Frontiers of Engineering Management
DOI 10.15302/J-FEM-2015048

ENGINEERING MANAGEMENT THEORIES AND METHODOLOGIES

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# Trapezoidal Intuitionistic Fuzzy Aggregation Operator Based on Choquet Integral and Its Application to Multi-Criteria Decision-Making Problems

**Abstract** The Choquet integral can serve as a useful tool to aggregate interacting criteria in an uncertain environment. In this paper, a trapezoidal intuitionistic fuzzy aggregation operator based on the Choquet integral is proposed for multi-criteria decision-making problems. The decision information takes the form of trapezoidal intuitionistic fuzzy numbers and both the importance and the interaction information among decision-making criteria are considered. On the basis of the introduction of trapezoidal intuitionistic fuzzy numbers, its operational laws and expected value are defined. A trapezoidal intuitionistic fuzzy aggregation operator based on the Choquet integral is then defined and some of its properties are investigated. A new multi-criteria decision-making method based on a trapezoidal intuitionistic fuzzy Choquet integral operator is proposed. Finally, an illustrative example is used to show the feasibility and availability of the proposed method.

**Keywords:** multi-criteria decision making, trapezoidal intuitionistic fuzzy numbers, Choquet integral, fuzzy measure, aggregation operator

## 1 Introduction

In the real decision-making problems, information available to decision-makers is often vague and imprecise. It is, thus, necessary to study fuzzy multi-criteria decision-making problems with an intuitionistic fuzzy setting. The Intuitionistic Fuzzy Set (IFS) theory, which was first introduced by Atanassov (1986), provides us with a powerful tool to tackle uncertain and vague information in real applications by allocating each element in intuitionistic fuzzy sets a membership degree and a non-membership degree. Since its appearance, it has received more and more attention. Recently, IFS theory has widely been applied to multi-criteria decision-making problems. For example, Chen and Tan (1994), Hong and Choi (2000), Li (2005), Xu and Yager (2006, 2008), Liu and Wang (2007), Xu (2007), Li, Wang, Liu, and Shan (2008), Wei (2009), Ye (2009, 2010), Wu and Zhang (2011) presented some new methods for handling multi-criteria decision-making problems based on intuitionistic fuzzy sets. However, the intuitionistic fuzzy set can only express the extent to which a criterion does or does not belong to a fuzzy concept "Excellence" or "Good" and IFS only use discrete domains. Fuzzy numbers are a special case of fuzzy sets and are of importance for fuzzy multi-attribute decision-making problems (Abbasbandy & Hajjari, 2009; Asady & Zendehnam, 2007; Dubois & Prade, 1980; Vencheh & Allame, 2010). Nehi and Maleki (2005) introduced the intuitionistic trapezoidal fuzzy numbers as the extension of intuitionistic triangular fuzzy numbers and proved some operation for them. The triangular intuitionistic fuzzy numbers and trapezoidal intuitionistic fuzzy numbers are the extension of intuitionistic fuzzy sets in another way, which extends discrete set to continuous set, and they are the extending of fuzzy numbers (Wang & Zhang, 2009; Ye, 2011).

Furthermore, all above mentioned studies only consider the situations that all of the elements in intuitionistic fuzzy sets are independent. However, for real decision-making problems, there always exist interactive characteristics among the elements in an uncertain situation. The Choquet integral is recognized as a useful tool to aggregate interacting criteria under uncertainty (Choquet, 1954). The aggregation based on the Choquet integral can deal with the decision information that may be correlative with each other. So, it is necessary to use the Choquet integral to overcome the limitation of independence assumption among criteria in multi-criteria decision making. Xu (2010) and Tan and Chen (2010) have paid attention to

Manuscript received March 25, 2015; accepted September 11, 2015

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the advantage of using the Choquet integral in real applications, and proposed some intuitionistic fuzzy Choquet integral operators using intuitionistic fuzzy sets.

Motivated by the advantages of intuitionistic trapezoidal fuzzy numbers and the Choquet integral in multi-criteria decision making, in this paper we propose a trapezoidal intuitionistic fuzzy aggregation operator based on the Choquet integral for multi-criteria decision-making problems. The decision information takes the form of trapezoidal intuitionistic fuzzy numbers and both the importance and interaction information among decision making criteria are considered.

This paper is organized as follows. In section 2, we define the trapezoidal intuitionistic fuzzy numbers and introduce some operational laws on trapezoidal intuitionistic fuzzy numbers. In section 3, based on a review of the Choquet integral, a trapezoidal intuitionistic fuzzy aggregation operator based on the Choquet integral is defined and some of its properties are investigated. In section 4, a kind of multi-criteria decision making method based on trapezoidal intuitionistic fuzzy Choquet integral operator is proposed. In section 5, an illustrative example shows the feasibility and availability of the proposed method. Finally, some concluding remarks are drawn in Section 6.

## 2 Trapezoidal intuitionistic fuzzy numbers

In this section, we briefly review some definitions related to intuitionistic fuzzy sets, trapezoidal intuitionistic fuzzy numbers, then the expected values and some operational laws of trapezoidal intuitionistic fuzzy numbers are defined.

Definition 1. Let $X$ be a universe of discourse. Then an intuitionistic fuzzy set $A$ in $X$ is given by [1]

$$A = \{\langle X, \mu_A(x), v_A(x) | x \in X \rangle\} \tag{1}$$

where $\mu_A(x)$: $X \to [0,1]$, $v_A(x)$: $X \to [0,1]$ with the condition

$$0 \leqslant \mu_A(x) + v_A(x) \leqslant 1, \text{ for all } x \in X. \tag{2}$$

The numbers $\mu_A(x)$ and $v_A(x)$ denote the degree of membership and non-membership of the element $x$ to the set $A$, respectively. In addition, the degree of hesitancy of $x$ can be computed as follows:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \text{ for all } x \in X \tag{3}$$

Obviously, if $\pi_A(x)=0$, then intuitionistic fuzzy set reduces to a fuzzy set.

A trapezoidal intuitionistic fuzzy number is a special intuitionistic fuzzy set. According to the studies of Nehi and Maleki (2005) and Wang and Zhang (2009), we introduce the definition of a trapezoidal intuitionistic fuzzy number as follows.

Definition 2. A trapezoidal intuitionistic fuzzy number $\tilde{a}$ with parameters $b_1 \leqslant a_1 \leqslant b_2 \leqslant a_2 \leqslant a_3 \leqslant b_3 \leqslant a_4 \leqslant b_4$ is denoted as $\tilde{a} = \langle (a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4); \mu_{\tilde{a}}, v_{\tilde{a}} \rangle$ on a real number set $R$, and its membership is defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} f_{\tilde{a}}^L(x), & a_1 \leqslant x < a_2 \\ \mu_{\tilde{a}}, & a_2 \leqslant x \leqslant a_3 \\ f_{\tilde{a}}^R(x), & a_3 < x \leqslant a_4 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

whereas its non-membership function is defined as follows:

$$v_{\tilde{a}}(x) = \begin{cases} h_{\tilde{a}}^L(x), & b_1 \leqslant x < b_2 \\ v_{\tilde{a}}, & b_2 \leqslant x \leqslant b_3 \\ h_{\tilde{a}}^R(x), & b_3 < x \leqslant b_4 \\ 1, & \text{otherwise} \end{cases} \tag{5}$$

where $f_{\tilde{a}}^L(x)$, $f_{\tilde{a}}^R(x)$, $h_{\tilde{a}}^L(x)$, $h_{\tilde{a}}^R(x)$ are the left and the right basis functions of the membership function and non-membership function, respectively, and

$$f_{\tilde{a}}^L(x) = \frac{x - a_1}{a_2 - a_1}\mu_{\tilde{a}}, \; f_{\tilde{a}}^R(x) = \frac{a_4 - x}{a_4 - a_3}\mu_{\tilde{a}}$$

$$h_{\tilde{a}}^L(x) = \frac{b_2 - x + v_{\tilde{a}}(x - b_1)}{b_2 - b_1}$$

$$h_{\tilde{a}}^R(x) = \frac{x - b_3 + v_{\tilde{a}}(b_4 - x)}{b_4 - b_3}$$

$0 \leqslant \mu_{\tilde{a}} \leqslant 1$, $0 \leqslant v_{\tilde{a}} \leqslant 1$, $\mu_{\tilde{a}} + v_{\tilde{a}} \leqslant 1$.

We call $\pi_{\tilde{a}} = 1 - \mu_{\tilde{a}} - v_{\tilde{a}}$ the degree of hesitancy of a trapezoidal intuitionistic fuzzy number $\tilde{a}$. Obviously, if $b_2 = b_3$ (hence $a_2 = a_3$) in a trapezoidal intuitionistic fuzzy number $\tilde{a}$, then $\tilde{a}$ reduces to a triangular intuitionistic fuzzy number.

A useful tool to deal with fuzzy numbers is the $\alpha$-cuts. Similarly, Nehi and Maleki (2005) and Ye (2011) proposed $\alpha$-cuts in the case of a intuitionistic fuzzy number. Motivated by these studies, we propose $\alpha$-cuts in the case of trapezoidal intuitionistic fuzzy numbers.

Definition 3. Let $\tilde{a}$ be a trapezoidal intuitionistic fuzzy number. The corresponding $\alpha$-cuts, i.e., $(\tilde{a}^+)_\alpha$ and $(\tilde{a}^-)_\alpha$, are, respectively, defined as

$$(\tilde{a}^+)_\alpha = \{x \in R | \mu_{\tilde{a}}(x) \geqslant \alpha\} \tag{6}$$

$$(\tilde{a}^-)_\alpha = \{x \in R | 1 - v_{\tilde{a}}(x) \geqslant \alpha\} \tag{7}$$

In view of the definition, each $\alpha$-cut is a closed interval. Hence, we have $(\tilde{a}^+)_\alpha = [\tilde{a}_L^+(\alpha), \tilde{a}_U^+(\alpha)]$ and $(\tilde{a}^-)_\alpha = [\tilde{a}_L^-(\alpha), \tilde{a}_U^-(\alpha)]$, where

$$\tilde{a}_L^+(\alpha) = \inf(x \in R | \mu_{\tilde{a}}(x) \geqslant \alpha) \tag{8}$$

$$\tilde{a}_U^+(\alpha) = \sup(x \in R | \mu_{\tilde{a}}(x) \geqslant \alpha) \quad (9)$$

$$\tilde{a}_L^-(\alpha) = \inf(x \in R | 1 - v_{\tilde{a}}(x) \geqslant \alpha) \quad (10)$$

$$\tilde{a}_U^-(\alpha) = \sup(x \in R | 1 - v_{\tilde{a}}(x) \geqslant \alpha) \quad (11)$$

Another important concept is the expected value of a trapezoidal intuitionistic fuzzy number, which is defined as follows:

Definition 4. Let $\tilde{a}$ be a trapezoidal intuitionistic fuzzy number in the set of real numbers $R$. Then, the expected value $EV(\tilde{a})$ of $\tilde{a}$ is the center of the expected interval of $\tilde{a}$, which is defined by

$$EV(\tilde{a}) = [EV_*(\tilde{a}) + EV^*(\tilde{a})]/2 \quad (12)$$

where $[EV_*(\tilde{a}),\ EV^*(\tilde{a})]$ is the expected interval, $EV_*(\tilde{a}) = \dfrac{\left(\int_0^{\mu_{\tilde{a}}} \tilde{a}_L^+(\alpha)\mathrm{d}\alpha + \int_0^{1-v_{\tilde{a}}} \tilde{a}_L^-(\alpha)\mathrm{d}\alpha\right)}{2}$, and $EV_*(\tilde{a}) = \dfrac{\left(\int_0^{\mu_{\tilde{a}}} \tilde{a}_U^+(\alpha)\mathrm{d}\alpha + \int_0^{1-v_{\tilde{a}}} \tilde{a}_U^-(\alpha)\mathrm{d}\alpha\right)}{2}$.

Theorem 1. Let $\tilde{a} = \langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4); \mu_{\tilde{a}}, v_{\tilde{a}}\rangle$ be a trapezoidal intuitionistic fuzzy number in the set of real numbers $R$. Thus, its expected value is obtained by

$$EV(\tilde{a}) = [\mu_{\tilde{a}}(a_1 + a_2 + a_3 + a_4) + (1 - v_{\tilde{a}})(b_1 + b_2 + b_3 + b_4)]/8 \quad (13)$$

Especially, if $\mu_{\tilde{a}} = 1$ (hence $v_{\tilde{a}} = 0$ and $(a_1,a_2,a_3,a_4) = (b_1, b_2,b_3,b_4)$, then

$$EV(\tilde{a}) = (a_1 + a_2 + a_3 + a_4)/4 \quad (14)$$

Thus, the above equation reduces to the expected value of a trapezoidal fuzzy number.

To compare two trapezoidal intuitionistic fuzzy numbers, Wang and Zhang (2009) proposed score function and accuracy function of two trapezoidal intuitionistic fuzzy numbers.

Definition 5. Let $\tilde{a}_1$ and $\tilde{a}_2$ be two trapezoidal intuitionistic fuzzy numbers, $S(\tilde{a}_1) = EV(\tilde{a}_1) \times (\mu_{\tilde{a}_1} - v_{\tilde{a}_1})$ and $S(\tilde{a}_2) = EV(\tilde{a}_2) \times (\mu_{\tilde{a}_2} - v_{\tilde{a}_2})$ be the score functions of $\tilde{a}_1$ and $\tilde{a}_2$, respectively. Let $H(\tilde{a}_1) = EV(\tilde{a}_1) \times (\mu_{\tilde{a}_1} + v_{\tilde{a}_1})$ and $H(\tilde{a}_2) = EV(\tilde{a}_2) \times (\mu_{\tilde{a}_2} + v_{\tilde{a}_2})$ be the accuracy functions of $\tilde{a}_1$ and $\tilde{a}_2$, respectively, then

If $S(\tilde{a}_1) > S(\tilde{a}_2)$, then $\tilde{a}_1 > \tilde{a}_2$;

If $S(\tilde{a}_1) = S(\tilde{a}_2)$, then

(1) If $H(\tilde{a}_1) > H(\tilde{a}_2)$, then $\tilde{a}_1 > \tilde{a}_2$;

(2) If $H(\tilde{a}_1) = H(\tilde{a}_2)$ then $\tilde{a}_1 = \tilde{a}_2$.

Definition 6. Let $\tilde{a}_1$ and $\tilde{a}_2$ be two trapezoidal intuitionistic fuzzy numbers

$$\tilde{a}_1 = \langle(a_{11},a_{12},a_{13},a_{14}),(b_{11},b_{12},b_{13},b_{14});\mu_{\tilde{a}_1},v_{\tilde{a}_1}\rangle$$

$$\tilde{a}_2 = \langle(a_{21},a_{22},a_{23},a_{24}),(b_{21},b_{22},b_{23},b_{24});\mu_{\tilde{a}_2},v_{\tilde{a}_2}\rangle$$

then we have

$$\tilde{a}_1 \oplus \tilde{a}_2 = \langle(a_{11} + a_{21},\ a_{12} + a_{22},\ a_{13} + a_{23},\ a_{14} + a_{24})\rangle,\ (b_{11} + b_{21},\ b_{12} + b_{22},\ b_{13} + b_{23},\ b_{14} + b_{24});\ \mu_{\tilde{a}_1} + \mu_{\tilde{a}_2} - \mu_{\tilde{a}_1}\mu_{\tilde{a}_2}, v_{\tilde{a}_1}v_{\tilde{a}_2}, \quad (15)$$

$$\lambda\tilde{a}_1 = \langle(\lambda a_{11},\lambda a_{12},\lambda a_{13},\lambda a_{14}),(\lambda b_{11},\lambda b_{12},\lambda b_{13},\lambda b_{14}); 1 - (1 - \mu_{\tilde{a}_1})^\lambda, v_{\tilde{a}_1}^\lambda\rangle, \lambda > 0. \quad (16)$$

Proposition 1. Let $\tilde{a}_1$ and $\tilde{a}_2$ be two trapezoidal intuitionistic fuzzy numbers, and $\tilde{a}_3 = \tilde{a}_1 \oplus \tilde{a}_2$, $\tilde{a}_4 = \lambda\tilde{a}_1$, where $\lambda$ is a positive real number; then both $\tilde{a}_3$ and $\tilde{a}_4$ are also trapezoidal intuitionistic fuzzy numbers.

Proposition 2. Let $\tilde{a}_1$ and $\tilde{a}_2$ be two trapezoidal intuitionistic fuzzy numbers and $\lambda_1$ and $\lambda_2$ are positive real numbers, one then obtains

$$\tilde{a}_1 \oplus \tilde{a}_2 = \tilde{a}_2 \oplus \tilde{a}_1$$

$$\lambda_1\tilde{a}_1 \oplus \lambda_2\tilde{a}_1 = (\lambda_1 + \lambda_2)\tilde{a}_1$$

$$\lambda_1\tilde{a}_1 \oplus \lambda_1\tilde{a}_2 = \lambda_1(\tilde{a}_1 + \tilde{a}_2)$$

According to the operational laws of Definition 3, Wang and Zhang (2009) extended the weighted averaging operator to trapezoidal intuitionistic fuzzy numbers, and proposed the trapezoidal intuitionistic fuzzy weighted averaging operator, which is defined as follows:

Definition 7. Let $\tilde{a}_i\,(i = 1,2,...,n)$ be a collection of trapezoidal intuitionistic fuzzy numbers, and $w = (w_1, w_2,...,w_n)^{\mathrm{T}}$ be the weight vector of $\tilde{a}_i$, we then define the trapezoidal intuitionistic fuzzy weighted averaging operator (TIFWA) as

$$\mathrm{TIFWA}(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n) = w_1\tilde{a}_1 \oplus w_2\tilde{a}_2 \oplus ... \oplus w_n\tilde{a}_n \quad (17)$$

Motivated by the order weighted averaging operator (OWA) proposed by Yager (1988), we extended the OWA operator to trapezoidal intuitionistic fuzzy numbers and propose a trapezoidal intuitionistic fuzzy order weighted averaging operator(TIFOWA), which is defined as follows:

Definition 8. Let $\tilde{a}_i\,(i = 1,2,...,n)$ be a collection of trapezoidal intuitionistic fuzzy numbers, and $w = (w_1, w_2,...,w_n)^{\mathrm{T}}$ be the weight vector associated with the TIFOWA, and then TIFOWA can be computed by the following expression:

$$\mathrm{TIFWA}(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n) = w_1\tilde{a}_{(1)} \oplus w_2\tilde{a}_{(2)} \oplus ... \oplus w_n\tilde{a}_{(n)} \quad (18)$$

where $(i)$ indicates a permutation on $X$ such that $\tilde{a}_{(1)} \leqslant \tilde{a}_{(2)} \leqslant ... \leqslant \tilde{a}_{(n)}$.

## 3 Trapezoidal intuitionistic fuzzy Choquet integral operator

### 3.1 Fuzzy measure and the Choquet integral

The philosophy of the Choquet integral was first introduced in capacity theory and used as a (fuzzy) integral with respect to a fuzzy measure proposed by Höhle (1982) and then rediscovered later by Murofushi and Sugeno (1989, 1991). The Choquet integral is defined to integrate functions with respect to the fuzzy measures (Murofushi & Sugeno, 1989).

As for the multi-criteria decision-making problems, in the Choquet integral model with criteria dependent, a fuzzy measure for criteria is used to define a weight on each combination of classifiers, thus making it possible to model the interaction among criteria. Fuzzy measure was first introduced by Sugeno (1974), which makes a monotonicity instead of additive property. The definitions of fuzzy measures and the Choquet integral are as follows (Murofushi & Sugeno, 1989).

Definition 9. Let $X$ be a finite set, and $P(X)$ be the power set of $X$. A fuzzy measure on $X$ is a set function $\psi$: $P(X) \to [0,1]$ if the following conditions hold:

(1) $\psi(\varphi)=0$, $\psi(X)=1$ (boundary conditions);

(2) If $A,B\in P(X)$ and $A \subseteq B$ then $\mu(A)\leqslant\mu(B)$ (monotonicity).

$\psi(C)$ in Definition 9 can be viewed as a measure on the grade of subjective importance of decision criteria set $C$. In addition to the usual weights on criteria taken separately, weights on any combination of criteria are also defined, as follows.

Definition 10. Let $\psi$ be a fuzzy measure on $X$. The Choquet integral of function $f$: $X\to R$ with respect to $\mu$ is defined as

$$C_\psi(f)=\sum_{i=1}^{n} f_{(i)}[\psi(A_{(i)})-\psi(A_{(i+1)})] \tag{19}$$

where $(i)$ indicates a permutation on $X$ such that $f_{(1)}\leqslant f_{(2)}\leqslant\ldots\leqslant f_{(n)}$ holds. In addition, $A_{(i)}=\{x_{(i)},\ldots,x_{(n)}\}$, $A_{(n+1)}=\phi$.

Since the Choquet integral model does not need to assume independency of one criterion from another, it can be used in nonlinear situations. The fuzzy integral of $f$ with respect to $\psi$ gives the overall evaluation of an alternative. When the criteria are independent, the fuzzy measure is additive, and the Choquet integral coincides with the weighted arithmetic average method. That is,

$$C_\psi(f)=\sum_{i=1}^{n} f_i\psi(\{x_i\}) \tag{20}$$

### 3.2 Trapezoidal intuitionistic fuzzy Choquet integral operator

Based on Definition 10, we use the Choquet integral to develop a novel operator on trapezoidal intuitionistic fuzzy numbers as follows:

Definition 11. Let $\psi$ be a fuzzy measure on $X$, and let $\tilde{a}_i\,(i=1,2,\ldots,n)$ be a collection of trapezoidal intuitionistic fuzzy numbers on $X$. The Choquet integral of $\tilde{a}_i$ with respect to $\psi$ is then defined as

$$\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)=\overset{n}{\underset{i=1}{\oplus}}\tilde{a}_{(i)}[\psi(A_{(i)})-\psi(A_{(i+1)})] \tag{21}$$

where $(i)$ indicates a permutation on $X$ such that $\tilde{a}_{(1)}\leqslant\tilde{a}_{(2)}\leqslant\ldots\leqslant\tilde{a}_{(n)}$. In addition, $A_{(i)}=\{x_{(i)},\ldots,x_{(n)}\}$, $A_{(n+1)}=\phi$ is satisfied.

Theorem 2. Let $\tilde{a}_i=\langle(a_{i1},a_{i2},a_{i3},a_{i4}),(b_{i1},b_{i2},b_{i3},b_{i4});\mu_{\tilde{a}_i},v_{\tilde{a}_i}\rangle$ $(i=1,2,\ldots,n)$ be a collection of trapezoidal intuitionistic fuzzy numbers on $X$ and $\psi$ be a fuzzy measure on $X$. Thus, the value by using TIFC operator in Eq. (21) is also a trapezoidal intuitionistic fuzzy numbers, and it can be transformed into the following form by induction on $n$:

$$\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)=\left\langle\begin{array}{l}(\sum_{i=1}^{n}\tau_i a_{(i)1},\sum_{i=1}^{n}\tau_i a_{(i)2},\sum_{i=1}^{n}\tau_i a_{(i)3},\sum_{i=1}^{n}\tau_i a_{(i)4}),(\sum_{i=1}^{n}\tau_i b_{(i)1},\sum_{i=1}^{n}\tau_i b_{(i)2},\\ \sum_{i=1}^{n}\tau_i b_{(i)3},\sum_{i=1}^{n}\tau_i b_{(i)4});1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i},\prod_{i=1}^{n}(v_{\tilde{a}_{(i)}})^{\tau_i}\end{array}\right\rangle \tag{22}$$

where $(i)$ indicates a permutation on $X$ such that $\tilde{a}_{(1)}\leqslant\tilde{a}_{(2)}\leqslant\ldots\leqslant\tilde{a}_{(n)}$ and $\tau_i=\psi(A_{(i)})-\psi(A_{(i+1)})$.

Proof. The first result can be easily proved by Definition 11 and Proposition 1. Now we prove Eq. (22) by using mathematical induction on $n$.

For $n=2$, according to the Definition 6, we have

$$\tau_1\tilde{a}_{(1)}=\langle(\tau_1 a_{(1)1},\tau_1 a_{(1)2},\tau_1 a_{(1)3},\tau_1 a_{(1)4}),(\tau_1 b_{(1)1},\tau_1 b_{(1)2},\tau_1 b_{(1)3},\tau_1 b_{(1)4});1-\left(1-\mu_{\tilde{a}_{(1)}}\right)^{\tau_1},v_{\tilde{a}_{(1)}}^{\tau_1}\rangle$$

$$\tau_2\tilde{a}_{(2)}=\langle(\tau_2 a_{(2)1},\tau_2 a_{(2)2},\tau_2 a_{(2)3},\tau_2 a_{(2)4}),(\tau_1 b_{(1)1},\tau_1 b_{(1)2},\tau_1 b_{(1)3},\tau_1 b_{(1)4});1-\left(1-\mu_{\tilde{a}_{(2)}}\right)^{\tau_2},v_{\tilde{a}_{(2)}}^{\tau_2}\rangle$$

Because the following relationship holds

$$\tilde{a}_1 \oplus \tilde{a}_2 = \langle (a_{11}+a_{21},a_{12}+a_{22},a_{13}+a_{23},a_{14}+a_{24}),(b_{11}+b_{21},b_{12}+b_{22},b_{13}+b_{23},b_{14}+b_{24});\mu_{\tilde{a}_1} + \mu_{\tilde{a}_2} - \mu_{\tilde{a}_1}\mu_{\tilde{a}_2},v_{\tilde{a}_1}v_{\tilde{a}_2}\rangle$$

we have

$$\mathrm{TIFC}(\tilde{a}_1,\tilde{a}_2) = \tilde{a}_1\tau_1 \oplus \tilde{a}_2\tau_2 = \left\langle \begin{array}{l} (\sum_{i=1}^{2}\tau_i a_{(i)1},\sum_{i=1}^{2}\tau_i a_{(i)2},\sum_{i=1}^{2}\tau_i a_{(i)3},\sum_{i=1}^{2}\tau_i a_{(i)4}),(\sum_{i=1}^{2}\tau_i b_{(i)1},\sum_{i=1}^{2}\tau_i b_{(i)2}, \\ \sum_{i=1}^{2}\tau_i b_{(i)3},\sum_{i=1}^{2}\tau_i b_{(i)4});1-\prod_{i=1}^{2}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i},\prod_{i=1}^{n}(v_{\tilde{a}_{(i)}})^{\tau_i} \end{array} \right\rangle$$

That is, for $n=2$, Eq. (22) holds. Assuming that for $n=k$, Eq. (20) holds, i.e.,

$$\mathrm{TIFC}\left(\tilde{a}_1,\ldots,\tilde{a}_k\right) = \left\langle \begin{array}{l} (\sum_{i=1}^{k}\tau_i a_{(i)1},\sum_{i=1}^{k}\tau_i a_{(i)2},\sum_{i=1}^{k}\tau_i a_{(i)3},\sum_{i=1}^{k}\tau_i a_{(i)4}),(\sum_{i=1}^{k}\tau_i b_{(i)1},\sum_{i=1}^{k}\tau_i b_{(i)2}, \\ \sum_{i=1}^{k}\tau_i b_{(i)3},\sum_{i=1}^{k}\tau_i b_{(i)4});1-\prod_{i=1}^{k}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i},\prod_{i=1}^{k}(v_{\tilde{a}_{(i)}})^{\tau_i} \end{array} \right\rangle$$

Then, for $n=k+1$, by Definition 6, we have

$$\tau_{k+1}\tilde{a}_{(k+1)} = \langle (\tau_{k+1}a_{(k+1)1},\tau_{k+1}a_{(k+1)2},\tau_{k+1}a_{(k+1)3},\tau_{k+1}a_{(k+1)4}),(\tau_{k+1}b_{(k+1)1},\tau_{k+1}b_{(k+1)2},\tau_{k+1}b_{(k+1)3},\tau_{k+1}b_{(k+1)4});$$

$$1-\left(1-\mu_{\tilde{a}_{(k+1)}}\right)^{\tau_{k+1}},v_{\tilde{a}_{(k+1)}}^{\tau_{k+1}}\rangle$$

According to Definition 11 and Definition 6, we have

$$\begin{aligned} \mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_{k+1}) &= \oplus_{i=1}^{k}\tilde{a}_i\tau_1 \oplus \tilde{a}_{k+1}\tau_{k+1} \\ &= \left\langle \begin{array}{l} (\sum_{i=1}^{k+1}\tau_i a_{(i)1},\sum_{i=1}^{k+1}\tau_i a_{(i)2},\sum_{i=1}^{k+1}\tau_i a_{(i)3},\sum_{i=1}^{k+1}\tau_i a_{(i)4}),(\sum_{i=1}^{k+1}\tau_i b_{(i)1},\sum_{i=1}^{k+1}\tau_i b_{(i)2}, \\ \sum_{i=1}^{k+1}\tau_i b_{(i)3},\sum_{i=1}^{k+1}\tau_i b_{(i)4});1-\prod_{i=1}^{k+1}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i},\prod_{i=1}^{k+1}(v_{\tilde{a}_{(i)}})^{\tau_i} \end{array} \right\rangle \end{aligned}$$

This means that for $n=k+1$, Eq. (22) holds. Therefore, for all $n$, Eq. (22) always holds.

In the following, we consider some special cases of TIFC operator.

Proposition 3. Let all elements $\tilde{a}_i$ $(i=1,2,\ldots,n)$ in the TIFC operator be independent, thus

$$\psi(a_{(i)}) = \psi(A_{(i)}) - \psi(A_{(i+1)}) \tag{23}$$

In this case, the TIFC operator reduces to the trapezoidal intuitionistic fuzzy weighted averaging operator (TIFWA), i.e.,

$$\mathrm{TIFWA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n) = \psi(\tilde{a}_1)\tilde{a}_1 \oplus \psi(\tilde{a}_2)\tilde{a}_2 \oplus \ldots \oplus \psi(\tilde{a}_n)\tilde{a}_n \tag{24}$$

Proposition 4. Let $\tilde{a}_i$ $(i=1,2,\ldots,n)$ be the element in the TIFC operator. If

$$\psi(B) = \sum_{i=1}^{|B|}\omega_i \text{ for all } B \subseteq \{\tilde{a}_1,\ldots,\tilde{a}_n\} \tag{25}$$

where $|B|$ is the cardinal number of $B$, then

$$\omega_i = (\psi(A_{(i)}) - \psi(A_{(i+1)})) \tag{26}$$

where $\omega_i \geqslant 0$, $i=1,\ldots,n$ and $\sum_{i=1}^{n}\omega_i = 1$. In this case, the TIFC operator reduces to the trapezoidal intuitionistic fuzzy ordered weighted averaging operator (TIFOWA), i.e.,

$$\mathrm{TIFOWA}(\tilde{a}_1,\ \tilde{a}_2,\ldots,\ \tilde{a}_n) = \omega_1\tilde{a}_{(1)} \oplus \tilde{a}_{(2)} \oplus \ldots \oplus \omega_n\tilde{a}_{(n)} \tag{27}$$

where $(i)$ indicates a permutation on $X$ such that $\tilde{a}_{(1)} \leqslant \tilde{a}_{(2)} \leqslant \ldots \leqslant \tilde{a}_{(n)}$.

Proposition 5. Let $\tilde{a}_i = \langle(a_{i1},a_{i2},a_{i3},a_{i4}),(b_{i1},b_{i2},b_{i3},b_{i4});\ \mu_{\tilde{a}_i},\ v_{\tilde{a}_i}\rangle$ $(i = 1,2,\ldots,n)$ be a collection of trapezoidal intuitionistic fuzzy numbers on $X$ and $\psi$ be a fuzzy measure on $X$. If the values of all $\tilde{a}_i$ are identical, i.e., $\tilde{a}_i = \tilde{a} = \langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4);\ \mu_{\tilde{a}},\ v_{\tilde{a}}\rangle$, then the value aggregated by using TIFC operator is as follows

$$\mathrm{TIFC}(\tilde{a}_1,\ldots,\ \tilde{a}_n) = \tilde{a} \tag{28}$$

Proof. According to Theorem 2, if $\tilde{a}_i = \tilde{a}$ $(i = 1,2,\ldots,n)$, we then have

$$\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n) = \left\langle \begin{array}{l} \left(\sum_{i=1}^{n}\tau_i a_1, \sum_{i=1}^{n}\tau_i a_2, \sum_{i=1}^{n}\tau_i a_3, \sum_{i=1}^{n}\tau_i a_4\right), \left(\sum_{i=1}^{n}\tau_i b_1, \sum_{i=1}^{n}\tau_i b_2,\right. \\ \left.\sum_{i=1}^{n}\tau_i b_3, \sum_{i=1}^{n}\tau_i b_4\right); 1-(1-\mu_{\tilde{a}})^{\sum_{i=1}^{n}\tau_i}, (v_{\tilde{a}})^{\sum_{i=1}^{n}\tau_i} \end{array} \right\rangle$$

Note that

$$\begin{aligned}\sum_{i=1}^{n}\tau_i &= [\psi(A_{(1)})-\psi(A_{(2)})] + [\psi(A_{(2)})-\psi(A_{(3)})] + \ldots + [\psi(A_{(n)})-\psi(A_{(n+1)})] \\ &= \psi(A_{(1)})-\psi(A_{(n+1)}) = 1\end{aligned}$$

Therefore, one obtains $\mathrm{TIFC}(\tilde{a}_1,\ldots,\ \tilde{a}_n) = \tilde{a}$.

Proposition 6. Let $\tilde{a}_i = \langle(a_{i1},a_{i2},a_{i3},a_{i4}),\ (b_{i1},b_{i2},b_{i3},b_{i4});\ \mu_{\tilde{a}_i},\ v_{\tilde{a}_i}\rangle$ $(i = 1,2,\ldots,n)$ be a collection of trapezoidal intuitionistic fuzzy numbers on $X$ and $\psi$ be a fuzzy measure on $X$. Let

$$\tilde{a}^- = \langle(\min_i(a_{i1}), \min_i(a_{i2}), \min_i(a_{i3}), \min_i(a_{i4})),(\min_i(b_{i1}), \min_i(b_{i2}), \min_i(b_{i3}), \min_i(b_{i4})); \min_i(\mu_{\tilde{a}_i}), \max_i(v_{\tilde{a}_i})\rangle$$

$$\tilde{a}^+ = \langle(\max_i(a_{i1}), \max_i(a_{i2}), \max_i(a_{i3}), \max_i(a_{i4})),(\max_i(b_{i1}), \max_i(b_{i2}), \max_i(b_{i3}), \max_i(b_{i4})); \max_i(\mu_{\tilde{a}_i}), \min_i(v_{\tilde{a}_i})\rangle$$

Then we have

$$\tilde{a}^- \leqslant \mathrm{TIFC}((\tilde{a}_1,\ldots,\tilde{a}_n) \leqslant \tilde{a}^+ \tag{29}$$

Proof. It is obvious that $\tilde{a}^-$ and $\tilde{a}^+$ are trapezoidal intuitionistic fuzzy numbers. By Theorem 1, we first computed the expected value and score function of $\tilde{a}^-$, $\tilde{a}^+$, and $\mathrm{TIFC}((\tilde{a}_1,\ldots,\ \tilde{a}_n)$ as follows:

$$\mathrm{EV}(\tilde{a}^-) = \frac{1}{8}\left[\min_i(\mu_{\tilde{a}_i})\sum_{j=1}^{4}\min_i(a_{ij}) + \left(1-\max_i(v_{\tilde{a}_i})\right)\sum_{j=1}^{4}\min_i(b_{ij})\right]$$

$$\mathrm{EV}(\tilde{a}^+) = \frac{1}{8}\left[\max_i(\mu_{\tilde{a}_i})\sum_{j=1}^{4}\max_i(a_{ij}) + \left(1-\min_i(v_{\tilde{a}_i})\right)\sum_{j=1}^{4}\max_i(b_{ij})\right]$$

$$\mathrm{EV}(\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)) = \frac{1}{8}\left[\left(1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i}\right)\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i a_{(i)j} + \left(1-\prod_{i=1}^{n}\left(v_{\tilde{a}_{(i)}}\right)^{\tau_i}\right)\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i b_{(i)j}\right]$$

$$S(\tilde{a}^-) = \mathrm{EV}(\tilde{a}^-) \times \left(\min_i(\mu_{\tilde{a}_i}) - \max_i(v_{\tilde{a}_i})\right) \qquad S(\tilde{a}^+) = \mathrm{EV}(\tilde{a}^+) \times \left(\max_i(\mu_{\tilde{a}_i}) - \min_i(v_{\tilde{a}_i})\right)$$

$$S(\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)) = \mathrm{EV}(\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)) \times \left(1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i}-\prod_{i=1}^{n}(v_{\tilde{a}_{(i)}})^{\tau_i}\right)$$

Since $A_{(i)} \supseteq A_{(i+1)}$, $\tau_i=\psi(A_{(i)})-\psi(A_{(i+1)})\geqslant 0$ holds. Thus, we have

$$1-\prod_{i=1}^{n}(1-\min_i(\mu_{\tilde{a}_i}))^{\tau_i}\leqslant 1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i} \leqslant 1-\prod_{i=1}^{n}(1-\max_i(\mu_{\tilde{a}_i}))^{\tau_i}$$

and

$$1-\prod_{i=1}^{n}(\max_i(v_{\tilde{a}_i}))^{\tau_i}\leqslant 1-\prod_{i=1}^{n}(v_{\tilde{a}_{(i)}})^{\tau_i}\leqslant 1-\prod_{i=1}^{n}(1-\min_i(v_{\tilde{a}_i}))^{\tau_i}$$

i.e.,

$$1-(1-\min_i(\mu_{\tilde{a}_i}))^{\sum_{i=1}^{n}\tau_i}\leqslant 1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i} \leqslant 1-\prod_{i=1}^{n}(1-\max_i(\mu_{\tilde{a}_i}))^{\sum_{i=1}^{n}\tau_i}$$

and

$$1-\max_i(v_{\tilde{a}_i}))^{\sum_{i=1}^{n}\tau_i}\leqslant 1-\prod_{i=1}^{n}(v_{\tilde{a}_{(i)}})^{\tau_i}\leqslant 1-\min(v_{\tilde{a}_i}))^{\sum_{i=1}^{n}\tau_i}$$

Note that

$$\sum_{i=1}^{n}\tau_i = (\psi(A_{(1)})-\psi(A_{(2)})) + (\psi(A_{(2)})-\psi(A_{(3)})) + \ldots +(\psi(A_{(n)})-\psi(A_{(n+1)})) = \psi(A_{(1)})-\psi(A_{(n+1)}) = 1$$

Thereby, we have

$$\min_i\left(\mu_{\tilde{a}_i}\right)\leqslant 1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i}\leqslant \max_i\left(\mu_{\tilde{a}_i}\right)$$

and

$$1-\max_i\left(v_{\tilde{a}_i}\right)\leqslant 1-\prod_{i=1}^{n}(1-v_{\tilde{a}_{(i)}})^{\tau_i}\leqslant 1-\min_i\left(v_{\tilde{a}_i}\right)$$

On the other hand, note that

$$\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i\min_i(a_{ij})\leqslant\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i a_{(i)j}\leqslant\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i\max_i(a_{ij})$$

and

$$\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i\min_i(b_{ij})\leqslant\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i b_{(i)j}\leqslant\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i\max_i(b_{ij})$$

i.e.,

$$\sum_{j=1}^{4}\min_i(a_{ij})\leqslant\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i a_{(i)j}\leqslant\sum_{j=1}^{4}\max_i(a_{ij})$$

and

$$\sum_{j=1}^{4}\min_i(b_{ij})\leqslant\sum_{j=1}^{4}\sum_{i=1}^{n}\tau_i b_{(i)j}\leqslant\sum_{j=1}^{4}\max_i(b_{ij})$$

Consequently, we have

$$\mathrm{EV}(\tilde{a}^-)\leqslant\mathrm{EV}(\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n))\leqslant\mathrm{EV}(\tilde{a}^+)$$

In view of the above, we have

$$\min_i\left(\mu_{\tilde{a}_i}\right)\leqslant 1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i}\leqslant \max_i\left(\mu_{\tilde{a}_i}\right)$$

and

$$1-\max_i\left(v_{\tilde{a}_i}\right)\leqslant 1-\prod_{i=1}^{n}(1-v_{\tilde{a}_{(i)}})^{\tau_i}\leqslant 1-\min_i\left(v_{\tilde{a}_i}\right)$$

Thus

$$\min_i\left(\mu_{\tilde{a}_i}\right)-\max_i\left(v_{\tilde{a}_i}\right)\leqslant 1-\prod_{i=1}^{n}(1-\mu_{\tilde{a}_{(i)}})^{\tau_i}-\prod_{i=1}^{n}(v_{\tilde{a}_{(i)}})^{\tau_i} \leqslant \max_i\left(\mu_{\tilde{a}_i}\right)-\min_i\left(v_{\tilde{a}_i}\right)$$

As a result, we have

$$\mathrm{EV}(\tilde{a}^-) \times\left(\min_i\left(\mu_{\tilde{a}_i}\right)-\max_i\left(v_{\tilde{a}_i}\right)\right)\leqslant\mathrm{EV}(\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)) \times\left(1-\prod_{i=1}^{n}\left(1-\mu_{\tilde{a}_{(i)}}\right)^{\tau_i}-\prod_{i=1}^{n}\left(v_{\tilde{a}_{(i)}}\right)^{\tau_i}\right)\leqslant\mathrm{EV}\left(\tilde{a}^+\right) \times\left(\max_i\left(\mu_{\tilde{a}_i}\right)-\min_i\left(v_{\tilde{a}_i}\right)\right)$$

this can be further written as

$$S(\tilde{a}^-)\leqslant S\left(\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)\leqslant S\left(\tilde{a}^+\right)\right.$$

According to Definition 5, we have

$$\tilde{a}^-\leqslant\mathrm{TIFC}(\tilde{a}_1,\ldots,\tilde{a}_n)\leqslant\tilde{a}^+$$

This implies that Eq. (29) holds. This completes the proof of the proposition.

## 4 Multi-criteria decision making method based on trapezoidal intuitionistic fuzzy Choquet integral operator

Multi-criteria decision making is the process of choosing the best alternative from some feasible alternatives based on multiple criteria. In each of these decisions, decision makers have several criteria to consider. Generally, in real decision-making problems, much information available to decision makers is vague and imprecise. Trapezoidal intuitionistic fuzzy numbers can effectively deal with uncertain and vague information in real applications. In the following, we fist describe the multi-criteria decision making problems with trapezoidal intuitionistic fuzzy setting.

For a multi-criteria decision making problem, let $A=\{a_1, a_2,...,a_m\}$ be a set of alternatives, $C=\{c_1,c_2,...,c_n\}$ are $n$ decision criteria. Assume the preference value of alternative $a_i$ on the criteria $c_j$ is a trapezoidal intuitionistic fuzzy number $t_{ij}=\langle(a_{ij1},a_{ij2},a_{ij3},a_{ij4}),(b_{ij1},b_{ij2},b_{ij3},b_{ij4});\mu_{t_{ij}},\nu_{t_{ij}}\rangle$ $(i=1,2,...,m$ and $j=1,2,...,n)$, thus the characteristic of alternatives can be expressed by the intuitionistic fuzzy numbers.

To handle this multi-criteria decision making problems with trapezoidal intuitionistic fuzzy setting, we propose the following procedure:

Step 1. With the preference value of alternative $a_i$ $(i=1,2,...,m)$ on the criteria $c_j$ $(j=1,2,...,n)$ which is described by a trapezoidal intuitionistic fuzzy number $\tilde{t}_{ij}=\langle(a_{ij1},a_{ij2},a_{ij3},a_{ij4}),(b_{ij1},b_{ij2},b_{ij3},b_{ij4});\mu_{t_{ij}},\nu_{t_{ij}}\rangle$ $(i=1,2,...,m$ and $j=1,2,...,n)$, the decision matrix $D=(\tilde{t}_{ij})_{m\times n}$ can be obtained.

Step 2. Standardize decision matrix $D$. One can obtain the normalized decision matrix $D'=(\tilde{r}_{ij})_{m\times n}$ using the following transformation.

$$\tilde{r}_{ij}=\left\langle\left(\frac{a_{ij1}}{\theta_j^+},\frac{a_{ij2}}{\theta_j^+},\frac{a_{ij3}}{\theta_j^+},\frac{a_{ij4}}{\theta_j^+}\right),\left(\frac{b_{ij1}}{\theta_j^+},\frac{b_{ij2}}{\theta_j^+},\frac{b_{ij3}}{\theta_j^+},\frac{b_{ij4}}{\theta_j^+}\right);\mu_{\tilde{t}_{ij}},\nu_{\tilde{t}_{ij}}\right\rangle \quad j\in\Omega_B \tag{30}$$

$$\tilde{r}_{ij}=\left\langle\left(\frac{\theta_j^-}{a_{ij4}},\frac{\theta_j^-}{a_{ij3}},\frac{\theta_j^-}{a_{ij2}},\frac{\theta_j^-}{a_{ij1}}\right),\left(\frac{\theta_j^-}{b_{ij4}},\frac{\theta_j^-}{b_{ij3}},\frac{\theta_j^-}{b_{ij2}},\frac{\theta_j^-}{b_{ij1}}\right);\mu_{\tilde{t}_{ij}},\nu_{\tilde{t}_{ij}}\right\rangle \quad j\in\Omega_C \tag{31}$$

where $\theta_j^+=\max_i(b_{ij4}),\theta_j^-=\min_i(b_{ij1})$, and $\Omega_B$ and $\Omega_C$ are the benefit and cost criteria, respectively.

Step 3. For each alternative $a_i$ $(i=1,...,m)$, according to Definition 5, by score functions $S$ and accuracy functions $H$, rank the normalized preference value $\tilde{r}_{ij}$ of alternative $a_i$ $(i=1,...,m)$ on the criteria $c_j$ $(j=1,2,...,n)$ such that $\tilde{r}_{i(j)}\leqslant\tilde{r}_{i(j+1)}$.

Step 4. Confirm the fuzzy measures on criteria of $C$. There are many methods that can be used to obtain fuzzy measures, such as linear methods (Marichal & Roubens, 1998), quadratic methods (Grabisch, 1996a, 1996b), statistics and neural networks methods (Wang, 1995; Wang, Klir, & Wang, 1998).

Step 5. Aggregate the normalized preference value $\tilde{r}_{ij}$ in the $i$th line of normalized decision matrix $D'=(\tilde{r}_{ij})_{m\times n}$ into an overall normalized preference value $\tilde{a}_i=\langle(a_{i1},a_{i2},a_{i3},a_{i4}),(b_{i1},b_{i2},b_{i3},b_{i4});\mu_{\tilde{a}_i},\nu_{\tilde{a}_i}\rangle$ $(i=1,2,...,m)$ by using trapezoidal intuitionistic fuzzy Choquet integral operator:

$$\mathrm{TIFC}(\tilde{r}_{i1},...,\tilde{r}_{in})=\mathop{\oplus}_{i=1}^{n}\tilde{r}_{i(j)}[\psi(A_{(j)})-\psi(A_{(j+1)})] \tag{32}$$

where $(j)$ indicates a permutation such that $\tilde{a}_{(1)}\leqslant\tilde{a}_{(2)}\leqslant...\leqslant\tilde{a}_{(n)}$. In addition, $A_{(j)}=\{c_{(j)},...,c_{(n)}\}$, $A_{(n+1)}=\phi$.

Step 6. According to Definition 5, compute the score function $S(\tilde{a}_i)$ and accuracy function $H(\tilde{a}_i)$, then compare the overall normalized preference value $\tilde{a}_i$ $(i=1,2,...,m)$ related to alternative $a_i$ $(i=1,2,...,m)$. The alternative with the biggest $\tilde{a}_i$ can be considered as the best alternative.

## 5 Illustrative examples

In this section, we apply the procedure that is proposed in the previous sections to the multi-criteria decision-making problem of computer software selection. It is assumed that a computer center wants to select a new information system to enhance work efficiency. Now suppose that after carefully consideration, there are three alternatives $\{a_1,a_2,a_3\}$ of which core competences can be evaluated by the following criteria:

(1) costs of software and hardware $(c_1)$;

(2) contribution to organizational performance improvement $(c_2)$; and

(3) outsourcing software developer's reliability $(c_3)$.

Step 1. A group of experts are invited to consider these alternatives $\{a_1,a_2,a_3\}$. After careful analysis of their characteristics, the experts give the following decision matrix $D=(\tilde{t}_{ij})_{3\times 3}$:

$$\tilde{t}_{11}=\langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.8);0.7,0.2\rangle$$

$$\tilde{t}_{21}=\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7);0.4,0.3\rangle$$

$$\tilde{t}_{31}=\langle(0.4,0.5,0.6,0.7),(0.3,0.4,0.6,0.8);0.5,0.3\rangle$$

$$\tilde{t}_{12}=\langle(0.6,0.7,0.8,0.9),(0.5,0.7,0.8,0.9);0.8,0.1\rangle$$

$$\tilde{t}_{22}=\langle(0.2,0.3,0.5,0.6),(0.2,0.3,0.5,0.6);0.7,0.1\rangle$$

$$\tilde{t}_{32}=\langle(0.2,0.4,0.5,0.6),(0.1,0.3,0.5,0.6);0.7,0.2\rangle$$

$$\tilde{t}_{13} = \langle(0.3,0.4,0.5,0.6),(0.3,0.4,0.5,0.7);0.4,0.5\rangle$$

$$\tilde{t}_{23} = \langle(0.5,0.6,0.7,0.8),(0.4,0.6,0.7,0.8);0.5,0.2\rangle$$

$$\tilde{t}_{33} = \langle(0.6,0.7,0.8,0.9),(0.6,0.7,0.8,0.9);0.7,0.2\rangle$$

Step 2. Among the criteria concerned, $c_1$ is of cost type, and $c_i$ ($i=2,3,4$) are the benefit criteria. According to Eqs. (30) and (31), $\theta_1^- = 0.2$, and $\theta_2^+ = \theta_3^+ = 0.9$, thus, the normalized decision matrix $D'=(\tilde{r}_{ij})_{3\times 3}$ can be computed as follows:

$$\tilde{r}_{11} = \langle(0.25,0.2857,0.3333,0.4),(0.25,0.2857,0.3333,0.5);0.7,0.2\rangle$$

$$\tilde{r}_{21} = \langle(0.3333,0.4,0.5,0.6667),(0.2857,0.4,0.5,0.1);0.4,0.3\rangle$$

$$\tilde{r}_{31} = \langle(0.2857,0.3333,0.4,0.5),(0.25,0.3333,0.5,0.6667);0.5,0.3\rangle$$

$$\tilde{r}_{12} = \langle(0.6667,0.7778,0.8889,1),(0.5556,0.7778,0.8889,1);0.8,0.1\rangle$$

$$\tilde{r}_{22} = \langle(0.2222,0.3333,0.5556,0.6667),(0.2222,0.3333,0.5556,0.6667);0.7,0.1\rangle$$

$$\tilde{r}_{32} = \langle(0.2222,0.4444,0.5556,0.6667),(0.1111,0.3333,0.5556,0.6667);0.7,0.2\rangle$$

$$\tilde{r}_{13} = \langle(0.3333,0.4444,0.5556,0.6667),(0.3333,0.4444,0.5556,0.7778);0.4,0.5\rangle$$

$$\tilde{r}_{23} = \langle(0.5556,0.6667,0.7778,0.8889),(0.4444,0.6667,0.7778,0.8889);0.5,0.2\rangle$$

$$\tilde{r}_{33} = \langle(0.6667,0.7778,0.8889,1),(0.6667,0.7778,0.88889,1);0.7,0.2\rangle$$

Step 3. For each alternative $a_i$ ($i=1,2,3$), according to Definition 5, the normalized preference value $\tilde{r}_{ij}$ is reordered such that $r_{ij} \leqslant r_{i(j+1)}$, as follows:

$$\tilde{r}_{1(1)} = \tilde{r}_{13},\ \tilde{r}_{1(2)} = \tilde{r}_{11},\ \tilde{r}_{1(3)} = \tilde{r}_{12},$$

$$\tilde{r}_{2(1)} = \tilde{r}_{21},\ \tilde{r}_{2(2)} = \tilde{r}_{23},\ \tilde{r}_{2(3)} = \tilde{r}_{22},$$

$$\tilde{r}_{3(1)} = \tilde{r}_{31},\ \tilde{r}_{3(2)} = \tilde{r}_{32},\ \tilde{r}_{3(3)} = \tilde{r}_{33}.$$

Step 4. Suppose the fuzzy measures on criteria of $C$ as follows:

$$\psi(c_1) = 0.4,\ \psi(c_2) = 0.3,\ \psi(c_3) = 0.3,\ \psi(c_1,c_2) = 0.8,\ \psi(c_1,c_3) = 0.6,\ \psi(c_2,c_3) = 0.7,\ \psi(c_1,c_2,c_3) = 1.$$

Step 5. According to Eqs. (21) and (22), aggregate the normalized preference value $\tilde{r}_{ij}$ in the $i$th line of normalized decision matrix $D'=(\tilde{r}_{ij})_{m\times n}$ into an overall normalized preference value $\tilde{a}_i = \langle(a_{i1},a_{i2},a_{i3},a_{i4}),(b_{i1},b_{i2},b_{i3},b_{i4});\ \mu_{\tilde{a}_i},\ v_{\tilde{a}_i}\rangle$ ($i=1,2,3$) by using trapezoidal intuitionistic fuzzy Choquet integral operator:

$$\tilde{a}_1 = \langle(0.7333,0.8847,1.0889,1.2867),(0.7,0.8847,1.0889,1.4778);0.8587,0.0692\rangle$$

$$\tilde{a}_2 = \langle(0.7889,0.9667,1.2111,1.4892),(0.6364,0.9667,1.2111,1.8222);0.7426,0.0487\rangle$$

$$\tilde{a}_3 = \langle(0.6143,0.8778,1.0556,1.2667),(0.5278,0.8,1.1556,1.4334);0.85,0.06\rangle$$

Step 6. According to Definition 5, compute the score function $S(\tilde{a}_i)$ ($i = 1,2,3$):

$$S(\tilde{a}_1) = 0.7198,\ S(\tilde{a}_2) = 0.6696,\ S(\tilde{a}_3) = 0.6837.$$

Then, we can obtain

$$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_2$$

Therefore, the order of the alternatives is $a_1$,$a_3$,$a_2$. Hence the best alternative is $a_1$.

## 6 Conclusions

As an extension of intuitionistic triangular fuzzy numbers and intuitionistic fuzzy sets, intuitionistic trapezoidal fuzzy numbers can express more adequate and flexible information and thus have a stronger expression ability to deal with the uncertain information. Most existing studies usually consider the situations with independent criteria. However, for real decision-making problems, there always exist interactive characteristics among the elements in an uncertain situation. In this paper, a trapezoidal intuitionistic fuzzy aggregation operator based on the Choquet integral is proposed for multi-criteria decision-making problems, in which the preference values for an alternative are expressed by trapezoidal intuitionistic fuzzy numbers. The proposed operator has the ability to deal with interaction information among decision making criteria. It is shown that the trapezoidal intuitionistic fuzzy weighted averaging operator (TIFWA) and the trapezoidal intuitionistic fuzzy ordered weighted averaging operator (TIFOWA) are special cases of the operator proposed in this paper. In addition, two operational laws and the expected value of the trapezoidal intuitionistic fuzzy number are introduced and some of properties of the proposed operator are investigated. An algorithm to tackle the multi-criteria decision making problems with trapezoidal intuitionistic fuzzy setting has been presented. For illustration purposes, the proposed procedure has been applied to a computer software selection problem.

Although the example has shown the stronger capability of the trapezoidal intuitionistic fuzzy numbers and some of their operators to deal with the uncertain information, they should be further illustrated in some actual fields, such as pattern recognition and medical diagnosis. We expect some new operators on trapezoidal intuitionistic fuzzy numbers to be investigated in future studies.

**Acknowledgements** This research was supported by the National Natural Science Foundation of China (Nos. 71431006, 71401184), Key Project of Philosophy and Social Sciences Research, Ministry of Education, PRC (No. 13JZD0016), China Postdoctoral Science Foundation (No. 2014M552169) and Central South University Business Management Postdoctoral Research Station.

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