

## Supporting Information

S1 The pseudo-codes for the improved Bayesian-MCMC, RWM, and GA

The flowchart of the improved Bayesian-MCMC, RWM and GA is presented in Fig. S1.

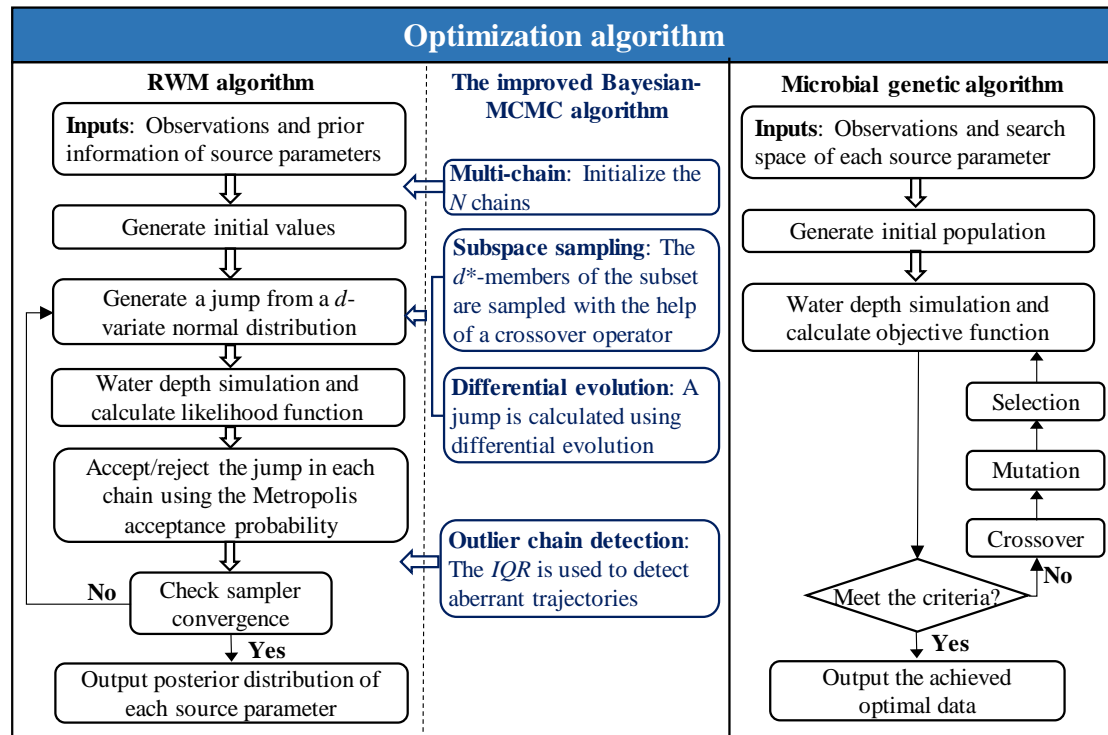


Fig. S1 Flowchart of the improved Bayesian-MCMC, RWM and microbial GA.

The pseudo-codes to perform the improved Bayesian-MCMC, RWM and GA are provided below. To obtain the complete source codes of the two cases in this manuscript, please contact us via E-mail address: [yinhailong@tongji.edu.cn](mailto:yinhailong@tongji.edu.cn), [m15801793070@163.com](mailto:m15801793070@163.com)

### 1) Pseudo-code for random walk metropolis (RWM) algorithm

```

Function [theta, p_theta] = RWM (prior, pdf, T, d)
theta (1, 1: d) = prior (1, d); % Initialize chain by sampling from prior
p_theta (1) = pdf (theta (1, 1: d)); % Compute density initial state chain
sigma = (2.38/sqrt (d))^2 * eye(d); % covariance matrix proposal distribution

```

```

for t = 2: T, % Dynamic part
    d_theta = mvnrnd (zeros (1, d), sigma); % d-variate normal proposal distribution.
    theta p = theta (t-1, 1: d) + d_theta; % Create candidate point
    p_theta p = pdf (theta p); % Calculate density proposal
    p_acc= min (1, p_theta p/ p_theta (t-1)); % Compute p_acc
    if p_acc > rand, % p_acc larger than U [0,1] ?
        theta (t, 1: d) = theta p; % accept proposal
        p_theta (t) = p_theta p;
    else
        theta (t, 1: d) = theta (t-1, 1: d); % reject proposal
        p_theta (t) = p_theta (t-1);
    end
end
end

```

## 2) Pseudo-code for the improved Bayesian-MCMC algorithm

```

Function [theta, p_theta] =Improved MCMC (prior, pdf, N, T, d)
theta = prior (N, d); % Create initial population
for i = 1:N, % Compute density initial population
    p_theta (i, 1) = pdf (theta (i, 1:d));
end
[delta, c, c_star, n_CR] = deal (3, 0.1, 1e-6, 3); % Default of algorithmic parameters
CR = [1: n_CR]/ n_CR; pCR = ones (1, n_CR) / n_CR;
for t = 2: T % Dynamic part: Evolution of N chains
    [-, draw] = sort(rand(N-1, N)); %Permute [1, ..., N-1] N times
    d theta = zeros (N,d); % Set N jump vectors to zero
        lambda = unifrnd (-c, c, N, 1); % Draw N lambda values
        for i = 1: N,
            D = randsample ([1: delta], 1, 'true'); % Set select. prob.
            a = R (i, draw(1: D, i)); b = R(i, draw(1: D, i)); % Extract vectors a and b not
            equal i
            id = randsample (1:nCR, 1, 'true', pCR); % Select index of crossover value
            z = rand (1, d); % Draw d values from U[0,1]
            A = find (z < CR (id)); % A must contain at least one value
            d_star = numel (A); % How many dimensions sampled?
            gamma_d = 2.38/sqrt (2*D*d_star); % Calculate jump rate
            d theta (i, A) = c_star*randn(1, d_star) + (1+lambda(i))* gamma_d*sum(theta (a,A)-
                theta(b,A),1); % Compute ith proposal
            p_theta p(i,1) = pdf (theta p (i, 1:d)); % Calculate density ith proposal
            p_acc = min (1, p_theta p(i,1)/ p_theta (i,1)); % Compute acceptance probability
            if p_acc > rand,
                theta (i, 1:d) = theta p(i, 1:d); % p_acc larger than U[0.1]?
                p_theta (i, 1:d) = p_theta p(i, 1:d); % Accept proposal
            else
                d theta (i, 1:d) =0; %Set jump back to zero for pCR
            end
        end
    end
end

```

```

end
    end
    [theta, p_theta] = check (theta, mean (log (p_theta(ceil(t/2):t, 1:N)))); % Outlier
detection and correction
end

```

### 3) Pseudo-code for the microbial genetic algorithm (MGA)

```

ga = MGA(dna_size, dna_bound, cross_rate, mutation_rate, pop_size)
[cross_rate, mutation_rate, pop_size] = deal (0.5, 0.5, 100); % Default of algorithmic
parameters
Generate a population P randomly;
generation = 1
for generation <= max_gen
    Use a fitness function f(.) to evaluate each individual in P;
    % Adapting crossover rate
    if CP_avg > MP_avg
theta = 0.01 * (max(f.)-avg(f.))/max(f.)-avg(f.);
        cross_rate = cross_rate + theta;
    else
        cross_rate = cross_rate - theta;
    end
    % Crossover
    for i: dna_size
if rand() < cross_rate
cross_idx(i) = True
else
cross_idx(i) = False
end
        loser_winner (0, cross_idx) = loser_winner (1, cross_idx) % assign winner gens
to loser
        end
        % Adapting mutation rate
        if CP_avg > MP_avg
theta = 0.01 * (max(f.)-avg(f.))/max(f.)-avg(f.)
            mutation_rate = mutation_rate + theta;
        else
            mutation_rate = mutation_rate - theta;
        end
        % Mutation
        for i: dna_size
if rand() < mutation_rate
mutation_idx(i) = True
else
mutation_idx(i) = False

```

```

end
    loser_winner (0, mutation_idx) = ~ loser_winner (0, cross_idx)
end
% evolve
    % tournSelect performs a tournament selection
    tournSize=options(2); % Get the number of tournaments
    loser_winner_idx = sort (fitness)
    loser_winner = crossover(loser_winner)
    loser_winner = mutate(loser_winner)
    % compute CP_avg
    CP = sum(f_child)-sum(f_parents); % sum(f_child) is the fitness sum of the two
    offspring
    CP_avg = 1/nc * sum(CP);
    % compute MP_avg
    MP = f_new - f_old %f_new is the fitness of the new offspring
    MP_avg = 1/nm * sum(MP);
    Generation = generation +1;
End

```

## S2 Running of the improved Bayesian-MCMC for the design case

For each of the four source parameters (i.e., source location  $x$ , source discharge  $q$ , starting discharge time  $T_1$ , and ending discharge time  $T_2$ ),  $\hat{R}$ -statistic used to measure the convergence of the sampled chains during the iterations is shown in Fig. S2. The evolutions of the parallel Markov chains for each source parameter during the iterations are presented in Fig. S3.

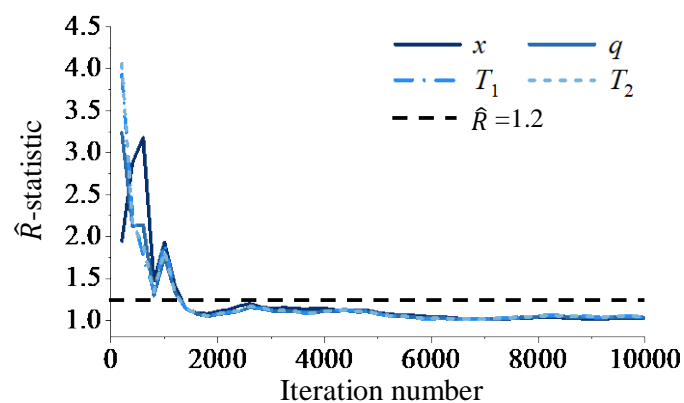


Fig. S2 The convergence of  $\hat{R}$  values of the four parameters in Case I.

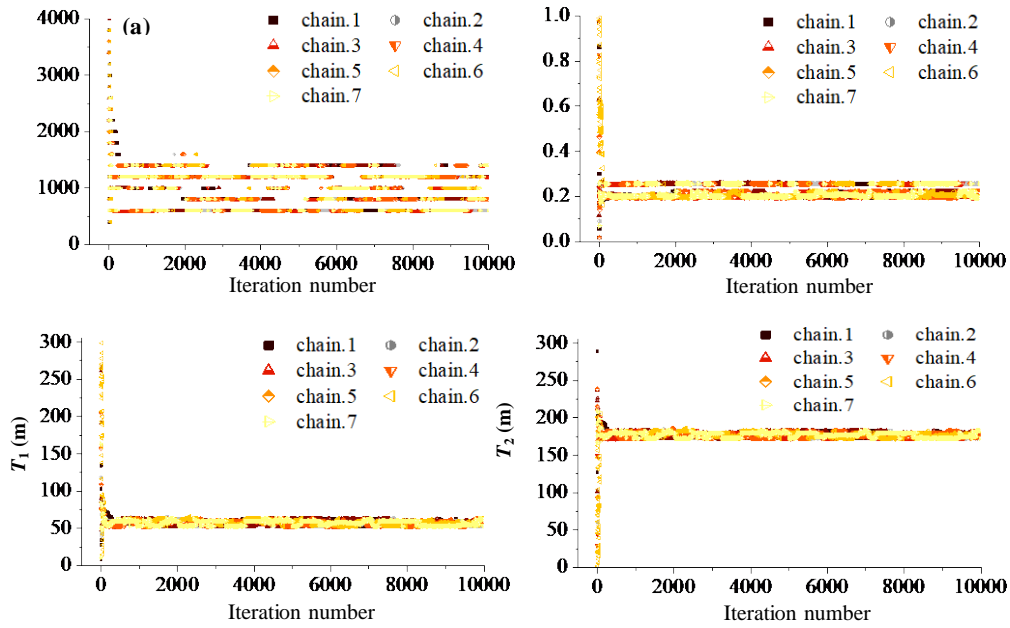


Fig. S3 Evolutions of the seven parallel Markov chains of the four source parameters with improved Bayesian-MCMC algorithm: (a) source discharge; (b) source location; (c) starting discharge time; (d) ending discharge time.

### S3 Running of the improved Bayesian-MCMC for the real case

For the identified four source parameters (i.e., sewage inputs of  $q_3, q_{11}, q_{18}, q_{22}$ ),  $\hat{R}$ -statistics to measure the convergence of the sampled chains during the iterations are presented in Fig. S4. The evolutions of the seven parallel Markov chains and the resulting posterior distributions are presented in Fig. S5. Figs. S4 and S5 correspond to the simulations under computational grid of 500 m.

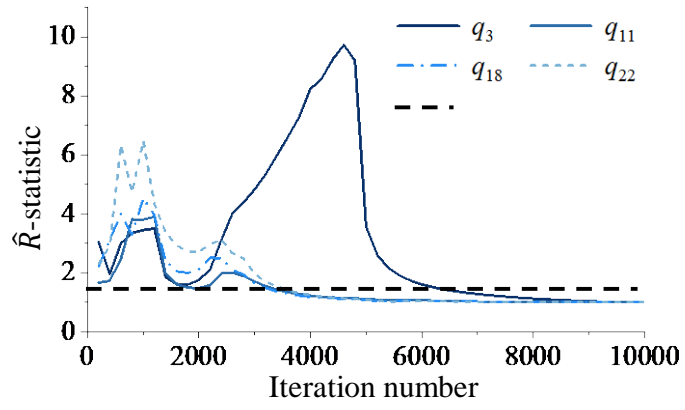


Fig. S4 Convergence of  $\hat{R}$  values of the four identified source flows.

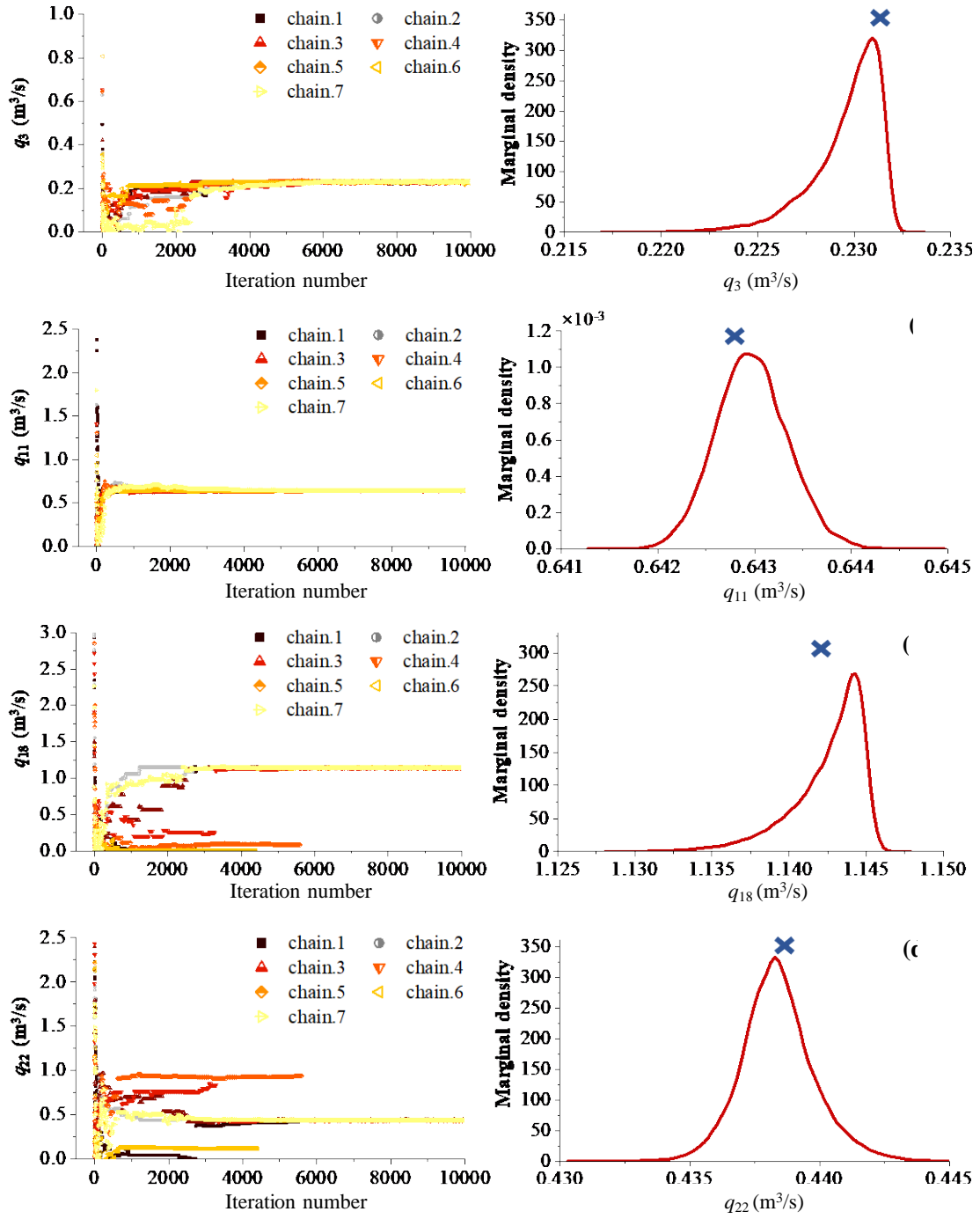


Fig. S5 Evolutions of the seven parallel Markov chains and posterior probability distributions of the four sewage inputs at (a) urban village, (b) Sili river, (c) Banqiao River and (d) municipal outlet.