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REVIEW ARTICLE

Shubin SI, Jiangbin ZHAO, Zhiqiang CAI, Hongyan DUI

Recent advances in system reliability optimization driven by importance measures

Abstract System reliability optimization problems have been widely discussed to maximize system reliability with resource constraints. Birnbaum importance is a well-known method for evaluating the effect of component reliability on system reliability. Many importance measures (IMs) are extended for binary, multistate, and continuous systems from different aspects based on the Birnbaum importance. Recently, these IMs have been applied in allocating limited resources to the component to maximize system performance. Therefore, the significance of Birnbaum importance is illustrated from the perspective of probability principle and gradient geometrical sense. Furthermore, the equations of various extended IMs are provided subsequently. The rules for simple optimization problems are summarized to enhance system reliability by using ranking or heuristic methods based on IMs. The importance-based optimization algorithms for complex or large-scale systems are generalized to obtain remarkable solutions by using IM-based local search or simplification methods. Furthermore, a general framework driven by IM is developed to solve optimization problems. Finally, some challenges in system reliability optimization that need to be solved in the future are presented.

Keywords importance measure, system performance, reliability optimization, optimization rules, optimization algorithms

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Shubin SI (✉), Jiangbin ZHAO, Zhiqiang CAI
School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an 710072, China; Ministry of Industry and Information Technology Key Laboratory of Industrial Engineering and Intelligent Manufacturing, Northwestern Polytechnical University, Xi'an 710072, China
E-mail: sisb@nwpu.edu.cn

Hongyan DUI
School of Management Engineering, Zhengzhou University, Zhengzhou 450001, China

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1 Introduction

The concept of reliability emerged after World War I to compare the operation safety of key weapons; this concept has been applied to technical systems for more than a century (Rausand and Høyland, 2003). A detailed history of reliability theory from the 1950s to 1990s was presented by summarizing some critical reliability progresses (Knight, 1991). Reliability is an important discipline applied in the entire product lifecycle of design, manufacturing, operation, maintenance, life extension, and end-of-life. System reliability optimization problems (ROPs) are popular topics in reliability engineering and play an essential part in engineering projects, such as in nuclear power plants (Čepin, 2019), energy storage systems (Mohamad and Teh, 2018), automotive systems (Yu et al., 2018), and traction drive systems of rail transit (Lin et al., 2018).

Generally, system reliability could be improved by increasing component reliability, adding parallel redundancy of components, reassigning the positions of interchangeable components, or combining these methods (Kuo and Prasad, 2000). System ROPs can be classified into five types, namely, ROP, redundancy allocation problem (RAP), reliability-RAP (RRAP), component assignment problem (CAP), and complex problem (CP). ROP (Gopal et al., 1980; Kuo et al., 1987; Coit and Smith, 1996; Mettas, 2000; Prasad and Kuo, 2000) is a general and straightforward system ROP that aims to achieve optimal reliability allocation to maximize system reliability considering the cost constraints. RAP (Liang and Smith, 2004; Ramirez-Marquez and Coit, 2004; Onishi et al., 2007; Kulturel-Konak et al., 2003) aims to achieve the maximum system reliability by allocating the redundant components suitably with the cost constraints. RRAP (Chern, 1992; Tian et al., 2008; Yeh and Hsieh, 2011; Garg and Sharma, 2013; Abouei Ardakan and Zeinal Hamadani, 2014) is a type of system ROP for obtaining the optimal solution with the highest system reliability by adjusting the component reliability and redundancy with consideration of the limitations (reliability, cost, weight, and volume). CAP (Derman et al., 1974; Zhu et al., 2011; Levitin et al., 2017b) aims to generate optimal assignment with maximum system reliability by assigning n available components into n positions. CP (Pant et al., 2015; Peng et al., 2016; Yu et al., 2017; Zhao et al., 2019a) is a type of difficult system ROP for complex systems with large-scale or complicated tasks to maximize system performance with limited resources.

Importance measures (IMs) are used to evaluate the effect of component reliability on system reliability (Birnbaum, 1969). IMs are useful tools in reliability engineering (Compare et al., 2017), risk analysis (Fang et al., 2017), and system reliability optimization (Fu et al., 2019a). These measures can help reliability engineers to find a better solution rapidly because they can identify the weakest links of the system, which are the premise and foundation of system design, maintenance, and resource configurations. During the system design period, component importance can help designers determine cost-effective design ideas with relatively high system reliability and low cost rapidly. The IMs can be applied in the design of communication systems (Liu et al., 2018), modern digital systems (Borgonovo et al., 2016), power industry (Espiritu et al., 2007), embedded systems (Aliee et al., 2016), and head-up display systems (Dui et al., 2017b). For the maintenance process, component importance can find the weakest components to improve its reliability, such as the integrated IM, considering the repair and failure rates (Si et al., 2012a; 2012b; 2013; Zhao et al., 2013; Dui et al., 2019). The IMs can also be found in wind turbine systems (Wu and Coolen, 2013; Dui et al., 2017c), electrical networks (Hilber and Bertling, 2004; Vu et al., 2016), control systems of computerized numerical control lathes (Xiahou et al., 2018), propeller plane systems (Dui et al., 2017a), and automobile tire systems (Fu et al., 2019b). For limited resources, IMs can determine the components that can generate the greatest system reliability improvement by arranging several resources reasonably to reduce system risk. The IMs are used for the bus test systems (Fang et al., 2016), critical infrastructure (Xu et al., 2020), navigation systems (Baroud and Barker, 2018), water distribution pipelines (He and Yuan, 2019), and resilient power systems (Wang et al., 2019).

IMs have been applied successfully in the complex engineering field to analyze reliability, safety, and risk in the past 60 years. The development of IMs in system reliability optimization can be classified into three stages.

(1) The first stage (before the 1990s) is an initial development of the system ROPs by applying the Birnbaum importance to solve these problems. The studies focused on the system reliability optimization for small-scale systems with series or parallel structures, which established the mathematical model of optimization problems and proposed some optimization rules based on the Birnbaum importance to solve these problems (Tillman et al., 1977). The extensions of the Birnbaum importance are used to solve optimization problems because assigning resources to the component with the highest importance can improve the system's performance significantly. In this stage, some optimization rules are proposed to attain better solutions for optimization problems considering some IMs, such as the criticality importance (Barlow and Proschan, 1975), redundant importance (Boland et al., 1988), and Δ-importance (Xie and Shen, 1989).

(2) The second stage (from 1990 to 2011) is a further exploration of the system reliability optimization based on the IMs. The heuristic methods are used to generate optimal results based on efficient heuristic rules. The IM-based heuristic rules are proposed to solve system ROPs (Lin and Kuo, 2002; Yao et al., 2011; Zuo and Kuo, 1990). These heuristic rules clarify the mechanism of IMs in solving system ROPs. Moreover, this stage is the solid foundation of system reliability optimization.

(3) The third stage (after 2011) is an in-depth exploration of IMs for solving system ROPs. The book *Importance Measures in Reliability, Risk, and Optimization* provided a comprehensive contribution of IMs in reliability engineering (Kuo and Zhu, 2012). The IMs are also used to deal with some system ROPs related to the design, maintenance, and resource allocation. This book made IM a popular topic in reliability research. The mathematical models of system reliability optimization have remarkably changed, including the following tendency: (a) the diversity and multilevel of the objective functions are considered (Abouei Ardakan and Rezvan, 2018; Bretas et al., 2018), such as system availability and the remaining useful time of the system; (b) the states of components obtain multiple levels (Yeh and Chu, 2018; Jiang et al., 2019; Zaretalab et al., 2020); (c) the system structure has become complicated (Su et al., 2018; Xiang and Yang, 2018); and (d) the relationship among the components becomes complicated (Li et al., 2018; Mi et al., 2018). Many intelligent algorithms, such as artificial bee colony algorithm (Ghambari and Rahati, 2018), directional bat algorithm (Chakri et al., 2018), boundary swarm optimization method (Yeh, 2019), and butterfly optimization algorithm (Arora et al., 2018), are introduced to solve the system ROPs. For the system ROPs of CP, some IMs are developed to deal with mathematical models, such as IMs for reconfigurable systems (Si et al., 2014) and IMs for the consecutive k-out-of-n systems with sparse d (Shen et al., 2015; Shen and Cui, 2015).

The methods for solving all problems can be divided into two categories. The first category refers to IM-based optimization rules, which can obtain the results rapidly based on IM analysis using ranking and heuristic methods. The former can obtain the optimization results directly by IM ranking, and the latter can obtain the results by using IM-based rules iteratively. The differences between these methods are presented as follows: The heuristic methods require many iterations, but the ranking methods only need one iteration. In the second category, the IM-based optimization algorithms are used to solve some CPs or large-scale problems based on the optimization rules. According to the purpose of the optimization rules, the IM-based optimization algorithms are divided into IM-based local search and simplification methods. IM-based local search methods are used in the sub-process of intelligent algorithms to improve the performance of the corresponding algorithms, while simplification methods can abridge the objectives or screen the critical factors of the optimization problems. Therefore, IM-based optimization rules are used to solve simple problems, whereas IM-based optimization algorithms are introduced to solve CPs.

The remainder of this work is organized as follows. Section 2 briefly illustrates the details of IMs, including their significance and extensions, based on Birnbaum importance. Section 3 summarizes the IM-based optimization rules for system ROPs. Section 4 investigates the IM-based optimization algorithms for system ROPs. Section 5 provides a general optimization framework driven by IMs for the system ROPs. Finally, Section 6 proposes the challenges in system reliability optimization in future research.

2 IMs for system ROPs

The Birnbaum importance of component i for binary systems was first proposed in 1969 to measure the impact of component reliability on the system reliability of binary state systems (Birnbaum, 1969). The Birnbaum importance of component i, which is denoted as $I^{\mathrm{BM}}(i)$, can be evaluated by:

$$I^{\mathrm{BM}}(i) = \frac{\partial R_\phi(\boldsymbol{P})}{\partial P_{i1}} = \Pr\{\Phi(\boldsymbol{X}) = 1|X_i = 1\} - \Pr\{\Phi(\boldsymbol{X}) = 1|X_i = 0\}, \quad (i = 1,\ldots, n), \tag{1}$$

where n is the number of components in the system; P_{i1} is the probability of component i at state 1, which is the reliability of component i in the binary state systems; $R_\phi(\boldsymbol{P})$ is the function of system reliability, in which $\boldsymbol{P} = (P_{11},\ldots, P_{i1},\ldots, P_{n1})$; X_i is the state of component i, where $X_i = 1$ indicates that component i is working, and $X_i = 0$ indicates that this component is failed in this binary state system; and $\Phi(\boldsymbol{X})$ is the system structure-function, in which $\boldsymbol{X} = \{X_1,\ldots, X_i,\ldots, X_n\}$, where $\Phi(\boldsymbol{X}) = 1$ illustrates that the system is functioning, and $\Phi(\boldsymbol{X}) = 0$ indicates that the system is failed.

2.1 Significance of IMs

Birnbaum importance can be demonstrated from two points of view (e.g., probability principle and the gradient geometrical meaning) to understand its significance better.

2.1.1 Probability principle of Birnbaum importance

We use random experiments to introduce the significance of the component importance for the IMs. A random experiment $\boldsymbol{E}$ has n basic events $\boldsymbol{\omega} = (\omega_1,\ldots, \omega_i,\ldots, \omega_n)$, and each basic event may or may not occur. The probability that event j will occur is represented by $\Pr(E_j) = f_j(\Pr(\omega_1), \Pr(\omega_2),\ldots, \Pr(\omega_n))$ $(j = 1,\ldots, 2^n)$. The random variable $x(E_j)$ represents the price of the event E_j, in which $x(E_j) = 1$ when E_j occurs and $x(E_j) = 0$ if E_j does not occur. According to the set value $x(E_j)$, x represents the average price of the random experiment $\boldsymbol{E}$ that consists of a set event $\{E_1, E_2,\ldots, E_{2^n}\}$. Then, x can be calculated by:

$$x = \sum_{j=1}^{2^n} x(E_j)\cdot\Pr(E_j) = \sum_{j=1}^{2^n} x(E_j)\cdot f_j(\Pr(\omega_1), \Pr(\omega_2),\ldots, \Pr(\omega_n)). \tag{2}$$

The effect of the basic event ω_i on the random experiment $\boldsymbol{E}$ can be represented by $\frac{\partial x}{\partial \Pr(\omega_i)}$, which represents the price change of the random experiment $\boldsymbol{E}$ with the probability change in the basic event ω_i. Thus, $\frac{\partial x}{\partial \Pr(\omega_i)}$ can also be used to represent the probability principle of Birnbaum importance.

2.1.2 Gradient geometrical meaning of Birnbaum importance

The gradient is a vector that represents the fastest directions of function change at a known point. In a rectangular coordinate system, the gradient of the function $f(y_1,\ldots, y_i,\ldots, y_n)$ refers to the partial derivative of f with respect to y_i (Dui et al., 2013):

$$\boldsymbol{\nabla} f = \mathbf{grad}f = \frac{\partial f}{\partial y_1}y_1 + \ldots + \frac{\partial f}{\partial y_i}y_i + \ldots + \frac{\partial f}{\partial y_n}y_n, \tag{3}$$

where y_i $(i = 1, 2,\ldots, n)$ is the orthogonal unit vector that points in the coordinate direction as y_i changes.

$\Pr\{\Phi(\boldsymbol{X}) = 1\} = f(P_{11}, P_{21},\ldots, P_{n1})$ can be considered a function of parameters $P_{11}, P_{21},\ldots, P_{n1}$ in Eq. (1). The gradient of $f(P_{11}, P_{21},..., P_{n1})$ is presented as follows (Dui et al., 2013):

$$\nabla f = \frac{\partial \Pr\{\Phi(\boldsymbol{X}) = 1\}}{\partial P_{11}} P_{11} + \frac{\partial \Pr\{\Phi(\boldsymbol{X}) = 1\}}{\partial P_{21}} P_{21} + ... + \frac{\partial \Pr\{\Phi(\boldsymbol{X}) = 1\}}{\partial P_{n1}} P_{n1}$$

$$= I^{\mathrm{BM}}(1) P_{11} + I^{\mathrm{BM}}(2) P_{21} + ... + I^{\mathrm{BM}}(n) P_{n1}, \quad (4)$$

where gradient is the sum of the product of the Birnbaum importance and the corresponding component reliability. The gradient at one point (P_{11}, P_{21},..., P_{n1}) shows the direction of improving the system reliability most rapidly, and the magnitude refers to the improvement of the system reliability in this direction.

2.2 Extensions of IMs for system ROPs

The Birnbaum importance was extended into many different forms to solve various engineering problems. All the extensions can be classified into three categories according to the system states: IMs for (1) binary state systems, (2) multistate systems, and (3) continuous systems.

2.2.1 IMs for binary state systems

Birnbaum structure importance is a structure importance that can be calculated by Birnbaum importance when all the reliability of components is equal to 0.5. Birnbaum structure importance is an extension of Birnbaum importance when all the component reliability is the same. The expression of Birnbaum structure importance $I_{\mathrm{S}}^{\mathrm{BM}}$ can be obtained by:

$$I_{\mathrm{S}}^{\mathrm{BM}}(i) = \frac{\sum_{\boldsymbol{X}} (\Pr\{\Phi(\boldsymbol{X}) = 1 | X_i = 1\} - \Pr\{\Phi(\boldsymbol{X}) = 1 | X_i = 0\})}{2^n}. \quad (5)$$

The criticality importance (Barlow and Proschan, 1975), considering system failure, refers to the ratio of the decrease in system reliability when component i fails to the probability of system failure. The criticality importance of component i, $I^{\mathrm{C}}(i)$, can be calculated as follows:

$$I^{\mathrm{C}}(i) = \Pr\{\Phi(1_i, \boldsymbol{X}) - \Phi(0_i, \boldsymbol{X}) = 1,\ X_i = 0 | \Phi(\boldsymbol{X}) = 0\}$$

$$= \frac{1 - P_{i1}}{1 - R(\boldsymbol{P})} \cdot I^{\mathrm{BM}}(i), \quad (6)$$

where 1_i and 0_i are the reliability vector of all components when the reliability of component i is one and zero, respectively.

The Birnbaum importance of component i for CAP $I_{\mathrm{CAP}}^{\mathrm{BM}}(i)$ is provided to determine the impact degree of the component on system reliability (Papastavridis, 1987). $I_{\mathrm{CAP}}^{\mathrm{BM}}(i)$ is the extension of Birnbaum importance in CAP, which has been derived from Eq. (1):

$$I_{\mathrm{CAP}}^{\mathrm{BM}}(i) = \frac{\partial R(n)}{\partial P_{i1}} = R(n | X_i = 1) - R(n | X_i = 0)$$

$$= \frac{R(i-1) R'(n-i) - R(n)}{1 - P_{i1}}, \quad (7)$$

where $R(j)$ is the reliability of the consecutive-k-out-of-j: F subsystem in consecutive-k-out-of-n: F system, which has components from position 1 to position j; and $R'(j)$ is the reliability of the consecutive-k-out-of-j: F subsystem, in which the components are from position $(n-j+1)$ to position n.

Δ-importance is proposed to represent the improvement of the system reliability when the reliability of component i changes from P_{i1} to P'_{i1} because the Birnbaum importance of component i is independent with the reliability of component i (Xie and Shen, 1989). The Δ-importance of component i, $I^{\Delta}(i)$, can be evaluated by:

$$I^{\Delta}(i) = \Pr\{\Phi(\boldsymbol{X}) = 1 | P'_{i1}\} - \Pr\{\Phi(\boldsymbol{X}) = 1\}$$

$$= (P'_{i1} - P_{i1}) \cdot I^{\mathrm{BM}}(i). \quad (8)$$

The parallel redundant importance of component i, $I^{\mathrm{PR}}(i)$, refers to the increase in system reliability by adding the parallel redundancy of component i (Shen and Xie, 1990), which can be calculated by:

$$I^{\mathrm{PR}}(i) = R(P_{11},..., P_{i1} + (1 - P_{i1}) P_{i1}^{*},..., P_{n1}) - R(P_{11},..., P_{i1},..., P_{n1})$$

$$= P_{i1}^{*} (1 - P_{i1}) I^{\mathrm{BM}}(i), \quad (9)$$

where P_{i1}^{*} is the reliability of the redundant component for component i.

For binary systems, the potential improvement importance of component i, $I^{\mathrm{IP}}(i)$, is the improvement of system reliability when component i is working, which can be calculated by (Aven and Jensen, 2000):

$$I^{\mathrm{IP}}(i) = \Pr\{\Phi(\boldsymbol{X}) = 1 | X_i = 1\} - \Pr\{\Phi(\boldsymbol{X}) = 1\}$$

$$= (1 - P_{i1}) \cdot I^{\mathrm{BM}}(i), \quad (10)$$

where the maximum potential improvement is the product of the maximum reliability improvement and the Birnbaum importance of this component.

The risk achievement worth of component i, $I^{\mathrm{RAW}}(i, t)$, is the ratio of the risk when component i is at the failed state to the nominal value of the risk (Zio and Podofillini, 2003a), indicating the relative increase in the risk because of the failure of component i, which can be calculated by:

$$I^{\mathrm{RAW}}(i, t) = \frac{F_i^+(t)}{F(t)}, \tag{11}$$

where $F(t)$ is the nominal value of the risk at time t, and $F_i^+(t)$ is the risk when component i fails.

The risk reduction worth of component i, $I^{\mathrm{RRW}}(i, t)$, is the ratio of the nominal value of the risk to the risk when component i is at the working state (Zio and Podofillini, 2003a) to evaluate the potential of component i in reducing the risk, which can be calculated by:

$$I^{\mathrm{RRW}}(i, t) = \frac{F(t)}{F_i^-(t)}, \tag{12}$$

where $F_i^-(t)$ is the risk when component i is working.

Considering the lifetime distribution of components, the Birnbaum importance can be represented by the conditional probability that difference exists between the conditional probability that system lifetime is larger than t when the component lifetime is larger than t and when component lifetime is not larger than t (da Costa Bueno, 2005). The Birnbaum importance of component i that considers lifetime at time t, $I_{\mathrm{T}}^{\mathrm{BM}}(i, t)$, is calculated by:

$$I_{\mathrm{T}}^{\mathrm{BM}}(i, t) = \Pr(\tau > t | S_i > t) - \Pr(\tau > t | S_i \leqslant t), \tag{13}$$

where τ is the lifetime of the system, and S_i is the lifetime of component i.

The Bayesian importance of component i, $I^{\mathrm{Bay}}(i)$, is the conditional probability that component i fails with the system (Singpurwalla, 2006):

$$I^{\mathrm{Bay}}(i) = \Pr\{X_i = 0 | \Phi(\boldsymbol{X}) = 0\}. \tag{14}$$

When a system fails, identifying which components have caused the failure, as well as the influence degree of components on the system failure, is important.

The availability importance of component i is the effect of the availability of component i on the availability of the entire system (Barabady and Kumar, 2007). Availability is a function related to the failure or repair rate. Therefore, the failure rate-based availability importance of component i, $I_\lambda^{\mathrm{A}}(i)$, is given by:

$$I_\lambda^{\mathrm{A}}(i) = \frac{\partial A(n)}{\partial A_i}\frac{\partial A_i}{\partial \lambda_i}, \tag{15}$$

where $A(n)$ is the system availability, A_i is the availability of component i, and λ_i is the failure rate of component i.

Similarly, the repair rate-based availability importance of component i, $I_\mu^{\mathrm{A}}(i)$, is given by:

$$I_\mu^{\mathrm{A}}(i) = \frac{\partial A(n)}{\partial A_i}\frac{\partial A_i}{\partial \mu_i}, \tag{16}$$

where μ_i is the repair rate of component i.

The cost-effective IM $I^{\mathrm{CEIM}}(i)$ is proposed by combining the Birnbaum importance with the total cost of failure (Gupta et al., 2013):

$$I^{\mathrm{CEIM}}(i) = \frac{\Delta g_i(P'_{i1})}{g(\boldsymbol{P})} \Big/ \frac{\sum_{i=1}^n E(C_i)}{E(C_i)}, \tag{17}$$

where $\Delta g_i(P'_{i1})$ is the change in the probability of system failure when the component reliability of component i changes from P_{i1} to P'_{i1}, $g(\boldsymbol{P})$ is the probability of system failure without the improvement of component reliability, and $E(C_i)$ is the expected cost when component i fails.

For reconfigurable systems, such as consecutive k-out-of-n systems, the approximation of the reconfigurable importance for component i, $I^{\mathrm{opt}}(i)$, can be calculated by (Si et al., 2014; Dui et al., 2018):

$$I^{\mathrm{opt}}(i) = \lim_{\Delta \to 0} \frac{R^{\mathrm{opt}}(P_{i1} + \Delta, \boldsymbol{P}) - R^{\mathrm{opt}}(P_{i1}, \boldsymbol{P})}{\Delta}, \tag{18}$$

where Δ is the change in component reliability; and $R^{\mathrm{opt}}(P_{i1}, \boldsymbol{P})$ and $R^{\mathrm{opt}}(P_{i1} + \Delta, \boldsymbol{P})$ are system reliabilities of the optimal configurations before and after improving the reliability of component i by Δ, respectively, $\boldsymbol{P} = (P_{11},\dots, P_{i1},\dots, P_{n1})$.

The Birnbaum importance of component i in mission k, $I_k^{\mathrm{PM}}(i, t)$, is an extension of Birnbaum importance for the phased-mission systems (Li et al., 2015):

$$I_k^{\mathrm{PM}}(i, t) = \frac{\partial R_k(t)}{\partial R_{k,i}(t)}, \tag{19}$$

where $R_{k,i}(t)$ is the component reliability of component i in mission k at time t; and $R_k(t)$ is the mission reliability for mission k at time t, represented by the reliabilities of components when $R_k(t) = f(R_{k,1}(t),\dots, R_{k,i}(t),\dots, R_{k,m_k}(t))$, in which m_k represents the number of components in the mission k.

The component maintenance priority $I_{j|i}^{\mathrm{M}}(t)$ is proposed to evaluate the importance of component j once component i fails (Wu et al., 2016):

$$I_{j|i}^{\mathrm{M}}(t) = H_{j|i}\frac{\partial R_\phi(\lambda_i, P_{i1}(t))}{\partial P_{j1}(t)}, \tag{20}$$

where

$$H_{j|i} = \begin{cases} 1 & \text{if } \Phi(1_1,\dots,1_{i-1},0_i,1_{i+1},\dots,1_n) = 0 \\ \Phi(0_i, 0_j, 1_{ij}) & \text{if } \Phi(1_1,\dots,1_{i-1},0_i,1_{i+1},\dots,1_n) = 1 \end{cases},$$

$(0_i, 0_j, 1_{ij})$ represents the states of the system when components i and j fail but other components are working;

$\lambda_i = \chi(\Phi(1_1,..., 1_{i-1}, 0_i, 1_{i+1},..., 1_n) = 0)$, where $\chi(\cdot)$ is an indicator function; and $P_{i1}(t)$ represents the reliability of component i at time t in the binary system.

The cost-based parallel redundancy importance $I_C^{\mathrm{PR}}(i)$ is introduced by combining the cost function with Birnbaum importance (Wu and Wu, 2017):

$$I_C^{\mathrm{PR}}(i) = I^{\mathrm{BM}}(i)\cdot\max(C(r_i, n_i))/C(r_i, n_i), \tag{21}$$

where $C(r_i, n_i)$ is the cost of adding redundancy for component i, r_i is the reliability of component i and its redundant components, and n_i is the redundant number of component i.

The global component importance of component i, $I^{\mathrm{GC}}(i)$, depends not only on the tasks of each phase but also on the components' nominal mean time between failure (MTBF) and their uncertain levels (Wu et al., 2018):

$$I^{\mathrm{GC}}(i) = \frac{E(D(R(\boldsymbol{M}|x_{\sim i})))}{D(R(\boldsymbol{M}))} = \frac{D(R(\boldsymbol{M}))-D(E(R(\boldsymbol{M}|x_{\sim i})))}{D(R(\boldsymbol{M}))}, \tag{22}$$

where $\boldsymbol{M}$ is the nominal MTBF vector of all components; $\sim i$ represents nearly all components, except for component i; $E(D(R(\boldsymbol{M}|x_{\sim i})))$ is the total effects caused by component i and its interaction effect with other components; $D(E(R(\boldsymbol{M}|x_{\sim i})))$ is the average decrement in the variance of the mission success probability without considering component i, depending on all the components themselves and the interaction among them except for component i; and $D(R(\boldsymbol{M}))$ is the variance of mission success probability.

The interval-valued Birnbaum importance of component i, $I_{[\cdot]}^{\mathrm{BM}}(i)$ is similar to that of Birnbaum importance, which uses the interval-valued reliability to calculate the Birnbaum importance (Qiu et al., 2018):

$$\begin{aligned} I_{[\cdot]}^{\mathrm{BM}}(i) &= [R(n|X_i = 1)]-[R(n|X_i = 0)] \\ &= [\underline{R}(n|X_i = 1)-\overline{R}(n|X_i = 0), \\ &\quad \overline{R}(n|X_i = 1)-\underline{R}(n|X_i = 0)], \end{aligned} \tag{23}$$

where $[R(n|X_i = 1)]$ is the interval-valued system reliability when component i is working; $[R(n|X_i = 0)]$ is the interval-valued system reliability when component i fails; $\underline{R}(\cdot)$ is the lower boundary of system reliability; and $\overline{R}(\cdot)$ is the upper boundary of system reliability.

The importance $I^{\mathrm{c-IM}}(i, t)$ based on the cut set, including edge i, is developed for the network with n edges when the edge failures follow a counting process $\{N(t),\ t\geqslant 0\}$ (Du et al., 2019). $I^{\mathrm{c-IM}}(i, t)$ is the conditional probability that a cut, including edge i, fails when the network fails, which is shown by:

$$I^{\mathrm{c-IM}}(i, t) = \frac{\Pr\{\text{a cut } C\in C_i \text{ is down at } t\}}{\Pr\{\text{the network is down at } t\}} = \frac{\sum_{k=0}^{n}\Pr(N(t) = k)F(k, 0_i)}{\sum_{k=0}^{n}\Pr(N(t) = k)F(k)}, \tag{24}$$

where $F(k)$ is the destruction spectrum (D-spectrum) of the network, which is the probability that the network fails when exactly k randomly selected edges fail; $F(k, 0_i)$ is the D-spectrum of edge i, which is equal to the probability that the network fails when randomly selected k edges fail, including edge i; and $\Pr(N(t) = k)$ is the probability that k edges fail at time t.

The importance based on the path set of edge i, $I^{\mathrm{p-IM}}(i, t)$, is the conditional probability that a path including edge i works when the network works at time t, which is shown by:

$$I^{\mathrm{p-IM}}(i, t) = \frac{\Pr\{\text{a path } P\in P_i \text{ is up at } t\}}{\Pr\{\text{the network is up at } t\}} = \frac{\sum_{k=0}^{n}\Pr(N(t) = k)F'(n-k, 1_i)}{\sum_{k=0}^{n}\Pr(N(t) = k)F'(n-k)}, \tag{25}$$

where $F'(k)$ is the probability that the network works if randomly selected k edges work; $F'(n-k, 1_i)$ (or $F'(n-k, 0_i)$) is the probability that the network works (fails) if randomly selected $n-k$ edges, including edge i, work (fail); and $F'(n-k)$ is equal to $F'(n-k, 0_i)+F'(n-k, 1_i)$.

The component reliability boundary, manufacturing difficulty, and feasibility are considered for improving the component reliability. A generalized Birnbaum IM (GBIM) $I^{\mathrm{GB}}(i)$ is proposed by considering the cost-reliability relation function to evaluate the contribution of individual components to improve the system reliability (Si et al., 2019):

$$I^{\mathrm{GB}}(i) = \frac{\partial R_{\mathrm{s}}}{\partial c_i} = \frac{\partial R_{\mathrm{s}}}{\partial P_{i1}}\frac{\partial P_{i1}}{\partial c_i}, \tag{26}$$

where R_{s} is the system reliability; and c_i is the cost of the component related to component reliability range, manufacturing complexity, and technology feasibility.

2.2.2 IMs for multistate systems

For the multistate systems, the concept of system performance is defined based on the expected utility U (Griffith, 1980), which can be calculated by:

$$U = \sum_{j=1}^{M} a_j\Pr(\Phi(\boldsymbol{X}) = j), \tag{27}$$

where $0 = a_0\leqslant a_1\leqslant ...\leqslant a_M$ represents the performance levels of the system at different states, which corresponds to the system state vector $\{0, 1,..., M\}$; $a_0 = 0$ is the state of the system with the lowest performance level.

The Griffith importance $I_m^{\mathrm{G}}(i)$ is the generalization of Birnbaum importance for evaluating the impact of component i on the system performance (Griffith, 1980):

$$I_m^{\mathrm{G}}(i) = \sum\nolimits_{j=1}^{M}(a_j - a_{j-1}) \times [\Pr(\Phi(m_i, \boldsymbol{X}) \geqslant j) - \Pr(\Phi((m-1)_i, \boldsymbol{X}) \geqslant j)], \tag{28}$$

where $\Phi(m_i, \boldsymbol{X})$ is the system state when the component i is at state m. $I_m^{\mathrm{G}}(i)$ evaluates the importance of component i at state m in multistate systems; it indicates the change in the system performance when the component i degrades from state m to state $m-1$.

The Birnbaum sensitivity measure for the generalized phased-mission systems $I^{\mathrm{BS}}(i)$ is proposed to consider the effect of the failure of component i on the unreliability of the system (Xing and Dugan, 2002):

$$I^{\mathrm{BS}}(i) = \frac{\partial Q(\boldsymbol{q})}{\partial q_i} = \frac{\partial\left(Q \cdot P_u + \overline{P}_u\right)}{\partial q_i} = -\overline{Q} \cdot \frac{\partial (P_u)}{\partial q_i} + P_u \frac{\partial (Q)}{\partial q_i}, \tag{29}$$

where q_i is the probability when component i fails; $Q(\boldsymbol{q})$ is the failure function of the corresponding perfect coverage model system, in which $\boldsymbol{q} = \{q_1,..., q_i,..., q_n\}$; Q is the probability that the system fails, and $\overline{Q} = 1 - Q$; and P_u is the probability that no single-point failure occurs, and $\overline{P_u} = 1 - P_u$.

The binary Birnbaum importance is generalized to the multistate, where the system or component works (or fails) when $\Phi(\boldsymbol{X}) \geqslant k_0$ (or $\Phi(\boldsymbol{X}) < k_0$) (Zio and Podofillini, 2003b), where k_0 is the required performance level. The multistate Birnbaum importance of component i in this system $I_{k_0}^{\mathrm{BM}}(i)$ is shown by:

$$I_{k_0}^{\mathrm{BM}}(i) = \Pr\{\Phi(\boldsymbol{X}) \geqslant k_0 | X_i \geqslant k_0\} - \Pr\{\Phi(\boldsymbol{X}) \geqslant k_0 | X_i < k_0\}. \tag{30}$$

Griffith importance only measures the effect of a particular component on the system performance. However, it does not determine the ranks of the components or states based on the important levels. Therefore, Wu importance of component i at state m, $I_m^{\mathrm{Wu}}(i)$, is proposed to overcome the deficiency of Griffith importance by considering the performance utility (Wu and Chan, 2003):

$$I_m^{\mathrm{Wu}}(i) = P_{im} \sum\nolimits_{j=0}^{M} a_j \Pr(\Phi(m_i, \boldsymbol{X}) = j), \tag{31}$$

where $I_m^{\mathrm{Wu}}(i)$ illustrates the contribution of component i at state m to the system performance U, which is the sum of $I_m^{\mathrm{Wu}}(i)$ (m = 0, 1,..., M_i), given that $U = \sum\nolimits_{m=0}^{M_i} I_m^{\mathrm{Wu}}(i)$.

The multistate redundancy importance of component i, $I^{\mathrm{MRI}}(i)$, is an IM based on the improvement of the system reliability (Ramirez-Marquez et al., 2006):

$$I^{\mathrm{MRI}}(i) = \Pr\{\Phi(x_i^+, \boldsymbol{X}) \geqslant d\} - \Pr\{\Phi(\boldsymbol{X}) \geqslant d\}, \tag{32}$$

where d represents the constant demand of system; and x_i^+ represents the addition of the same components for component i during the system design. $I^{\mathrm{MRI}}(i)$ is an estimation of the profit earned by adding redundancy or leveling up the state.

The risk reduction worth of component i in the multistate system is extended by considering the required component condition α (Zio et al., 2007), called the generalized performance achievement worth measure, $I_\alpha^{\mathrm{PAW}}(i)$, which is shown by:

$$I_\alpha^{\mathrm{PAW}}(i) = \frac{L}{L_i^{>\alpha}}, \tag{33}$$

where L is the nominal loss of the system; α is the determined and known required component condition, and the system operates better when the component condition is more than α; and $L_i^{>\alpha}$ is the expected loss of the entire system when the condition of component i remains above level α.

The mean absolute deviation (MAD) importance $I^{\mathrm{MAD}}(i)$ is proposed to measure the expected absolute deviation of reliability for the multistate systems with multistate components caused by the various performance levels and corresponding probabilities of a specified component (Ramirez-Marquez and Coit, 2007), which emphasizes the probabilities of the operation state of the system and the failure state of a component:

$$I^{\mathrm{MAD}}(i) = \sum\nolimits_l \Pr(l_i) |\Pr(\Phi(l_i, \boldsymbol{X}) \geqslant d) - \Pr(\Phi(\boldsymbol{X}) \geqslant d)|, \tag{34}$$

where l_i represents the state of component i.

Griffith importance studies the situation that a component degrades from the state m to the state $m-1$. However, it does not consider the case that a component degrades from the state m to any other lower states $\{m-1, m-2,..., 0\}$. For that reason, the integrated IM of component states $I_{m,l}^{\mathrm{IIM}}(i)$ is introduced to extend the Griffith importance by considering the transition rates among different component states (Si et al., 2012c):

$$\begin{aligned} I_{m,l}^{\mathrm{IIM}}(i) &= P_{im} \cdot \lambda_{m,l}^i \sum\nolimits_{j=1}^{M}(a_j - a_{j-1}) \times [\Pr(\Phi(m_i, \boldsymbol{X}) \geqslant j) - \Pr(\Phi(l_i, \boldsymbol{X}) \geqslant j)] \\ &= P_{im} \cdot \lambda_{m,l}^i \sum\nolimits_{j=1}^{M} a_j \times [\Pr(\Phi(m_i, \boldsymbol{X}) = j) - \Pr(\Phi(l_i, \boldsymbol{X}) = j)], \ (m > l), \end{aligned} \tag{35}$$

where $\lambda_{m,l}^{i}$ is the failure rate of component i transferring from state m to state l. $I_{m,l}^{\mathrm{IIM}}(i)$ is the expected change in the system performance when component i degrades from state m to state l.

The multistate component performance importance for component i at state j, $I_j^{\mathrm{MCP}}(i)$, can be calculated by (Roychowdhury and Bhattacharya, 2019):

$$I_j^{\mathrm{MCP}}(i)=\frac{\partial h(\boldsymbol{P})}{\partial P_{ij}}=\frac{\partial h(P_{1\cdot},\ldots,P_{i\cdot},\ldots,P_{n\cdot})}{\partial P_{ij}}, \tag{36}$$

where $h(\boldsymbol{P})$ is the system performance function; $P_{i\cdot}$ is the probability set of all states for component i; and P_{ij} represents the probability of component i at state j.

2.2.3 IMs for continuous systems

The continuum structure system is introduced by considering that the system or component states are any value in a segment [0, 1] (Baxter, 1984; 1986). The structure function $\Phi:[0,1]^n\rightarrow[0,1]$ of a continuous system is nondecreasing in each argument, satisfying $\Phi(0_1,0_2,\ldots,0_n)=0$ and $\Phi(1_1,1_2,\ldots,1_n)=1$.

For continuous systems, $[0,\beta)$ represents the failure states of the system and $[\beta,1]$ represents the working states. The Birnbaum importance of component i at a level $\beta\in(0,1]$, $I_\beta^{\mathrm{CS}}(i,t)$, is evaluated by (Kim and Baxter, 1987):

$$I_\beta^{\mathrm{CS}}(i,t)=\Pr(\Phi(\boldsymbol{X}(t)\geqslant\beta|X_i(t)\geqslant\delta_i^\beta))-\Pr(\Phi(\boldsymbol{X}(t)\geqslant\beta|X_i(t)<\delta_i^\beta)), \tag{37}$$

where δ_i^β denotes the key element of component i, and $0<\delta_i^\beta<1$ for all $\beta\in(0,1]$.

The lower variance of the importance for all components can obtain a balance system, which can eliminate bottlenecks or overly reliable components (Zio and Podofillini, 2007). The balanced IM I^{B} can be calculated by:

$$I^{\mathrm{B}}=\frac{1}{\sqrt{\overline{I}^2-\overline{I'}^2}},$$

$$\overline{I}^2=\frac{1}{n}\sum\nolimits_{i=1}^{n}I_i^2,\quad \overline{I'}=\frac{1}{n}\sum\nolimits_{i=1}^{n}I_i, \tag{38}$$

where I_i is the importance of component i. I^{B} can be used in binary, multistate, or continuous systems.

The maintenance-based IM for the continuous system is the system performance improvement by maintaining the component i for the unit maintenance time (Cai et al., 2018). Therefore, the maintenance-based importance of component i, $I^{\mathrm{U}}(i,t)$, can be expressed as follows:

$$I^{\mathrm{U}}(i,t)=\frac{\mathrm{d}\Delta U_i(t)}{\mathrm{d}t}=\int_{m_i=0}^{\infty}p(m_i)\int_{l_i>m_i}^{\infty}\mu_{m_i,l_i}(t)\int_{u=0}^{\infty}a(u)[p_{l_i}(u)-p_{m_i}(u)]\mathrm{d}u\mathrm{d}l_i\mathrm{d}m_i, \tag{39}$$

where $\Delta U_i(t)$ is the performance improvement of system when considering time t to maintain the component i; $p(m_i)$ represents the probability when the component i is at state m; $\mu_{m_i,l_i}(t)$ is the density function when the state of component i transits from m to l at maintenance time t; $a(u)$ is the utility function of the system when its state is at u; and $p_{l_i}(u)$ and $p_{m_i}(u)$ represent the probability of the system at state u when component i is at state l_i and m_i, respectively.

The extended importance indexes have a relatively close relationship with the Birnbaum importance. Most IMs for binary systems can be derived by the Birnbaum importance directly, such as the $I_{\mathrm{S}}^{\mathrm{BM}}(i)$, $I^{\mathrm{C}}(i)$, $I_{\mathrm{CAP}}^{\mathrm{BM}}(i)$, $I^{\Delta}(i)$, $I^{\mathrm{PR}}(i)$, $I^{\mathrm{IP}}(i)$, $I^{\mathrm{opt}}(i)$, $I_k^{\mathrm{PM}}(i,t)$, $I_{j|i}^{\mathrm{M}}(t)$, and $I_{[\cdot]}^{\mathrm{BM}}(i)$. Some cost-related IMs can also be derived by Birnbaum importance by introducing the change in system performance for unit cost, such as $I_\lambda^{\mathrm{A}}(i)$, $I_\mu^{\mathrm{A}}(i)$, $I^{\mathrm{CEIM}}(i)$, $I_C^{\mathrm{PR}}(i)$, $I^{\mathrm{GC}}(i)$, and $I^{\mathrm{GB}}(i)$. Birnbaum importance considers the relationships between the system performance when component i is perfect, the system performance when component i fails, and the current system performance. However, some importance indexes consider these factors to analyze the importance level of component i, such as $I^{\mathrm{RAW}}(i,t)$, $I^{\mathrm{RRW}}(i,t)$, $I_{\mathrm{T}}^{\mathrm{BM}}(i,t)$, $I^{\mathrm{Bay}}(i)$, $I^{\mathrm{c-IM}}(i,t)$, and $I^{\mathrm{p-IM}}(i,t)$. Griffith importance has extended the Birnbaum importance into multistate systems considering the system performance. Almost all IMs for multistate systems and continuous systems are derived by the Griffith importance.

The development of IMs extended by the Birnbaum importance can be summarized in Fig. 1. The selected IMs are used to optimize system ROPs and can be divided into three categories based on the system states as binary systems, multistate systems, and continuous systems.

3 IM-based optimization rules for system reliability optimization

Some scholars have proposed the IM-based optimization rules to solve the system ROPs, in which its validity is proven mathematically or through numerical experiment results. Optimization rules can be classified into two categories, namely, optimization rules by IM-based ranking and heuristic methods. The references for system reliability optimization can be analyzed in Table 1 by considering the problems, system states, IMs, and optimization rules.

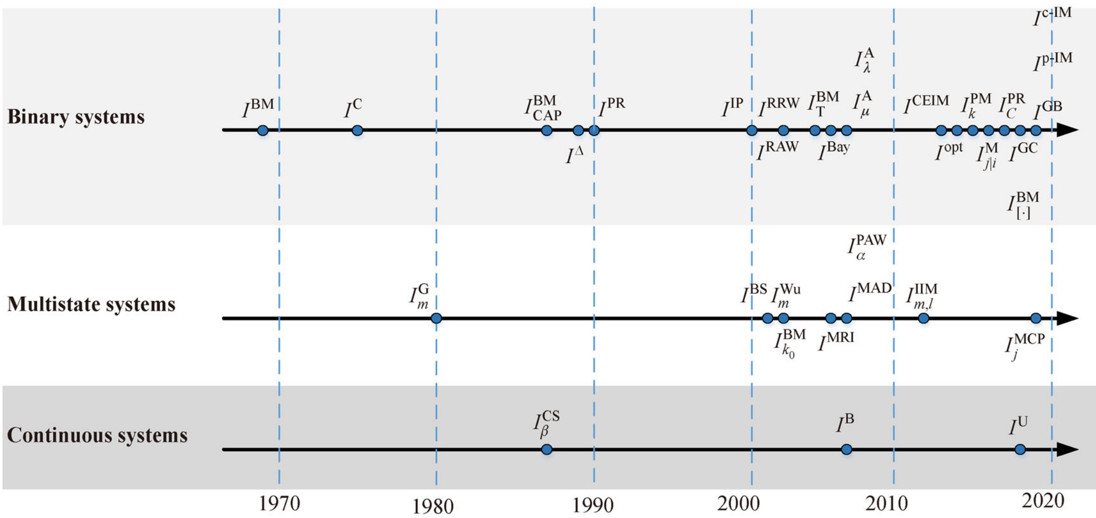

Fig. 1 Extensions and classifications of Birnbaum importance for system reliability optimization.

Table 1 Reference analysis of optimization rules based on IMs

References	Problems	Systems	IMs	Rules
Barabady and Kumar (2007)	ROP	Binary	I_λ^{A} or I_μ^{A}	Ranking
Zio et al. (2007)	ROP	Multistate	I_α^{PAW}	Ranking
Gupta et al. (2013)	ROP	Binary	I^{CEIM}	Ranking
Wu et al. (2016)	ROP	Binary	$I_{j\|i}^{\mathrm{M}}$	Ranking
Roychowdhury and Bhattacharya (2019)	ROP	Multistate	I_j^{MCP}	Ranking
Boland et al. (1988)	RAP	Binary	I^{PR}	Ranking
Shen and Xie (1990)	RAP	Binary	I^{PR}	Ranking
da Costa Bueno (2005)	RAP	Binary	$I_{\mathrm{T}}^{\mathrm{BM}}$	Ranking
Ramirez-Marquez and Coit (2007)	RAP	Multistate	I^{MAD}	Ranking
Bhattacharya and Roychowdhury (2014)	RAP	Binary	$I_{\mathrm{S}}^{\mathrm{BM}}$ or I^{BM}	Ranking
Bhattacharya and Roychowdhury (2016)	RAP	Binary	I^{Bay}	Ranking
Zuo and Kuo (1990)	CAP	Binary	$I_{\mathrm{CAP}}^{\mathrm{BM}}$	Heuristic
Lin and Kuo (2002)	CAP	Binary	$I_{\mathrm{CAP}}^{\mathrm{BM}}$	Heuristic
Yao et al. (2011)	CAP	Binary	$I_{\mathrm{CAP}}^{\mathrm{BM}}$	Heuristic
Zhu et al. (2017)	CAP	Binary	$I_{\mathrm{CAP}}^{\mathrm{BM}}$	Heuristic
Qiu et al. (2018)	CAP	Binary	$I_{[\cdot]}^{\mathrm{BM}}$	Heuristic

3.1 Optimization rules for ROP

The availability allocation problems are solved to determine the optimal solution related to reliability and maintainability; the constraints of these problems are the cost for maximizing system availability. For repairable systems, the availability IMs (I_λ^{A} and I_μ^{A}) of component i based on failure and repair rates are the partial derivative of system availability with respect to the component availability and failure and repair rates (Barabady and Kumar, 2007). The strategy indicates that the component with the largest available IM can have the greatest effect on system availability. The research indicates that the proposed strategy can be applied in the availability allocation problems based on the ranking of IM, wherein performance is demonstrated through numerical examples.

The multistate railway network is modeled, in which each rail section can remain at different states as a component, and the speed depends on the degradation tracks and traffic conditions (Zio et al., 2007). The generalized performance achievement worth can be represented by the delay decrement when the traveling speed in the rail section is higher than the determined level. The results of different cases indicate that relaxing the speed requirements on the sections with high importance can result in a large decrement of the overall delay. Therefore, the order of relaxing sections should be determined by the ranking of $I_{\alpha}^{\mathrm{PAW}}$.

Maintenance cost plays a critical role in improving system performance, and the IM that combines reliability with cost is useful in determining which components can generate the highest improvement in the system performance using the same cost constraints. I^{CEIM} can be used to rank the primary events based on the cost-effective approach for inspection, replacement of components, and maintenance in the engineering systems (Gupta et al., 2013). If the system performance needs to be improved with limited budget, then the component with high I^{CEIM} must be prioritized.

Performing preventive maintenance (PM) frequently may require a great deal of time and can reduce system availability because of the increase in downtime. Therefore, engineers perform PM on several components simultaneously. A component maintenance priority $I_{j|i}^{\mathrm{M}}$ is a typical IM for selecting components in PM (Wu et al., 2016). The maintenance policy is introduced by selecting some components with higher $I_{j|i}^{\mathrm{M}}$, and the proposed policy can determine the optimal number of components to minimize the expected cost with a given time based on the ranking of $I_{j|i}^{\mathrm{M}}$.

A multistate component performance importance is presented to evaluate the effect of the probability of a component at one state on the system performance (Roychowdhury and Bhattacharya, 2019). I_j^{MCP} is proven useful for improving system performance. The multistate component performance importance can maximize the system performance at the design stage by improving the component performance with the largest importance repeatedly. The proposed method is based on the ranking of I_j^{MCP}, and its effectiveness is demonstrated by numerical experiments.

By analyzing the references of the optimization rules for ROP, most of the optimization rules are based on importance ranking, and heuristic-based optimization rules are seldom used to solve the ROP. As such, the procedures of ranking-based optimization rules for ROP are summarized in Fig. 2. For ROP, the proper IM should be selected first by analyzing the optimization model. Second, the IM is calculated, and the components are ranked in decreasing order of IM values. Third, the reliability improvement of the component with the largest importance is determined. Then, the remaining resource is checked. If no resource remains, the final solution is outputted; otherwise, the second step is repeated until all resources are exhausted.

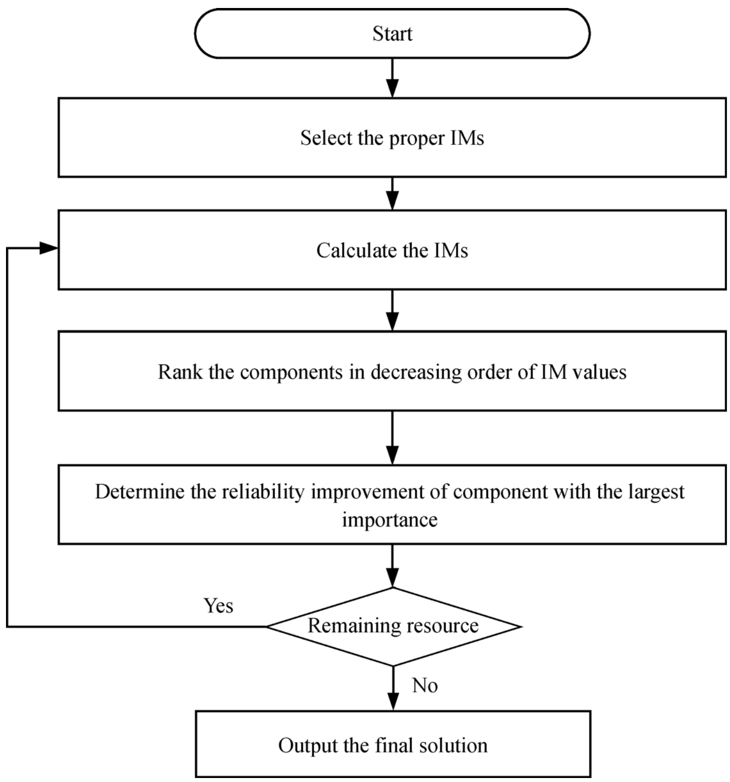

Fig. 2 Procedures of ranking-based optimization rules for ROP.

3.2 Optimization rules for RAP

The redundancy importance that considers active redundancy is introduced into coherent systems to evaluate the changes in system reliability by adding one redundant component (Boland et al., 1988). The parallel redundancy importance I^{PR} is used to identify the priority of components in improving system reliability. The redundant component should be assigned to the component with the largest I^{PR}. The proposed rule is based on the rank of component reliability. Its effectiveness is verified by mathematical derivation and numerical experiments for the series-parallel systems and parallel-series systems.

To obtain the largest improvement of system reliability, parallel redundancy should be added to the components that generate the largest improvement in terms of system reliability, especially when the cost, space, and weight are limited. When the cost of each redundant component is equal, the component, which can generate the largest system reliability increment, results in parallel redundancy (Shen and Xie, 1990). Ranking the components based on I^{PR} to identify the most profitable component is a reasonable approach.

The system lifetime optimization problem allocates a redundant component for maximizing the system lifetime. The problem in allocating a spare part into a k-out-of-n: F system with dependent components is investigated because adding the redundancy to different components can result in additional lifetimes (da Costa Bueno, 2005). The main contribution of this work is that, for a series (parallel) system, performing a minimal standby (active) redundancy operation and allocating it to the weakest component with the largest $I_{\mathrm{T}}^{\mathrm{BM}}$ is stochastically recommended.

For coherent systems, the optimal solution with maximum reliability improvement is generated by allocating a redundant component to the appropriate components based on the IMs (Bhattacharya and Roychowdhury, 2014). The proposed optimal method of iteratively adding one redundant component is extended to solve the problem with unknown information about the components' reliabilities. For situation 1, in which we do not know the component reliabilities, the redundant component with the same reliability should be assigned to the component with the highest Birnbaum structure importance $I_{\mathrm{S}}^{\mathrm{BM}}$. For situation 2, in which the reliabilities of all components are the same, the redundant component should be allocated to the component with the highest Birnbaum importance I^{BM}. For situation 3, supposing that all component reliabilities are different, the redundant component should be added to component with the largest $(1-P_{i1})I^{\mathrm{BM}}(i)$. For situation 4, in which the reliabilities of redundant components may be not the same but the reliabilities of components are known, a redundant component should be added to the component with the highest $P_{i1}'(1-P_{i1})$ $I^{\mathrm{BM}}(i)$.

Bayesian importance is introduced to assign redundant components to maximize system reliability (Bhattacharya and Roychowdhury, 2016). Some rules are applied to optimize RAP, as follows: (1) For series systems, system reliability can be maximized by allocating redundancy to the component with the highest I^{Bay}; (2) For any coherent systems with non-overlapping subsystems, in which components' reliabilities in the same subsystem are the same, adding redundancy to the subsystem with high I^{Bay} can improve system reliability.

By analyzing the optimization rules that are based on the ranking of importance, the procedures of ranking-based optimization rules for RAP can be summarized in Fig. 3. For RAP, the proper IM should be selected first by analyzing the optimization model. Second, we calculate and rank the IM in decreasing order according to the importance of components. Third, we determine the redundancy allocation of the component with the largest importance. Then, we check the remaining resource. If no resources remain, the final solution is outputted; otherwise, we return to the second step until all resources are exhausted.

Some heuristic-based optimization rules are used to maximize system performance. A heuristic approach based on MAD is developed to maximize the reliability of multistate systems with multistate components (Ramirez-Marquez and Coit, 2007). The RAP for multistate systems with multistate components is to the consideration of the allocation of redundant components with a limited budget to maximize system reliability with the given demand. The procedures of MAD-based heuristics are summarized in Table 2. The results of numerical experiments with different system complexities illustrate that the MAD-based heuristic can solve this type of optimization problem effectively by using the optimization rules based on heuristic methods.

3.3 Optimization rules for CAP

The optimal invariant design of consecutive k-out-of-n systems aims to determine the optimal assignment once the ranking of components' reliabilities is known. On the one hand, the optimal invariant design can identify the optimal invariant assignment of some typical consecutive-k-out-of-n systems; on the other hand, the non-existence of optimal invariant assignment in other consecutive k-out-of-n systems should also be proven (Zuo and Kuo, 1990). Systems without optimal invariant assignments can generate at least suboptimal designs based on heuristic methods, and the key idea is to assign the component with high reliability to the position with large IM. For CAP in linear consecutive-k-out-of-n systems, the reliability pattern should match the IM pattern. The initial assignments of the two heuristics should be generated randomly. Heuristic 1 (ZKA) assigns the components with low

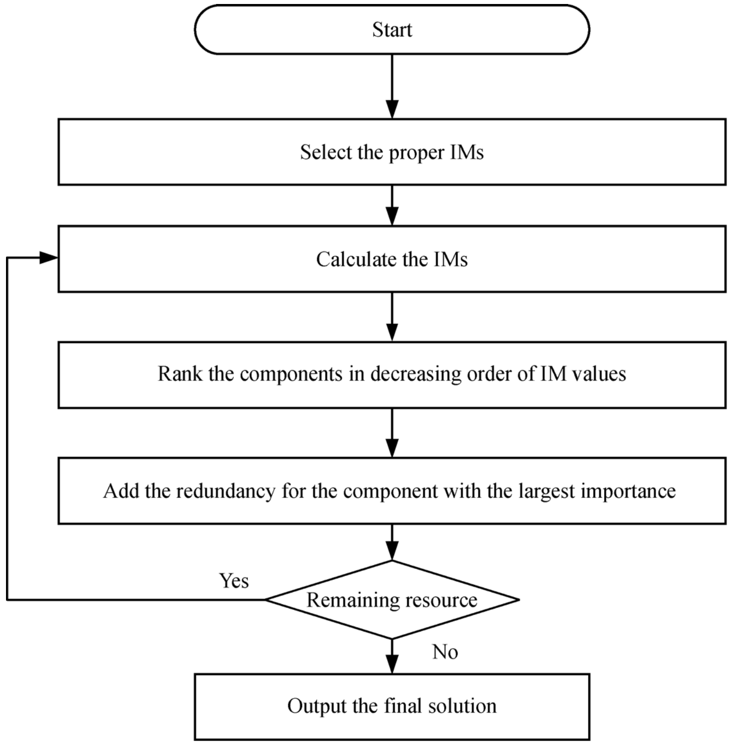

Fig. 3 Procedures of ranking-based optimization rules for RAP.

Table 2 Process of the MAD-based heuristic

Steps	Description
I	Evaluate the I^{MAD} of each component by simulation with the max-flow min-cut algorithm
II	(a) Determine the number of redundant components based on the cost per unit increase in the value of I^{MAD} (b) Update each binary minimal cut vector
III	Judge the stopping rules, if the rules are not satisfied, the process goes to Step I; otherwise, stop this heuristic

reliability to the position with low IM. Heuristic 2 (ZKB) assigns the components with high reliability to the position with high IM. If the component reliability pattern does not match with the IM pattern, the component should exchange with other components, thereby making these two patterns consistent and improving the system reliability after the exchange. The ZKA heuristic process is summarized in Table 3.

A greedy heuristic method called LKA heuristic is developed to search for an ideal assignment according to reliability importance (Lin and Kuo, 2002). If an optimal invariant design is present, the optimal assignment can be determined by the ranking of the component reliability, and the optimal assignment generated by the LKA heuristic is the same with the optimal assignment. LKA heuristic arranges the components with the lowest reliability to all positions as the initialization. Then, it assigns the available components with the highest reliability to the unassigned positions with the largest IM iteratively. The LKA heuristic process is shown in Table 4.

Table 3 Process of the ZKA heuristic

Steps	Description
I	Generate an initial arrangement randomly, $\pi = (\pi_1, \pi_2, \ldots, \pi_i, \ldots, \pi_n)$
II	Calculate $I_{\mathrm{CAP}}^{\mathrm{BM}}(i)$ for all positions from position 1 to position n by Eq. (7)
III	For $k = 1$ to $n-1$, do the loop (a) Find positions m and r such that $\pi_m = k$ and $\pi_r = k+1$ (b) If $I_{\mathrm{CAP}}^{\mathrm{BM}}(m) > I_{\mathrm{CAP}}^{\mathrm{BM}}(r)$ and $R(P, \pi) > R(P, \pi(m, r))$, exchange the assignments of components π_m and π_r
IV	If there is no exchange in Step III, output the final assignment; otherwise, go to Step II

Table 4 Process of LKA heuristic

Steps	Description
I	Assign component 1 to all positions that are set $P_{i1} = P_{11}$ and $\pi_i = 1$ for $i = 1, 2, \ldots, n$
II	Let $S = \{1, 2\ldots, n\}$, which is the set of available positions that could receive other components
III	For $k = n$ to 2, do the loop (a) Calculate $I_{\text{CAP}}^{\text{BM}}(i)$ for all $i \in S$ by Eq. (7) (b) Find the position $m \in S$, which meets that $I_{\text{CAP}}^{\text{BM}}(m) = \max_{i \in S} I_{\text{CAP}}^{\text{BM}}(i)$ (c) Let $S = S/\{m\}$, assign component k to position m
IV	If there are no components in S, output the final assignment; otherwise, go to Step II

The Birnbaum importance is used to design heuristics for solving CAP. The ZKA and ZKB heuristics start pairwise exchanges from the least reliable component, while the ZKC and ZKD heuristics start pairwise exchanges from the most reliable component with the revised Alternatives 1 and 2, respectively (Yao et al., 2011). The new Alternative 1 in ZKC heuristic compares the component $\pi_m = k$ with the component $\pi_r = k-1$, and the new Alternative 2 in ZKD heuristic compares component $\pi_m = k$ with the component with the largest Birnbaum importance among the positions whose reliabilities are lower than P_{i1}. The differences between ZK-type heuristics are listed in Table 5.

Three new LK-type heuristics are proposed by modifying the initialization and/or the assignment rule of the LKA heuristic (Yao et al., 2011). The LKB heuristic assigns the component according to the increasing order of component reliability and initializes the components with the lowest reliability in all positions. The LKC heuristic has the same initialization as the LKA heuristic but uses a different assignment rule. That is, the assignment of position with the smallest Birnbaum importance is retained, and the least reliable unassigned component is allocated to all other positions. The LKD heuristic uses the same assignment rule as the LKC heuristic, but it iterates from the component with high reliability to the component with the lowest reliability. The differences between LK-type heuristics are listed in Table 6.

The Birnbaum importance-based two-stage (BIT) approach is proposed by integrating the simulation results of ZK- and LK-type heuristics for solving the CAP (Yao et al., 2011). First, we generate two initial assignments by using both LKA and LKB heuristics. Second, we select the ZKB heuristic if all the reliabilities of components are less than 0.2; otherwise, we select the ZKD heuristic. Finally, we select the better one as the final solution. The procedures of the BIT heuristic are summarized in Table 7.

A single type of component is used in CAP, and this problem is extended to a multi-type CAP by dividing the components into different types of components that should be assigned to the corresponding type of position (Zhu et al., 2017). The BIT method for the multi-type CAP (BITSM) is developed after the simulation based on the extended ZK- and LK-type heuristics. BITSM can be summarized as follows. In the first stage, we generate two initial assignments based on the parallel iterative assignment. In the second stage, we perform the parallel pairwise exchange separately on these two assignments. In the third stage, we should improve the solution.

Uncertainties are inevitable in many real engineering projects because of insufficient data and complex relationships. The extension of the LKA heuristic using the

Table 5 Differences between ZK-type heuristics

Heuristics	Step III	Step III(a)	Step III(b)
ZKA	1 to $n-1$	$\pi_r = k+1$	$I_{\text{CAP}}^{\text{BM}}(m) > I_{\text{CAP}}^{\text{BM}}(r)$
ZKB	1 to $n-1$	$I_{\text{CAP}}^{\text{BM}}(r) = \min_{i:P_{i1}>P_{m1}} I_{\text{CAP}}^{\text{BM}}(i)$	$I_{\text{CAP}}^{\text{BM}}(m) > I_{\text{CAP}}^{\text{BM}}(r)$
ZKC	n to 2	$\pi_r = k-1$	$I_{\text{CAP}}^{\text{BM}}(m) < I_{\text{CAP}}^{\text{BM}}(r)$
ZKD	n to 2	$I_{\text{CAP}}^{\text{BM}}(r) = \max_{i:P_{i1}<P_{m1}} I_{\text{CAP}}^{\text{BM}}(i)$	$I_{\text{CAP}}^{\text{BM}}(m) < I_{\text{CAP}}^{\text{BM}}(r)$

Table 6 Differences between LK-type heuristics

Heuristics	Step I	Step III	Step III(b)	Step III(d)
LKA	Component 1 to all positions	n to 2	$I_{\text{CAP}}^{\text{BM}}(m) = \max_{i \in S} I_{\text{CAP}}^{\text{BM}}(i)$	Component k to position m
LKB	Component n to all positions	1 to $n-1$	$I_{\text{CAP}}^{\text{BM}}(m) = \min_{i \in S} I_{\text{CAP}}^{\text{BM}}(i)$	Component k to position m
LKC	Component 1 to all positions	2 to n	$I_{\text{CAP}}^{\text{BM}}(m) = \min_{i \in S} I_{\text{CAP}}^{\text{BM}}(i)$	Component k to positions in S
LKD	Component n to all positions	$n-1$ to 1	$I_{\text{CAP}}^{\text{BM}}(m) = \max_{i \in S} I_{\text{CAP}}^{\text{BM}}(i)$	Component k to positions in S

Table 7 Process of the BIT heuristic

Steps	Description
I	Generate two initial arrangements by both LKA and LKB heuristics
II	(a) Select the ZKB heuristic if all the components have low reliability; otherwise, select ZKD heuristic (b) Stop by giving the final arrangement with higher system reliability

evidential network and interval-valued IM solves CAP under uncertain parameters and models (Qiu et al., 2018). The procedures of the proposed heuristic method are similar to those of the original LKA heuristic, but the relations of interval order are introduced to compare the bounded system reliability and the interval-valued Birnbaum importance. The system reliability and the Birnbaum importance are interval-valued when the parameter is uncertain, and the interval order should be conducted in Steps III(a) and III(b) in the LKA heuristic process.

3.4 Summary of the optimization rules

(1) Optimization rules by IM-based ranking methods

By analyzing the optimization rules of ROPs, RAPs, and CAPs, all the ROPs and almost all the RAPs are solved by the IM-based ranking methods, but none of the CAPs are solved using the IM-based ranking methods. The general procedures of the IM-based ranking methods for ROPs are similar to those of RAPs. We choose the proper IMs to analyze the optimization model at first. Subsequently, we rank the components in decreasing order according to IM values. Then, we improve the reliability or redundancy of the component with the largest importance. Finally, we check the remaining resource. If no resources remain, we output the final solution; otherwise, we return to the previous step to rank the components.

(2) Optimization rules by IM-based heuristic methods

Some RAPs and all the CAPs are solved by the IM-based heuristic methods with the consideration of the complexity of the corresponding problems by analyzing the optimization rules of ROPs, RAPs, and CAPs. For RAPs, redundancy should be assigned to the component with the highest IM, whereas the highest (lowest) reliable component should be assigned to the position with the highest (lowest) IM. The general procedures of IM-based heuristic methods can be summarized as follows. First, we generate an initial solution. Second, we assign the resources to the components with the highest IM. Third, we update the results. Finally, we check the remaining resources and stop the IM-based heuristic methods.

4 IM-based optimization algorithms for system reliability optimization

System optimization problems consistently minimize the cost subject to the requirement of system reliability or maximize the system reliability under resource constraints. Considering the complexity of the system ROPs, many scholars have proved that the system ROPs are NP (non-deterministic polynomial)-hard (Lin and Chen, 1997). Many problems are complicated. Thus, obtaining the optimal solution by using the enumeration method is difficult. However, IM-based optimization algorithms can generate an effective solution. On the one hand, IM-based rules are combined with evolutionary algorithms to improve the performance of the proposed algorithms. On the other hand, the IMs are used to screen the critical factors to simplify the optimization problems. Both approaches are IM-based optimization algorithms, and they are known as the IM-based local search and simplification methods. The references for optimization algorithms based on IMs are summarized in Table 8 by considering the problems, system states, IMs, and algorithms.

4.1 Optimization algorithms for ROP

To solve the ROP, Birnbaum importance and Δ-importance are introduced into the genetic algorithm to obtain the optimal solution with constraints on cost (Wang et al., 2018). The optimization algorithm combines the local search method with the advantages of IM. The mechanism of IMs is that the components with larger IM should focus on improving its reliability. Genetic algorithms based on Birnbaum importance or Δ-importance are proposed to illustrate the effectiveness of I^{BM} and I^{Δ} for solving the ROP. The performance of the genetic algorithm based on Δ-importance is better than that based on Birnbaum importance via the comparison of the generations of convergence and system reliability.

Considering the limited design resources, increasing the reliability of some components can maximize system reliability. I^{GB} can evaluate the contribution of component reliability to system reliability considering the boundary of component reliability and the difficulty of increasing component reliability (Si et al., 2019). The GBIM-based genetic algorithm is presented to solve the ROP by increasing the reliability of the component with the highest I^{GB} during local search. The GBIM-based genetic algorithm could find a better near-global solution with a faster convergence speed compared with genetic algorithms that are based on Birnbaum importance and Δ-importance, and the effectiveness of the proposed algorithm is illustrated by a mixed system with 17 components.

Table 8 Reference analysis of optimization algorithms based on IMs

References	Problems	Systems	IMs	Algorithms
Wang et al. (2018)	ROP	Binary	I^{BM} and I^{Δ}	Local search
Si et al. (2019)	ROP	Binary	I^{GB}	Local search
Zio and Podofillini (2007)	ROP	Any states	I^{B}	Simplification
Cai et al. (2018)	ROP	Continuous	I^{U}	Local search
Xiong et al. (2017)	RAP	Binary	I^{PRW}	Simplification
Shojaei and Mahani (2019)	RAP	Binary	I^{BM}	Simplification
Zhao et al. (2019c)	RAP	Binary	I^{BM}	Local search
Yao et al. (2014)	CAP	Binary	I^{BM}	Local search
Cai et al. (2016)	CAP	Binary	I^{BM}	Local search
Zhang et al. (2019)	CAP	Binary	I^{BM}	Local search
Dui et al. (2018)	CAP	Binary	I^{opt}	Simplification
Zhao et al. (2019b)	CAP	Binary	I^{BM}	Simplification
Nguyen et al. (2017)	CP	Binary	$I_{\mathrm{S}}^{\mathrm{BM}}$	Simplification
Du et al. (2019)	CP	Binary	$I^{\mathrm{c-IM}}$ and $I^{\mathrm{p-IM}}$	Simplification
Xing and Dugan (2002)	CP	Multistate	I^{BS}	Simplification
Li et al. (2015)	CP	Binary	I_k^{PM}	Local search
Wu and Wu (2017)	CP	Binary	I_C^{PR}	Local search
Wu et al. (2018)	CP	Binary	I^{GC}	Simplification

The performance improvement of multistate systems with multistate components is a CP because system performance depends on the combinations of components' performance levels and is related to the component performance and their positions. Balanced IM I^{B} is considered an objective in the mathematical model for the multi-objective ROP (Zio and Podofillini, 2007), which can simplify the model because maximizing the balanced IMs can make the system fully balanced to avoid bottlenecks or overly reliable components.

Continuous systems can describe component situations more accurately than discrete-state systems. However, solving optimization problems with continuous systems using conventional optimization algorithms is complicated. The performance improvement problem for a continuous system is to arrange the resources optimally to maximize system performance after the maintenance, which can be solved by the performance improvement-based genetic algorithm (Cai et al., 2018). The local search method based on I^{U} is proposed to rearrange the maintenance time of components. When the maintenance information of other components is known and identical, the components with the largest I^{U} should provide the optimal value, and the maintenance time of other components is balanced based on the proportion of previous maintenance time.

4.2 Optimization algorithms for RAP

Many scholars have investigated the optimization algorithms for RAP because some extended RAPs are developed by establishing complicated mathematical models. The RAPs are extended by considering the abnormal external failures when designing a system that maximizes the system reliability considering the uncertain abnormal external failures under normal and worst cases (Xiong et al., 2017). A multicomponent IM can identify the amounts of abnormal external failures for the worst case when the system configuration is provided. As proven, the effect of component failures on the system can be simplified by considering the effect of component failures on the subsystems, which can determine the system reliability under worst cases with different amounts of the abnormal external failures and simplify the complexity of calculating the objective. This extended RAP is a multi-objective optimization problem that maximizes system reliability under normal and abnormal external failures and minimizes cost. I^{PRW} can be used to qualify the importance of component combinations with arbitrary amounts based on the effect of their failures on system reliability. Therefore, I^{PRW} can solve the optimization model of RAPs.

This multi-objective nonlinear RRAP maximizes system reliability and minimizes the variance of IMs (Shojaei and Mahani, 2019). The variance of IMs in the latter objective can evaluate the dependent degree of the system. The system is more dependable when the variance of IMs for subsystems decreases, indicating that the IMs of each component is very close. The variance of IMs is used to simplify the objective functions of the RRAP.

A multi-objective particle swarm optimization algorithm that combines IM and harmony search is developed to deal with multi-objective RRAP for serial parallel-series systems considering system reliability and design cost (Zhao et al., 2019c). Two interesting IM-based local searches are developed on the basis of two reliability adjustment strategies, namely, increasing component reliability after the redundancy decrement and decreasing component reliability after the redundancy increment. Birnbaum importance can evaluate the improvement of system reliability. The IM-based local search method can improve the effectiveness of solving the multi-objective RRAP, and the procedures are summarized in Table 9.

4.3 Optimization algorithms for CAP

A Birnbaum importance-based genetic local search (BIGLS) algorithm is proposed to solve the CAP by combining an IM-based local search method with the genetic algorithm (Yao et al., 2014). The idea of the Birnbaum importance-based three-way exchange is to adjust the position of three components to make their component reliability consistent with their importance. If the exchange is not beneficial for increasing system reliability, then the assignment does not change. The effectiveness of the BIGLS algorithm is compared with those of BIT and the general genetic algorithm.

Linear consecutive-k-out-of-n systems are a type of typical system of CAP. A Birnbaum importance-based genetic algorithm (BIGA), which combines a genetic algorithm with BIT-based local search method, is proposed to obtain the near-global optimal solution of CAP (Cai et al., 2016). The BIGA is applied to deal with the system reliability optimization of the circular consecutive-k-out-of-n systems (Zhang et al., 2017). Birnbaum importance-based quantum genetic algorithm (BIQGA) is developed by combining the quantum genetic algorithm with the Birnbaum importance-based local search method to solve the CAP efficiently and accurately based on the advantages of quantum computing and IMs (Zhang et al., 2019).

The changes in the optimal assignment for linear consecutive-k-out-of-n systems are analyzed by considering the changes in component reliability and IM of components during the system's lifetime (Dui et al., 2018). These results show the relationships between the changes in component reliability and IM with the changes in the optimal assignment. The importance of a specific component represents the improvement of the largest system reliability when the component reliability changes. The proposed importance can simplify the optimization process by considering the relationship between the optimal arrangement and the changes in component reliability.

An integrated method that combines the rearrangement method with the replacement method, is proposed to maximize the reliability of the reconfigurable system and minimize the cost during the reconfiguration (Zhao et al., 2019b). The BIT heuristic is used to generate the current optimal assignment, which is essential in measuring the optimal system reliability and the reconfiguration cost. Therefore, the BIT heuristic can be used to evaluate the system reliability when the reconfigurable cost is the largest to simplify the calculation of the fitness value.

4.4 Optimization algorithms for CP

Advanced technology in optimization theory can solve the system ROPs for complex systems in different fields (Coit and Zio, 2019). Network designs are used to determine the redundant configurations and select different available components to form the system structure. For fixed network topology, the reliability of a network system is improved by allocating redundancy or adjusting the links' reliability between node pairs with the constraints on a budget (Marseguerra et al., 2005). Sometimes, obtaining the optimal configuration directly in case the network structure is complex is difficult. Multiple phases are widely used in many practical applications, and many practical systems are phased-mission systems (PMS), which are multiple, consecutive, non-overlapping phases (Levitin et al., 2012). The PMSs need to accomplish some specified tasks during each phase. The features of system performance and the component degradation may change from phase to phase, in which the relationships between components in different phases are complicated. Therefore, the systems of the CP can be classified into two categories: Systems with complex structures, such as network systems (Compare et al., 2019; Xiao et al., 2018);

Table 9 Procedures of the IM-based local search method

Procedures	Description
I	Select the component modules with the highest $I^{\mathrm{BM}}(i)$
II	Perform the reliability adjustment strategy 1 or 2 randomly (1) Strategy 1: Increasing component reliability after decreasing the component redundancy (2) Strategy 2: Decreasing component reliability after increasing the component redundancy
III	Identify the solution after the adjustment

and systems with a complicated relationship, such as PMS (Levitin et al., 2017a; 2017c).

A joint PM and inventory strategy of systems with complex structures can minimize the cost rate (Nguyen et al., 2017). The Birnbaum structure importance of components is evaluated on the basis of predictive reliability, which is used in deciding the ordering of spare parts and PM at a regular time. The structure importance decreased the number of decision parameters to three, regardless of the number of components.

When the component failure follows the counting process, two IMs are proposed to evaluate the effect of one edge on network failure or working (Du et al., 2019). These IMs depend on the structure of the network and the failure distribution of edges instead of the probabilistic information of the individual edge. The ranking of the proposed IMs can be potentially used to arrange resources of networks during the design stage and maintenance process.

An analysis method for a generalized PMS is proposed to calculate the probability of mission success with high computation efficiency (Xing and Dugan, 2002). The Birnbaum sensitivity measure I^{BS} can be used to estimate the effect of parameter-change on the result without considering a full model-solution as all parameters change. I^{BS} over the entire phased mission is a weighted sum of sensitivity in each phase. The more unreliable a phase is, the more sensitive the component in the phase should be. The proposed sensitivity measure can upgrade suitable candidates who can improve the system performance effectively to simplify the objective of the optimization problem.

For multi-mission networked avionics, a mission reliability allocation method is proposed to maximize avionic mission reliability (Li et al., 2015). The proposed allocation principles are summarized as follows: (1) adjusting the complex component with low reliability and (2) assigning the important component with high reliability. The IM-based heuristic algorithm is regarded as the local search method combined with the algorithm provided by the Advisory Group on the Reliability of Electronic Equipment (AGREE) to solve the problem with the network structure.

A hybrid algorithm that combines IM-based heuristic with the genetic algorithm is presented to solve the RRAP in PMS (Wu and Wu, 2017). The cost-based parallel redundancy importance is proposed by combining the Birnbaum importance with the cost function. The IM-based heuristic method increases the reliability of components with higher I_C^{PR} and decreases the reliability of components with lower I_C^{PR}, thereby possibly reducing the total system cost. The hybrid algorithm takes advantage of the higher global search ability of the genetic algorithm and the local search ability of the proposed heuristic.

Considering the uncertainty of components' parameters caused by the changing environments in each phase and the insufficient information, a reliability allocation model is introduced to incorporate component uncertainty during the reliability allocation process (Wu et al., 2018). The global component importance I^{GC} can evaluate the importance level of a component for mission reliability because components' reliabilities vary randomly. The improved procedure based on global importance is that decreasing the MTBF of the component with the highest cost and setting the new MTBF based on the global importance value. I^{GC} is an effective way to generate the initial feasible solution, which can simplify optimization.

4.5 Summary of the optimization algorithms

(1) Optimization algorithms by IM-based local search methods

The IM-based optimization algorithms vary because the optimization rules of the local search methods are proposed on the basis of the ranking of IM or the IM-based heuristic algorithms. Therefore, the optimization algorithms based on the local search methods are analyzed in Table 10 to illustrate the differences among different optimization algorithms.

The general procedures of optimization algorithms by IM-based local search methods based on the analysis of the procedures of the previous research are proposed in Fig. 4.

Table 10 Reference analysis of the optimization algorithms by IM-based local search methods

References	Problems	Systems	Algorithms	Optimization rules
Wang et al. (2018)	ROP	Binary	Genetic algorithm	Ranking
Si et al. (2019)	ROP	Binary	Genetic algorithm	Ranking
Cai et al. (2018)	ROP	Continuous	Genetic algorithm	Ranking
Zhao et al. (2019c)	RAP	Binary	Particle swarm algorithm	Heuristic
Yao et al. (2014)	CAP	Binary	Genetic algorithm	Heuristic
Cai et al. (2016)	CAP	Binary	Genetic algorithm	Heuristic
Zhang et al. (2019)	CAP	Binary	Genetic algorithm	Ranking
Li et al. (2015)	CP	Binary	AGREE method	Heuristic
Wu and Wu (2017)	CP	Binary	Genetic algorithm	Heuristic

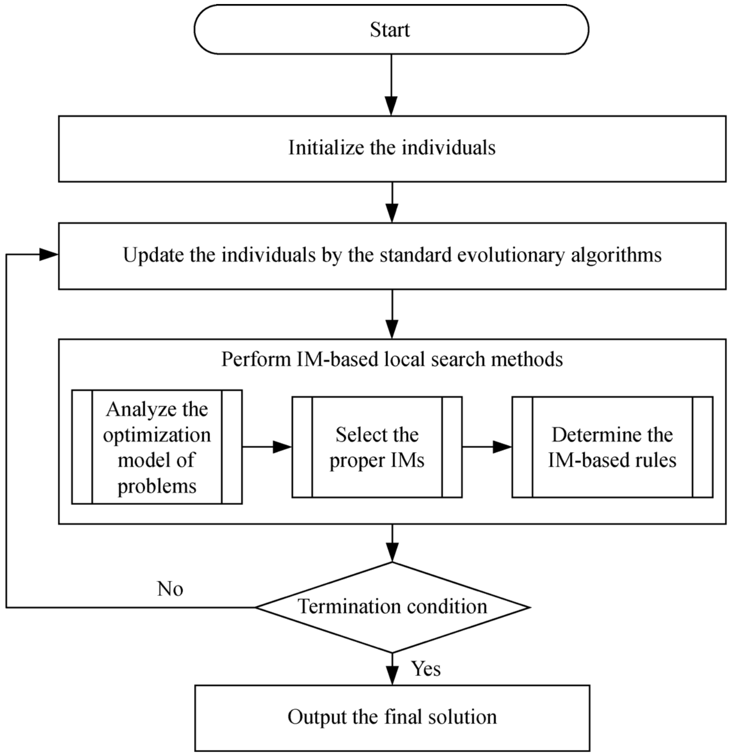

Fig. 4 General procedures of optimization algorithms by IM-based local search method.

The termination conditions are used to stop the optimization algorithms, which are optimization algorithm parameters used to determine whether the algorithms should be terminated. Thus, the conditions can be the maximum generations, the running time, or the convergence conditions, and they depend on the optimization algorithms, the requirements of the practical engineering, and the accuracy of the solution.

(2) Optimization algorithms by IM-based simplification methods

The optimization algorithms by the IM-based simplification method mainly use IM in the mathematical models or the optimization process to simplify the objectives or screening the critical factors to simplify the optimization problems. The analysis of the simplification method focuses on the sub-process to illustrate where the IM can be used to simplify the optimization models. Therefore, we summarized the usage of IMs during simplification in Table 11, and the general procedures of optimization algorithms using IM-based simplification method is proposed in Fig. 5.

5 Optimization framework driven by IMs

Various IMs clarify the effect of one component's performance on the overall system performance from different perspectives to distinguish the weak link of the systems. If the weak link is found, then giving additional resources to the weakest component can improve the

Table 11 Reference analysis of optimization algorithms by IM-based simplification methods

References	Problems	Systems	Sub-processes	Strategies
Zio and Podofillini (2007)	ROP	Any states	Objective	Use importance as the objective
Xiong et al. (2017)	RAP	Binary	Objective	Simplify the solving method
Shojaei and Mahani (2019)	RAP	Binary	Objective	Use importance as the objective
Dui et al. (2018)	CAP	Binary	Decision variable	Obtain the solution effectively
Zhao et al. (2019b)	CAP	Binary	Fitness	Simplify the complexity of the objective calculation
Nguyen et al. (2017)	CP	Binary	Decision parameters	Use the importance ranking to screen critical factors
Du et al. (2019)	CP	Binary	Decision variable	Obtain the solution effectively
Xing and Dugan (2002)	CP	Multistate	Objective	Evaluate the objective effectively
Wu et al. (2018)	CP	Binary	Initialization	Obtain the initial feasible solution

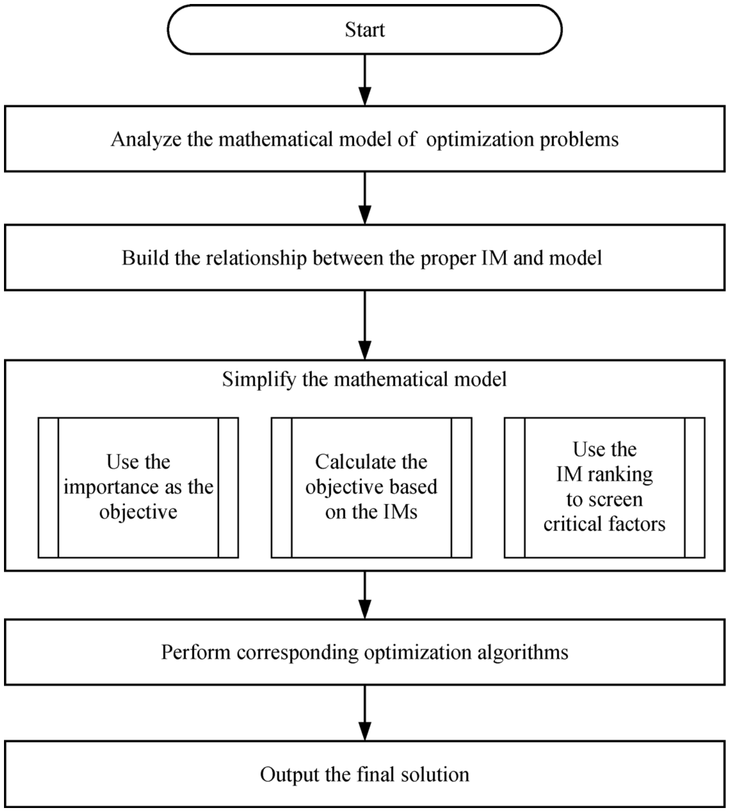

Fig. 5 General procedures of optimization algorithms by IM-based simplification methods.

system performance significantly. Sometimes, this approach is a shortcut for improving system performance through IM ranking. Thus, the merits of IMs are to find the weakest parts in the system, possibly improving system performance cost-effectively.

The previous solutions of the ROPs focused on the mathematical methods or algorithms of the operation research, but the contributions of IMs are ignored. The weakest link in the system is the premise and basis of the optimization problems; improving the component performance based on IM is an effective method to improve the system performance. The IM-based methods can summarize some optimization rules easily to solve the optimization problems effectively. This simplification is the benefit of practical engineering. Moreover, many future research topics are related to IMs. We will discuss these methods in Section 6.

According to the state-of-the-art about the system reliability optimization based on the IMs in Sections 3 and 4, the general optimization framework driven by IMs is summarized in Fig. 6. Clearly, the selection of solving methods mainly depends on the optimization problems and the characteristics of IMs.

For the mentioned optimization problems (ROP, RAP or RRAP, CAP, and CP), the IMs can maximize the system performance by arranging resources for the components with higher IM. The solving method should be proposed by analyzing the complexity of problems, choosing the proper IM, and checking the effectiveness of the IM rankings. After the analysis of the system ROP and IMs, if the IM rankings are useful in solving the problem, then the IM-based rules should be selected to deal with the system optimization problem; otherwise, the IM-based algorithm is selected.

The solving methods based on IMs include the IM-based optimization rules and the IM-based optimization algorithms. The IM-based rules are proposed on the basis of the following principles: (1) improving the component performance with the highest importance by the IM-based ranking methods to maximize the system performance with the constraints on the limited resources and (2) assigning the resources to the components by the IM-based heuristic methods for maximizing system performance. The IM-based optimization algorithms are introduced to generate an improved solution for CPs with a large-scale system or complicated tasks in the system. The IM-based optimization algorithms consistently combine the IM-based optimization rules with the evolution algorithms. Sometimes, IM-based optimization algorithms also use the IM-based rules to simplify the mathematical model by using the IMs to replace the objectives or screen the critical factors.

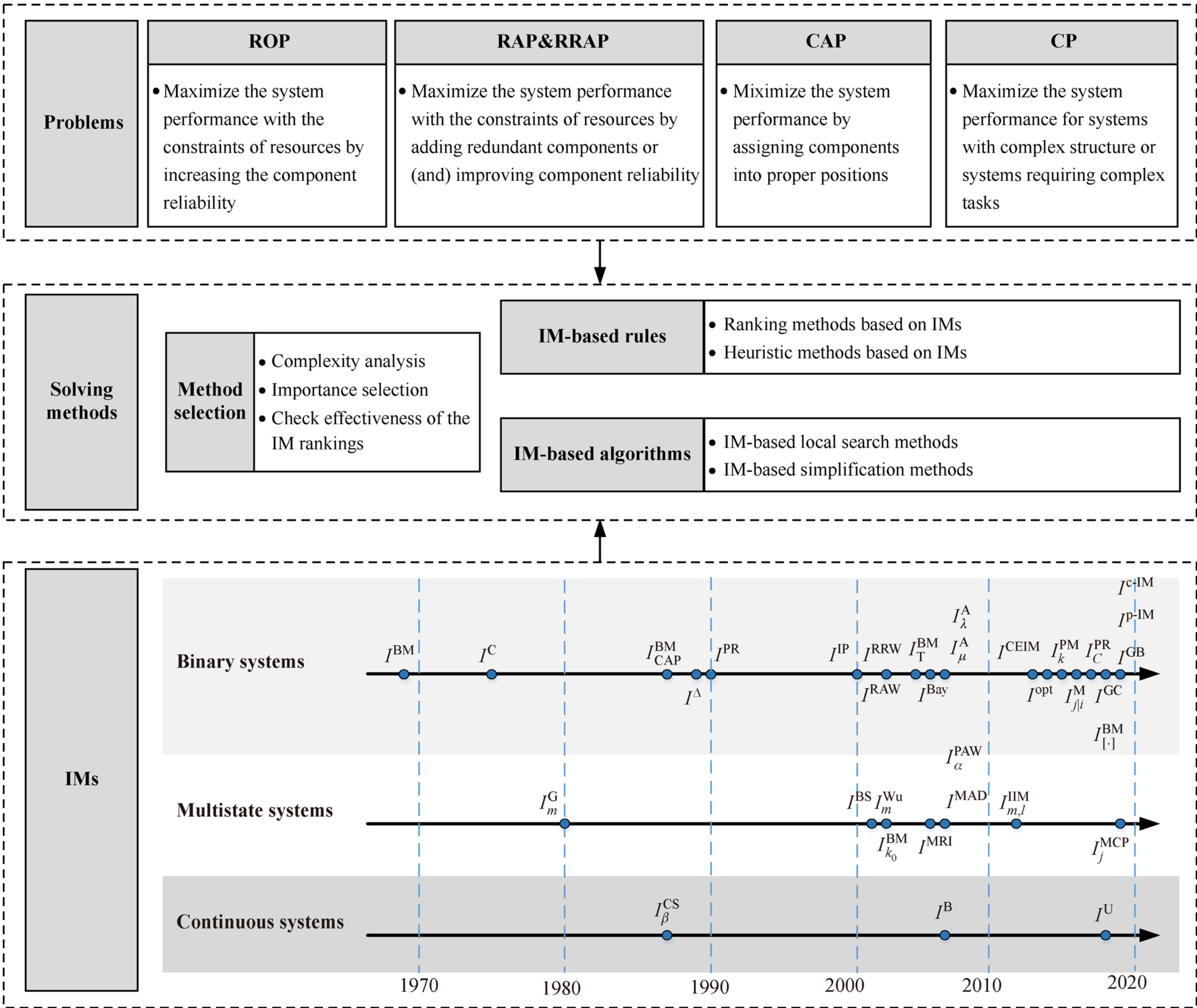

Fig. 6 General optimization framework driven by IMs.

6 Future research directions

Some challenges on the system reliability optimization driven by IMs remain for contemporary researchers. Therefore, the following problems should be a focus in the future.

(1) How can the mechanism of IMs in system ROPs be summarized? The mechanism of IMs can be used for solving system ROPs effectively.

(2) How be the IMs defined with the consideration of the relationship between different objectives and constraints in the optimization model? The proper definition of IMs is the basis of a suitable heuristic method for solving the system ROPs.

(3) How can the matching degree between an IM and a specific optimization problem be evaluated? Such a matching degree could be valuable for engineers to deal with the system ROPs driven by IMs.

(4) How can the IMs of nodes in the complex networks be evaluated? This problem is one of the most complicated problems in many practical engineering projects for considering the transition mechanism and the structure feature comprehensively. The results would simplify the networks' control, deposition, and defense by identifying some critical nodes during optimization.

References

Abouei Ardakan M, Zeinal Hamadani A (2014). Reliability-redundancy allocation problem with cold-standby redundancy strategy. Simulation Modelling Practice and Theory, 42: 107–118

Abouei Ardakan M, Rezvan M T (2018). Multi-objective optimization of reliability-redundancy allocation problem with cold-standby strategy using NSGA-II. Reliability Engineering & System Safety, 172: 225–238

Aliee H, Vitzethum S, Glaß M, Teich J, Borgonovo E (2016). Guiding Genetic Algorithms using importance measures for reliable design of embedded systems. In: Proceedings of International Symposium on Defect and Fault Tolerance in VLSI and Nanotechnology Systems. Storrs, CT: IEEE, 53–56

Arora S, Singh S, Yetilmezsoy K (2018). A modified butterfly optimization algorithm for mechanical design optimization problems. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 40(1): 21

Aven T, Jensen U (2000). Stochastic Models in Reliability, 2nd ed. New York: Springer

Barabady J, Kumar U (2007). Availability allocation through importance measures. International Journal of Quality & Reliability Management, 24(6): 643–657

Barlow R E, Proschan F (1975). Importance of system components and fault tree events. Stochastic Processes and Their Applications, 3(2): 153–173

Baroud H, Barker K (2018). A Bayesian kernel approach to modeling resilience-based network component importance. Reliability Engineering & System Safety, 170: 10–19

Baxter L A (1984). Continuum structures I. Journal of Applied Probability, 21(4): 802–815

Baxter L A (1986). Continuum structures II. Mathematical Proceedings of the Cambridge Philosophical Society, 99(2): 331–338

Bhattacharya D, Roychowdhury S (2014). Redundancy allocation using component-importance measures for maximizing system reliability. American Journal of Mathematical and Management Sciences, 33(1): 36–54

Bhattacharya D, Roychowdhury S (2016). Bayesian importance measure-based approach for optimal redundancy assignment. American Journal of Mathematical and Management Sciences, 35(4): 335–344

Birnbaum Z W (1969). On the importance of different components in a multicomponent system. Multivariate Analysis, II: 581–592

Boland P J, El-Neweihi E, Proschan F (1988). Active redundancy allocation in coherent systems. Probability in the Engineering and Informational Sciences, 2(3): 343–353

Borgonovo E, Aliee H, Glaß M, Teich J (2016). A new time-independent reliability importance measure. European Journal of Operational Research, 254(2): 427–442

Bretas A S, Cabral R J, Leborgne R C, Ferreira G D, Morales J A (2018). Multi-objective MILP model for distribution systems reliability optimization: A lightning protection system design approach. International Journal of Electrical Power & Energy Systems, 98: 256–268

Cai Z Q, Si S B, Liu Y, Zhao J B (2018). Maintenance optimization of continuous state systems based on performance improvement. IEEE Transactions on Reliability, 67(2): 651–665

Cai Z Q, Si S B, Sun S D, Li C T (2016). Optimization of linear consecutive-*k*-out-of-*n* system with a Birnbaum importance-based genetic algorithm. Reliability Engineering & System Safety, 152: 248–258

Čepin M (2019). Evaluation of the power system reliability if a nuclear power plant is replaced with wind power plants. Reliability Engineering & System Safety, 185: 455–464

Chakri A, Yang X S, Khelif R, Benouaret M (2018). Reliability-based design optimization using the directional bat algorithm. Neural Computing & Applications, 30(8): 2381–2402

Chern M S (1992). On the computational complexity of reliability redundancy allocation in a series system. Operations Research Letters, 11(5): 309–315

Coit D W, Smith A E (1996). Reliability optimization of series-parallel systems using a genetic algorithm. IEEE Transactions on Reliability, 45(2): 254–260

Coit D W, Zio E (2019). The evolution of system reliability optimization. Reliability Engineering & System Safety, 192: 106259

Compare M, Bellani L, Zio E (2019). Optimal allocation of prognostics and health management capabilities to improve the reliability of a power transmission network. Reliability Engineering & System Safety, 184: 164–180

Compare M, Bellora M, Zio E (2017). Aggregation of importance measures for decision making in reliability engineering. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 231(3): 242–254

da Costa Bueno V (2005). Minimal standby redundancy allocation in a *k*-out-of-*n*: F system of dependent components. European Journal of Operational Research, 165(3): 786–793

Derman C, Lieberman G J, Ross S M (1974). Optimal allocations in the construction of *k*-out-of-*n* reliability systems. Management Science, 21(3): 241–250

Du Y J, Si S B, Jin T D (2019). Reliability importance measures for network based on failure counting process. IEEE Transactions on Reliability, 68(1): 267–279

Dui H Y, Chen L W, Wu S M (2017a). Generalized integrated importance measure for system performance evaluation: Application to a propeller plane system. Maintenance and Reliability, 19(2): 279–286

Dui H Y, Li S M, Xing L D, Liu H L (2019). System performance-based joint importance analysis guided maintenance for repairable systems. Reliability Engineering & System Safety, 186: 162–175

Dui H Y, Si S B, Sun S D, Cai Z Q (2013). Gradient computations and geometrical meaning of importance measures. Quality Technology & Quantitative Management, 10(3): 305–318

Dui H Y, Si S B, Wu S M, Yam R C M (2017b). An importance measure for multistate systems with external factors. Reliability Engineering & System Safety, 167: 49–57

Dui H Y, Si S B, Yam R C M (2017c). A cost-based integrated importance measure of system components for preventive maintenance. Reliability Engineering & System Safety, 168: 98–104

Dui H Y, Si S B, Yam R C M (2018). Importance measures for optimal structure in linear consecutive-*k*-out-of-*n* systems. Reliability Engineering & System Safety, 169: 339–350

Espiritu J F, Coit D W, Prakash U (2007). Component criticality importance measures for the power industry. Electric Power Systems Research, 77(5–6): 407–420

Fang C, Marle F, Xie M (2017). Applying importance measures to risk analysis in engineering project using a risk network model. IEEE Systems Journal, 11(3): 1548–1556

Fang Y, Pedroni N, Zio E (2016). Resilience-based component importance measures for critical infrastructure network systems. IEEE Transactions on Reliability, 65(2): 502–512

Fu Y, Yuan T, Zhu X (2019a). Importance-measure based methods for component reassignment problem of degrading components. Reliability Engineering & System Safety, 190: 106501

Fu Y, Yuan T, Zhu X (2019b). Optimum periodic component reallocation and system replacement maintenance. IEEE Transactions on Reliability, 68(2): 753–763

Garg H, Sharma S P (2013). Multi-objective reliability-redundancy allocation problem using particle swarm optimization. Computers & Industrial Engineering, 64(1): 247–255

Ghambari S, Rahati A (2018). An improved artificial bee colony algorithm and its application to reliability optimization problems. Applied Soft Computing, 62: 736–767

Gopal K, Aggarwal K K, Gupta J S (1980). A new method for solving reliability optimization problem. IEEE Transactions on Reliability, R-29(1): 36–37

Griffith W S (1980). Multistate reliability models. Journal of Applied Probability, 17(3): 735–744

Gupta S, Bhattacharya J, Barabady J, Kumar U (2013). Cost-effective importance measure: A new approach for resource prioritization in a production plant. International Journal of Quality & Reliability Management, 30(4): 379–386

He X, Yuan Y (2019). A framework of identifying critical water distribution pipelines from recovery resilience. Water Resources Management, 33(11): 3691–3706

Hilber P, Bertling L (2004). Monetary importance of component reliability in electrical networks for maintenance optimization. In: Proceedings of International Conference on Probabilistic Methods Applied to Power Systems. Ames, IA: IEEE, 150–155

Jiang T, Liu Y, Zheng Y X (2019). Optimal loading strategy for multi-state systems: Cumulative performance perspective. Applied Mathematical Modelling, 74: 199–216

Kim C, Baxter L A (1987). Reliability importance for continuum structure functions. Journal of Applied Probability, 24(3): 779–785

Knight C R (1991). Four decades of reliability progress. In: Proceedings of Annual Reliability and Maintainability Symposium. Orlando, FL: IEEE, 156–160

Kulturel-Konak S, Smith A E, Coit D W (2003). Efficiently solving the redundancy allocation problem using tabu search. IIE Transactions, 35(6): 515–526

Kuo W, Lin H H, Xu Z K, Zhang W X (1987). Reliability optimization with the Lagrange-multiplier and branch-and-bound technique. IEEE Transactions on Reliability, R-36(5): 624–630

Kuo W, Prasad V R (2000). An annotated overview of system-reliability optimization. IEEE Transactions on Reliability, 49(2): 176–187

Kuo W, Zhu X (2012). Importance Measures in Reliability, Risk and Optimization: Principles and Applications. Chichester: John Wiley & Sons

Levitin G, Finkelstein M, Dai Y (2017a). Redundancy optimization for series-parallel phased-mission systems exposed to random shocks. Reliability Engineering & System Safety, 167: 554–560

Levitin G, Xing L, Amari S V (2012). Recursive algorithm for reliability evaluation of non-repairable phased-mission systems with binary elements. IEEE Transactions on Reliability, 61(2): 533–542

Levitin G, Xing L, Dai Y (2017b). Optimization of component allocation/distribution and sequencing in warm standby series-parallel systems. IEEE Transactions on Reliability, 66(4): 980–988

Levitin G, Xing L, Dai Y (2017c). Reliability versus expected mission cost and uncompleted work in heterogeneous warm standby multi-phase systems. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 47(3): 462–473

Li R Y, Wang J F, Liao H T, Huang N (2015). A new method for reliability allocation of avionics connected via an airborne network. Journal of Network and Computer Applications, 48: 14–21

Li X Y, Huang H Z, Li Y F (2018). Reliability analysis of phased-mission system with non-exponential and partially repairable components. Reliability Engineering & System Safety, 175: 119–127

Liang Y C, Smith A E (2004). An ant colony optimization algorithm for the redundancy allocation problem (RAP). IEEE Transactions on Reliability, 53(3): 417–423

Lin F H, Kuo W (2002). Reliability importance and invariant optimal allocation. Journal of Heuristics, 8(2): 155–171

Lin M S, Chen D J (1997). The computational complexity of the reliability problem on distributed systems. Information Processing Letters, 64(3): 143–147

Lin S, Fang X, Lin F, Yang Z, Wang X (2018). Reliability of rail transit traction drive system: A review. Microelectronics and Reliability, 88–90: 1281–1285

Liu S Y, She R, Fan P Y, Letaief K B (2018). Non-parametric message importance measure: Storage code design and transmission planning for big data. IEEE Transactions on Communications, 66(11): 5181–5196

Marseguerra M, Zio E, Podofillini L, Coit D W (2005). Optimal design of reliable network systems in presence of uncertainty. IEEE Transactions on Reliability, 54(2): 243–253

Mettas A (2000). Reliability allocation and optimization for complex systems. In: Proceedings of Annual Reliability and Maintainability Symposium. International Symposium on Product Quality and Integrity. Los Angeles, CA: IEEE, 216–221

Mi J H, Li Y F, Peng W, Huang H Z (2018). Reliability analysis of complex multi-state system with common cause failure based on evidential networks. Reliability Engineering & System Safety, 174: 71–81

Mohamad F, Teh J (2018). Impacts of energy storage system on power system reliability: A systematic review. Energies, 11(7): 1749

Nguyen K A, Do P, Grall A (2017). Joint predictive maintenance and inventory strategy for multi-component systems using Birnbaum's structural importance. Reliability Engineering & System Safety, 168: 249–261

Onishi J, Kimura S, James R J W, Nakagawa Y (2007). Solving the redundancy allocation problem with a mix of components using the improved surrogate constraint method. IEEE Transactions on Reliability, 56(1): 94–101

Pant S, Anand D, Kishor A, Singh S B (2015). A particle swarm algorithm for optimization of complex system reliability. International Journal of Performability Engineering, 11(1): 33–42

Papastavridis S (1987). The most important component in a consecutive-*k*-out-of-*n*: F system. IEEE Transactions on Reliability, R-36(2): 266–268

Peng R, Zhai Q Q, Xing L D, Yang J (2016). Reliability analysis and optimal structure of series-parallel phased-mission systems subject to fault-level coverage. IIE Transactions, 48(8): 736–746

Prasad V R, Kuo W (2000). Reliability optimization of coherent systems. IEEE Transactions on Reliability, 49(3): 323–330

Qiu S Q, Sallak M, Schön W, Ming H X G (2018). Extended LK heuristics for the optimization of linear consecutive-*k*-out-of-*n*: F systems considering parametric uncertainty and model uncertainty. Reliability Engineering & System Safety, 175: 51–61

Ramirez-Marquez J E, Coit D W (2004). A heuristic for solving the redundancy allocation problem for multi-state series-parallel systems. Reliability Engineering & System Safety, 83(3): 341–349

Ramirez-Marquez J E, Coit D W (2007). Multi-state component criticality analysis for reliability improvement in multi-state systems. Reliability Engineering & System Safety, 92(12): 1608–1619

Ramirez-Marquez J E, Rocco C M, Gebre B A, Coit D W, Tortorella M (2006). New insights on multi-state component criticality and importance. Reliability Engineering & System Safety, 91(8): 894–904

Rausand M, Høyland A (2003). System Reliability Theory: Models, Statistical Methods and Applications, 2nd ed. Hoboken: John Wiley & Sons

Roychowdhury S, Bhattacharya D (2019). Performance improvement of a multi-state coherent system using component importance measure. American Journal of Mathematical and Management Sciences, 38(3): 312–324

Shen J, Cui L (2015). Reliability and Birnbaum importance for sparsely connected circular consecutive-*k* systems. IEEE Transactions on Reliability, 64(4): 1140–1157

Shen J, Cui L, Du S (2015). Birnbaum importance for linear consecutive-*k*-out-of-*n* systems with sparse *d*. IEEE Transactions on Reliability, 64(1): 359–375

Shen K, Xie M (1990). On the increase of system reliability by parallel redundancy. IEEE Transactions on Reliability, 39(5): 607–611

Shojaei M, Mahani A (2019). Efficient reliability-redundancy allocation with uniform importance measure in presence of correlated failure. International Journal of Computers and Applications, 41(5): 378–391

Si S B, Dui H Y, Cai Z Q, Sun S D (2012a). The integrated importance measure of multi-state coherent systems for maintenance processes. IEEE Transactions on Reliability, 61(2): 266–273

Si S B, Dui H Y, Cai Z Q, Sun S D, Zhang Y F (2012b). Joint integrated importance measure for multi-state transition systems. Communications in Statistics—Theory and Methods, 41(21): 3846–3862

Si S B, Dui H Y, Zhao X B, Zhang S G, Sun S D (2012c). Integrated importance measure of component states based on loss of system performance. IEEE Transactions on Reliability, 61(1): 192–202

Si S B, Levitin G, Dui H Y, Sun S D (2013). Component state-based integrated importance measure for multi-state systems. Reliability Engineering & System Safety, 116: 75–83

Si S B, Levitin G, Dui H Y, Sun S D (2014). Importance analysis for reconfigurable systems. Reliability Engineering & System Safety, 126: 72–80

Si S B, Liu M L, Jiang Z Y, Jin T D, Cai Z Q (2019). System reliability allocation and optimization based on generalized Birnbaum importance measure. IEEE Transactions on Reliability, 68(3): 831–843

Singpurwalla N D (2006). Reliability and Risk: A Bayesian Perspective. Hoboken: John Wiley & Sons

Su H, Zhang J J, Zio E, Yang N, Li X Y, Zhang Z J (2018). An integrated systemic method for supply reliability assessment of natural gas pipeline networks. Applied Energy, 209: 489–501

Tian Z G, Zuo M J, Huang H Z (2008). Reliability-redundancy allocation for multi-state series-parallel systems. IEEE Transactions on Reliability, 57(2): 303–310

Tillman F A, Hwang C L, Kuo W (1977). Optimization techniques for system reliability with redundancy: A review. IEEE Transactions on Reliability, R-26(3): 148–155

Vu H C, Do P, Barros A (2016). A stationary grouping maintenance strategy using mean residual life and the Birnbaum importance measure for complex structures. IEEE Transactions on Reliability, 65 (1): 217–234

Wang L H, Ma D H, Han X, Wang W (2019). Optimization for postearthquake resilient power system capacity restoration based on the degree of discreteness method. Mathematical Problems in Engineering, 5489067

Wang N, Zhao J B, Jiang Z Y, Zhang S (2018). Reliability optimization of systems with component improvement cost based on importance measure. Advances in Mechanical Engineering, 10(11): 1–15

Wu S M, Chan L Y (2003). Performance utility-analysis of multi-state systems. IEEE Transactions on Reliability, 52(1): 14–21

Wu S M, Chen Y, Wu Q T, Wang Z L (2016). Linking component importance to optimization of preventive maintenance policy. Reliability Engineering & System Safety, 146: 26–32

Wu S M, Coolen F P A (2013). A cost-based importance measure for system components: An extension of the Birnbaum importance. European Journal of Operational Research, 225(1): 189–195

Wu X Y, Wu X Y (2017). An importance based algorithm for reliability-redundancy allocation of phased-mission systems. In: Proceedings of International Conference on Software Quality, Reliability and Security Companion. Prague: IEEE, 152–159

Wu X Y, Wu X Y, Balakrishnan N (2018). Reliability allocation model and algorithm for phased-mission systems with uncertain component parameters based on importance measure. Reliability Engineering & System Safety, 180: 266–276

Xiahou T F, Liu Y, Jiang T (2018). Extended composite importance measures for multi-state systems with epistemic uncertainty of state assignment. Mechanical Systems and Signal Processing, 109: 305–329

Xiang S H, Yang J (2018). Performance reliability evaluation for mobile ad hoc networks. Reliability Engineering & System Safety, 169: 32–39

Xiao R S, Xiang Y M, Wang L F, Xie K G (2018). Power system reliability evaluation incorporating dynamic thermal rating and network topology optimization. IEEE Transactions on Power Systems, 33(6): 6000–6012

Xie M, Shen K (1989). On ranking of system components with respect to different improvement actions. Microelectronics and Reliability, 29 (2): 159–164

Xing L D, Dugan J B (2002). Analysis of generalized phased-mission system reliability, performance, and sensitivity. IEEE Transactions on Reliability, 51(2): 199–211

Xiong G Z, Zhang C, Zhou F (2017). A robust reliability redundancy allocation problem under abnormal external failures guided by a new importance measure. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 231(2): 180–199

Xu Z P, Ramirez-Marquez J E, Liu Y, Xiahou T F (2020). A new resilience-based component importance measure for multi-state networks. Reliability Engineering & System Safety, 193: 106591

Yao Q, Zhu X, Kuo W (2011). Heuristics for component assignment problems based on the Birnbaum importance. IIE Transactions, 43 (9): 633–646

Yao Q, Zhu X, Kuo W (2014). A Birnbaum-importance based genetic local search algorithm for component assignment problems. Annals of Operations Research, 212(1): 185–200

Yeh W C (2019). A novel boundary swarm optimization method for reliability redundancy allocation problems. Reliability Engineering & System Safety, 192: 106060

Yeh W C, Chu T C (2018). A novel multi-distribution multi-state flow network and its reliability optimization problem. Reliability Engineering & System Safety, 176: 209–217

Yeh W C, Hsieh T J (2011). Solving reliability redundancy allocation problems using an artificial bee colony algorithm. Computers & Operations Research, 38(11): 1465–1473

Yu H, Yang J, Lin J, Zhao Y (2017). Reliability evaluation of non-repairable phased-mission common bus systems with common cause failures. Computers & Industrial Engineering, 111: 445–457

Yu J W, Zheng S L, Pham H, Chen T (2018). Reliability modeling of multi-state degraded repairable systems and its applications to automotive systems. Quality and Reliability Engineering International, 34(3): 459–474

Zaretalab A, Hajipour V, Tavana M (2020). Redundancy allocation problem with multi-state component systems and reliable supplier selection. Reliability Engineering & System Safety, 193: 106629

Zhang S, Zhao J B, Li H G, Wang N (2017). Reliability optimization and importance analysis of circular-consecutive *k*-out-of-*n* system. Mathematical Problems in Engineering, 1831537

Zhang S, Zhao J B, Zhu W J, Du L (2019). Reliability optimization of linear consecutive *k*-out-of-*n*: F systems with Birnbaum importance-based quantum genetic algorithm. Advances in Mechanical Engineering, 11(4): 1–16

Zhao J B, Cai Z Q, Si W T, Zhang S (2019a). Mission success evaluation of repairable phased-mission systems with spare parts. Computers & Industrial Engineering, 132: 248–259

Zhao J B, Si S B, Cai Z Q (2019b). A multi-objective reliability optimization for reconfigurable systems considering components degradation. Reliability Engineering & System Safety, 183: 104–115

Zhao J B, Si S B, Cai Z Q, Su M, Wang W (2019c). Multiobjective optimization of reliability-redundancy allocation problems for serial parallel-series systems based on importance measure. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 233(5): 881–897

Zhao X B, Si S B, Dui H Y, Cai Z Q, Sun S D (2013). Integrated importance measure for multi-state coherent systems of *k* level. Journal of Systems Engineering and Electronics, 24(6): 1029–1037

Zhu X Y, Yao Q Z, Kuo W (2011). Birnbaum importance in solving component assignment problems. In: Proceedings of Annual Reliability and Maintainability Symposium. Lake Buena Vista, FL: IEEE, 1–6

Zhu X Y, Fu Y Q, Yuan T, Wu X Y (2017). Birnbaum importance based heuristics for multi-type component assignment problems. Reliability Engineering & System Safety, 165: 209–221

Zio E, Marella M, Podofillini L (2007). Importance measures-based prioritization for improving the performance of multi-state systems: Application to the railway industry. Reliability Engineering & System Safety, 92(10): 1303–1314

Zio E, Podofillini L (2003a). Importance measures of multi-state components in multi-state systems. International Journal of Reliability Quality and Safety Engineering, 10(3): 289–310

Zio E, Podofillini L (2003b). Monte Carlo simulation analysis of the effects of different system performance levels on the importance of multi-state components. Reliability Engineering & System Safety, 82 (1): 63–73

Zio E, Podofillini L (2007). Importance measures and genetic algorithms for designing a risk-informed optimally balanced system. Reliability Engineering & System Safety, 92(10): 1435–1447

Zuo M, Kuo W (1990). Design and performance analysis of consecutive-*k*-out-of-*n* structure. Naval Research Logistics, 37(2): 203–230