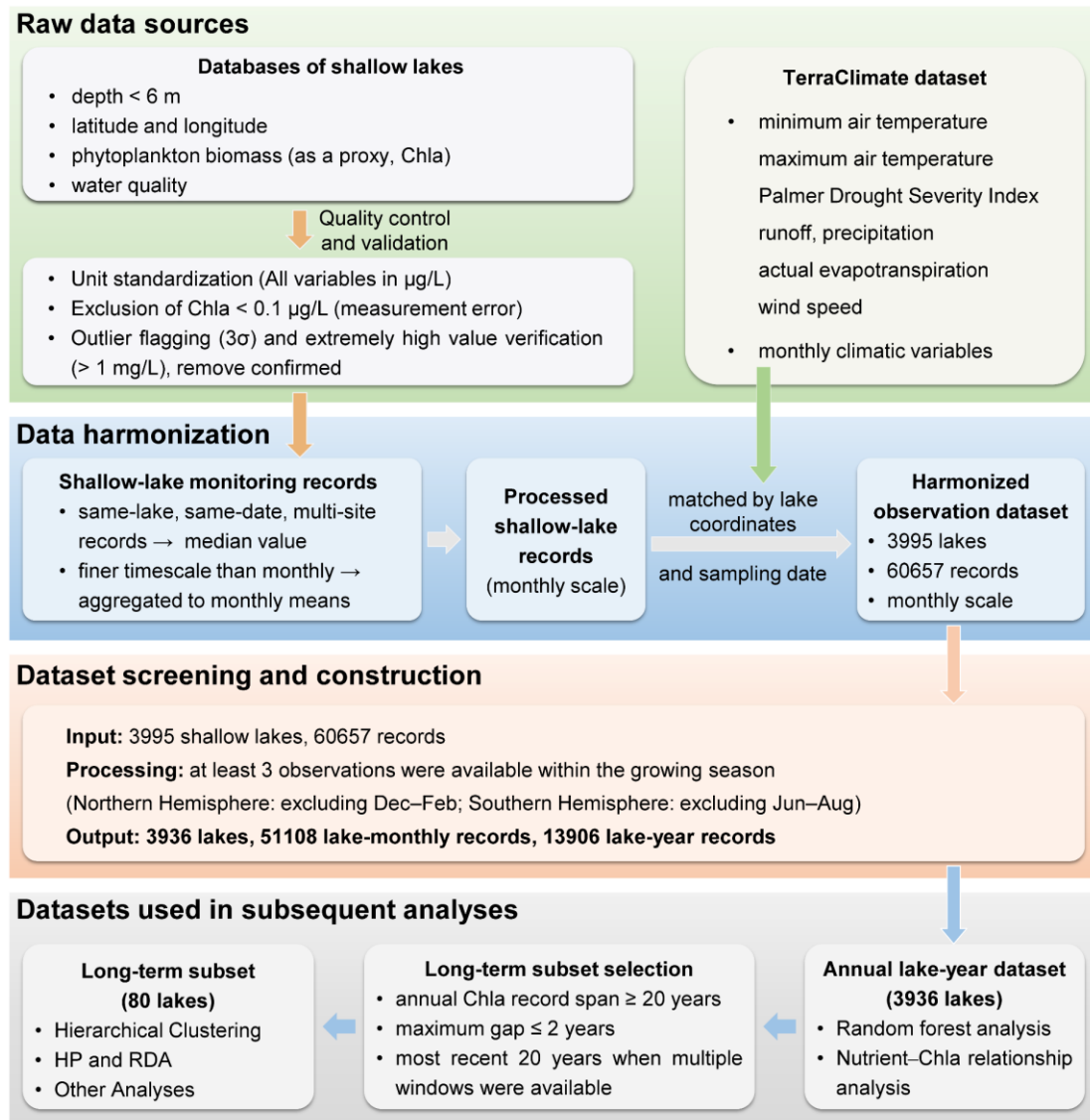
### **Regulatory pathways of phytoplankton regime shifts in shallow lakes: a new insight from non-equilibrium statistical mechanics**

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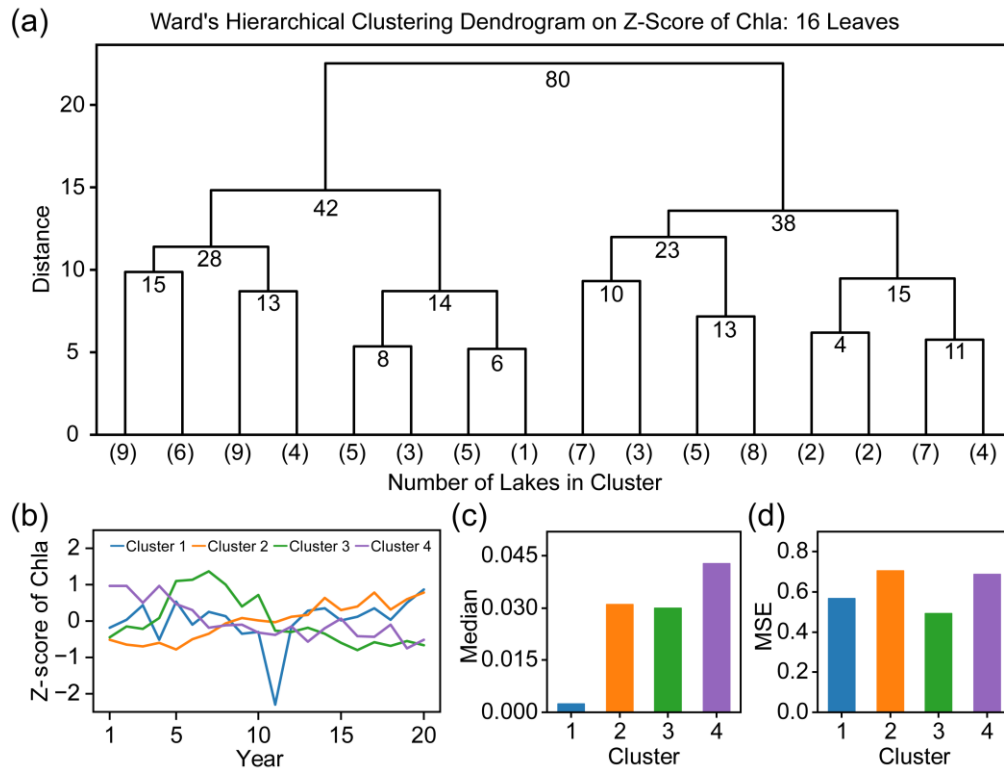


**Figure S1 Workflow of data harmonization, screening, and construction of analysis datasets.** During quality control and validation, all water quality variables (Chla, TN, TP) were recorded in consistent units ( $\mu\text{g/L}$ ). Chla measurements below 0.1  $\mu\text{g/L}$  were excluded due to the possibility of measurement error, as analytical equipment used within studies rarely had better precision. For all water chemistry variables, observations exceeding three standard deviations from the mean were flagged and cross-checked against the original data sources; extremely high values (> 1 mg/L) were additionally traced back to the original datasets to identify records potentially resulting from unit conversion errors or compilation errors. All flagged records confirmed as unreliable were subsequently discarded.

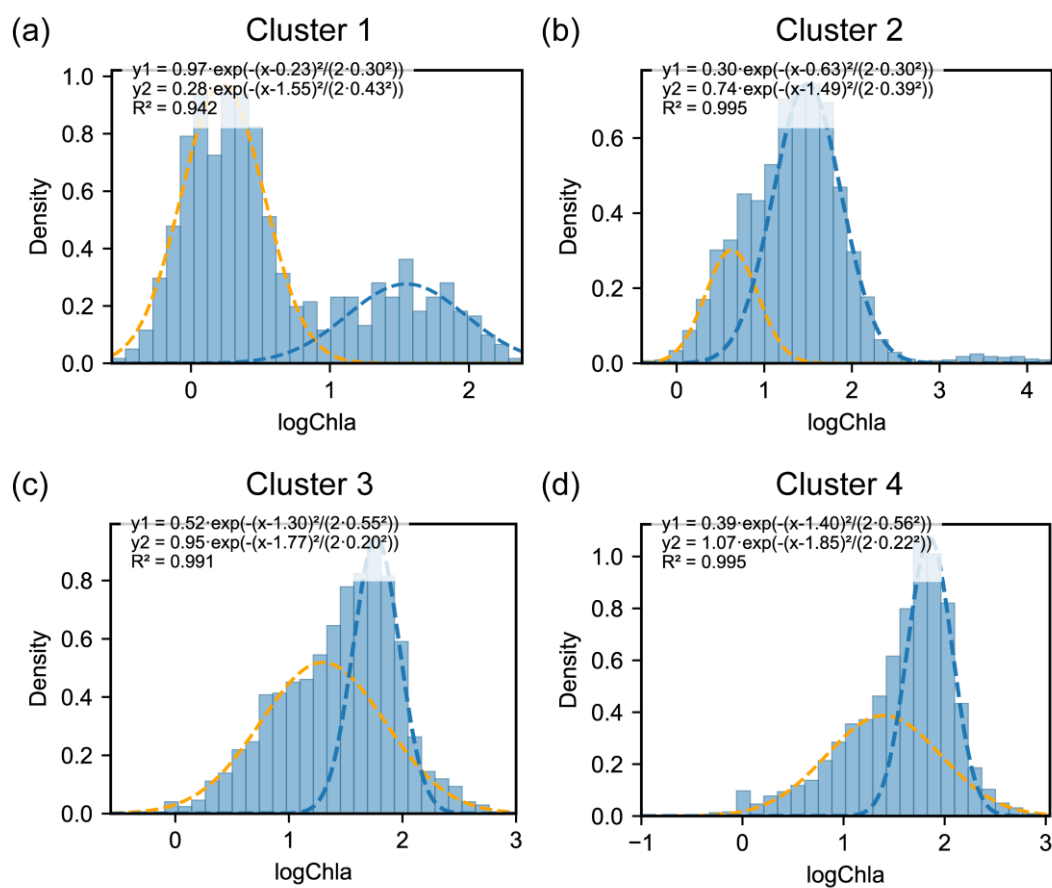
Water-quality observations from global shallow-lake databases, including phytoplankton biomass (as a proxy, chlorophyll-a), water quality, latitude and longitude, and climatic variables, were preprocessed and aligned to monthly resolution. Same-lake, same-date observations from multiple sampling sites were merged using the median value, and records collected at finer-than-monthly resolution were aggregated to monthly means; records without month information were excluded. The processed lake monitoring records were then matched with monthly climatic variables from the TerraClimate dataset according to lake coordinates and sampling month/year, yielding a harmonized lake–climate dataset containing 60657 records from 3995 lakes.

The harmonized monthly dataset was then screened to retain only growing-season records from lake-years with at least three observations within the growing season. This screening yielded 51108 growing-season lake-month records from 3936 lakes, from which 13906 lake-year records were generated by annual averaging. The growing season was defined as all months except December–February in the Northern Hemisphere and June–August in the Southern Hemisphere. The lake-year dataset was used for the random forest analysis.

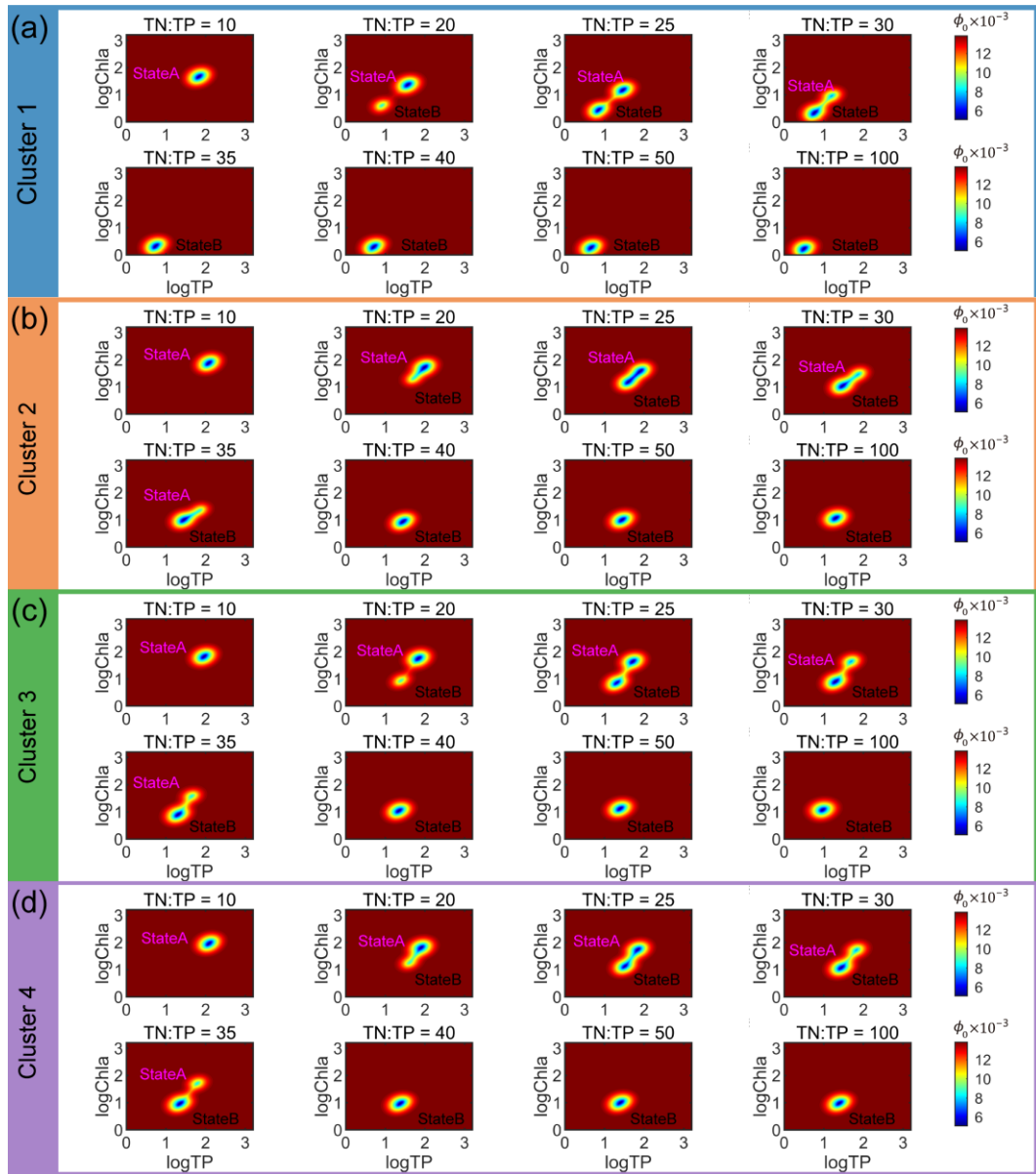
For long-term analyses, 80 lakes were further selected by requiring annual Chla records spanning at least 20 years with a maximum gap of 2 years; when multiple eligible windows were available, the most recent 20-year window was retained. Annual records from these 80 lakes were used for hierarchical clustering, whereas the corresponding growing-season monthly records were used for HP, RDA, and other analyses unless otherwise specified.



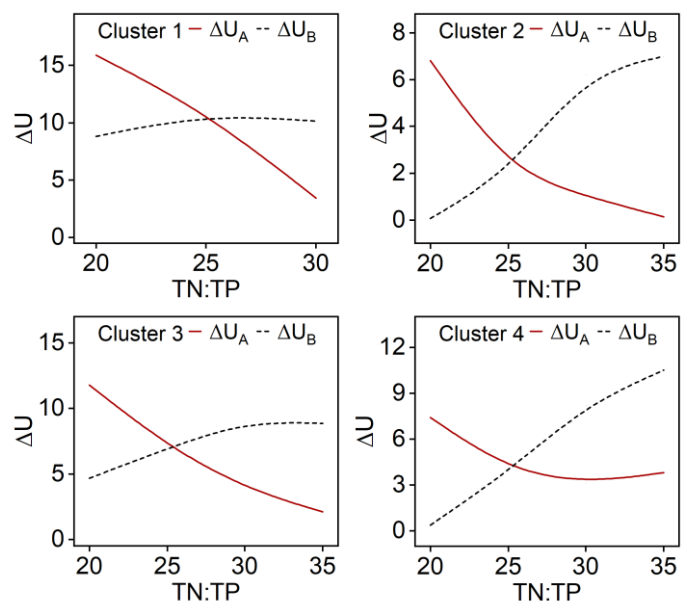
**Figure S2** Hierarchical clustering tree (a), clustering trend (b), median of each cluster (c), mean square error of each cluster (d).



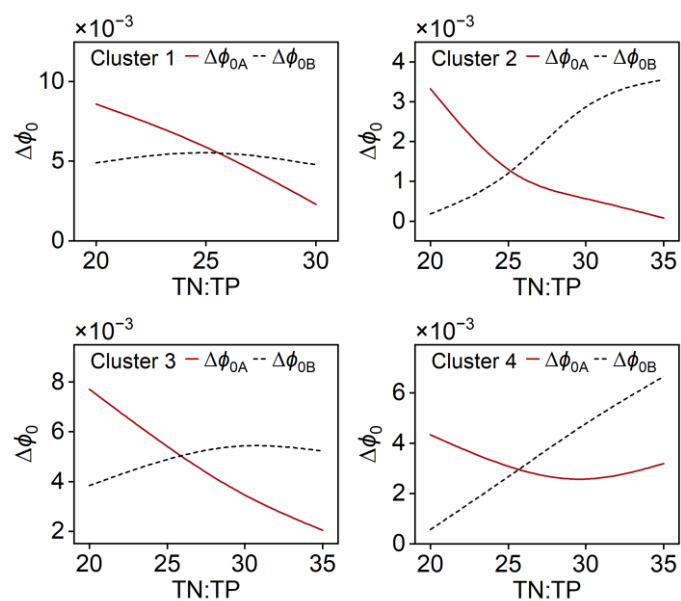
**Figure S3** The frequency distributions of logChla for each cluster (a-d). Yellow dotted line, y1; blue dotted line, y2.



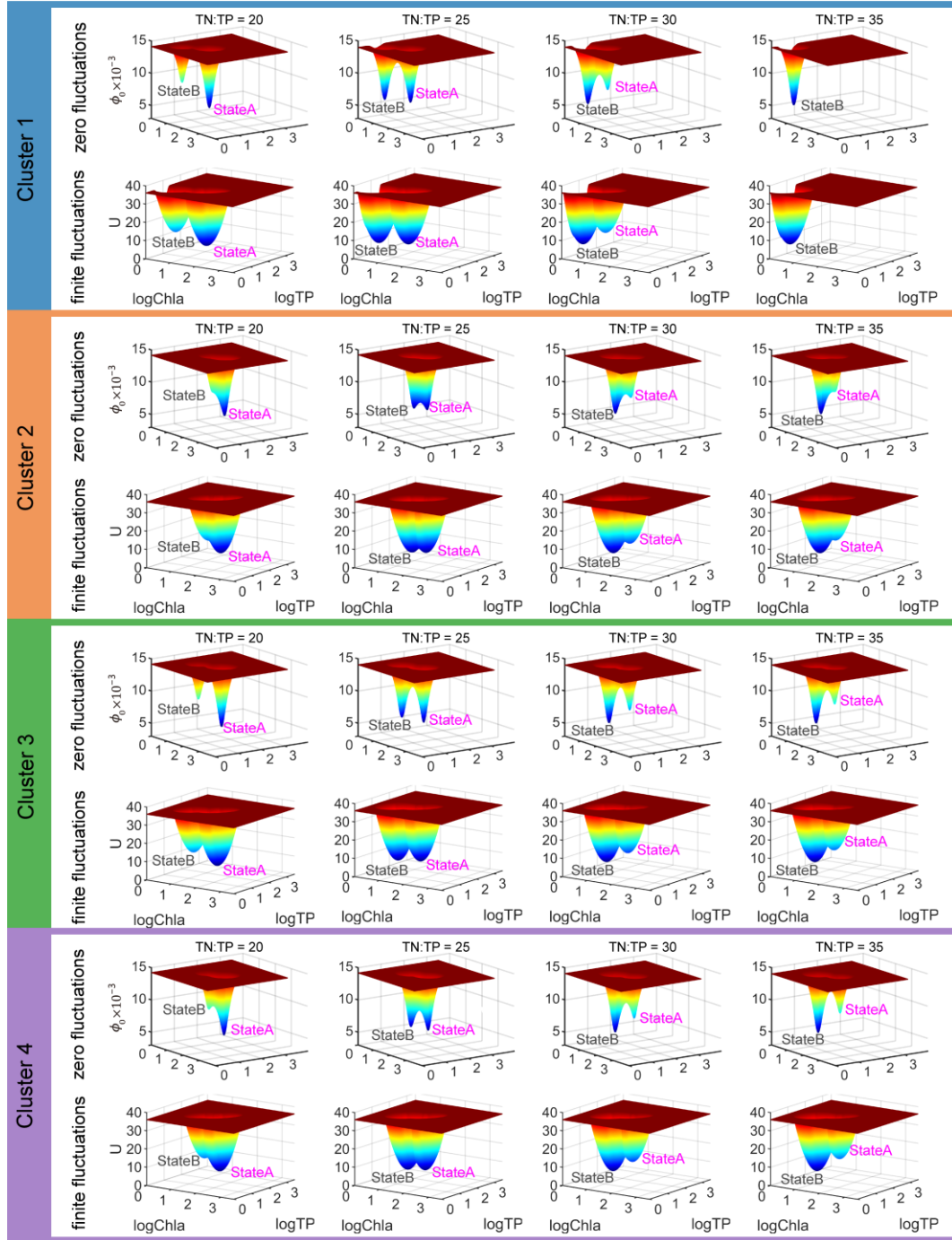
**Figure S4** 2D intrinsic potential landscapes under zero fluctuations ( $\phi_0$ ) for the SLPNF model across different clusters (a-d) along the TN:TP gradient (10–100). Colorbar on the right, ranging from blue to red, represents potential energy decreasing from high to low.



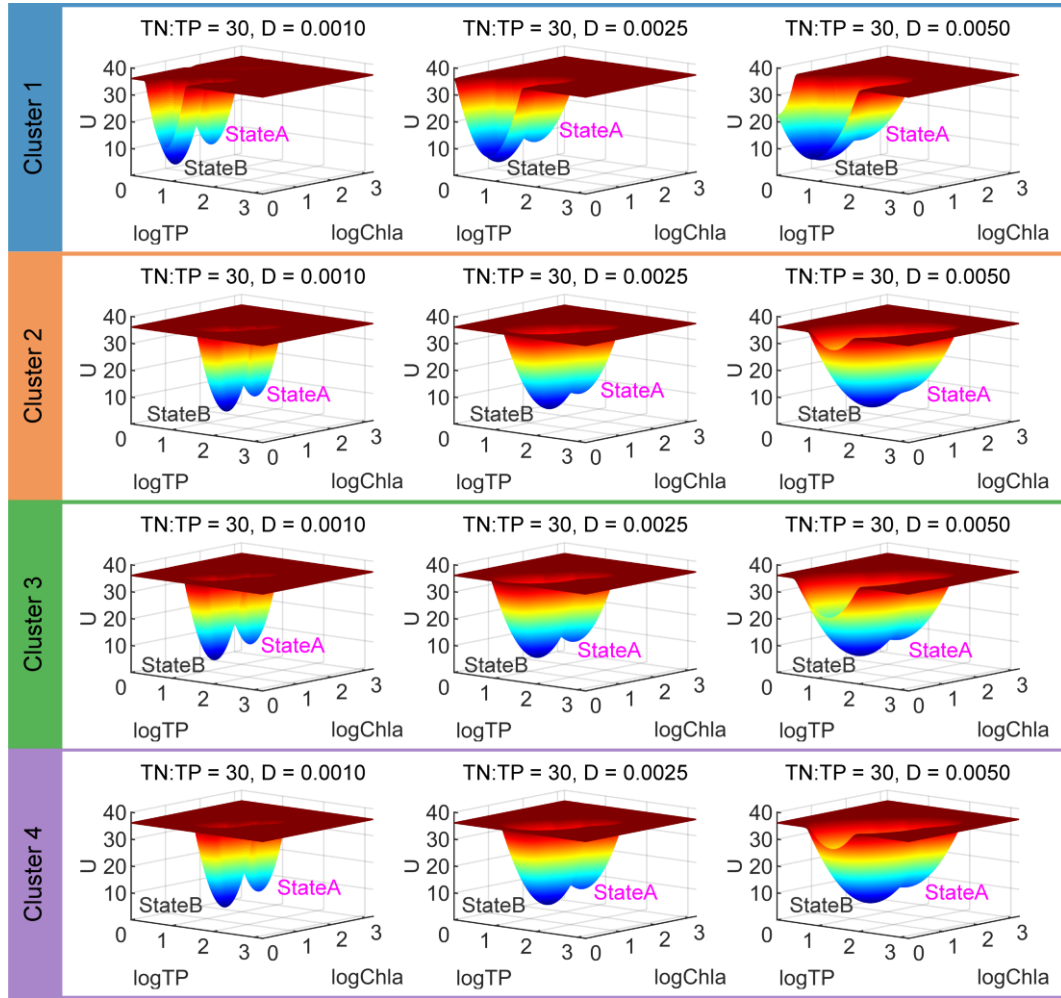
**Figure S5** The barrier heights ( $\Delta U$ ) versus TN:TP for each cluster.  $\Delta U_A$ , the barrier height from StateA to StateB;  $\Delta U_B$ , the barrier height from StateB to StateA.



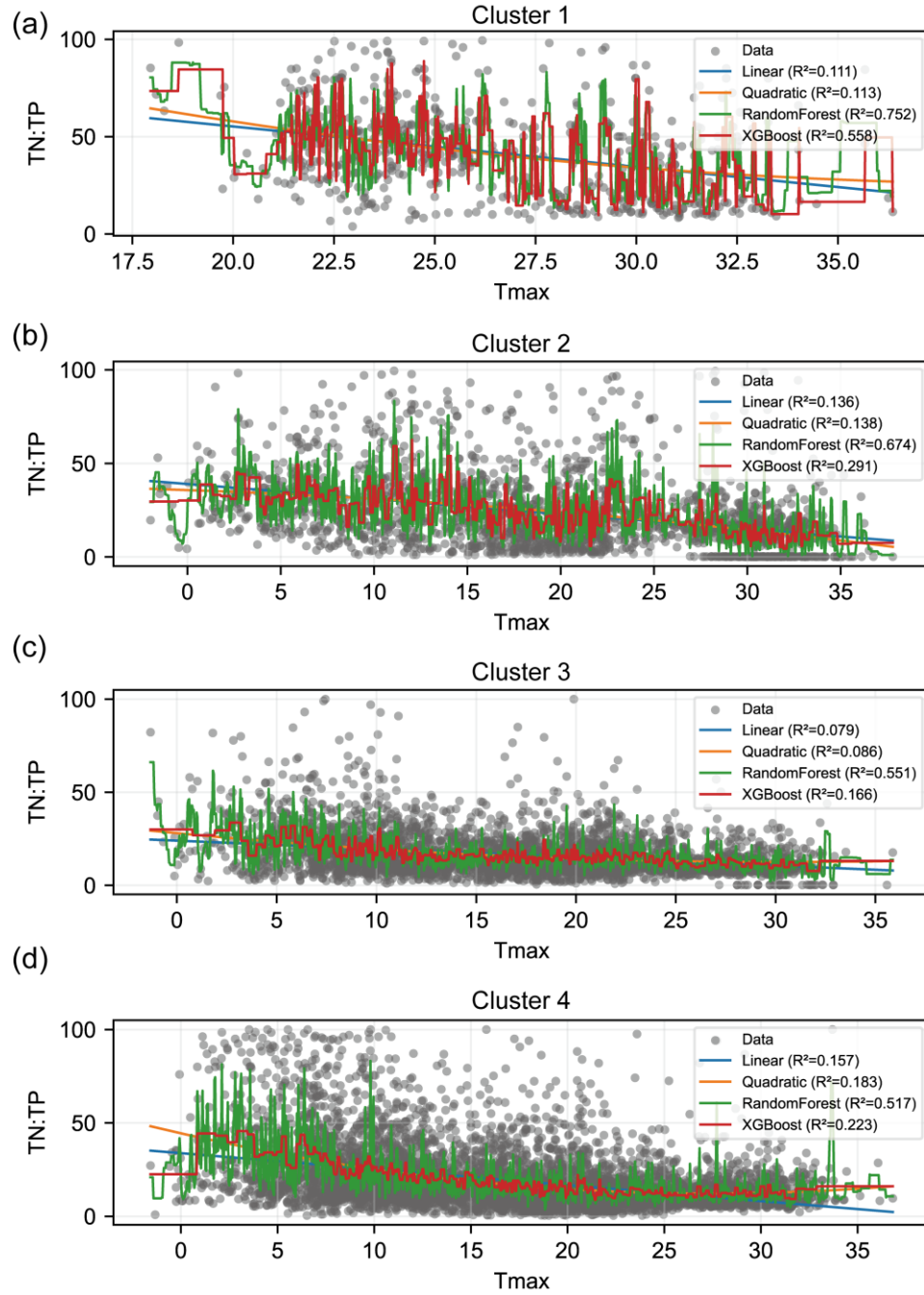
**Figure S6** The intrinsic barrier heights ( $\phi_0$ ) versus TN:TP for each cluster.  $\Delta\phi_{0A}$ , the intrinsic barrier height from StateA to StateB;  $\Delta\phi_{0B}$ , the intrinsic barrier height from StateB to StateA.



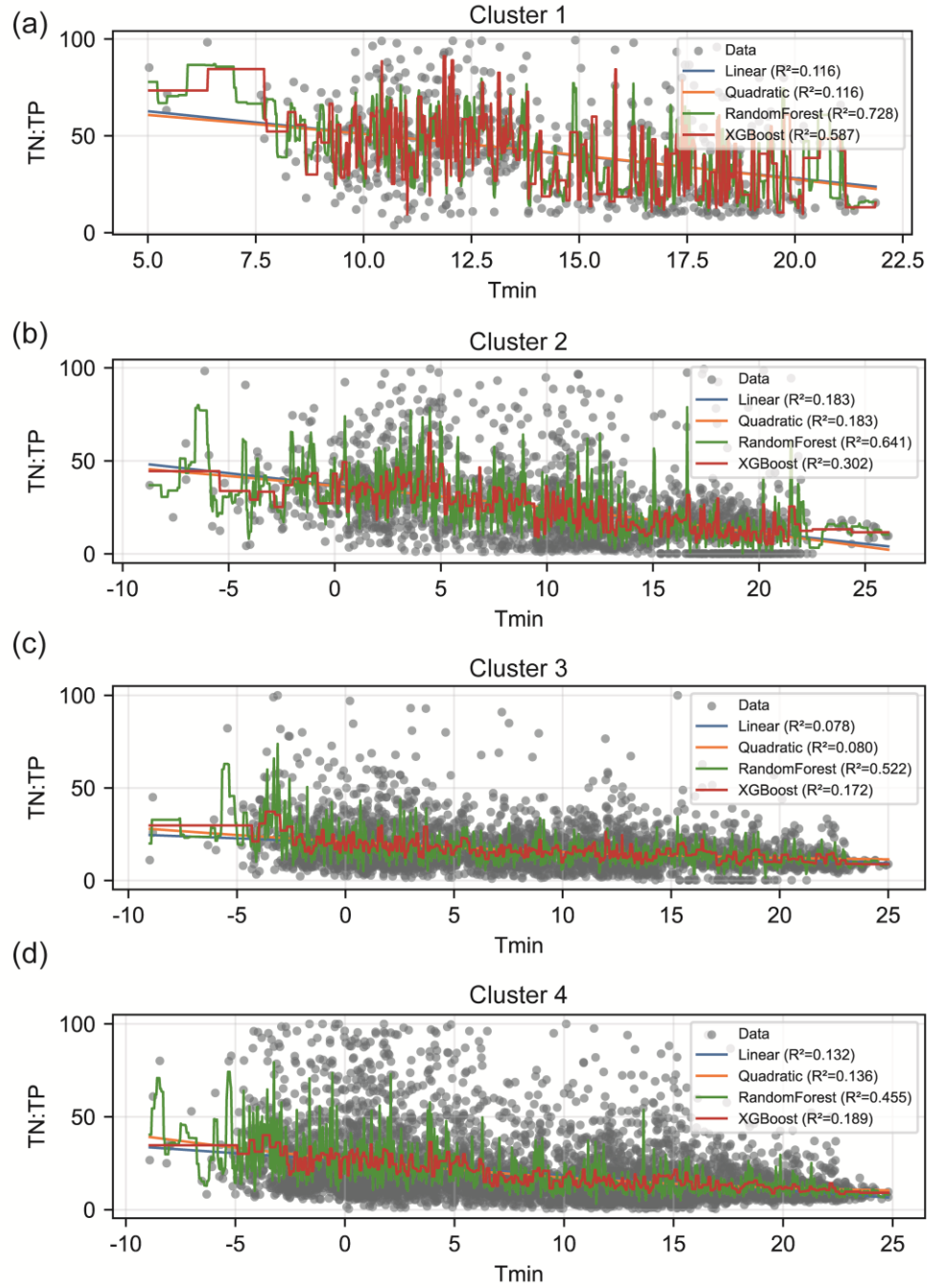
**Figure S7** 3D potential landscapes under zero fluctuations ( $\phi_0$ ) and finite fluctuations ( $U$ ) for the SLPNF model under different clusters along the TN:TP gradient (20–35).



**Figure S8** 3D potential landscapes for the SLPNF model under different clusters with different parameters  $D = 0.001, 0.0025, 0.005$  and  $TN:TP = 30$ . Parameter  $D$  was used to measure the strength of perturbations, which was defined as the diffusion coefficient of fluctuation intensity.



**Figure S9** The model fitting performance for predicting the relationship between maximum temperature (Tmax) and TN:TP was evaluated to select the best surrogate model under different clusters (a-d), respectively. The random forest model was selected as the best surrogate model for subsequent sensitivity analysis.



**Figure S10** The model fitting performance for predicting the relationship between minimum temperature (Tmin) and TN:TP was evaluated to select the best surrogate model under different clusters (a-d), respectively. The random forest model was selected as the best surrogate model for subsequent sensitivity analysis.

**Table S1** Information associated with each of the online databases.

| Data type          | Name                                                                                                         | Data sources                                                                                                                                                    |
|--------------------|--------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Water quality data | A water quality database for global lakes                                                                    | Naderian et al. (2024)                                                                                                                                          |
|                    | The Danish Centre for Environment at the Aarhus University                                                   | Davidson et al. (2023)<br><a href="https://odaforalle.au.dk/">https://odaforalle.au.dk/</a>                                                                     |
|                    | Environmental Information Data Centre (UK)                                                                   | <a href="https://catalogue.ceh.ac.uk/eidc/documents">https://catalogue.ceh.ac.uk/eidc/documents</a>                                                             |
|                    | Long term and high frequency monitoring of lakes in the English Lake District                                | <a href="https://catalogue.ceh.ac.uk/documents/bf30d6aa-345a-4771-8417-ffb8c08c28">https://catalogue.ceh.ac.uk/documents/bf30d6aa-345a-4771-8417-ffb8c08c28</a> |
|                    | Ministry of Land, Infrastructure, Transport and Tourism, Japan                                               | <a href="https://www1.river.go.jp/">https://www1.river.go.jp/</a>                                                                                               |
|                    | The National Institute for Environmental Studies (NIES), Japan                                               | <a href="https://www.nies.go.jp/chiiki/en/project10_en.html">https://www.nies.go.jp/chiiki/en/project10_en.html</a>                                             |
|                    | Government of New Brunswick(Canada) - Department of Environment and Local Government Lake Water Quality Data | <a href="https://www.elgegl.gnb.ca/LakesNB-NBLacs/en/SamplingLocation/Index">https://www.elgegl.gnb.ca/LakesNB-NBLacs/en/SamplingLocation/Index</a>             |
| Climatic data      | A comprehensive time-series dataset linked to cyanobacterial blooms in Lake Taihu                            | Xue et al. (2024)                                                                                                                                               |
|                    | TerraClimate, a high-resolution global dataset of monthly climate and climatic water balance from 1958–2015  | Abatzoglou et al. (2018)                                                                                                                                        |

**Table S2** Distribution of collected shallow lake data by country.

| Country        | Lakes | Country                      | Lakes |
|----------------|-------|------------------------------|-------|
| Canada         | 114   | Norway                       | 3     |
| Switzerland    | 4     | Netherlands                  | 27    |
| China          | 9     | New Zealand                  | 2     |
| Czech Republic | 4     | Poland                       | 38    |
| Germany        | 13    | The Portuguese Republic      | 1     |
| Denmark        | 2254  | Sweden                       | 20    |
| Spain          | 11    | Slovakia                     | 1     |
| Estonia        | 3     | Slovenia                     | 3     |
| Finland        | 19    | Turkey                       | 13    |
| Japan          | 9     | The United States of America | 1235  |
| Mexico         | 153   |                              |       |

**Table S3** Distribution of the 80 long-term monitoring shallow lakes by continent.

| Continent     | Lakes |
|---------------|-------|
| North America | 41    |
| Asia          | 9     |
| Oceania       | 2     |
| Europe        | 28    |

**Table S4** Summary of the parameters interpretation and default values for SLPNF model.

| Symbol      | Default values <sup>a</sup> | Definition and interpretation                                                      | Reference                                       |
|-------------|-----------------------------|------------------------------------------------------------------------------------|-------------------------------------------------|
| A           |                             | Concentration of phytoplankton biomass ( $\log_{10}(Chla(ug / L))$ )               |                                                 |
| N           |                             | Concentration of total nitrogen ( $\log_{10}(TN(ug / L))$ )                        |                                                 |
| P           |                             | Concentration of total phosphorus ( $\log_{10}(TP(ug / L))$ )                      |                                                 |
| R           |                             | Total nitrogen to total phosphorus ratios                                          |                                                 |
| $\mu_{max}$ | 0.85                        | Maximum growth rate of phytoplankton ( $1 / day$ )                                 | Reynolds (2006a), Laws (2013)                   |
| $K_N$       | 0.15                        | Half saturation constant of nitrogen concentration to phytoplankton ( $mg / L$ )   | Mulder and Hendriks (2014)                      |
| $K_P$       | 0.08                        | Half saturation constant of phosphorus concentration to phytoplankton ( $mg / L$ ) | Mulder and Hendriks (2014)                      |
| $\lambda_1$ | 0.25                        | Intrinsic decay coefficient of phytoplankton ( $1 / day$ )                         | Tilman (1981), Reynolds (2006b)                 |
| $\lambda_2$ | 0.10                        | Competitive mortality coefficient of phytoplankton $1 / (mg * L^{-1} * day)$       | Reynolds (2006b)                                |
| $\alpha_N$  | 0.50                        | Uptake rate of nitrogen by phytoplankton                                           | Tilman (1981), Reynolds (2006c)                 |
| $\alpha_P$  | 0.60                        | Uptake rate of phosphorus by phytoplankton                                         | Tilman (1981), Reynolds (2006c)                 |
| $\theta_N$  | 0.36                        | Phytoplankton conversion into nitrogen after death                                 | Reynolds (2006b), Tiwari et al. (2019)          |
| $\theta_P$  | 0.55                        | Phytoplankton conversion into phosphorus after death                               | Reynolds (2006b), Tiwari et al. (2019)          |
| $I_N$       | 0.002                       | Input rate of nitrogen ( $ug * L^{-1} * day$ )                                     | Janse and Proefschrift (2005)                   |
| $I_P$       | 0.0015                      | Input rate of phosphorus ( $ug * L^{-1} * day$ )                                   | Jørgensen (1988), Janse and Proefschrift (2005) |

<sup>a</sup> For each cluster, due to differences in features, the default values were calibrated to ensure the accuracy and adaptability of the SLPNF model.

**Table S5** Results of hierarchical partitioning for the RDA model.

| Clusters  | RDA maximum<br>variance explained | Variable                      | Independent<br>Contribution | <i>p</i> |
|-----------|-----------------------------------|-------------------------------|-----------------------------|----------|
| Cluster 1 | 29.2%                             | minimum temperature           | 11.29%                      | 0.001    |
|           |                                   | maximum temperature           | 11.03%                      | 0.001    |
|           |                                   | wind speed                    | 3.74%                       | 0.001    |
|           |                                   | actual evapotranspiration     | 1.66%                       | 0.001    |
|           |                                   | Palmer Drought Severity Index | 0.65%                       | 0.985    |
|           |                                   | precipitation                 | 0.42%                       | 0.891    |
|           |                                   | runoff                        | 0.37%                       | 0.072    |
| Cluster 2 | 21.62%                            | minimum temperature           | 9.02%                       | 0.001    |
|           |                                   | maximum temperature           | 5.58%                       | 0.001    |
|           |                                   | actual evapotranspiration     | 2.43%                       | 0.001    |
|           |                                   | runoff                        | 2.02%                       | 0.001    |
|           |                                   | wind speed                    | 1.13%                       | 0.001    |
|           |                                   | Palmer Drought Severity Index | 0.72%                       | 0.001    |
|           |                                   | precipitation                 | 0.72%                       | 0.001    |
| Cluster 3 | 9.2%                              | maximum temperature           | 3.52%                       | 0.001    |
|           |                                   | minimum temperature           | 2.79%                       | 0.001    |
|           |                                   | actual evapotranspiration     | 1.16%                       | 0.001    |
|           |                                   | wind speed                    | 0.98%                       | 0.001    |
|           |                                   | Palmer Drought Severity Index | 0.32%                       | 0.001    |
|           |                                   | precipitation                 | 0.28%                       | 0.001    |
|           |                                   | runoff                        | 0.20%                       | 0.002    |
| Cluster 4 | 10.51%                            | minimum temperature           | 4.65%                       | 0.001    |
|           |                                   | maximum temperature           | 3.12%                       | 0.001    |
|           |                                   | actual evapotranspiration     | 1.26%                       | 0.001    |
|           |                                   | wind speed                    | 0.56%                       | 0.001    |
|           |                                   | precipitation                 | 0.44%                       | 0.001    |
|           |                                   | Palmer Drought Severity Index | 0.31%                       | 0.001    |
|           |                                   | runoff                        | 0.18%                       | 0.001    |

**Table S6** The regulation threshold for each cluster.

| Cluster   | Regulation threshold |
|-----------|----------------------|
| Cluster 1 | 22.7820              |
| Cluster 2 | 23.0681              |
| Cluster 3 | 23.0681              |
| Cluster 4 | 23.2583              |

## References

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