

# Transport of a volatile contaminant in a free-surface wetland flow

Jue YUAN<sup>1</sup>, Li ZENG (✉)<sup>1</sup>, Yijun ZHAO<sup>1</sup>, Yihong WU<sup>1</sup>, Ping JI<sup>1</sup>, Bin CHEN<sup>2</sup>

<sup>1</sup> State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin,  
China Institute of Water Resources and Hydropower Research, Beijing 100038, China

<sup>2</sup> State Key Laboratory of Water Environment Simulation, School of Environment, Beijing Normal University, Beijing 100875, China

© Higher Education Press and Springer-Verlag Berlin Heidelberg 2013

**Abstract** Presented in this paper is an analytical study of a pulsed volatile contaminant emission into a free-surface wetland flow. A simplified model is given for contaminant transport under the combined action of advection, mass dispersion, apparent reaction, and volatilization at the free water surface. The effect of periodic apparent reaction on contaminant transport is separated from the hydraulic effect via an extended transformation, with a limiting case covering the known transformation for constant apparent reaction rate. The analytical solutions of zeroth and first order concentration moments are rigorously derived and illustrated. It was found that the amount of contaminant decreases from the bottom bed to the free-surface under volatilization, and the total amount of contaminant decays with time. It was also found that the moving speed of the mass center of the whole contaminant cloud increases, as the ratio of volatilization coefficient to vertical effective mass dispersivity increases.

**Keywords** contaminant transport, volatilization, reaction, wetland hydraulics

## 1 Introduction

The mechanism of contaminant transport in wetland flows has received increasing attention, due to its significant value for environmental risk assessment associated with wetlands (Mitsch and Gosselink, 1993; Costanza et al., 1997; Chen et al., 2009; Shao et al., 2012).

Contaminant transport in wetland flows is dependent on various factors, such as velocity distribution, apparent reaction, volatilization at the interface of water and air,

infiltration at the bottom bed, etc. With respect to contaminant transport due to pure velocity distribution, some efforts have been made (Lightbody and Nepf, 2006a, b; Murphy et al., 2007; Nepf et al., 2007; Chen et al., 2010, 2012; Zeng, 2010; Wu et al., 2011a, b, 2012; Zeng et al., 2011, 2012a, b) based on Taylor's classic analysis of dispersion (Taylor, 1953), the method of concentration moments (Aris, 1956), and the multi-scale analysis method (Mei et al., 1996). In realistic wetland flows, there are various complex reactions, such as absorption, desorption, hydrolysis, etc., which are important to contaminant transport. Zeng and Chen (2011) analyzed the effect of apparent reaction rate on contaminant dispersion based on an ecological transformation, in which apparent reaction rate is assumed to be constant. However, the apparent reaction rate may change due to changes of temperature, salinity, etc. The effect of variations in apparent reaction rate over time on contaminant transport in wetland flows has not been well understood.

Municipal sewage and industrial wastewater may contain various volatile contaminants, such as dimethylsulfide, dimethyldisulfide, etc. When the contaminants are discharged into a wetland flow, they may escape from the water into the air by volatilization. Up to now, the effects of volatilization on the transport of contaminants in wetland flows has not been well understood.

This paper presents a theoretical analysis of the transport of volatile contaminants for a typical case involving a pulsed volatile contaminant emission into a free-surface wetland flow. The specific objectives are: (i) to obtain a transformation to separate the effect of periodic apparent reaction from the hydraulic effect, (ii) to present analytical solutions for the zeroth and first order concentration moments for volatile contaminants, and (iii) to illustrate the effect of characteristic parameters, associated with apparent reaction and volatilization, on contaminant transport.

## 2 Formulation

The governing equation for the mass transport of a volatile contaminant in typical wetland flows is shown below (Liu and Masliyah, 2005; Zeng and Chen, 2011).

$$\phi \frac{\partial C}{\partial t} + \nabla \cdot (uC) = \nabla \cdot (\kappa \lambda \phi \nabla C) + \kappa \nabla \cdot (K \cdot \nabla C) - \phi k_a C, \quad (1)$$

where  $\phi$  is porosity,  $t$  is time,  $C$  is contaminant concentration,  $u$  is velocity,  $\kappa$  is tortuosity,  $\lambda$  is mass diffusivity,  $K$  is mass dispersivity tensor, and  $k_a$  is apparent reaction rate. In realistic wetland flow situations, the apparent reaction rate may change with time. In this paper, we express a periodic apparent reaction rate with

$$k_a = k_s + k_o f(t), \quad (2)$$

where  $k_s$  is the steady component of the apparent reaction rate,  $k_o$  is the amplitude of the oscillatory component of the apparent reaction rate, and  $f(t)$  is a periodic function with periodic average equal to zero.

Consider the transport of a volatile contaminant in a fully developed free-surface wetland flow in a Cartesian coordinate system, as shown in Fig. 1. The longitudinal  $x$ -axis is parallel to the flow direction, the vertical  $z$ -axis is upward, and the origin is located at the bottom bed. The solution for velocity distribution is (Zeng and Chen, 2011).

$$u(z) = U \frac{\cosh \alpha - \cosh [\alpha(1-z/H)]}{(\alpha \cosh \alpha - \sinh \alpha)/\alpha}, \quad (3)$$

where  $U$  is the depth-averaged velocity, and  $\alpha$  is a dimensionless characteristic parameter.

Consider a uniform instantaneous discharge of a volatile contaminant at the position of  $x=0$  at time  $t=0$ . The corresponding initial condition can be written as (Zeng and Chen, 2011)

$$C(x,z,t)|_{t=0} = \frac{Q}{\phi H} \delta(x), \quad (4)$$

where  $Q$  is the mass per unit width, and  $\delta(x)$  is the Dirac delta function.

For the two-dimensional process, Eq. (1) reduces to

$$\frac{\partial C}{\partial t} + \frac{u}{\phi} \frac{\partial C}{\partial x} = \kappa \left[ \lambda + \frac{K_L}{\phi} \right] \frac{\partial^2 C}{\partial x^2} + \kappa \left[ \lambda + \frac{K_V}{\phi} \right] \frac{\partial^2 C}{\partial z^2} - k_s [1 + \sigma f(t)] C, \quad (5)$$

where  $K_L$  and  $K_V$  are the longitudinal and vertical mass dispersivities, respectively; and  $\sigma = k_o/k_s$  is the ratio of the steady component to the amplitude of the oscillatory component.

Since the discharge amount of the volatile contaminant is definite, the concentration boundary conditions at  $x = \pm \infty$  are

$$C(\pm \infty, z, t) = 0, \quad (6)$$

assuming that the ambient concentration of the volatile contaminant in the air is equal to zero. For the impermeable bottom bed at  $z=0$ , and the free surface at  $z=H$ , the concentration boundary conditions are

$$\left. \frac{\partial C}{\partial z} \right|_{z=0} = 0, \quad \left[ -\kappa \left( \lambda + \frac{K_V}{\phi} \right) \frac{\partial C}{\partial z} \right] \Big|_{z=H} = (h_s C)|_{z=H}, \quad (7)$$

where  $h_s$  is the volatility coefficient, which is similar to a heat transfer coefficient.

The governing equation, as well as the boundary and initial conditions, can be rewritten in dimensionless form as

$$\frac{\partial \Omega}{\partial \tau} + \frac{Pe_x R_x^K}{\phi} \psi \frac{\partial \Omega}{\partial \xi} = R_x^K \frac{\partial^2 \Omega}{\partial \xi^2} + \frac{\partial^2 \Omega}{\partial \zeta^2} - N [1 + \sigma \theta(\tau)] \Omega, \quad (8)$$

$$\Omega(\pm \infty, \zeta, \tau) = 0, \quad (9)$$

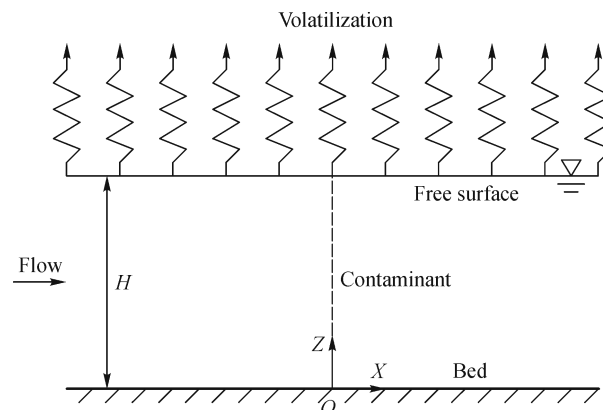


Fig. 1 Sketch for a free-surface wetland flow under volatilization.

$$\left. \frac{\partial \Omega}{\partial \zeta} \right|_{\zeta=0} = 0, \quad \left[ R_s \Omega + \frac{\partial \Omega}{\partial \zeta} \right] \Big|_{\zeta=1} = 0, \quad (10)$$

$$\Omega(\xi, \zeta, \tau)|_{\tau=0} = H\delta(H\xi), \quad (11)$$

where:

$$\begin{aligned} \xi &= \frac{x}{H}, \quad \zeta = \frac{z}{H}, \quad \tau = \frac{\kappa(\lambda + K_V/\phi)t}{H^2}, \quad \Omega(\xi, \zeta, \tau) = \frac{\phi CH^2}{Q}, \\ Pe_x &= \frac{UH}{\kappa(\lambda + K_L/\phi)}, \quad R_x^K = \frac{\lambda + K_L/\phi}{\lambda + K_V/\phi}, \\ \psi &= \frac{\alpha \{ \cosh \alpha - \alpha \cosh [\alpha(\zeta - 1)] \}}{\alpha \cosh \alpha - \sinh \alpha}, \\ \theta(\tau) &= f\left(\frac{H^2 \tau}{\kappa(\lambda + K_V/\phi)}\right), \quad R_s = \frac{Hh_s}{\kappa(\lambda + K_V/\phi)}. \end{aligned} \quad (12)$$

The dimensionless parameter  $R_s$  reflects the relative strength of volatilization, and the effective mass dispersion in the vertical direction.

### 3 Separation of the effect of periodic apparent reaction from the hydraulic effect

The dimensionless concentration determined by Eqs. (8)–(11) is dependent on advection, mass dispersion, apparent reaction, and volatilization. For the case of constant apparent reaction rate, the effect of apparent reaction can be separated from the hydraulic effect by a simple exponential transformation (Zeng and Chen, 2011). For the case of periodic apparent reaction rate, an extended exponential transformation equation,

$$\Omega(\xi, \zeta, \tau) = \Omega^*(\xi, \zeta, \tau) \exp\left(\int_0^\tau -N[1 + \sigma\theta(\tau_1)]d\tau_1\right), \quad (13)$$

was used to separate the effect of apparent reaction from the hydraulic effect, where  $\Omega^*(\xi, \zeta, \tau)$  is only dependent on advection, mass dispersion, and volatilization. The transformation given by Zeng and Chen (2011) can be included as a special case of  $\sigma=0$ . With Eq. (13), we have

$$\frac{\partial \Omega^*}{\partial \tau} + \frac{Pe_x R_x^K}{\phi} \psi \frac{\partial \Omega^*}{\partial \xi} = R_x^K \frac{\partial^2 \Omega^*}{\partial \xi^2} + \frac{\partial^2 \Omega^*}{\partial \zeta^2}, \quad (14)$$

$$\Omega^*(\pm\infty, \zeta, \tau) = 0, \quad (15)$$

$$\left. \frac{\partial \Omega^*}{\partial \zeta} \right|_{\zeta=0} = 0, \quad \left( R_s \Omega^* + \frac{\partial \Omega^*}{\partial \zeta} \right) \Big|_{\zeta=1} = 0, \quad (16)$$

$$\Omega^*(\xi, \zeta, \tau)|_{\tau=0} = H\delta(H\xi). \quad (17)$$

To illustrate the effect of time-dependent apparent reaction rate on contaminant transport, three kinds of apparent reaction rate were considered. The first case is

where  $\theta(\tau) = |\cos(\tau)|$ , which means that ambient factors strengthen the apparent reaction all the time. The second case is where  $\theta(\tau) = -|\cos(\tau)|$ , which means that the ambient factors weaken the apparent reaction all the time. The third case is where  $\theta(\tau) = \cos(\tau)$ , which means that the ambient factors strengthen and weaken the apparent reaction periodically.

Figure 2 shows the variation of  $\Lambda_1$  with  $\tau$  for  $N=1$ , and  $\sigma=0.1, 0.2$ , and  $0.3$ , where  $\Lambda_1$  is expressed as

$$\Lambda_1 = \frac{\Omega_1}{\Omega_0} = \frac{\exp\left(\int_0^\tau -N[1 + \sigma|\cos(\tau_1)]d\tau_1\right)}{\exp\left(\int_0^\tau -Nd\tau_1\right)}. \quad (18)$$

$\Omega_0$  is the contaminant concentration corresponding to  $\theta(\tau)=0$ , and  $\Omega_1$  is the contaminant concentration corresponding to  $\theta(\tau) = |\cos(\tau)|$ . It is shown that  $\Lambda_1$  decreases with time. For a given  $N$ ,  $\Lambda_1$  decreases with the increase of  $\sigma$ .

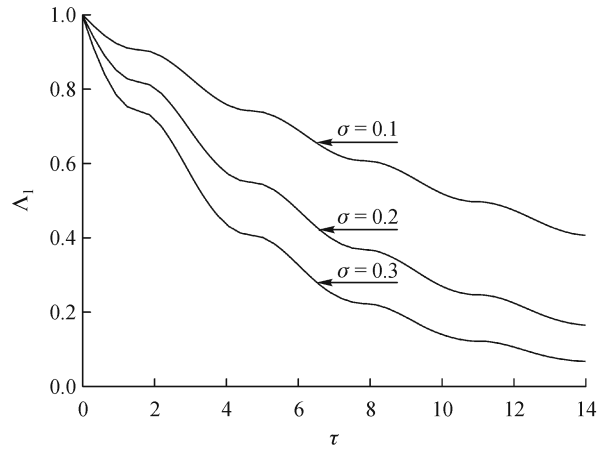


Fig. 2 Variation of  $\Lambda_1$  with  $\tau$  for  $N=1$ , and  $\sigma=0.1, 0.2$ , and  $0.3$ .

Figure 3 shows the variation of  $\Lambda_2$  with  $\tau$  for  $N=1$ , and  $\sigma=0.1, 0.2$ , and  $0.3$ , where  $\Lambda_2$  is expressed as

$$\Lambda_2 = \frac{\Omega_2}{\Omega_0} = \frac{\exp\left(\int_0^\tau -N[1 - \sigma|\cos(\tau_1)]d\tau_1\right)}{\exp\left(\int_0^\tau -Nd\tau_1\right)}, \quad (19)$$

and  $\Omega_2$  is the contaminant concentration corresponding to  $\theta(\tau) = -|\cos(\tau)|$ . It is shown that  $\Lambda_2$  increases with time. For a given  $N$ ,  $\Lambda_2$  increases with increases of  $\sigma$ .

Figure 4 shows the variation of  $\Lambda_3$  with  $\tau$  for  $\sigma=0.1, 0.2, 0.3$ , and  $N=1$ , where  $\Lambda_3$  is expressed as

$$\Lambda_3 = \frac{\Omega_3}{\Omega_0} = \frac{\exp\left(\int_0^\tau -N[1 + \sigma\cos(\tau_1)]d\tau_1\right)}{\exp\left(\int_0^\tau -Nd\tau_1\right)}, \quad (20)$$

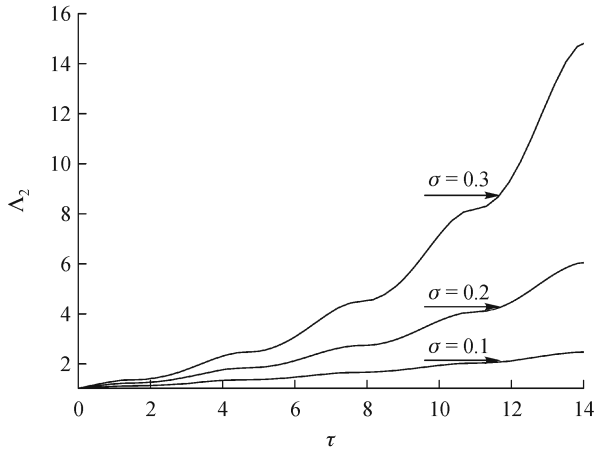


Fig. 3 Variation of  $\Lambda_2$  with  $\tau$  for  $N=1$ , and  $\sigma=0.1, 0.2$ , and  $0.3$ .

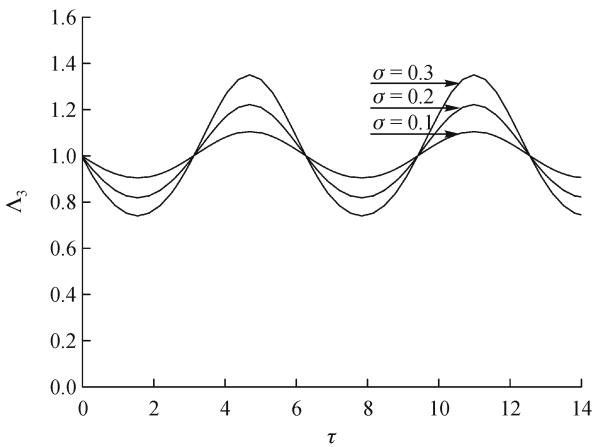


Fig. 4 Variation of  $\Lambda_3$  with  $\tau$  for  $N=1$ , and  $\sigma=0.1, 0.2$ , and  $0.3$ .

and  $\Omega_3$  is the contaminant concentration corresponding to  $\theta(\tau)=\cos(\tau)$ . It is shown that  $\Lambda_3$  changes periodically with time. For a given  $N$ , the amplitude of  $\Lambda_3$  increases with increases of  $\sigma$ .

Figure 5 shows the variation of  $\Lambda_3$  with  $\tau$  for  $\sigma=0.2$ , and  $N=0.5, 1$ , and  $2$ . It is shown that as  $N$  increases, the amplitude of  $\Lambda_3$  increases.

#### 4 Concentration moment analysis

The method of concentration moments (Aris, 1956) was adopted to analyze the transport of a volatile contaminant in a free-surface wetland flow. The zeroth order concentration moment  $m_0$  and the first order concentration moment  $m_1$  are defined respectively as

$$m_0 \equiv \int_{-\infty}^{\infty} \Omega^*(\xi, \zeta, \tau) d\xi, \quad (21)$$

and

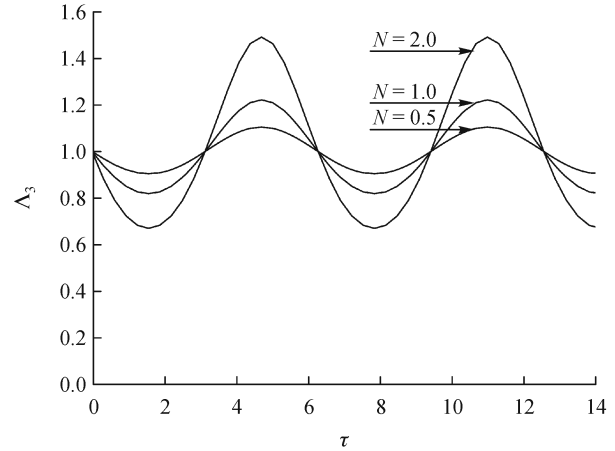


Fig. 5 Variation of  $\Lambda_3$  with  $\tau$  for  $\sigma=0.2$ , and  $N=0.5, 1$ , and  $2$ .

$$m_1 \equiv \int_{-\infty}^{\infty} \Omega^*(\xi, \zeta, \tau) \xi d\xi. \quad (22)$$

The zeroth and first order concentration moments reflect the amount of volatile contaminant, and the movement of the mass center of the contaminant cloud, respectively.

The concentration moments satisfy (Fried, 1975; Barton, 1983)

$$\xi \Omega^* \Big|_{\xi=\pm\infty} = \frac{\partial \Omega^*}{\partial \xi} \Big|_{\xi=\pm\infty} = \xi \frac{\partial \Omega^*}{\partial \xi} \Big|_{\xi=\pm\infty} = 0. \quad (23)$$

Applying the operator  $\int_{-\infty}^{\infty} (\cdot) d\xi$  to Eqs. (8), (10), and (11), with the aid of Eqs. (9) and (23), we have

$$\frac{\partial m_0}{\partial \tau} = \frac{\partial^2 m_0}{\partial \zeta^2}, \quad (24)$$

$$\frac{\partial m_0}{\partial \zeta} \Big|_{\zeta=0} = 0, \quad \left( R_s m_0 + \frac{\partial m_0}{\partial \zeta} \right) \Big|_{\zeta=1} = 0, \quad (25)$$

$$m_0(\zeta, \tau) \Big|_{\tau=0} = 1. \quad (26)$$

Solving Eqs. (24)–(26), we have

$$m_0(\zeta, \tau) = \sum_{i=1}^{\infty} \frac{2(\beta_i^2 + R_s^2)}{\beta_i^2 + R_s^2 + R_s} \frac{\sin \beta_i}{\beta_i} \cos(\beta_i \zeta) \exp(-\beta_i^2 \tau), \quad (27)$$

where  $\beta_i$  is determined by

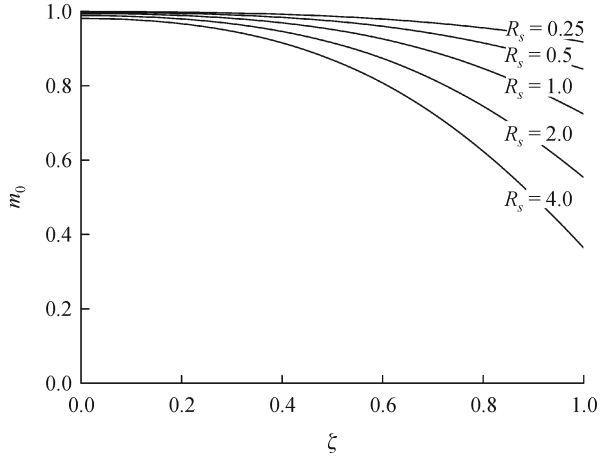
$$\beta_i \tan \beta_i = R_s. \quad (28)$$

Table 1 presents the eigenvalues  $\beta_i$  ( $i = 1, 2, 3, 4$ , and  $5$ ) for  $R_s = 0.25, 0.5, 1.0, 2.0$ , and  $4.0$ .

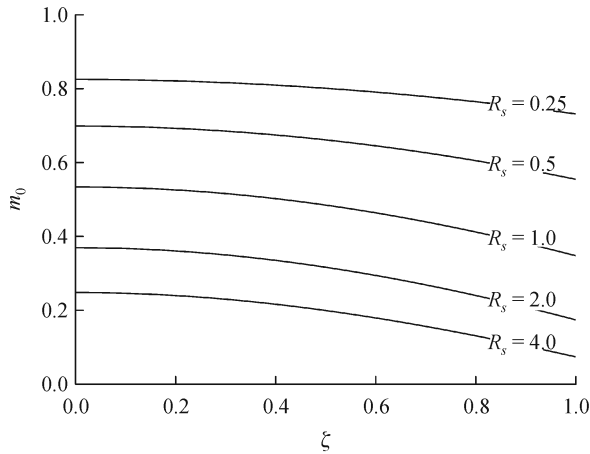
Figure 6 shows the variation of  $m_0$  with  $\xi$  for  $\tau=0.1$ , and  $R_s = 0.25, 0.5, 1.0, 2.0$ , and  $4.0$ . Figure 7 shows the variation of  $m_0$  with  $\xi$  for  $\tau=1$ , and  $R_s = 0.25, 0.5, 1.0, 2.0$ , and  $4.0$ . It is shown that  $m_0$  decreases as  $\xi$  increases, which

**Table 1** Eigenvalues  $\beta_i$  ( $i = 1, 2, 3, 4$  and  $5$ ) for  $R_s = 0.25, 0.5, 1.0, 2.0,$  and  $4.0$

$R_s$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$
0.25	0.480094	3.219099	6.322705	9.451223	12.586231
0.5	0.653271	3.292311	6.361621	9.477486	12.606013
1.0	0.860334	3.425618	6.437299	9.529334	12.645287
2.0	1.076874	3.643597	6.578334	9.629560	12.722299
4.0	1.264592	3.935162	6.814010	9.811878	12.867756



**Fig. 6** Variation of  $m_0$  with  $\xi$  for  $\tau = 0.1$ , and  $R_s = 0.25, 0.5, 1.0, 2.0$  and  $4.0$ .



**Fig. 7** Variation of  $m_0$  with  $\xi$  for  $\tau = 1$ , and  $R_s = 0.25, 0.5, 1.0, 2.0$  and  $4.0$ .

means that the amount of contaminant near the free-water surface is less than that near the bottom bed. The

distribution of  $m_0$  for the volatile contaminant is different from that for the non-volatile contaminant in the vertical direction, which stays constant (Zeng and Chen, 2011). At the initial stage, the volatilization has less effect on  $m_0$  near the bottom bed. The effect of volatilization on  $m_0$  gradually increases with time, and finally has an important impact on the whole cross-section.

The depth average of  $m_0$  is

$$\bar{m}_0 = \sum_{i=1}^{\infty} \frac{2(\beta_i^2 + R_s^2)}{\beta_i^2 + R_s^2 + R_s} \left( \frac{\sin \beta_i}{\beta_i} \right)^2 \exp(-\beta_i^2 \tau). \quad (29)$$

Figure 8 shows the variation of  $\bar{m}_0$  with  $\tau$  for  $R_s = 0.25, 0.5, 0.75, 1.0,$  and  $2.0$ . It is shown that  $\bar{m}_0$  gradually decreases to zero. The dimensionless time scale for  $m_0$  to decay fully is given by

$$\tau_c \gg \frac{1}{\beta_1^2}, \quad (30)$$

and the corresponding dimensioned time scale is given by

$$t_c \gg \frac{H^2}{\beta_1^2 \kappa (\lambda + K_V / \phi)}. \quad (31)$$

As  $R_s$  increases,  $\bar{m}_0$  decreases. This result means that the total amount of volatile contaminant in the water body decreases, as volatilization increases.

Applying the operator  $\int_{-\infty}^{\infty} (\cdot) \xi d\xi$  to Eqs. (8), (10), and (11), with the aid of Eqs. (9) and (23), we have

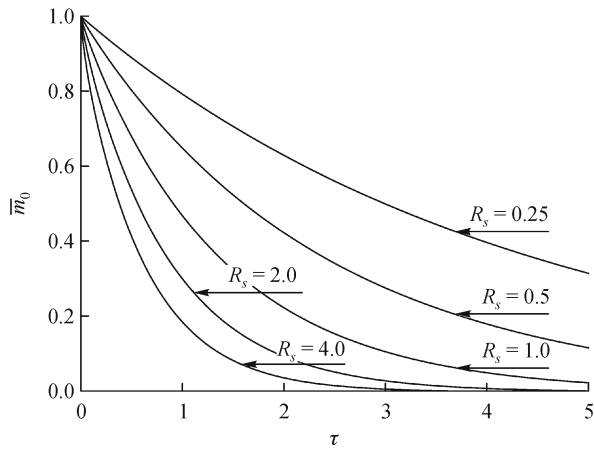
$$\frac{\partial m_1}{\partial \tau} = \frac{\partial^2 m_1}{\partial \zeta^2} + \frac{Pe_x R_x^K \psi}{\phi} m_0, \quad (32)$$

$$\left. \frac{\partial m_1}{\partial \zeta} \right|_{\zeta=0} = 0, \quad \left( R_s m_1 + \frac{\partial m_1}{\partial \zeta} \right) \Big|_{\zeta=1} = 0, \quad (33)$$

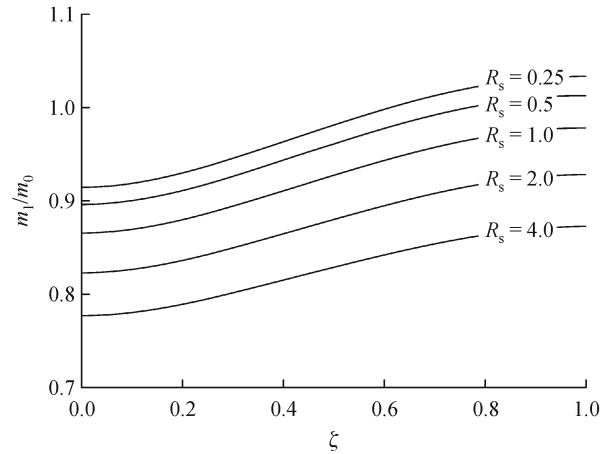
$$m_1(\zeta, \tau) \Big|_{\tau=0} = 0. \quad (34)$$

Solving Eqs. (32)–(34), we have

$$m_1(\zeta, \tau) = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \frac{Pe_x R_x^K}{\phi} \frac{2(\beta_j^2 + R_s^2)}{\beta_j^2 + R_s^2 + R_s} \frac{2(\beta_i^2 + R_s^2)}{\beta_i^2 + R_s^2 + R_s} \frac{\sin \beta_i}{\beta_i} \cdot (P_1 + P_2) P_3 \cos(\beta_j \zeta), \quad (35)$$



**Fig. 8** Variation of  $\bar{m}_0$  with  $\tau$  for  $R_s = 0.25, 0.5, 0.75, 1.0,$  and  $2.0$ .



**Fig. 9** Variation of  $m_1/m_0$  with  $\xi$  for  $Pe_x R_x^K / \phi = 1.0, \alpha = 1.0, \tau = 1.0$  and  $R_s = 0.25, 0.5, 1.0, 2.0$  and  $4.0$ .

where  $P_1, P_2$  and  $P_3$  can be expressed respectively as

$$P_1 = \begin{cases} \frac{(\beta_i + \beta_j) \sin(\beta_i - \beta_j) + (\beta_i - \beta_j) \sin(\beta_i + \beta_j)}{2(\beta_i^2 - \beta_j^2) (\cosh \alpha - \sinh \alpha / \alpha) / \cosh \alpha} & (i \neq j) \\ \frac{\sin(2\beta_i) + 2\beta_i}{4\beta_i (\cosh \alpha - \sinh \alpha / \alpha) / \cosh \alpha} & (i = j) \end{cases}, \tag{36}$$

$$P_2 = - \frac{(\beta_i + \beta_j) [(\beta_i - \beta_j)^2 + \alpha^2] \sin(\beta_i + \beta_j)}{[2\alpha^4 + 2(\beta_i^2 - \beta_j^2)^2 + 4\alpha^2(\beta_i^2 + \beta_j^2)] (\cosh \alpha - \sinh \alpha / \alpha)} - \frac{(\beta_i - \beta_j) [(\beta_i + \beta_j)^2 + \alpha^2] \sin(\beta_i - \beta_j)}{[2\alpha^4 + 2(\beta_i^2 - \beta_j^2)^2 + 4\alpha^2(\beta_i^2 + \beta_j^2)] (\cosh \alpha - \sinh \alpha / \alpha)} - \frac{2\alpha(\beta_i^2 + \beta_j^2) \sinh \alpha + 2\alpha^3 \sinh \alpha}{[2\alpha^4 + 2(\beta_i^2 - \beta_j^2)^2 + 4\alpha^2(\beta_i^2 + \beta_j^2)] (\cosh \alpha - \sinh \alpha / \alpha)}, \tag{37}$$

and

$$P_3 = \begin{cases} \frac{-\exp(-\beta_j^2 \tau) + \exp(-\beta_i^2 \tau)}{\beta_j^2 - \beta_i^2} & (i \neq j) \\ \tau \exp(-\beta_j^2 \tau) & (i = j) \end{cases}. \tag{38}$$

Then we have

$$\bar{m}_1(\tau) = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} \frac{Pe_x R_x^K}{\phi} \frac{2(\beta_j^2 + R_s^2)}{\beta_j^2 + R_s^2 + R_s} \frac{2(\beta_i^2 + R_s^2)}{\beta_i^2 + R_s^2 + R_s} \frac{\sin \beta_i \sin \beta_j}{\beta_i \beta_j} \cdot (P_1 + P_2) P_3. \tag{39}$$

Figure 9 shows the variation of  $m_1/m_0$  with  $\xi$  for  $Pe_x R_x^K / \phi = 1.0, \alpha = 1.0, \tau = 1.0,$  and  $R_s = 0.25, 0.5, 1.0, 2.0,$  and  $4.0$ . It is shown that  $m_1/\bar{m}_0$  increases with  $\xi$ , which means that the mass center of a contaminant moves

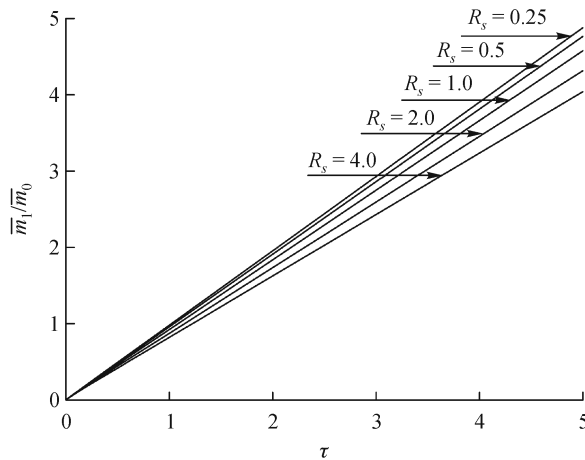
quickly on the streamline through  $\xi = 1.0$ .

Figure 10 shows the variation of  $\bar{m}_1/\bar{m}_0$  with  $\tau$  for  $Pe_x R_x^K / \phi = 1.0, \alpha = 1.0,$  and  $R_s = 0.25, 0.5, 1.0, 2.0,$  and  $4.0$ . The mass center of the whole contaminant cloud will move longitudinally. The slope of the curves in Fig. 10 reflects the moving speed of the mass center. As volatilization increases, the moving speed decreases.

## 5 Conclusions

For free-water surface wetland flows, a simplified model has been given to reflect the effects of advection, mass dispersion, apparent reaction, and volatilization on contaminant transport.

The effect of periodic apparent reaction on contaminant transport is separated from the hydraulic effect via an extended transformation. The transformation for constant



**Fig. 10** Variation of  $\bar{m}_1/\bar{m}_0$  with  $\tau$  for  $Pe_x R_x^K/\phi = 1.0$ ,  $\alpha = 1.0$ , and  $R_s = 0.25, 0.5, 1.0, 2.0$  and  $4.0$ .

apparent reaction rate can be included as a special case of  $\sigma = 0$ . For three cases,  $\theta(\tau) = |\cos(\tau)|$ ,  $-|\cos(\tau)|$ , and  $\cos(\tau)$ , the effects of periodic reaction on contaminant transport are illustrated. It is shown that the oscillatory component of periodic apparent reaction rate may increase or decrease the contaminant concentration.

The method of concentration moments was used to obtain the zeroth and first order concentration moments. The distribution of  $m_0$  for the volatile contaminant was different from that for the non-volatile contaminant in the vertical direction. The former kept constant, while the latter decreased from the bottom bed to the free water surface. The total amount of contaminant cloud decays with time. At the initial stage, the influenced range of volatilization for  $m_0$  is limited to the bottom bed. For the long term evolution of a volatile contaminant, the influenced range spreads to the whole cross-section. It was found that the moving speed of the mass center of the contaminant on the streamline increases from the bottom bed to the free water surface. The moving speed of the mass center for the whole contaminant cloud decreases with increases of the ratio of volatilization coefficient to vertical effective mass dispersivity.

**Acknowledgements** This work is supported by the Special Foundation of China Institute of Water Resources and Hydropower Research (Contract No. Shuji1345).

## References

- Aris R (1956). On the dispersion of a solute in a fluid flowing through a tube. *P Roy Soc Lond A Mat*, 235(1200), 67–77
- Barton N G (1983). On the method of moments for solute dispersion. *J Fluid Mech*, 126(1): 205–218
- Chen G Q, Wu Z, Zeng L (2012). Environmental dispersion in a two-

layer wetland: analytical solution by method of concentration moments. *Int J Eng Sci*, 51: 272–291

- Chen G Q, Zeng L, Wu Z (2010). An ecological risk assessment model for a pulsed contaminant emission into a wetland channel flow. *Ecol Model*, 221(24): 2927–2937
- Chen Z M, Chen G Q, Chen B, Zhou J B, Yang Z F, Zhou Y (2009). Net ecosystem services value of wetland: environmental economic account. *Commun Nonlinear Sci Numer Simul*, 14(6): 2837–2843
- Costanza R, d'Arge R, de Groot R, Farber S, Grasso M, Hannon B, Limburg K, Naeem S, O'Neill R V, Paruelo J, Raskin R G, Sutton P, van den Belt M (1997). The value of the world's ecosystem services and natural capital. *Nature*, 387(6630): 253–260
- Fried J J (1975). *Groundwater Pollution: Theory, Methodology, Modelling and Practical Rules*. Amsterdam: Elsevier Scientific Publishing Company
- Lightbody A F, Nepf H M (2006a). Prediction of velocity profiles and longitudinal dispersion in emergent salt marsh vegetation. *Limnol Oceanogr*, 51(1): 218–228
- Lightbody A F, Nepf H M (2006b). Prediction of near-field shear dispersion in an emergent canopy with heterogeneous morphology. *Environ Fluid Mech*, 6(5): 477–488
- Liu S, Masliyah J H (2005). Dispersion in porous media. In: Vafai K ed. *Handbook of Porous Media*. Boca Raton: CRC Press, 81–140
- Mei C C, Auriault J L, Ng C O (1996). Some applications of the homogenization theory. In: Hutchinson J W, Wu T Y eds. *Advances in Applied Mechanics*. California: Academic Press, 277–348
- Mitsch W J, Gosselink J G (1993). *Wetlands* (2nd ed). New York: Van Nostrand Reinhold
- Murphy E, Ghisalberti M, Nepf H (2007). Model and laboratory study of dispersion in flows with submerged vegetation. *Water Resour Res*, 43(5): W05438
- Nepf H, Ghisalberti M, White B, Murphy E (2007). Retention time and dispersion associated with submerged aquatic canopies. *Water Resour Res*, 43(4): W04422
- Shao L, Wu Z, Zeng L, Chen Z M, Zhou Y, Chen G Q (2012). Embodied energy assessment for ecological wastewater treatment by a constructed wetland. *Ecol Model*, doi: 10.1016/j.ecolmodel.2012.09.004
- Taylor G (1953). Dispersion of soluble matter in solvent flowing slowly through a tube. *P Roy Soc Lond A Mat*, 219(1137): 186–203
- Wu Z, Chen G Q, Zeng L (2011a). Environmental dispersion in a two-zone wetland. *Ecol Modell*, 222(3): 456–474
- Wu Z, Li Z, Chen G Q (2011b). Multi-scale analysis for environmental dispersion in wetland flow. *Commun Nonlinear Sci Numer Simul*, 16(8): 3168–3178
- Wu Z, Zeng L, Chen G Q, Li Z, Shao L, Wang P, Jiang Z (2012). Environmental dispersion in a tidal flow through a depth-dominated wetland. *Commun Nonlinear Sci Numer Simul*, 17(12): 5007–5025
- Zeng L (2010). *Analytical Study on Environmental Dispersion in Wetland Flow*. Dissertation for Ph D degree. Beijing: Peking University (in Chinese)
- Zeng L, Chen G, Wu Z, Li Z, Wu Y H, Ji P (2012a). Flow distribution and environmental dispersivity in a tidal wetland channel of rectangular cross-section. *Commun Nonlinear Sci Numer Simul*, 17

- (11): 4192–4209
- Zeng L, Chen G Q (2011). Ecological degradation and hydraulic dispersion of contaminant in wetland. *Ecol Modell*, 222(2): 293–300
- Zeng L, Chen G Q, Tang H S, Wu Z (2011). Environmental dispersion in wetland flow. *Commun Nonlinear Sci Numer Simul*, 16(1): 206–215
- Zeng L, Wu Y, Ji P, Chen B, Zhao Y J, Chen G Q, Wu Z (2012b). Effect of wind on contaminant dispersion in a wetland flow dominated by free-surface effect. *Ecol Modell*, 237–238: 101–108