

River basin water resource compensation characteristics by set pair analysis: the Dongjiang example

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Abstract Flood and drought coexist in many river basins, thus analyses of water resource compensation characteristics become important, since they are the foundation for rational utilization of floodwaters. In this research, set pair analysis (SPA), a relatively new uncertainty analysis method, is used to study the dry and wet compensation characteristics of water resource parameters. In addition, fuzzy membership and grey correlation degree are adopted to test the result of set pair analysis. The Dongjiang River is taken as an example and the analyzed parameters include precipitation and mean discharge from different hydrological stations. The results show that there is a high homeotype-encountering chance for precipitation and mean discharge between different stations for both dry and wet conditions; thus the compensation capacity is small. Although the mean discharge is synchronous with the precipitation in the river basin, there exists a certain degree of shift, indicating possible utilization of floodwater on a small scale. The results from SPA are consistent with that from a traditional analysis method, showing that SPA is a promising alternative method for studying river basin water resource compensation characteristics, in particular for exploring potential complements embedded in non-complementary general features.

Keywords water resources, compensation characteristics, set pair analysis (SPA), Dongjiang River basin

1 Introduction

The analysis of regional water resource compensation characteristic is to identify the possibility of encountering

between dry and wet conditions in time or space. The analyses could not only be used to understand the changes in water resources, but also benefit the management and utilization of water, which is essential to sustainable economic development and eco-environmental conservation. Flood and drought usually coexist in many river basins, thus analyses of water resource complementary characteristics become important, since they are the foundation for rational utilization of water resources.

It is well known that China is suffering from serious water resource scarcity because of uneven distribution of precipitation in space and time. Flood occurs frequently, while drought exists at the same time. Rational harvesting of floodwater is receiving great attentions from society, demanding special attention to the temporal-spatial distribution characteristics of dry and wet conditions. Water resource compensation analysis provides the basis for floodwater utilization, and identifies the possibility of encountering dry and wet conditions (Stakhiv, 2003; Al-Houri and Barber, 2011), the effects of climate change on precipitation patterns and hence the runoff yield (Mehta et al., 2011; Gallant et al., 2012), long-term variation of averaged stream flow (Kiem and Franks, 2004; van Ogtrop et al., 2011) and the consequent adaptive management (Davis, 2007; MacDonald et al., 2010). However, combined analyses of both precipitation and runoff are rarely reported, as indicated in recent literature (Kiem and Verdon-Kidd, 2010; Kirono et al., 2010; Rientjes et al., 2013).

Set pair analysis (SPA), which was proposed by Zhao (2000), is a relatively new mathematic theory and systematic analysis method for treating uncertainty. In hydrology and water resources, there are various set pairs that create preconditions for the application of set pair analysis. SPA has been increasingly applied to hydrology and water resources (Feng et al., 2004; Wan et al., 2006),

environmental assessment (Fleiner, 2001), selection of analogous basins (Deng et al., 2006a), and spatial anomaly analysis of river runoff (Wooldridge et al., 2002; Deng et al., 2006b).

Given the merit of SPA for exploring potential complements embedded in non-complementary general features, which is exactly the case for the Dongjiang River, this study applied the method to identify the complementary characteristics of both precipitation and mean discharge. Meanwhile, fuzzy membership and grey correlation degree are used to justify the results of set pair analysis. The research provides useful clues to the regional floodwater utilization and water resource management, and also demonstrates that set pair analysis is a promising method to study river basin water resources' complementary characteristics.

2 Materials and methods

2.1 Set pair analysis

SPA is a systematic analysis method for treating uncertainty problems. Set pair means a pair of set A and set B that have some linkage, which is $H(A, B)$. For example, variables of a designed basin and variables of a similar basin could be set as a pair. The key point of set pair is to analyze synchronization, difference, and opposition characteristics of two sets. The expression of correlation grade of two sets μ_{A-B} is given as

$$\mu_{A-B} = \frac{S}{N} + \frac{F}{N}i + \frac{P}{N}j, \quad (1)$$

in which N is total number of set characteristics; S is the number of identical characteristics (e.g., dry-dry); F is the number of difference characteristics (e.g., wet-normal, normal-dry); P is the number of opposition characteristics (e.g., wet-dry); i is coefficient of uncertain difference, taking values in $[-1, 1]$; j is opposition coefficient and $j = -1$. To simplify, $a = S/N$ is called identical degree; $b = F/N$ is called difference degree; $c = P/N$ is called opposition degree. Then, Eq. (1) can be modified as

$$\mu_{A-B} = a + bi + cj, \quad (2)$$

where a , b and c are non-negative, and satisfy the condition of normalization. a indicates a positive correlation trend, b indicates neither a positive nor a negative uncertainty correlation relationship, and c indicates a negative correlation trend. If $a > c$, the two sets have a positive correlation relationship; if $a < c$, the two sets have a negative correlation relationship; if b is large, the correlation is highly uncertain. The procedures to conduct set pair analysis are:

(1) Take for example sets A , B , C as objects to be analyzed, and define any two of them as a set pair (A, B),

(A, C) and (B, C).

(2) According to the sorting standard, divide each element into classes, such as wet, normal and dry (denoted by I, II and III respectively), then the symbolization sets A' , B' and C' can be obtained. Define the same set pairs for A' , B' and C' .

(3) Compare the elements in each set pair, and count the number with the same class symbol (denoted as S); and count the number with difference (denoted as F), that is one grade apart such as II and I, III and II; and count the number with opposition (denoted as P), that is two grades apart, such as III and I. Use Eq. (1) to calculate the correlation degree of each set pair.

2.2 Study area

The Dongjiang River is one of the three tributaries of the Pearl River, which flows from northeast to southwest with an average bed slope of 0.35 ‰. The river channel length to Shilong is 520 km, and to Lion lagoon is 562 km. The total drainage area is 35,340 km², and that of the area upstream of Shilong is 27,040 km².

The annual water supply in the Dongjiang River basin was 89.85×10^9 m³ in 2005, which is 27.5% of the basin's annual average volume. According to international guidance about water demand for river ecosystems in humid regions, the water usage from a stream should not exceed 25% of the total river runoff. The water resources exploitation degree in the Dongjiang River basin has exceeded this value, in particular in dry seasons.

In this study, the data of precipitation and average discharge from the Heyuan, Lingxia, and Boluo hydrological stations (Fig. 1) in a 24 month period from 1997 to 1998 are used to analyze the water resources complemen-

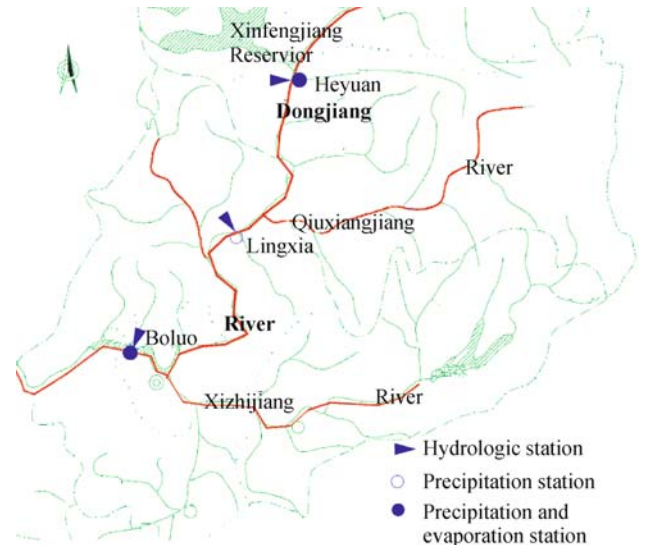


Fig. 1 The middle and lower reach of the Dongjiang River basin.

tary characteristics of the middle and lower reaches of the Dongjiang River basin, in order to support flood management and water resource development. The data are classified into wet and dry types.

3 Results and discussion

3.1 Set pair analysis results

Take the precipitation series of Heyuan, Lingxia, Boluo as set H_r, L_r, B_r , then form three pairs (H_r, L_r) , (H_r, B_r) and (L_r, B_r) . Count the number of identical characteristics S , difference characteristics F , and opposition characteristics P . Applying Eq. (2), the correlation degree of each pair can be calculated:

$$\begin{cases} \mu_{L_r-B_r} = 0.917 + 0.042i + 0.042j \\ \mu_{H_r-B_r} = 0.875 + 0.083i + 0.042j \\ \mu_{H_r-L_r} = 0.875 + 0.042i + 0.083j. \end{cases} \quad (3)$$

From Eq. (3), the identical degrees are the largest and all are over 0.8, while the opposition degrees are the smallest.

Take the average discharge series of Heyuan, Lingxia, Boluo as sets H_f, L_f, B_f , then three set pairs (H_f, L_f) , (H_f, B_f) and (L_f, B_f) are formed. The correlation degrees of each pair are calculated through SPA:

$$\begin{cases} \mu_{L_f-B_f} = 1 \\ \mu_{H_f-B_f} = 0.667 + 0.333i \\ \mu_{H_f-L_f} = 0.667 + 0.333i. \end{cases} \quad (4)$$

Eq. (4) shows that among the correlation degrees, the identical degrees are the largest and all are over 0.6, but the opposition correlation degrees are all zero.

Precipitation is the direct driving factor of runoff, and they inherently have a close relationship. However, in different regions, the runoff has different response to precipitation. Due to temporal and spatial variations of precipitation, it is usually difficult to determine the precise result of precipitation influence; however, the qualitative influence degree can be analyzed. Taking the precipitation and the average discharge of Heyuan, Lingxia, Boluo in the same period to conduct combined analysis, the calculated correlation degrees are:

$$\begin{cases} \mu_{L_r-L_f} = 0.792 + 0.083i + 0.125j \\ \mu_{B_r-B_f} = 0.708 + 0.125i + 0.167j \\ \mu_{H_r-H_f} = 0.541 + 0.292i + 0.167j, \end{cases} \quad (5)$$

where μ_L means the correlation degree of average discharge and precipitation in Lingxia; μ_B means the correlation degree of average discharge and precipitation in Boluo; μ_H means the correlation degree of average discharge and precipitation in Heyuan.

In Eq. (5), among the three correlation degrees, the identical degrees are the largest. However, it is also seen there are some difference characteristics, and even opposition characteristics. For example, in September 1997, the precipitation in Lingxia, Boluo and Heyuan gauges are all dry type; however, the corresponding average discharges are normal type, normal type and wet type. Also in August 1998, the precipitation in Lingxia, Boluo and Heyuan are all wet type; however the corresponding average discharges are wet type, wet type and normal type.

Finally, taking the precipitation in the upstream gauge and the average discharge in the downstream gauge for pair analyses, and the correlation degrees are

$$\begin{cases} \mu_{H_r-L_f} = 0.667 + 0.125i + 0.208j \\ \mu_{H_r-B_f} = 0.667 + 0.125i + 0.208j \\ \mu_{L_r-H_f} = 0.792 + 0.083i + 0.125j. \end{cases} \quad (6)$$

From Eq. (6), it is observed that $\mu_{H_r-L_f}$ and $\mu_{H_r-B_f}$ are equal.

3.2 Discussion

Eq. (3) indicates that the synchronization characteristics of precipitation among the three gauges are comparatively dominant, while the possibility of shifting between dry and wet is rather small. Because the river basin is controlled by the same meteorological system, the wet and dry conditions between the stations have a high synchronization feature. Therefore, there is little compensation capacity in the Dongjiang River basin regarding precipitation.

The high identical degree in Eq. (4) means the average discharges among the three gauges have similar features, while the zero value of opposition degrees means the chance of advanced placement in dry and wet pattern is almost negligible. It also means that the average discharges of the three gauges have synchronization for both wet and dry conditions.

In Eq. (5), the identical degrees are the largest, which means precipitation has an important and direct effect on average discharge. The significant difference characteristics and even opposition characteristics imply that the average discharges of the three stations have some degree of uncertainty with precipitation, probably due to the adjustment of the upstream reservoir and other hydraulic structures.

From Eq. (6), the precipitation of Heyuan gauge has the same effect on the averaged discharges at the Lingxia and Boluo stations. The precipitation effect of the Lingxia gauge on the discharge at the Boluo station (the set pair identical degree was 0.792) is stronger than that of the Heyuan gauge on the Boluo station (the set pair identical degree was 0.667).

4 Tests on the set pair analysis results

To test the results of set pair analyses and evaluate the applicability of the method, fuzzy membership degree and grey correlation degree are used to the data set.

4.1 Fuzzy membership degree test

Fuzzy membership can be used to indicate the correlation degree between two sets. For several series without special standard, a series denoted as X^* could be selected as the reference, then the difference percentage of the other series for example Y to the reference series is calculated by $Y = \frac{2|Y-X^*|}{|Y+X^*|}$. In this research, the ‘descending semi trapezium distribution function’ is adopted:

$$R_i = \begin{cases} 1 & Y_i \leq n \\ \frac{m-Y_i}{m-n} & n < Y_i < m \\ 0 & Y_i \geq m \end{cases} \quad (7)$$

where m is maximum difference percentage, and n is the minimum percentage. Therefore, the fuzzy membership degree between two series is $B = \sum_{i=1}^k R_i \cdot W_i$, where W_i is the weight of each element and is usually assigned to $W_i = \frac{1}{k}$, in which k is the length of the series. The larger the fuzzy membership degree is, the closer relationship the two series have.

Taking the precipitation series of Boluo gauge as the referencing series, one can calculate the fuzzy membership degree between it and the precipitation of Lingxia gauge as well as Heyuan gauge. The results are $P_{L-B} = 0.845$ and $P_{H-B} = 0.780$. This means the correlation of precipitation between Lingxia gauge and Boluo gauge is higher than that of Heyuan gauge and Boluo gauge. The result is consistent with the identical degree calculated by SPA (Eq. (3)).

Taking the average discharge series of Boluo station as the reference series, one can calculate the fuzzy membership degree between it and the average discharge of Lingxia station as well as Heyuan station. The results are $D_{L-B} = 0.845$ and $D_{H-B} = 0.780$, indicating that the correlation of average discharge between Lingxia station and Boluo station is higher than that of Heyuan station and Boluo station. The result agrees well with the identical degree calculated by SPA (Eq. (4)).

4.2 Grey correlation degree test

The correlation analysis in grey system theory (Deng, 2002; Karmakar and Mujumdar, 2007; Alvisi and Franchini, 2012; Alvisi et al., 2013) determines the degree

of similarity or difference of the developing trend between variables. The method does not have a strict requirement of sample size or typical distribution type of variables. In fact, the correlation degree only reflects the interaction between factors; it is more important to look at the sorted order.

For a data set comprising m variable and n samples, $x_i = \{x_i(k) | k = 1, \dots, n; i = 0, 1, \dots, m-1\}$, let x_0 be the reference series, and the others be comparison series. After standardization, a new series x'_i is obtained. Denoting the absolute difference between x'_0 and x'_i at time k as $\Delta_i(k) = |x'_0(k) - x'_i(k)|$, the correlation coefficient ξ between $x'_0(k)$ and $x'_i(k)$ is given as

$$\xi_i(k) = \frac{\min_i \min_k \Delta_i(k) + \rho \max_i \max_k \Delta_i(k)}{\Delta_i(k) + \rho \max_i \max_k \Delta_i(k)}, \quad (8)$$

where \min_k is primary minimum difference indicating the minimum distance from each point on curve x'_i to that on curve x'_0 ; $\min_i \min_k$ is secondary minimum difference indicating the basis of minimum difference \min , searching in every curve in the sequence of $i = 1, 2, \dots, m-1$ for the minimum difference of all curves. $\max_i \max_k$ is secondary maximum difference; $\rho \in [0, 1]$ is resolution coefficient; the smaller ρ is, the higher the resolution is, and ρ is usually set at 0.5. Finally, the grey correlation degree is calculated by

$$\gamma_i = \frac{1}{n} \sum_{k=1}^n \xi_i(k). \quad (9)$$

The grey correlation degree is then sorted. If the order is $\gamma_1 > \gamma_2 > \dots > \gamma_{m-1}$, it means the influence of X_1 on X_0 is higher than X_2 on X_0 .

The standardized series of precipitation and averaged runoff of the three stations are given in Fig. 2 and Fig. 3 respectively. It is easily seen that they exhibit similar variation patterns.

The grey correlation degrees of precipitation between Heyuan and Boluo, and between Lingxia and Boluo were

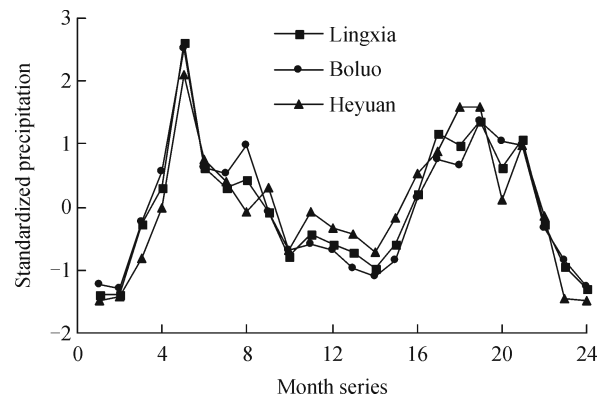


Fig. 2 Standardized series of monthly precipitation.

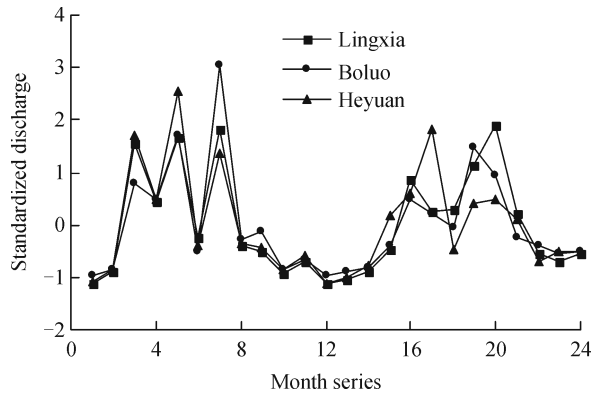


Fig. 3 Standardized series of monthly average discharge.

analyzed. The results are $\gamma_{H-B}^p = 0.766$ and $\gamma_{L-B}^p = 0.806$, indicating that the correlation of precipitation between Lingxia gauge and Boluo gauge is higher than that of Heyuan gauge and Boluo gauge, which further confirms the result by SPA.

The grey correlation degrees of averaged discharge between Heyuan and Boluo, and between Lingxia and Boluo were analyzed. The results are $\gamma_{H-B}^D = 0.616$ and $\gamma_{L-B}^D = 0.796$, showing the correlation of average discharge between Lingxia station and Boluo station is higher than that of Heyuan station and Boluo station, which also further confirms the result by SPA.

Finally, the method was used to investigate the influence of precipitation at Heyuan, Lingxia and Boluo on the average discharge at Boluo. The results are $\gamma_{H-B} = 0.719$, $\gamma_{L-B} = 0.738$ and $\gamma_{B-B} = 0.756$, which is consistent with the identical degree calculated by SPA (Eqs. (5) and (6)).

5 Conclusions

From the study, it is found because they are controlled by the same meteorological system, the precipitation for Heyuan, Lingxia, and Boluo gauges of the Dongjiang River basin are in general varying synchronously, and thus the wet and dry conditions between the stations have high homeotype-encountering chances. Therefore, the compensation capacity in the river basin is small. The same conclusion is drawn for the runoff.

Because the discharge is manipulated by the upstream hydraulic structures such as Xinfengjiang reservoir, there exist some different and even opposite characteristics between the discharges and precipitations of each station. This creates a possibility for regional floodwater utilization and improves water resource management in the river basin.

Being a systematic approach for uncertainty study, SPA proves to be a promising alternative method to investigate the correlation characteristics of hydrological series. It can reveal the correlation and uncertainty of hydrological

parameters in detail. Although fuzzy membership degree and grey correlation degree are broadly applied, they are comparatively complicated for straightforward interpretation.

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