Front. Earth Sci. 2021, 15(4): 947–955
https://doi.org/10.1007/s11707-020-0835-z

RESEARCH ARTICLE

A high precision model for the terminal settling velocity of drops in fluid medium

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Abstract The terminal settling velocity (TSV) calculation of drops and other spherical objects in fluid medium is a classical problem, which has important application values in many fields such as the study of cloud and precipitation processes, the evaluation of soil erosion, and the determination of fluid viscosity coefficient etc. In this paper, a new explicit approximation model of TSV is established, which combines the theoretical solution of N-S equation about fluid motion around spherical objects and the statistical regression of solution dimensionless coefficients with measurement data. This new model can adapt to different values of drop parameters and medium parameters in a large range of *Re*. By this model, the relative and absolute calculation errors of TSV are in range of −3.42%–+4.34% and −0.271 m/s–+0.128 m/s respectively for drop radius 0.005–2.9 mm. Their corresponding root mean square values are 1.77% and 0.084 m/s respectively, which are much smaller than that of past theoretical and empirical models.

Keywords terminal settling velocity, drag coefficient, viscous resistance, drop, fluid medium

1 Introduction

The determination of the terminal settling velocity (TSV) of a drop or other spherical object in viscous fluid involves the solution of N-S equation for the steady flow of the fluid medium around the drop, which has important application value in many fields. The TSV, the drag coefficient, and the viscous resistance are different concerns of one problem. The collision caused by the TSV difference of water droplets is an important cloud microphysical process, and the falling of raindrops and hail stones are related to precipitation estimation in weather monitoring. Raindrops and hail are important factors of soil erosion, which can cause dispersion of soil masses and splatter of soil particles (Lv et al., 1997). The roof insulation design of high-speed trains needs to consider the influence of falling drops on the electric field distortion of roof surface to prevent the breakdown and ablation of the roof caused by electric discharge (Sun et al., 2014; Wu et al., 2017). It is an important method to calculate the fluid viscosity coefficient by measuring the TSV of a solid sphere in fluid medium (so-called falling sphere method) (Cao and Zhen, 2013; Wang et al., 2017; Cheng, 2018).

The TSV of a drop (note: for simplicity, we use the word “drop” to stand for any spherical objects.) in fluid medium depends on the geometric and physical parameters of drop and medium, such as the drop radius, the drop density, the fluid viscosity coefficient and the fluid density etc.. The methods for establishing the TSV calculation model include physical experiment, numerical simulation and approximate solution of N-S equation. The physical experimental data with high precision is usually applied to verify the numerical simulation result and the approximate solution of N-S equation.

The measurement experiment about the TSV of water drops in static air dates back to Laws (1941). Gunn and Kinzer (1949) measured the TSV of water drops with diameters of less than 5.8 mm in static air. Pruppacher and Steinberger (1968) measured the TSV of solid spheres in a fuel tank. Beard and Pruppacher (1969) measured the resistance of water drops falling in an adjustable wind tunnel filled with saturated air. The results of these experiments are summarized in Mason’s (1978) famous book “cloud physics”. As pointed out by Mason (1978), the quantitative relationship between the drag coefficient and the Reynolds number (*Re*) for water drops and that for solid spheres is identical when *Re* is smaller than 400. Karamanev (1996) analyzed the empirical formulas

Received May 8, 2020; accepted September 17, 2020

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proposed in the literature for calculating the drag coefficient and terminal velocity of a solid sphere, and developed a new relationship between the drag coefficient and the Archimedes, instead of *Re*, to make the relationship explicit for terminal velocity. Ceylan et al. (2001) summarized the widely used empirical formulas for estimating the drag coefficient of solid spherical particles and cylindrical particles, and presented new relationships between the drag coefficient and *Re*, which are inexplicit for the terminal velocity. Riazi et al. (2020) focused on carbonate sands which can be represented by ellipsoids, and presented new empirical formulas estimating the drag coefficient and settling velocity of sands. The numerical simulation to study the TSV of drops began in the 1990s, such as the simulation about raindrops done by Lv et al. (1997), Sun et al. (2011) and Guo et al. (2018).

According to the results of physical experiment and numerical simulation, it is a fact that with the increase of drop radius, its TSV undergoes a transformation of being proportional to the square radius, to the radius, and subsequently to the root radius. So, it is possible to establish a piecewise empirical statistical TSV formula such as given in Sheng et al. (2003). However, since physical experiments and numerical simulations are always carried out under specific parameters of drop and fluid medium, how to adapt to other parameters is a problem.

The most classical approximate solution based on N-S equation is the work of Stokes (1851), which deleted the nonlinear advection term $\vec{u}\cdot\nabla\vec{u}$ in the N-S equation, proving that the TSV of a drop is proportional to its square radius as the drop radius approaches zero. Oseen (1910) pointed out that even if the drop radius is very small, the effect of the nonlinear advection term cannot be ignored completely. He replaced $\vec{u}\cdot\nabla\vec{u}$ with $\vec{u_t}\cdot\nabla\vec{u}$ (where, $\vec{u_t}$ is the TSV of drop) and solved the N-S equation approximately by series expansion. Thereafter, Chester and Breach (1969), Chen (1975) and others obtained higher order series expansion approximation through the iteration of the Oseen solution. These improvements on Stoke approximation extended the applicable *Re* range of theoretical approximate solution of N-S equation from $Re < 0.5$ for Stokes solution to $Re < 6$.

Although the quantitative accuracy of the theoretical approximate solution is low, the advantage is that these solutions have clear physical meaning and strong adaptability to the changes of drop parameters and fluid medium parameters. At the same time, these works reflect the very limited potential of trying to expand the applicable *Re* range by increasing the order of series expansion, and the approximation process and results are complex and impractical. So, other ways must be found.

Yin and Xu (1991) replaced the nonlinear advection term $\vec{u}\cdot\nabla\vec{u}$ of N-S equation with $a\vec{u_t}\cdot\nabla\vec{u}$ (a is an undetermined dimensionless constant to realize the linear approximation, and multiplied the viscous term $\nu\nabla^2\vec{u}$ with an undetermined dimensionless constant b to avoid the possible imbalance of the equation caused by the linear approximation. The explicit general solution of drop TSV was obtained by solving the approximate linear N-S equation. This approximate solution perfectly explains the pattern from physical experiments that, with the increase of drop radius r, the change of drop TSV is proportional to r^2, r, and $r^{1/2}$ successively. By optimizing the values for these two undetermined dimensionless constants, the approximation accuracy of Yin and Xu (1991) is comparable to the previous piecewise empirical statistical formula in the range of $Re = 0.002$–3549.

As described by Karamanev (1996), the formula form of drag coefficient must be quite elegant in order to obtain the explicit approximate solution of TSV. There has been no new progress on this issue in more than two decades. In fact, the Stokes aproximation solution, which was obtained supposing that the drop radius tends to zero, is applied for determining the viscosity coefficient of oil by the falling sphere method even when the sphere scale is millimeters.

In this paper, we will study how to improve the approximate solution of N-S equation to make the drop TSV can be calculated with a precision about several percent ($< 5\%$) from the point of view of relative error.

The approximation model of drop TSV with high precision, which adapts to the variation of drop parameters and fluid parameters in large range of *Re* (correspondingly, in large range of drop scale), can provide better support than ever before for the calculation of cloud drop velocity, the estimation of rain and hail intensity, the determination of drag coefficient of small spheres in a fluid medium, and measurement of fluid viscosity coefficient, etc. Therefore, such a model is of great application values in the studies of meteorology, soil and water conservation, and oil lubricating property.

2 The approximate solution of N-S equation about fluid motion around spherical object

Assuming that the spherical object such as a drop is an ideal constant sphere, the medium is an incompressible viscous fluid, and the center of the reference coordinate system is set at the center of the sphere, the sphere settling in the fluid can be treated as the motion of the fluid around the sphere, and the equation is N-S equation:

$$\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\nabla\vec{u}=-\frac{1}{\rho}\nabla p-\mathrm{g}\vec{z}+\nu\nabla^2\vec{u}, \tag{1}$$

$$\nabla\cdot\vec{u}=0, \tag{2}$$

where $\vec{u}$ is the around velocity of flow, which equals to zero on the sphere surface, and to the settling velocity of the sphere at a sufficient distance. $\frac{\partial \vec{u}}{\partial t}$ is the acceleration term, and since we only care about the sphere TSV, this term is zero. $\vec{u} \cdot \nabla \vec{u}$ is the nonlinear advection term; $-\frac{1}{\rho}\nabla p$ is the pressure gradient term; ρ is the fluid density; p is the fluid pressure; $-g\vec{z}$ is the gravity term; g is the gravity acceleration; $\vec{z}$ is the unit vector that goes vertically up; $\nu\nabla^2\vec{u}$ is the viscous term; $\nu = \mu/\rho$ is the kinematic viscosity coefficient; and μ is the dynamic viscosity coefficient.

So far, the complete strict solution of N-S equation is impossible. Fortunately, we only focus on the solutions of physical quantities such as TSV, viscous resistance and drag coefficient, rather than the detailed distribution structure of the flow field. Similar to Yin and Xu (1991), Eq. (1) is linearized approximately by the TSV,

$$\vec{u_t} = u_t\vec{z},$$

$$\left(\frac{2}{9}a\vec{u_t}\right)\cdot\nabla\vec{u} = -\frac{1}{\rho}\nabla p - g\vec{z} + b\nu\nabla^2\vec{u}, \tag{3}$$

where a and b are dimensionless coefficients.

Different from the treatment of Yin and Xu (1991) which requires the dimensionless coefficient a to be a constant, we allow that a can be affected by the sphere radius, but independent of the physical parameters such as the sphere density, the medium density, and the medium viscosity coefficient, i.e.,

$$a = a(r). \tag{4}$$

The treatment of a as a function of r does not change the physical nature of Eq. (3) as the approximation of Eq. (1). We hope that:

1) After $a(r)$ is adopted to linearizing the nonlinear advection term of N-S equation, the pattern from physical experiments, "with the increase of drop radius, its TSV undergoes a transformation of being proportional to the square radius, to the radius, and to the root radius subsequently," is still followed.

2) The correction coefficient b of the viscous term, which is used to avoid the unbalance of the left and right of the approximate linearized N-S equation, is very close to 1, since it conforms to the physical reality.

Similar to the derivation process of Yin and Xu (1991), the approximate solution of the viscous resistance F_μ can be obtained

$$F_\mu = 12\pi\rho r^2 u_t^2\left\{\frac{a(r)}{24} + \frac{b}{Re}\right\}, \tag{5}$$

where a is a undetermined functional coefficient of sphere radius, b is an undetermined constant coefficient, and Re is the Reynolds number:

$$Re = \frac{2ru_t}{\nu}. \tag{6}$$

Then, the drag coefficient C_D is

$$C_D \equiv \frac{F_\nu}{(\pi r^2/2)\rho u_t^2} = a(r) + \frac{24}{Re}b = \frac{24}{Re}\left\{b + \frac{Re}{24}a(r)\right\}. \tag{7}$$

Equation (7) yeilds the explicit formula of TSV. The details are as follows.

For a sphere, its settling velocity reaches terminal only when the viscous resistance and the buoyant force balance with the gravity force:

$$F_\mu = \frac{4}{3}\pi r^3(\rho_s - \rho)g. \tag{8}$$

From Eqs. (6)–(8), the sphere TSV, u_t, can be obtained

$$u_t = \kappa\frac{b}{a(r)}\frac{1}{r}\left\{\sqrt{1+\zeta\frac{a(r)}{b^2}r^3} - 1\right\}, \tag{9}$$

where

$$\kappa = 6\nu, \tag{10}$$

$$\zeta = \frac{2}{27}\frac{\rho_s - \rho}{\rho}\frac{g}{\nu^2}. \tag{11}$$

From Eq. (9), it can be seen that if $a(r)$ can keep its value limited when r is very small, and its value changes little when r is very large, then

$$u_t = \begin{cases} \frac{\kappa\zeta}{2b}r^2 & (r \to 0) \\ \kappa\sqrt{\zeta\frac{r}{a(r)}} & (r \to \infty) \end{cases}. \tag{12}$$

Equation (12) means,

1) If the sphere radius is very small, its TSV is proportional to the square of the radius, and if the sphere radius is very large, its TSV is proportional to the root of the radius. Therefore, there must be a radius range between the two, in which the sphere TSV is proportional to the radius, in line with the regular characteristics of experimental results.

2) When the sphere radius is very small, its TSV is related to the value of b, while independent the value of $a(r)$. When the sphere radius is very large, its TSV is related to the value of $a(r)$, while independent the value of b.

If taking $a(r) = 0$ and $b = 1$, Eq. (5), Eq. (7) and Eq. (9) are converted to

$$\begin{cases} F_\mu = \frac{12\pi\rho r^2 u_t^2}{\mathrm{Re}} = 6\pi\mu r u_t \\ C_D = \frac{24}{Re} = \frac{12\mu}{\rho r u_t} \\ u_t = \frac{\kappa\zeta}{2}r^2 = \frac{2}{9}g\frac{\rho_s - \rho}{\mu}r^2 \end{cases}. \tag{13}$$

That is Strokes approximate solution.

If taking $a(r) = 9/2$ and $b = 1$, Eq. (5), Eq. (7) and Eq. (9) are converted to

$$\begin{cases} F_\mu = 12\pi\rho r^2 u_t^2 \left\{ \dfrac{3}{16} + \dfrac{1}{Re} \right\} = 6\pi\mu r u_t \left\{ 1 + \dfrac{3}{16} Re \right\} \\ C_D = \dfrac{9}{2} + \dfrac{24}{Re} = \dfrac{24}{Re}\left(1 + \dfrac{3}{16} Re \right) = \dfrac{9}{2} + \dfrac{12\mu}{\rho r u_t} \\ u_t = \kappa \dfrac{2}{9r} \left\{ \sqrt{1 + \zeta \dfrac{9}{2} r^3} - 1 \right\} \end{cases} . \tag{14}$$

That is Oseen approximate solution.

For a more general situation, the two dimensionless coefficients $a(r)$ and b can be determined by reliable experimental data. Whether the formulas Eq. (5), Eq. (7) and Eq. (9) about the viscous resistance F_μ, the drag coefficient C_D, and the terminal velocity u_t are suitable for an undeformed spherical object or a deformable spherical object depends on the experimental data set.

3 The determination of *a*(*r*) and *b* for liquid drops

In this paper, the measurement data on TSV of water drops in air summarized by Mason (1978) is used to evaluate the accuracy of TVS approximation. There are 43 sets of measurement results, covering drop radius 0.005–2.9 mm, drop TSV 0.003–9.17 m/s, and *Re* 0.002–3549.

Considering that the water drop density, air density, and air viscosity coefficient assumed by previous literature when using this measurement data are not completely consistent, the changes of *Re* value given by Mason (1978) is applied to make the values of these parameters more reasonable. Finally, the specific parameters are drop density $\rho_s = 1.0 \times 10^3$ kg/m^3, air density $\rho = 1.2$ kg/m^3, and dynamic viscosity coefficient $\mu = 1.8 \times 10^{-5}$ kg/(m·s).

3.1 Update of the constant coefficients *a* and *b* of Yin and Xu (1991)

The constant coefficients *a* and *b* of Yin and Xu (1991) are recalculated by the specific parameters of water drops and air given above. The results are as follows:

If the RMS relative approximation error of the calculated drop TSV is required to be the minimum, then

$$\begin{cases} a = 0.6058 \\ b = 1.1432 \end{cases} . \tag{15}$$

At this point, the variations of the absolute error and relative error of drop TSV with the radius are shown in Fig. 1(a). From Fig. 1(a), the relative error is in the range of -11.94%–$+26.09\%$ with the RMS value of 10.23%, and the absolute error is in the range of -0.867–$+0.977$ m/s with the RMS value of 0.478 m/s.

If the RMS absolute approximation error of the calculated drop TSV is required to be the minimum, then

$$\begin{cases} a = 0.5767 \\ b = 1.3122 \end{cases} . \tag{16}$$

At this point, the absolute and relative approximation

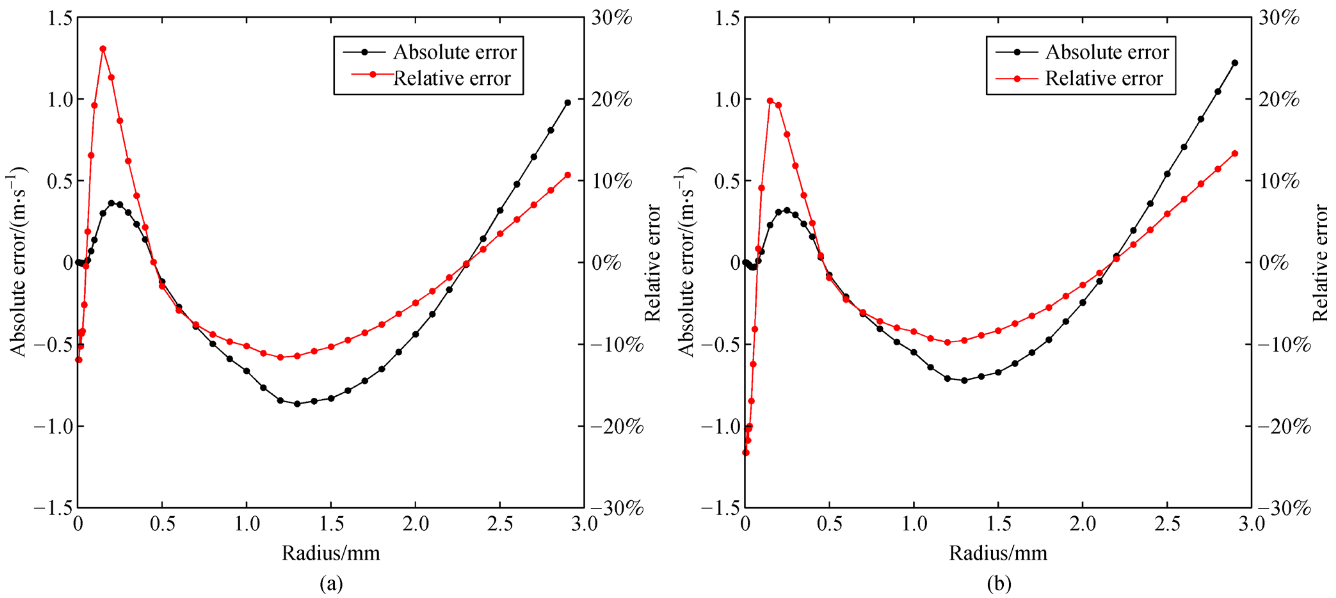

Fig. 1 The change of the relative and absolute errors of drop TSV with drop radius when *a* and *b* are supposed to be constant as Yin and Xu (1991). (a) Minimizing the RMS relative approximation error; (b) Minimizing the RMS absolute approximation error.

errors of TSV at different radii are shown in Fig. 1(b). From Fig. 1(b), the relative error is in the range of –23.27%–+19.77% with the RMS value of 11.77%, and the absolute error is in the range of −0.723–+1.218 m/s with the RMS value of 0.459 m/s.

Overall, the calculation accuracy obtained by minimizing RMS relative error and that by minimizing the RMS absolute error are at same level. Here, the former is adopted, which is referred to as the Yin-Xu approximate formula for short.

3.2 The determination of $a(r)$ and b and the corresponding calculation accuracy of drop TSV

From Eq. (9), when considering the change of a with r, no matter what function model of $a(r)$ is supposed to be, it is a complex nonlinear problem to determine the optimal model parameters of $a(r)$ by the measurement data of u_t vs r.

After a large number of tests, the determination of $a(r)$ and b in the following ways can achieve good results:

1) The first step: inversion of a at different r by measurement data.

Based on the measurement data of Mason (1978), take $b = 1$, and inverse the values of a at different drop radii by Eq. (9). The results are shown in Fig. 2, where the horizontal-coordinate is the drop radius with unit mm, and the vertical-coordinate is ln$a$.

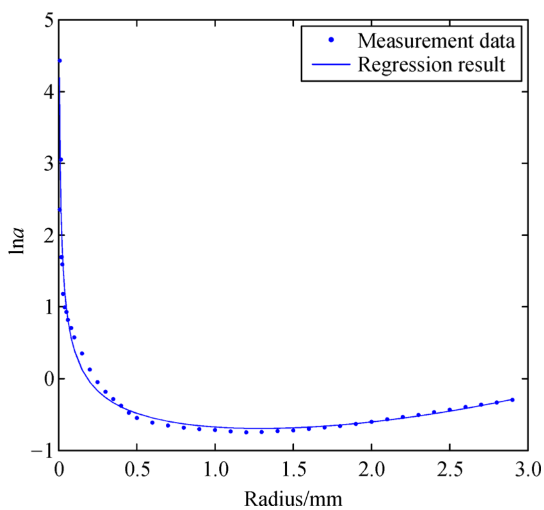

Fig. 2 The values of a at different drop radii inversed by the measurement data and the optimized ln$a$–r fitting curve ($b = 1$).

It can be seen from Fig. 2 that, when the radius is greater than 1.2 mm, ln$a$ increases slowly with the radius from −0.7482 at $r = 1.2$ mm to −0.2965 at $r = 2.9$ mm, which may trace to the deformation of liquid drops. When the radius is less than 1.2 mm, ln$a$ increases as the radius decreases from −0.7482 at $r = 1.2$ mm to 4.4338 at $r = 0.005$ mm, and the increasing slope also increases as the radius decreases.

As mentioned in the Section 2, the drop TSV is actually not related to a when the radius is very small, which means the inversion value of a is unstable by Eq. (9). This fact is reflected from the value jump of a for the smallest three radii. Therefore, it is not necessary to worry about the exact value of a when the radius is very small.

2) The second step: regression analysis of ln$a$ vs r by least square fitting.

A variety of ln$a$ vs r regression models are tried from the point of view of least square fitting, and the results show that the statistical regression model $\ln a = c_0 + c_1 r^2 + c_2 r^{-d_2}$ can well control the approximation error of drop TSV. The concrete results are

$$\begin{cases} \ln a = -1.7438 + 0.092 r^2 + 0.9757 \gamma^{-0.3409} \\ b = 1 \end{cases}, \tag{17}$$

where the unit of radius r is millimeter.

The $a(r)$ and b of Eq. (17) are applied by Eq. (9) to determine the absolute and relative approximation errors of TSV with radius. The results are shown in Fig. 3.

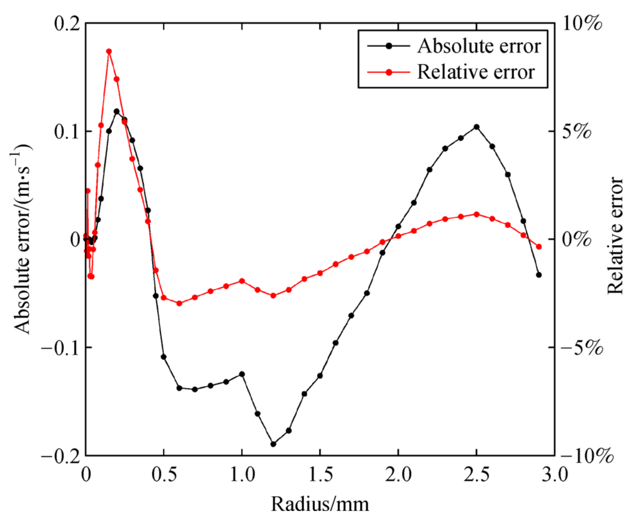

Fig. 3 The change of absolute and relative errors of TSV with radius when $a(r)$ and b are given by Eq. (17).

It can be seen from Fig. 3 that by applying Eq. (17) instead of Eq. (15), the approximation errors of drop TSV are significantly reduced. The range of relative error reduces from −11.94%–+26.09% with the RMS value of 10.23% to −2.97%–+8.69% with the RMS value of 2.65%, and the range of absolute error reduces from −0.867–+0.977 m/s with the RMS value of 0.478 m/s to −0.189–+0.118 m/s with the RMS value of 0.090 m/s, respectively.

3) The third step: optimization of $a(r)$ and b by minimizing the relative error of TSV.

With $a(r)$ and b given by Eq. (17) as the initial $a(r)$ and b, the values of all parameters c_0, c_1, c_2, d_2 of $\ln a = c_0 + c_1 r^2 + c_2 r^{-d_2}$ and b are redetermined by minimizing the RMS relative approximation error of drop TSV. The results are as follows,

$$\begin{cases} \ln a = -2.4970 + 0.1187r^2 + 1.6681r^{-0.2616} \\ b = 0.9872 \end{cases}, \quad (18)$$

where the unit of radius r is millimeter.

Applying $a(r)$ and b of Eq. (18), the relative and absolute approximation errors of TSV at different radius are calculated and shown in Fig. 4.

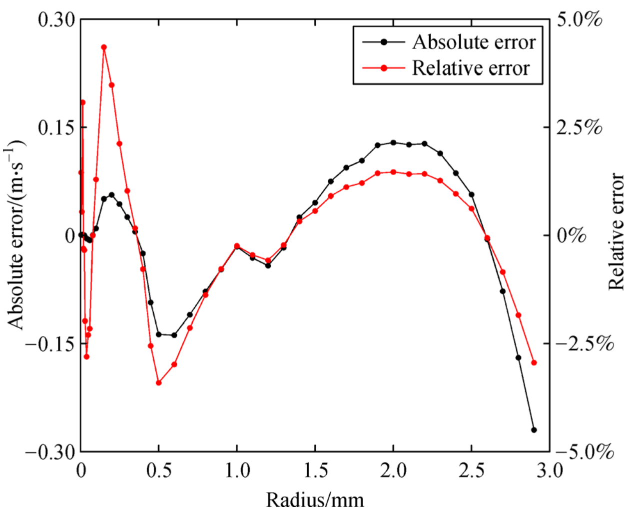

Fig. 4 The change of absolute and relative errors of TSV with radius when $a(r)$ and b are given by Eq. (18).

From Fig. 4, when applying Eq. (18) instead of Eq. (17), the range of relative error of TSV narrows from –2.97%–+8.69% to –3.42%–+4.34%, and the range of absolute error changes from –0.189–+0.118 m/s to –0.271–+0.128 m/s mainly due to the enlargement of absolute error at very large drop radius. Correspondingly, the RMS relative error of TSV is reduced further from 2.65% for Eq. (17) to 1.77% for Eq. (18), while the RMS absolute error of TSV is changed little from 0.090 m/s for Eq. (17) to 0.084 m/s for Eq. (18).

We also try to take $a(r)$ and b of Eq. (17) as initial values to redetermine the parameters c_0, c_1, c_2, d_2 of $\ln a = c_0 + c_1 r^2 + c_2 r^{-d_2}$ and b, so as to minimize the RMS absolute approximation error of drop TSV. The result shows that, although the RMS absolute error is reduced from 0.090 m/s to 0.046 m/s, the RMS relative error is increased significantly from 2.65% to 20.77%, which is undesirable.

So, Eq. (18) is adopted to couple with Eq. (9) as the ultimate approximation TSV model of liquid drops.

Note that liquid drops will deform as radius increases. Take the atmospheric raindrop as an example (Andsager et al. 1999), when its radius less than 140 μm, it can be considered as an ideal sphere, slight deformation for drops with radius 140–500 μm, and the deformation gradually increases with the increase of raindrop radius. The axial ratio of raindrops with $r = 1$ mm and $r = 2.5$ mm is approximately 0.98 and 0.6 respectively. Raindrops with radius greater than 2.5 mm may break up, and most raindrops with radius greater than 3 mm will almost always break up (Mason 1978).

Moreover, as the regression model of $a(r)$ is $\ln a = c_0 + c_1 r^2 + c_2 r^{-d_2}$, if r is very large, $\frac{\mathrm{d}a}{\mathrm{d}r} > \frac{a}{r}$, that is $\frac{\mathrm{d}(r/a)}{\mathrm{d}r} < 0$, and the calculated TVS of liquid drops will change from increase to decrease with the increase of radius. Specifically, the turning point appears at the radius near $r = 2.5$ mm. Therefore, it is suggested that the application range of Eq. (18) is $r \leqslant 2.5$ mm for liquid drops and $r \leqslant 1$ mm for solid spheres.

It is a suggestion from theoretical inference, which hasn't been verified by measurement data, Eq. (17) and Eq. (18) can also be used for solid sphere with radius larger than 1 mm if the r in Eq. (17) and Eq. (18) be replaced by $r^* = \min(r, 1\ \text{mm})$.

4 Comparison with other theoretical and empirical approximate TVS formulas

4.1 Comparison with Strokes approximate solution and Oseen approximate solution

According to the previous views shown in section 1, the applicable range of Re for the Stokes approximate solution is $Re < 0.5$ and that for other past approximate solutions except Yin and Xu (1991) is $Re < 6$.

The calculated TSV of drops with the new approximation model of this paper (Eq. (9) and Eq. (18)), Stokes approximate solution (Eq. (13)) and Oseen approximate solution (Eq. (14)) are compared with the measurement data, which are shown in Fig. 5 and Table 1.

As can be seen from Fig. 5 and Table 1,

1) The calculation result of the new approximation model is in good agreement with the measurement data. Within the range of $Re < 6$, the RMS relative and absolute errors are 1.84% and 3.50×10^{-3} m/s, respectively.

2) In the range of $Re < 6$, the calculation accuracy of the Oseen approximate solution is much lower than that of this paper. Its RMS relative and absolute errors are 8.29% and 3.69×10^{-2} m/s respectively. If focusing on $Re < 0.5$, the

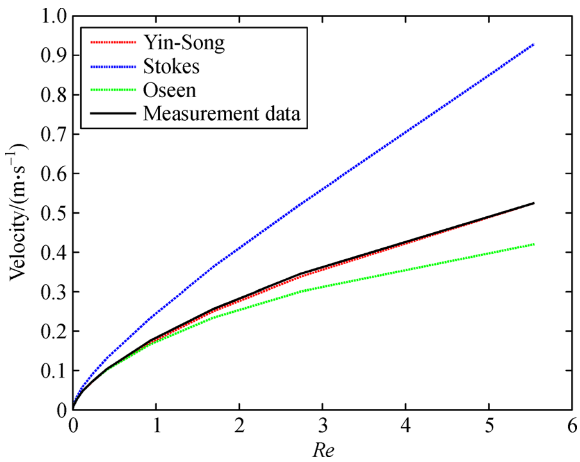

Fig. 5 Comparison of the approximation models of this paper, Stokes approximate solution and Oseen approximate solution with the measurement data in $Re < 6$.

calculation accuracy of the Oseen approximate solution is at the same level of the new approximation model.

3) The calculation result of the Stokes approximate solution is larger than the measurement data, while the calculation result of the Oseen approximate solution is smaller than the measurement data. Moreover, the larger the radius is (equally, the larger the *Re*), the larger the deviation between them and the measurement data. Within the range of $Re < 0.5$, the RMS absolute error of the two approximate solutions are 1.46×10^{-2} m/s and 8.75×10^{-4} m/s respectively. While in the range of $Re < 6$, the RMS absolute error of the two approximate solutions are 1.45×10^{-1} m/s and 3.69×10^{-2} m/s respectively.

4) The Oseen approximate solution is better than the Stokes approximate solution, even in the range of $Re < 0.5$. Within the range of $Re < 6$, the RMS relative errors of the former and the latter are 8.29% and 38.43% respectively. Within the range of $Re < 0.5$, the RMS relative errors of the former and the latter are 1.68% and 23.98% respectively.

4.2 Comparison with the Yin-Xu approximation model and the piecewise empirical model

Within the range of r = 0.005–2.5 mm, the calculation results of the new approximation model of this paper, the Yin-Xu approximation model (Eq. (9) and Eq. (15)) and the piecewise empirical model given in Sheng et al. (2003) are compared with the measurement data. The details are shown in Fig. 6 and Table 2.

As shown in Fig. 6 and Table 2,

1) In the whole range of r = 0.005–2.5 mm and its sub-ranges, the calculated results of the new approximation model are in good agreement with the measurement data, which is obviously better than that of the Yin-Xu approximation model and the piecewise empirical model. The RMS relative and absolute errors of the new approximation model are 1.77% and 7.06×10^{-2} m/s for r = 0.005–2.5 mm, respectively, which is about 1/5 that of Yin-Xu approximation model and the piecewise empirical model.

2) In the whole range of r = 0.005–2.5 mm and its sub-ranges, the calculation accuracy of Yin-Xu approximation model and that of the piecewise empirical model are at the same level with the former is slightly inferior to the latter. For r = 0.005–2.5 mm, their RMS relative errors are 10.42% and 9.22%, and RMS absolute errors are 4.42×10^{-1} m/s and 3.86×10^{-1} m/s, respectively.

3) For all the three models, their RMS relative errors for the range of r = 0.005–1.0 mm are all larger than that for the range of r = 1.0–2.5 mm, while their RMS absolute errors for the range of r = 0.005–1.0 mm are smaller than that for the range of r = 1.0–2.5 mm.

Table 1 The RMS errors of the approximation models of this paper, Stokes approximate solution and Oseen approximate solution

RMS error	$Re < 0.5$		$Re < 6$	
	Relative error/%	Absolute error /(m·s^{-1})	Relative error/%	Absolute error /(m·s^{-1})
Yin-Song	1.63	9.07×10^{-4}	1.84	3.50×10^{-3}
Stokes	23.98	1.46×10^{-2}	38.43	1.45×10^{-1}
Oseen	1.68	8.75×10^{-4}	8.29	3.69×10^{-2}

Table 2 The RMS errors of the approximation model of this paper, the Yin-Xu approximation model (Eq. (9) and Eq. (15)) and the piecewise empirical model given in Sheng et al. (2003) for different ranges of radius

RMS error	$0.005 < r < 2.5$		$0.005 < r < 1.0$		$1.0 < r < 2.5$	
	Relative error/%	Absolute error /(m·s^{-1})	Relative error/%	Absolute error /(m·s^{-1})	Relative error/%	Absolute error /(m·s^{-1})
Yin-Song	1.77	7.06×10^{-2}	2.11	5.66×10^{-2}	0.98	8.58×10^{-2}
Yin-Xu	10.42	4.42×10^{-1}	11.72	2.75×10^{-1}	8.06	6.24×10^{-1}
Piecewise empirical model	9.22	3.86×10^{-1}	10.28	1.95×10^{-1}	7.28	5.69×10^{-1}

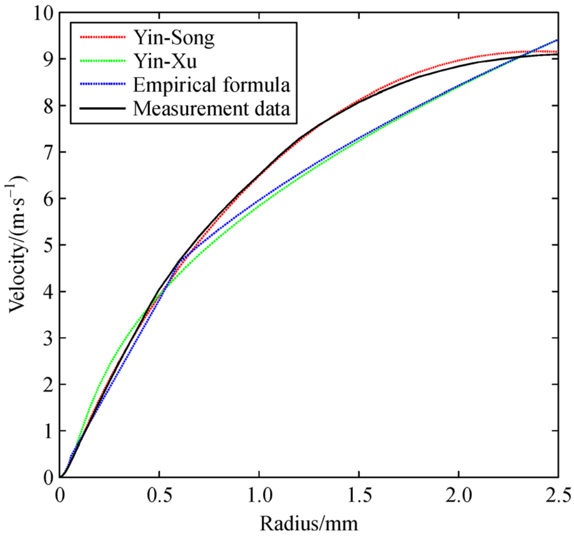

Fig. 6 Comparison of the approximation model of this paper, the Yin-Xu approcximation model (Eq. (9) and Eq. (15)) and the piecewise empirical model given in Sheng et al. (2003) with the measurement data in r = 0.005–2.5 mm.

5 Summary

In this paper, the approximate TSV solution of liquid drops in fluid medium given by Yin and Xu (1991) is improved to obtain a high-precision approximation model. The main work and conclusions are as follows.

Assuming that the drops and other spherical objects are ideal spheres, the N-S equation is solved approximately by linearizing its nonlinear advection term with the product of the far-field velocity and a dimensionless functional coefficient of drop radius, meanwhile, adjusting its viscous term by a dimensionless constant coefficient to avoid the equation imbalance caused by the linearization of advection term. By this means, the general approximate explicit solutions of the terminal settling velocity, drag coefficient and viscous resistance of drops and other spherical objects in viscous fluid medium are obtained.

The two dimensionless coefficients of the general solutions are determined in the sense of statistical optimization by the iteration of least squares fitting with the experimental terminal settling velocity data of water drops in air. Thus, a new explicit approximation model of drop terminal settling velocity is established, which combines the theoretical general solution of N-S equation about fluid motion around spherical object and the statistical regression of solution coefficients with measurement data of drops.

For drops in the range of radius r = 0.005–2.9 mm (or equally Re = 0.002–3549 for water drops in air), the relative and absolute errors of their terminal settling velocity calculated by the new approximation model vary in −3.42%–+4.34% and −0.271–+0.128 m/s respectively. Their corresponding statistical root-mean-square values are 1.77% and 0.084 m/s respectively, which are about 1/5 that of the approximation models introduced by Yin and Xu (1991) and the piecewise empirical model given in Sheng et al. (2003). For small Re ($Re < 6$), these two statistical quantities are about 1/5 and 1/10 that of Oseen approximate solution and about 1/20 and 1/40 that of Stokes approximate solution respectively.

The new approximation model can adapt to the change of the parameters of drop and fluid medium in a large range, and is not limited to small Re. Meanwhile, the calculated terminal settling velocities of liquid drops are in good consistent with the measurement results. So, this model can meet the needs of calculating the terminal settling velocity, the drag coefficient and the viscous resistance of liquid drops with high-precision in various application fields.

The dimensionless functional coefficient and the dimensionless constant coefficient of the new approximation model are independent of the physical parameters of drop and medium. Since the measurement data applied in this paper for determining these two dimensionless coefficients are the terminal settling velocity data of water drops in air, the deformation of drop will affect results of determination, especially when the drop radius is greater than 1 mm. Therefore, if the new approximation model is applied to undeformed spheres with radius larger than 1 mm, it is suggested setting the dimensionless functional coefficient equal to its value at sphere radius 1 mm

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