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RESEARCH ARTICLE

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A delay-time model for inspector team assignment and non-periodic inspection intervals

Abstract Maintenance models based on delay-time have been extensively used in industry. However, some models still impose strong assumptions, e.g., most models do not pay attention on determining who is responsible for performing maintenance actions. Even when models do consider this, decisions regarding the moment for these actions and who is responsible for them are typically separately optimized. This paper sets out to tackle the problem of optimizing the moments for maintenance actions and the assignment of inspection teams responsible for each task, such that both decisions are jointly optimized. Thus, we propose a hybrid policy that combines inspections and age-based replacement. Due to the complexity of the problem, we propose an Adaptive Simulated Annealing algorithm, which presents percentages of optimization of up to 4.4% when compared to a general "black-box" algorithm. Our numerical results indicate that neglecting to whom the inspection would be assigned could generate worse solutions. Finally, we developed an user-friendly online app for assessing the cost-rate of the policy.

Keywords maintenance, inspector team assignment, aperiodic inspections, simulated annealing, imperfect inspections

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1 Introduction

In complex and large-scale industrial environments, such as nuclear power plants (Wu et al., 2020), wind turbines (Shafiee and Finkelstein, 2015), oil and gas industries (Cai et al., 2022), steel mills (Yang et al., 2019) and mining companies (Sánchez Herguedas et al., 2021), maintenance decisions are crucial for operational continuity, safety, and efficiency maximization. Traditionally, maintenance policies have predominantly focused on determining when to inspect, repair, or replace components, based on optimized or predefined rules (Pham, 2024). However, the reality of maintenance engineering and the complexity of daily operations reveal a critical dimension often neglected by existing models: the management and allocation of human resources and teams.

In practice, many organizations operate with multiple inspection and maintenance teams, each possessing distinct characteristics in terms of performance, experience, available tools, and associated costs. For example, water or energy distribution companies frequently concentrate their maintenance teams in various centers, and each team may have significant variations in their ability to detect defects (false-positive and false-negative probabilities) or in the costs of hiring and executing tasks. Furthermore, logistical considerations play a vital role, where choosing a cheaper, locally available team might seem advantageous in the short-term, but its long-term efficiency may be compromised by the quality of the actions performed (Scarf and Cavalcante, 2012).

Existing maintenance models, despite their widespread application, often impose restrictive assumptions. Most notably, they frequently fail to explicitly address who is responsible for undertaking maintenance actions. Even when resource allocation is considered, decisions regarding the timing of these actions and the assignment of responsibilities are typically optimized in a dissociated manner (Zhang et al., 2021a). This fragmented approach can inevitably lead to suboptimal solutions, as the quality of the executing team and the associated costs are intrinsically linked to the overall effectiveness and cost-benefit of a maintenance policy. Numerical results indeed demonstrate that neglecting the assignment of inspectors can generate suboptimal, or even worse, solutions. The integrated consideration of these decisions is therefore an undeniable necessity in contemporary engineering practice, enabling maintenance managers to more efficiently plan activities, optimize workforce utilization and ensure the proper and timely execution of each maintenance action, aligning with the technical skills of the personnel.

Therefore, the central motivation of this work stems from this critical gap between theoretical maintenance models and the multifaceted complexities of engineering practice. By introducing a novel delay-time model (DTM) (Christer, 1973; Wang, 2012) that facilitates the joint optimization of both the timing for maintenance actions and the assignment of specific inspection teams responsible for each task, while explicitly accounting for the heterogeneity and capabilities of these teams, this study aims to provide a more realistic, robust and effective decision-support tool for maintenance managers. Such an integrated approach not only substantially increases operational efficiency and effectiveness in preventing failures but also offers invaluable managerial insights for optimizing resource allocation, enhancing operational planning, informing strategic decision-making and streamlining service routes. This model represents a pivotal innovation in DTM by aligning human resource quality with optimal maintenance timings, redefining maintenance management to improve equipment reliability and streamline operations.

1.1 Literature review

According to de Almeida et al. (2015), maintenance planning plays a critical role in achieving goals by determining (i) the actions for each asset, (ii) their frequency and (iii) the resources to be used. However, many existing models predominantly focus on the first two aspects, neglecting the third one (Scarf et al., 2009; Sinisterra et al., 2023; Cao et al., 2025; Lv et al., 2024). Conversely, others concentrate solely on resource allocation by selecting the maintenance suppliers (Ren et al., 2020; Sheikhalishahi and Torabi, 2014; Wang, 2009; de Almeida, 2001).

By highlighting the importance of the resources to be used, the need to define maintenance contracts becomes apparent. Martin (1997) identified favorable conditions for outsourcing maintenance, such as instances when the scale of operations is relatively small or when technological advances necessitate extensive training for in-house maintenance teams. Common types of maintenance contract include work packages, performance and facilitator contracts, as outlined by Martin (1997). In his review of the literature, Wang (2012) emphasized the lack of studies on outsourcing and underscored the importance of strategically positioning inspections once the contract is determined. Contrary to this perspective, we argue for the joint consideration of both decisions on determining moments for maintenance actions together with whoever is responsible for conducing them.

Existing papers that address all three decision purposes together often tend to separate the decision-making on the responsibility for task execution from other aspects, thus leading to suboptimal solutions (Scarf and Cavalcante, 2012; Rodrigues et al., 2023a). For instance, Scarf and Cavalcante (2012) assess maintenance quality by first selecting the maintenance provider and then optimizing the moments of maintenance actions. Additionally, Wang (2010) explores this issue from both the supplier's and the customer's perspectives, considering contracts that vary in terms of responsibility for inspections, repairs at inspections and corrective repairs. In the context of Wind Turbine Systems (WTS), Qiu et al. (2017) optimize a periodic inspection policy to maximize the net revenue from equipment, considering maintenance errors, soft and hard failures, imperfect maintenance and performance-based contracts. In summary, all previous papers treat the problem of selecting inspection providers as a separate decision in relation to the maintenance policy. In our innovative approach, policy and team assignment decisions are done jointly, leading to even better solutions, whereas the dissociated decision achieves sub-optimal solutions. In other words, the main innovation lies in considering quality of human resources and maintenance actions aligned with the optimal timings.

A performance-based contract ensures a minimum level for the performance of a system, and in this study, it offers the added benefit of potential rewards aimed at maximizing the availability of the system. Wang et al. (2019) also proposed a model to maximize a supplier's profit under performance-based contracts which considers economic dependencies between components for WTS. As previously highlighted, studies have concentrated on the three decisions. Yet, they have typically approached the problem by separating these decisions. In this paper, we expand upon this perspective by considering decisions on what actions perform, their frequencies and the resources to be used simultaneously, where inspector team selection depends on the quality of actions executed by the team and their costs.

This article presents a pivotal innovation in the use of DTM, which enhances maintenance decision-making by aligning the quality of human resources and maintenance actions with the optimal timing for execution. By ensuring the proper and timely execution of each maintenance action, which aligns with the technical skills of the personnel, this model also significantly increases operational efficiency and effectiveness in addressing and preventing failures. The model redefines maintenance management by adding team quality and readiness considerations into the planning process, resulting in improved equipment reliability and streamlined operations. Differently from simulation procedures (Dui et al., 2025a), here we discuss all analytical expressions for computing expected costs and lifetime of the policy.

Since the first paper on DTM (Christer, 1973), many contributions that incorporated essential features in maintenance have been added. For example, Wang and Christer (1997) tackled the problem when there is a finite time horizon in the context of DTM. The consideration of imperfect inspections, particularly the “false-negative” probability of not detecting a defect during inspection, was examined by Baker and Wang (1991). Okumura et al. (1996) further explored imperfect inspections by introducing the probability of “false-positive” outcomes. Scarf et al. (2009) developed a hybrid policy to manage situations where the component population consists of two types: “strong” components that fail later and “weak” components that fail early. Building on this hybrid policy, Cavalcante et al. (2018) incorporated an opportunistic perspective, drawing inspiration from economic and structural interdependencies.

Other authors have also explored in depth opportunistic view of the theme (Berrade et al., 2017; Scarf et al., 2019; Liu et al., 2021; Zhang, 2019). Recent advances in DTM and maintenance policies have focused on mission abortion (Levitin et al., 2019b and 2019a), the consideration of shocks (Zhang et al., 2021b; Yang et al., 2017; Qiu and Cui, 2018), more than two stages of degradation (Kou et al., 2023) and multiple defect types (Mahmoudi et al., 2017). For a comprehensive overview of DTM, readers are recommended to refer to the review paper by Wang (2012).

The challenge of determining when to perform maintenance actions essentially can be formulated as a Mixed Integer Nonlinear Problem (MINLP). When we incorporate inspector team assignments, additional binary variables are introduced, in a similar way to other maintenance-related problems (Khatab et al., 2018; Chaabane et al., 2020; Lima et al., 2025). Given the complexity of the problem, metaheuristic approaches are well-suited to tackling it. Simulated Annealing (SA) (Kirkpatrick et al., 1983) algorithms, for instance, have been successfully used to generate high-quality solutions for analogous problems, such as chemical problems modeled as Nonlinear problems (Cardoso et al., 1996) and Mixed-Integer Nonlinear problems (Cardoso et al., 1997). The first SA for MINLP was discussed by Cardoso et al. (1997), using the Nelder-Mead algorithm (Nelder and Mead, 1965) for continuous variables and a stochastic procedure for integer variables. This algorithm, when compared with evolutionary algorithms (Costa and Oliveira, 2001), produced nearly equally good solutions. Furthermore, SAs have found application in production and storage planning contexts (Mohammadi et al., 2014) and reservoir systems (Teegavarapu and Simonovic, 2002), which also present as MINLPs.

Since the seminal work of Kirkpatrick et al. (1983), numerous variants (Shi et al., 2023; Shin et al., 2023; Lim et al., 2023) and hybridizations (Fontes et al., 2023) have been proposed. Cheng et al. (2023) introduced an Adaptive Simulated Annealing algorithm tailored for an MINLP. This algorithm includes mechanisms for adaptively controlling temperature, neighborhood operators, and search sizes to facilitate local escapes and rapid convergence. In this adaptive approach, the selection probabilities of operators are influenced by their historical performance, tracked by indicators and predefined bounds. Inspired by the good results for a problem that is similar to ours, we propose a similar algorithm but with the addition of a freezing mechanism for operators with recent poor performances.

1.2 Technical contributions and managerial implications

Concerning technical contributions, the model is particularly effective in supporting team composition and contract formulation for inspections, including those requiring technical certification. It identifies specific demands for each inspector profile, optimizing workforce utilization and enabling more efficient labor allocation. Additionally, it facilitates scenario analysis, where improvements in component reliability reduce inspection frequency, cutting costs and allowing for more strategic investment priorities.

The proposed model also contributes to inspection grouping by providing specific inspection schedules for each component. Once the inspection moments and the teams responsible for them are predefined, the maintenance manager can leverage this single-component information to identify relationships among different components, such as grouping inspections of multiple components that are scheduled closely and assigned to the same team. This approach helps reduce travel distances, thereby lowering logistical costs (by minimizing departures from the inspection center) and accelerating the inspection process (Colledani and Tolio, 2012). Furthermore, when integrated with other management tools, the proposed model indirectly supports the transformation of management practices into a more strategic and integrated framework. By offering a holistic perspective, it aids organizations to adapt swiftly to evolving market dynamics, transforming flexibility and agility into competitive advantages (Pinjala et al., 2006).

The managerial impacts of the proposed model extend well beyond addressing specific maintenance issues. By indicating the inspector assignments, the model provides valuable insights to optimize resource allocation, operational planning, strategic decision-making and even service routes (Si et al., 2023). The model enables the development of more efficient contracts by aligning resources with demands, whether through teams with varying levels of expertise or external suppliers (Chaabane et al., 2018).

By reducing scheduling conflicts, minimizing workload imbalances and improving team scalability, the model integrates maintenance decisions into broader strategies, such as cost reduction and competitiveness enhancement. Its wide applicability benefits sectors where inspection activities are critical, including manufacturing (Ferreira Neto et al., 2021), logistics, food industries (Jiménez et al., 2023) and energy generation (Yeter et al., 2020; Dui et al., 2025b), fostering efficiency, regulatory compliance, and organizational resilience.

To address all the expected impacts, we formulated the problem of inspection planning integrated with inspector assignment, providing the mathematical foundations and equations for calculating probabilities, expected costs and component lifetimes. We validated the proposed model through numerical experiments and sensitivity analyses, identifying relationships between the time intervals between inspections and the quality of inspections. Finally, due to the computational complexity of the problem, we developed a Simulated Annealing (SA) algorithm with problem-specific mechanisms to achieve high-quality solutions efficiently and an online application for assessing the cost-rate of the proposed policy, thereby enhancing the model's accessibility to non-expert managers. Although these last ones also represent contributions of the study, our primary focus lies in the joint optimization of the maintenance policy and the inspector team.

Therefore, the following sections will explore in depth the mathematical formulations used in our problem (Section 2). Section 3 will elaborate on the newly proposed algorithm, while numerical examples illustrating its application will be presented in Section 4. Finally, Section 5 will draw some conclusions and summarize the main findings and contributions.

2 Modeling

In this section, we outline the characteristics of the problem under study (Subsection 2.1) and present the computations of key variables based on the principles of renewal-reward theory, categorized by different types of renewals (Subsection 2.2). Finally, we give the Mixed-Integer Nonlinear Programming (MINLP) formulation for finding the optimal policy, assuming a fixed number of inspections (Subsection 2.3). The notations used throughout are given in Table 1.

Table 1 Notations

Notation	Description
H	Delay-time
X	Defect arrival time
f_X	P.d.f. of X (Weibull distribution)
f_H	P.d.f. of H (Exponential distribution)
F_H	C.d.f. of H
R_X	Survival function of X
R_H	Survival function of H
η_1	Scale parameter for the defect arrival distribution of "strong" components
β_1	Shape parameter for the defect arrival distribution of "strong" components
η_2	Scale parameter for the defect arrival distribution of "weak" components
β_2	Shape parameter for the defect arrival distribution of "weak" components
p	Mixing parameter
λ	Rate of the exponential distribution for delay-time
M	Number of different inspector teams
$\alpha_r, r = 1, \ldots, M$	False-positive probability of inspector team r
$\epsilon_r, r = 1, \ldots, M$	False-negative probability of inspector team r
$C_r^I, r = 1, \ldots, M$	Inspection cost for inspector team r
$C_r^H, r = 1, \ldots, M$	Hiring cost for inspector team r
C^R	Replacement cost
C^F	Cost of failure
K	Number of inspections to be carried out
$Y_{ir}, i = 1, \ldots, K, r = 1, \ldots, M$	Binary variable for indicating if inspector r is responsible for ith inspection
$\Delta_i, i = 1, \ldots, K$	Time for the ith inspection
T	Time for the age-based replacement

2.1 Problem description

First, we consider a single-component system under the DTM framework, i.e., the system can exist in three distinct states: “good,” “defective” and “failed” (the system is operational in the first two states). Components are categorized into two mutually exclusive groups: “weak” ones which tend to develop defects early and “strong” ones which typically exhibit the occurrences of defects later (both following Weibull distributions). The proportion of each sub-population is governed by the mixing parameter p. The transition from the defective to failed state follows an exponential distribution with rate λ. However, this transition can be halted at the moment of an inspection upon detection of the defect, leading to preventive replacements. There are M distinct inspector teams, each associated with probabilities of making misclassification errors: detecting a defect when it does not exist (α_r, $r=1,2,...,M$) and failing to detect a defect when it does exist ($\epsilon_r, r=1,\ 2,...,\ M$). These probabilities of indicating a defect when none exists, and of failing to detect an actual defect, are commonly referred in maintenance contexts as false-positive and false-negative probabilities, respectively. For example, during the visual inspection of valves in a water distribution system, an inspector may observe a humid area and incorrectly assume it is caused by a leak. However, the moisture may simply result from recent rainfall (false-positive). Conversely, a false-negative may occur when a small, real leak exists but goes unnoticed because it has not yet produced visible signs. Moreover, team r conducts an inspection incurring a cost of C_r^I. Additionally, there are fixed costs for hiring these teams denoted by C_r^H, which encompass expenses incurred when forming a new team, including costs for establishing new contracts, purchasing additional auxiliary tools, and other related expenses. Finally, replacements during inspection moments or at the age-based replacement incur costs of C^R, while corrective actions cost C^F.

Also, the model incorporated some specific features. Preventive and corrective maintenance are treated as perfect, restoring the system to an “as good as new” condition. Additionally, the model does not account for inventory or human resource constraints, implying that spare parts and maintenance teams are available as needed. All costs are considered constant and known in advance and the time required for maintenance actions is assumed to be negligible. We have not considered the possibility of the inspector inducing defects when inspecting the component. However, we have accounted for a heterogeneous component population and this models scenarios of different manufacturers or instances of poor installation, which could transform a strong component into a weaker one. While these considerations simplify the current analysis, they highlight areas that could be further developed and investigated in future research.

Therefore, the decision-making process for this problem revolves around three main issues: (i) determining the number of inspections to be conducted, (ii) scheduling the maintenance actions (inspections and the age-based replacement) and (iii) assigning inspection teams to inspections. Our objective is to find the optimal policy that minimizes the cost-rate. So, we enumerate all possible renewal scenarios, utilizing the principles of renewal-reward theory (Ross, 1996), which accounts for 10 distinct scenarios.

2.2 Cost-rate computation

To enhance clarity, we present the scenarios based on the type of renewal action, whether due to failure (Subsection 2.2.1), by a defect detection (Subsection 2.2.2) or by reaching the age-based threshold (Subsection 2.2.3), and present the respective equations for the probability (P_i), expected cost (EC_i) and expected lifetime (EL_i) associated with each scenario i.

2.2.1 Failure occurrence

For scenarios ending in failure (Fig. 1), we consider four distinct cases: failure occurring between inspections without false-negative errors (Fig. 1(a)), failure occurring after the Kth inspection but before the age-based preventive action (Fig. 1(b)), failure occurring between inspections with at least one false-negative error (Fig. 1(c)) and failure occurring between inspections and before the age-based action (Fig. 1(d)). The corresponding Eqs. (1)–(12), define these scenarios in detail. These metrics are essential to formulating the objective function, which is expressed as the total expected cost divided by the expected lifetime of the component. It is worth noting that in these equations and subsequent calculations, when $i=0$, the product returns 1 as it iterates from $k=1$ to 0.

$$P_1=\sum_{i=0}^{K-1}\left(\prod_{k=1}^{i}\sum_{r=1}^{M}(1-\alpha_r)Y_{kr}\right)\int_{\Delta_i}^{\Delta_{i+1}}f_X(x)\int_0^{\Delta_{i+1}-x}f_H(h)\,\mathrm{d}h\mathrm{d}x, \tag{1}$$

$$EC_1=\sum_{i=0}^{K-1}\left(C^F+\sum_{k=1}^{i}\sum_{r=1}^{M}C_r^IY_{kr}\right)\left(\prod_{k=1}^{i}\sum_{r=1}^{M}(1-\alpha_r)Y_{kr}\right)\int_{\Delta_i}^{\Delta_{i+1}}f_X(x)\int_0^{\Delta_{i+1}-x}f_H(h)\,\mathrm{d}h\mathrm{d}x, \tag{2}$$

$$EL_1=\sum_{i=0}^{K-1}\left(\prod_{k=1}^{i}\sum_{r=1}^{M}(1-\alpha_r)Y_{kr}\right)\int_{\Delta_i}^{\Delta_{i+1}}f_X(x)\int_0^{\Delta_{i+1}-x}(x+h)f_H(h)\mathrm{d}h\mathrm{d}x, \tag{3}$$

$$P_2=\left(\prod_{k=1}^{K}\sum_{r=1}^{M}(1-\alpha_r)Y_{kr}\right)\int_{\Delta_K}^{T}f_X(x)\int_0^{T-x}f_H(h)\,\mathrm{d}h\mathrm{d}x, \tag{4}$$

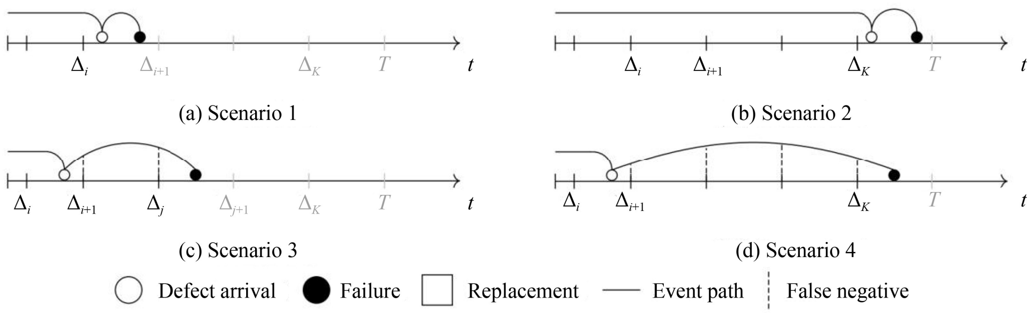

Fig. 1 Failure associated scenarios.

$$EC_2 = \left(C^F + \sum_{i=1}^{K} \sum_{r=1}^{M} C_r^I Y_{ir}\right)\left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \int_{\Delta_K}^{T} f_X(x) \int_0^{T-x} f_H(h)\,\mathrm{d}h\mathrm{d}x, \quad (5)$$

$$EL_2 = \left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \int_{\Delta_K}^{T} f_X(x) \int_0^{T-x} (x+h) f_H(h)\mathrm{d}h\mathrm{d}x, \quad (6)$$

$$P_3 = \sum_{i=0}^{K-2} \sum_{j=i+1}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\prod_{k=i+1}^{j} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) \int_{\Delta_j - x}^{\Delta_{j+1}-x} f_H(h)\,\mathrm{d}h\mathrm{d}x, \quad (7)$$

$$EC_3 = \sum_{i=0}^{K-2} \sum_{j=i+1}^{K-1} \left(C^F + \sum_{k=1}^{j} \sum_{r=1}^{M} C_r^I Y_{kr}\right) \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\prod_{k=i+1}^{j} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) \int_{\Delta_j - x}^{\Delta_{j+1}-x} f_H(h)\,\mathrm{d}h\mathrm{d}x, \quad (8)$$

$$EL_3 = \sum_{i=0}^{K-2} \sum_{j=i+1}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\prod_{k=i+1}^{j} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) \int_{\Delta_j - x}^{\Delta_{j+1}-x} (x+h) f_H(h)\mathrm{d}h\mathrm{d}x, \quad (9)$$

$$P_4 = \sum_{i=0}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\prod_{k=i+1}^{K} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) \int_{\Delta_K - x}^{T-x} f_H(h)\,\mathrm{d}h\mathrm{d}x, \quad (10)$$

$$EC_4 = \left(C^F + \sum_{k=1}^{K} \sum_{r=1}^{M} C_r^I Y_{kr}\right) \sum_{i=0}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\prod_{k=i+1}^{K} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) \int_{\Delta_K - x}^{T-x} f_H(h)\,\mathrm{d}h\mathrm{d}x, \quad (11)$$

$$EL_4 = \sum_{i=0}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\prod_{k=i+1}^{K} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) \int_{\Delta_K - x}^{T-x} (x+h) f_H(h)\mathrm{d}h\mathrm{d}x. \quad (12)$$

2.2.2 Defect detection

For renewals during inspections, the scenarios are illustrated in Fig. 2. Specifically: Scenario 1 represents the correct detection of a defect without any false-negative errors; Scenario 2 depicts the correct detection of a defect with at least one false-negative error occurring beforehand; and Scenario 3 captures the occurrence of a false-positive, which results in an immediate preventive replacement. The set of Eqs. relative to these scenarios are described in Eqs. (13)–(21).

$$P_5 = \sum_{i=0}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\sum_{r=1}^{M} (1-\epsilon_r) Y_{(i+1)r}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) (1 - F_H(\Delta_{i+1} - x))\,\mathrm{d}x, \quad (13)$$

$$EC_5 = \sum_{i=0}^{K-1} \left(C^R + \sum_{k=0}^{i} \sum_{r=1}^{M} C_r^I Y_{(k+1)r}\right) \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\sum_{r=1}^{M} (1-\epsilon_r) Y_{(i+1)r}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) (1 - F_H(\Delta_{i+1} - x))\,\mathrm{d}x, \quad (14)$$

$$EL_5 = \sum_{i=0}^{K-1} \Delta_{i+1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\sum_{r=1}^{M} (1-\epsilon_r) Y_{(i+1)r}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) (1 - F_H(\Delta_{i+1} - x))\,\mathrm{d}x, \quad (15)$$

$$P_6 = \sum_{i=0}^{K-2} \sum_{j=i+1}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\prod_{k=i+1}^{j-1} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right)\left(\sum_{r=1}^{M} (1-\epsilon_r) Y_{jr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) R_H(\Delta_j - x)\mathrm{d}x, \quad (16)$$

$$EC_6 = \sum_{i=0}^{K-2} \sum_{j=i+1}^{K-1} \left(C^R + \sum_{k=1}^{j} \sum_{r=1}^{M} C_r^I Y_{ir}\right) \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\prod_{k=i+1}^{j-1} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \left(\sum_{r=1}^{M} (1-\epsilon_r) Y_{jr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) R_H(\Delta_j - x)\mathrm{d}x, \quad (17)$$

$$EL_6 = \sum_{i=0}^{K-2} \sum_{j=i+1}^{K-1} \Delta_j \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\prod_{k=i+1}^{j-1} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right)\left(\sum_{r=1}^{M} (1-\epsilon_r) Y_{jr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) R_H(\Delta_j - x)\mathrm{d}x, \quad (18)$$

$$P_7 = \sum_{i=1}^{K} \left(\prod_{k=1}^{i-1} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\sum_{r=1}^{M} \alpha_r Y_{ir}\right) R_X(\Delta_i), \quad (19)$$

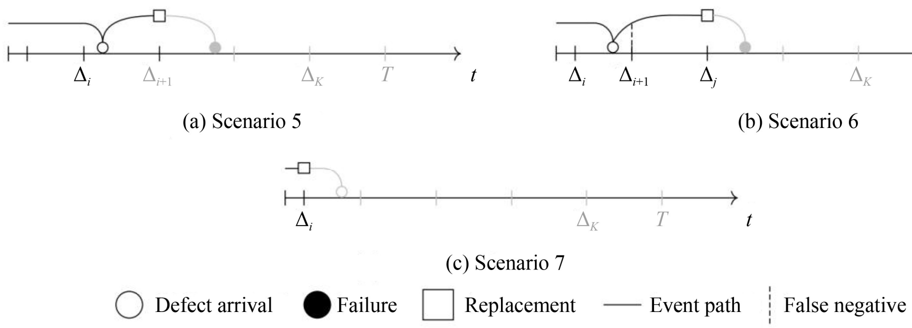

Fig. 2 Scenarios with inspection renewals.

$$EC_7 = \sum_{i=1}^{K} \left(C^R + \sum_{k=1}^{i} \sum_{r=1}^{M} C_r^I Y_{ir}\right)\left(\prod_{k=1}^{i-1} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\sum_{r=1}^{M} \alpha_r Y_{ir}\right) R_X(\Delta_i), \tag{20}$$

$$EL_7 = \sum_{i=1}^{K} \Delta_{i+1} \left(\prod_{k=1}^{i-1} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\sum_{r=1}^{M} \alpha_r Y_{ir}\right) R_X(\Delta_i). \tag{21}$$

2.2.3 Age-based replacement

Finally, Fig. 3 consolidates all scenarios involving preventive replacements at the age-based threshold. Scenario 3.(a) illustrates a defect occurrence after the Kth inspection but before the age-based threshold, whereas scenario 3.(b) depicts the situation where no defect occurs and the component is replaced preventively at the age-based threshold. Scenario 3.(c) represents the case of a defect appearing between inspections, accompanied by at least one false-negative error, with the component surviving until T. The corresponding equations are those from Eq. (22) to Eq. (30).

$$P_8 = \left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \int_{\Delta_K}^{T} f_X(x) (1 - F_H(T-x))\, \mathrm{d}x, \tag{22}$$

$$EC_8 = \left(C^R + \sum_{i=1}^{K} \sum_{r=1}^{M} C_r^I Y_{ir}\right)\left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \int_{\Delta_K}^{T} f_X(x)(1 - F_H(T-x))\, \mathrm{d}x, \tag{23}$$

$$EL_8 = T\left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \int_{\Delta_K}^{T} f_X(x)(1 - F_H(T-x))\, \mathrm{d}x, \tag{24}$$

$$P_9 = \left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) R_X(T), \tag{25}$$

$$EC_9 = \left(C^R + \sum_{i=1}^{K} \sum_{r=1}^{M} C_r^I Y_{ir}\right)\left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) R_X(T), \tag{26}$$

$$EL_9 = T\left(\prod_{k=1}^{K} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) R_X(T), \tag{27}$$

$$P_{10} = \sum_{i=0}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\prod_{k=i+1}^{K} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) R_H(T-x) \mathrm{d}x, \tag{28}$$

$$EC_{10} = \left(C^R + \sum_{k=1}^{K} \sum_{r=1}^{M} C_r^I Y_{kr}\right) \sum_{i=0}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right) \left(\prod_{k=i+1}^{K} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) R_H(T-x) \mathrm{d}x, \tag{29}$$

$$EL_{10} = T \sum_{i=0}^{K-1} \left(\prod_{k=1}^{i} \sum_{r=1}^{M} (1-\alpha_r) Y_{kr}\right)\left(\prod_{k=i+1}^{K} \sum_{r=1}^{M} \epsilon_r Y_{kr}\right) \int_{\Delta_i}^{\Delta_{i+1}} f_X(x) R_H(T-x) \mathrm{d}x. \tag{30}$$

With all previous EC_i and EL_i for $i = 1, 2, ..., 10$, we can construct the objective function of the policy, i.e., the cost-rate. For a given feasible policy, non-expert managers can easily assess the cost-rate using an online application developed by us. This application provides a user-friendly interface where users can input the parameters and it calculates the cost-rate based on the provided data. This approach makes it accessible for users without deep expertise in mathematical models to evaluate the financial impact of different maintenance policies.

2.3 Policy optimization problem

Our goal is to determine the optimal number of inspections (K), the timing of these inspections ($\Delta_i, i = 0, ..., K$), the team responsible for each inspection ($Y_{ir}, i = 0, ..., K, r = 1, ..., M$) and the timing for the age-based preventive replacement (T). The binary variables Y_{ir} are represented in matrix form by Y, where the number of columns is equal to K, and each row corresponds to an available inspection team. A value of 1 in Y_{ir} indicates that the i-th inspection team is assigned to the rth inspection. It is important to note that for each column, only one variable can be nonzero. The objective function aims to minimize the cost-rate, which is the expected cost divided by the expected length of the renewal cycle.

Once K is kept constant, the MINLP that describes the problem of finding the optimal policy is modeled by Eqs.

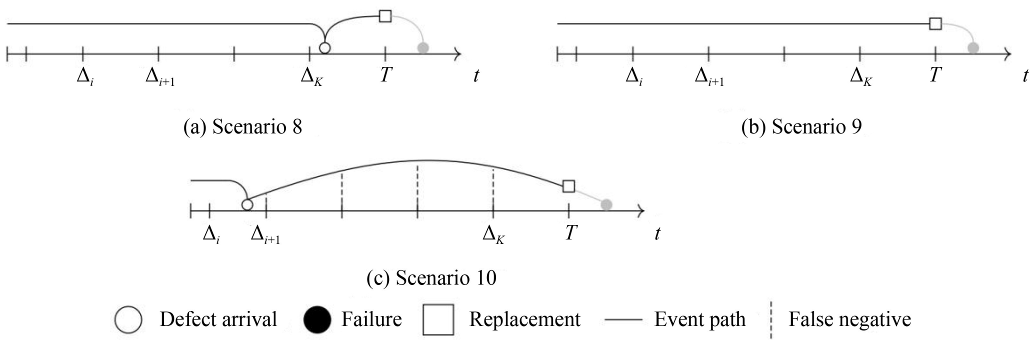

Fig. 3 Scenarios with age-based renewals.

(31) – (40).

$$\text{Min}\frac{\sum_{r=1}^{M} z_r C_r^H + \sum_{i=1}^{10} EC_i}{\sum_{i=1}^{10} EL_i}. \tag{31}$$

Subject to:

$$\Delta_0 = 0, \tag{32}$$

$$\Delta_i < \Delta_{i+1}, \quad i = 0, \ldots, K-1, \tag{33}$$

$$\Delta_K < T, \tag{34}$$

$$\textstyle\sum_{r=1}^{M} Y_{ir} = 1, \quad i = 0, \ldots, K, \tag{35}$$

$$Y_{ir} \leqslant z_r, \quad i = 1, \ldots, K, r = 1, \ldots, M, \tag{36}$$

$$\Delta_i \in \mathbb{R}^+, \quad i = 0, \ldots, K, \tag{37}$$

$$T \in \mathbb{R}^+, \tag{38}$$

$$Y_{ir} = \{0, 1\}, \quad i = 1, \ldots, K, \tag{39}$$

$$z_r = \{0, 1\}, \quad r = 1, \ldots, M. \tag{40}$$

The objective function (31) minimizes the cost-rate, incorporating both maintenance and personnel costs related to the inspector teams. Constraint (32) ensures that Δ_0 equals 0, while UnEq. (33) imposes that each inspection occurs after the previous one and UnEq. (34) specifies the age-based action should occur after the last inspection. Moreover, Eq. (35) guarantees that each inspection is assigned exactly one inspector, while UnEq. (36) ensures inspectors can only perform inspections if their contract costs are covered. The remaining conditions in Eqs. (37)–(40) describe natural constraints for the decision variables.

The MINLP formulation described above requires the previous determining of the optimal number of inspections, K^*. However, finding this optimal value is not trivial, as solving the problem in Eqs. (31)–(40) for different values of K can be computationally intensive. To tackle this, we propose using a Simulated Annealing (SA) algorithm, which has proven effective for solving complex MINLP problems (Cheng et al., 2023; Mohammadi et al., 2014; Teegavarapu and Simonovic, 2002). This algorithm combines SA with adaptive rules, improving convergence and efficiency, making it an appropriate choice for addressing this challenge.

3 Adaptive simulated annealing

Our algorithm is classified as a Simulated Annealing algorithm (Kirkpatrick et al., 1983), with its flowchart summarized in Fig. 4. In the figure, green and red circles denote the beginning and the end of the algorithm, respectively. The procedure Initial Solution() generates a feasible solution, with its steps outlined in Algorithm 1. We employed standard stopping criteria, namely the ending temperature level, while the objective function, computed from Eqs. (1)–(30), is represented by the cost-rate. Candidate solutions are derived from five operators, which are explained later. One of the enhancements of our method is the incorporation of a freezing mechanism for ineffective operators over a set number of iterations, rendering them inactive during this period.

Thus, Algorithm 1 is rooted in the Multi-Start meta-heuristic approach, initiating with a randomly generated solution and then iterating through a series of successive refinements. Each continuous variable is optimized by using Sequential Least Squares Programming (SLSQP), with due consideration being given to the constraints of the problem. Notably, SLSQP exhibited a superior performance across all the tests conducted.

The generation of candidate solutions is made through five neighborhood operators: (i) inserting one inspection, (ii) removing one inspection, (iii) changing the inspector for an inspection, (iv) interchanging inspectors for two inspections and (v) optimizing two continuous variables. Unlike the Adaptive Large Neighborhood Search approach (Mara et al., 2022; Hajad et al., 2019), our

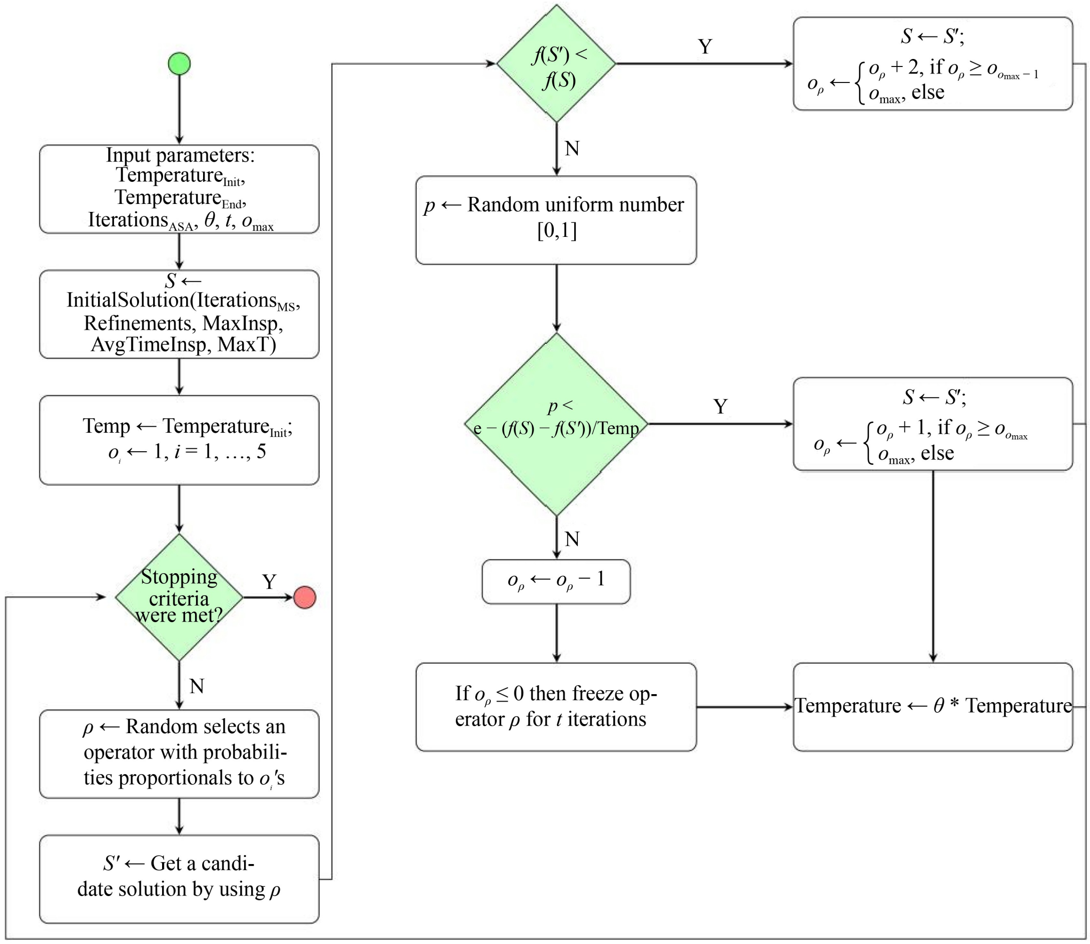

Fig. 4 Proposed algorithm.

method does not involve the destruction and subsequent reconstruction of the current solution. Rather, our operators move from one feasible solution to another, ensuring continuous feasibility throughout. The following section outlines the five operators used:

3.1 Insert inspection

In this operator, we examine all intervals between inspections or between the Kth inspection and T by inserting a new inspection into these intervals. For each interval, we optimize the best inspection moment using SLSQP, considering the optimal inspector (determined by taking an enumeration approach). Additionally, we optimize the preceding and succeeding inspections (or T) one at a time in an arbitrary sequence.

3.2 Remove inspection

All inspections are potentially removed, and the preceding and succeeding inspections are optimized using SLSQP in a randomly selected sequence. The optimal removal is then selected.

3.3 Change the inspector for an inspection

Each inspection is evaluated with respect to its assigned inspector, considering all possible inspectors. For each inspector, the inspection moment is optimized using SLSQP. The optimal change in inspector, along with the corresponding inspection moment if needed, is then chosen.

3.4 Interchange inspectors for two inspections

For each pair of inspections, their maintenance teams are swapped, and the inspection moments are optimized using SLSQP in an arbitrary sequence. The pair resulting in the greatest cost-rate saving is then selected.

3.5 Simultaneous optimization of two continuous variables

In this operator, two continuous variables (Δ_i or T) are jointly optimized using SLSQP. Although more computationally intensive, this approach often yields significant improvements.

Algorithm 1 InitialSolution()

Input: Problem data, *Iterations, Refinements, MaxInsp, AvgTimeInsp, MaxT*
Output: An initial solution $K_{Best} \cup \Delta_{Best} \cup Y_{Best} \cup T_{\{Best\}}$
$K_{Best} \leftarrow 0, \Delta_{Best} \leftarrow \emptyset, Y_{Best} \leftarrow \emptyset, T_{Best} \leftarrow MaxT, f_{Best} \leftarrow \infty$
for k = 1→ *Iterations* **do**
  $K \leftarrow \text{Integer}(0, MaxInsp)$
  $\Delta_0 \leftarrow 0$
  for i = 1→ K **do**
    $\Delta_i \leftarrow \text{Uniform}(\Delta_{i-1}, \Delta_{i-1} + AvgTimeInsp)$
  endfor
  $T \leftarrow \text{Uniform}(\Delta_K, MaxT)$
  $Y_0 \leftarrow 0$
  for i = 1→ K **do**
    $Y_i \leftarrow \text{Integer}(1, M)$
  endfor
  for k = 0 → $Refinements$ **do**
  $s \leftarrow$ Selects a random variable (Δ_i, T or Y_i)
   if $s \in \{\Delta, T\}$ **then**
    $\Delta', T' \leftarrow SLSQP(s/K, \Delta, T, Y)$
    $Y' \leftarrow Y$
   else
    $Y' \leftarrow$ Test all inspectors for variable *s*
    $\Delta' \leftarrow \Delta, T' \leftarrow T$
   endif
   $\Delta \leftarrow \Delta', T \leftarrow T', Y \leftarrow Y'$
  endfor
  $f_k \leftarrow K\Delta_K T(K, \Delta, T, Y)$
  if $f_k < f_{Best}$ **then**
   $K_{Best} \leftarrow K, \Delta_{Best} \leftarrow \Delta, Y_{Best} \leftarrow \text{Y}, T_{Best} \leftarrow T$
  endif
  endfor

From the generation of the initial solution to the refinement of new solutions using the aforementioned operators, the continuous optimization component is handled by the SLSQP algorithm. The problem involves both integer and continuous decision variables: while the integer decisions are managed by the five discrete operators, the continuous variables are optimized using SLSQP. This algorithm is widely applied in maintenance policy optimization problems (Rodrigues et al., 2023b; Scarf et al., 2024). Not surprisingly, in preliminary tests comparing it with Nelder-Mead, Powell, and other methods, SLSQP demonstrated superior performance. Its gradient-based nature, unlike the other derivative-free methods tested, is likely a key factor contributing to its effectiveness in this context.

During each generation of a candidate solution, one of these operators is probabilistically selected. The selection dynamics of these operators are governed by their historical performances, leading to an adaptive approach known as Adaptive Simulated Annealing. Each operator is associated with a historical indicator $o_i (i = 1, ..., 5)$, initially set to 1. Upon the generation of each candidate solution in the Simulated Annealing (SA) process, one of three outcomes can occur: (i) the new solution is superior to the previous one, (ii) the new solution is worse but still acceptable according to the Metropolis criterion, or (iii) the new solution is worse and not accepted.

In these scenarios, the indicator o_i is adjusted accordingly: it is incremented by 2 for a better solution, by 1 for an acceptable solution, and decremented by 2 for an unacceptable solution. Additionally, we set a maximum threshold $o_{\max}$ for these indicators to prevent the repeated selection of a specific operator, which could lead to local optima. If an indicator reaches a value of $o_i \leqslant 0$, it no longer reflects a meaningful probability for selection. In such cases, the operator is "frozen" for t iterations, preventing its selection during this period. Afterward, the indicator is reset to 1, allowing the operator to be considered for selection again.

In detail, the Adaptive Simulated Annealing (ASA) algorithm starts from an initial solution generated by a Multi-Start metaheuristic, which creates several random solutions and improves them through successive local searches. Over a fixed number of iterations, ASA seeks better solutions by applying one of five operators in each iteration. These operators act on the maintenance policy and inspector team assignment variables, always getting feasible solutions. The choice of operator at each step is guided by historical performance indicators. Operators that lead to improved solutions are rewarded, while those that result in worse solutions have their performance scores penalized. If an operator's score falls below the lower bound, it is temporarily frozen, i.e. excluded from selection, for a few iterations. This mechanism helps avoid wasting computational effort on unpromising operators during that phase of the search.

4 Numerical examples

At this stage, we present the computational experiments conducted on the proposed model. This section is divided into three Subsections. Subsection 4.1 focuses on analyzing the impact of mixed contracts (cohorts) and assessing the model's robustness under variations in inspector costs and quality. Subsection 4.2 evaluates the performance of the proposed algorithm, while Subsection 4.3 conducts a sensitivity analysis on key parameters to derive practical maintenance insights for a more real-size instance. All tests were conducted on a personal computer with the following specifications: Intel Core i7-3770, 3.40GHz processor and 16GB RAM. Furthermore, the parameters used in the experiments are primarily based on the maintenance data for bearings reported by Scarf et al. (2009). Table 2 summarizes the common parameters of the base case adopted across all subsequent tests.

4.1 Impact of mixed cohorts and robustness

In this Subsection, we analyze the impact of mixed contracts and assess the model's sensitivity to variations in inspector costs and error probabilities. To ensure the determination of optimal solutions, we consider a simplified scenario involving two inspector teams and periodic inspections. Under these assumptions, it is possible to fully enumerate all feasible solutions. The procedure for identifying the optimal solution in this case is as follows: we vary the number of inspections from 0 up to an empirically defined upper bound, $\overline{K}$. For each value of K, all possible combinations of inspector assignments are generated. Then, for each configuration of number of inspections and inspector assignment, we optimize $\Delta_i, i = 0, ..., K$ and T. Note that $\Delta_i - \Delta_j = \delta$ for $i > j$ and $i = j + 1$. As a result, each optimization only involves determining the values of δ and T, which significantly reduces the problem's complexity and requires less than 60 s per run. The upper bound K is defined empirically, such that increasing it beyond this value leads to worse solutions due to over-inspection. In total, $\sum_{i=0}^{U} M^i$ optimization problems are solved to determine the global optimum.

Considering two inspectors with parameters detailed in Table 3, the optimal solution corresponds to $\Delta = \{2.22, 4.44, 6.66, 8.88, 11.11\}, Y' = \{2, 2, 1, 1, 1\}$ and $T = 12.07$, where Y_i' denotes the assignment of inspectors to each inspection. This configuration yields a cost-rate of \$0.1845 per time unit. Compared to the use of simplex (single-type) cohorts, the cost-rate shows deviations of 2.33% and 2.34%, respectively. In these comparisons, the decision variables differ in the following ranges: $K = [2,5]$, $\delta = [2.221, 2.251]$ and $T = [10.97, 12.07]$. In one specific case, a notable change occurs in the number of maintenance interventions and the scheduled time for the age-based action. This leads to a reduction in the component's usage duration, such that preventive replacement is performed earlier. These results highlight the efficiency gains achieved through a mixed inspector team, as opposed to homogeneous team compositions.

To further assess the robustness of this optimal configuration, we conduct a sensitivity analysis by varying the inspector error probabilities and inspection costs. Specifically, we test variations of ± 5% and ± 10% in each parameter. Figure 5 presents the impact of variations in the false-positive probability.

A quasi-linear trend can be observed across all variations, with the cost-rate changing by up to 0.572% in comparison with the base case solution. Except for one instance, all scenarios resulted in the same number of inspections and inspector assignments as in the original optimal solution. The inspection intervals varied by up to 19%, while the age-based replacement time differed by no more than 2.5%. In summary, despite noticeable changes in inspection intervals, the remaining decision variables, particularly the number of inspections and inspector assignments, exhibited no variation. This indicates a strong robustness of the proposed solution with respect to changes in false-positive errors. Finally, in comparison with the situation of single cohorts, the mixed solution on over all cases tested reaches up to 3.14% of improvement in cost-rate with the lower bound in 1.29%. In other words, for the variations tested, in all cases the mixed cohort presents improvements in cost-rate when compared to the situation of having only one team available. This highlights even more the impact of the proposed model.

Figure 6 shows a pattern similar to that observed for variations in the false-positive rate, but with an even smaller maximum variation in the cost-rate: only 0.29%. Regarding the decision variables, all cases maintained the same number of inspections and team assignments as in the base case. The maximum variation in the inspection intervals was 0.076%, while the variation in the age-based replacement time was just 0.135%. These results further demonstrate the strong robustness of the solution with respect to changes in false-negative probabilities.

Table 2 Data for "weak" and "strong" components

η_1(t.u.)	β_1	η_2(t.u.)	β_2	λ(events/t.u.)	**p**	C^R (\$)	C^F (\$)
3	2.5	18	5.0	0.5	0.1	1	10

Table 3 Data for inspectors for the periodic tests

Inspector	α	ε	C^I (\$)	C^H (\$)
1	0.10	0.20	0.0150	0.0
2	0.02	0.04	0.0875	0.0

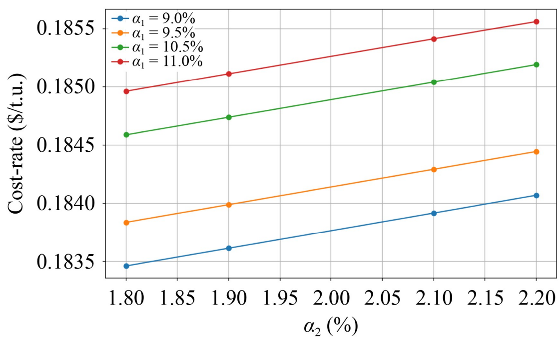

Fig. 5 Variations on the false-positive probabilities for both inspectors.

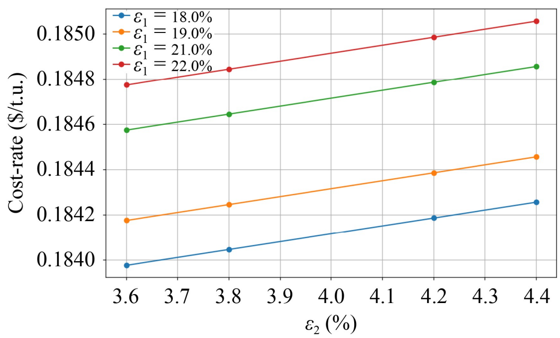

Fig. 6 Variations on the false-negative probabilities for both inspectors.

The maximum gain over the simplex contracts reaches up to 2.84% with lower bound in 1.83%. Like for the false-positive variations, again the mixed contracts are more beneficial than the simplex case.

Also, we analyzed the variations on the inspection costs, which are indicated in Fig. 7.

The cost-rate variation reaches up to 1.1%, and in three instances, the optimal solution involves three inspections with altered configurations in inspectors used, but still keeping the usage of higher quality inspector (inspector 2) followed by inspections assigned inspector 1. In all other cases, the solution remains identical to the base case. Due to these more substantial changes in the number of inspections, the inspection intervals also show increased variability, with differences reaching up to 55%. However, in scenarios where the number of inspections remains unchanged, the variation in inspection intervals is limited to just 0.16%. As for the age-based replacement time, the maximum variation observed is 3.8%. In comparison with the simplex contracts, the mixed case causes improvements in cost-rate in the range of 1.26%–3.39%.

Our final analysis focuses on the impact of fixed costs, with all results summarized in Table 4. As expected, the presence of fixed costs discourages the use of mixed inspector teams, even when the fixed costs are identical across teams. Additionally, variations in the cost-rate become more pronounced under these conditions, often

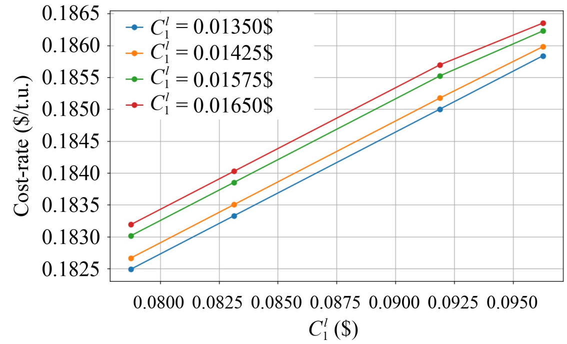

Fig. 7 Variations on the inspection cost for both inspectors.

Table 4 Variations on fixed costs for both inspectors

C_1^H	C_2^H	K^*	δ^*	Y'	T^*	ϕ
0.00	0.00	5	2.22	{2, 2, 1, 1, 1}	12.07	0.1845
0.05	0.05	2	2.25	{2, 2}	11.03	0.1939
0.05	0.10	4	2.71	{1, 1, 1, 1}	11.94	0.1943
0.10	0.05	2	2.25	{2, 2}	11.02	0.1939
0.10	0.10	2	2.25	{2, 2}	11.09	0.1989

leading to changes in both the number of inspections and the timing of the age-based replacement.

In conclusion, the optimal solution for the base case demonstrated strong robustness across most of the tested scenarios. For variations in false-positive and false-negative probabilities, as well as inspection costs, the maximum observed deviation in cost-rate was only 1.1%. Furthermore, key decision variables, such as the number of inspections, inspector assignments and the timing of the age-based replacement, showed minimal sensitivity, with only minor variations. On the other hand, mixed contracts were consistently part of the optimal solution in all cases, except when the hiring cost of inspection teams was varied. In these instances, the presence of fixed costs did not favor the use of mixed cohorts.

4.2 Algorithm performance validation

In this Subsection, we validate the performance of the proposed metaheuristic algorithm. Unlike the previous analysis, we now consider non-periodic inspections. This added complexity makes it infeasible to obtain the optimal solution through the exhaustive enumeration approach described earlier. Therefore, we generated a set of instances by changing certain parameters of the problem to validate our model against "black-box" algorithms available in software packages. We used an enumeration procedure to benchmark our SA algorithm. Thus, we systematically enumerated through $K = 0, ..., \overline{K}$ and optimized all remaining variables (Δ, Y and T) using SLSQP implemented in the *scipy.optimize* package of the Python language. Binary variables were treated as continuous, rounded to the nearest integer to determine the inspector assignment. Also, to mitigate computational demands on the complete enumeration, we introduced a stopping criterion: if three consecutive iterations (number of inspections) yielded no improvement in the best solution, we halted the enumeration.

For the following tests we considered the parameters presented in Table 2 with a different set of inspectors that are described in Table 5. Additionally, Table 6 brings the set of instances created from the base case together with performance for the benchmark algorithm. In these tests, the running time was allowed to run freely, that is, it was not used as a stopping criterion. For each instance, the best cost-rate, running time and the solution are depicted.

Concerning our algorithm, because of the stochastic nature of the proposed algorithm, we run it 10 times for each test. Also, we calibrated all parameters needed and their values were presented in appendix A (Table A1), while the results for the same cases presented in Table 6 are shown in Table 7 for the proposed SA. In Table 7, we include a column labeled *Gap*, which reports the relative difference computed as $\frac{b-a}{a}\times 100\%$, where b is the cost-rate obtained by the benchmark method and a is the cost-rate achieved by the proposed ASA.

Table 5 Data for inspectors

Inspector	α	ε	C^I ($)	C^H ($)
1	0.2	0.2	0.02	0
2	0.2	0.3	0.01	0

Across all tested cases, our algorithm outperformed the benchmark in 78% of them. For the remaining cases where it did not yield the best solution, the maximum deviation was only 0.61%. In contrast, the benchmark algorithm showed a deviation around 8.6% at the worst occasion. An average gap of 3.28% was observed between the methods. Looking at the computational results, in the worst-case tested, the benchmark algorithm is more than 5 times faster than ours. On the other hand, in our best-case scenario, our algorithm is 318 times faster than the benchmark. In summary, our algorithm consistently offers good solutions, and even when it is not the best, it does so within a very feasible runtime. The same cannot be said for the benchmark algorithm. We also emphasize that when comparing Tables 6 and Table 7, apart from the disparity in cost rates, the solutions exhibit significant differences in terms of number of inspections, inspection moments, inspectors used and the age-based replacement.

Finally, we depicted in Figs. 8 and 9 the candidate solutions found at each iteration and the best solution found for both algorithms, respectively, for the base-case in a single running. Our algorithm converges to the best solution found in a few iterations and continues to try to escape from this local optimum for the subsequent iterations.

Our final analysis of ASA's performance focuses on

Table 6 Numerical results for benchmark

Change	Cost-rate ($/u.t.)	RunTime (s)	Solution			
			K	Δ (t.u.)	Y	T (t.u.)
–	0.2001	931.1	3	{3.63, 10.09, 11.16}	{1, 2, 1}	12.18
$C_1^H = 0.1$	0.2085	352.6	1	{3.67}	{2}	11.17
$C_1^H = 0.2$	0.2085	322.6	1	{3.67}	{2}	11.17
$C_1^H = 0.3$	0.2085	366.6	1	{3.67}	{2}	11.17
$C_1^H = 0.4$	0.2085	300.6	1	{3.67}	{2}	11.17
$C_2^H = 0.1$	0.2152	136.2	0	{}	{}	11.22
$C_2^H = 0.2$	0.2152	135.9	0	{}	{}	11.22
$C_2^H = 0.3$	0.2152	183.4	0	{}	{}	11.22
$C_2^H = 0.4$	0.2055	2643.8	1	{3.66}	{1}	11.14
$C_1^I = 0.01$	0.2045	292.6	1	{3.66}	{1}	11.13
$C_1^I = 0.03$	0.2021	70057.4	5	{3.63, 9.98, 10.95, 11.68, 12.33}	{2, 2, 2, 1, 1}	13.00
$C_1^I = 0.05$	0.2087	343.0	1	{3.68}	{1}	11.19
$C_1^I = 0.005$	0.1967	272954.5	9	{3.62, 9.91, 10.95, 11.56, 12.05, 12.50, 12.95, 13.33, 13.62}	{1, 1, 2, 2, 2, 1, 1, 2, 2}	14.00
$C_2^I = 0.015$	0.2037	66636.9	5	{3.03, 4.58, 10.21, 11.25, 11.90}	{1, 1, 1, 2, 2}	12.63
$\lambda = 0.25$	0.1684	10583.0	6	{4.00, 10.19, 11.52, 12.34, 13.05, 13.56}	{1, 1, 2, 1, 2, 2}	14.20
$\lambda = 0.75$	0.2211	243.3	1	{3.39}	{1}	10.93
p = 0.05	0.1623	5199.0	6	{9.40, 10.39, 11.15, 11.70, 12.11, 12.47}	{2, 1, 1, 2, 2, 2}	12.92
p = 0.15	0.2308	81318.9	9	{2.82,4.23,10.28,11.27,12.05,12.66,13.13,13.49,13.85}	{1,1,2,1,1,1,2,2,1}	14.32

Table 7 Numerical results for our algorithm

Change	Cost-rate ($/u.t.)	RunTime (s)	Best Cost-rate	Solution			
				K	Δ (t.u.)	Y	T (t.u.)
–	0.2037±0.0068	1187.2±726.6	0.1980	7	{3.63, 10.20, 11.30, 11.91, 12.37, 12.72, 12.99}	{1,1,1,1,2,2,2}	13.44
$C_1^H = 0.1$	0.2164±0.0026	8331.7±1389.5	0.2010	9	{3.64, 10.21, 11.37, 11.86, 12.22, 12.50, 12.73, 12.85, 13.10}	{2,2,2,2,2,2,2,2,2}	13.58
$C_1^H = 0.2$	0.2037±0.0059	984.5±64.5	0.2008	8	{3.64, 10.27, 11.01, 11.62, 12.01, 12.34, 12.64, 12.75}	{2,2,2,2,2,2,2,2}	13.34
$C_1^H = 0.3$	0.2065±0.0106	895.7±285.0	0.2003	9	{3.63, 9.96, 10.86, 11.47, 11.95, 12.35, 12.73, 13.04, 13.22}	{2,2,2,2,2,2,2,2,2}	13.67
$C_1^H = 0.4$	0.2022±0.0022	1015.1±365.6	0.2004	9	{3.68, 10.12, 11.18, 11.83, 12.27, 12.68, 12.90, 13.37, 13.63}	{2,2,2,2,2,2,2,2,2}	14.00
$C_2^H = 0.1$	0.2070±0.0103	904.7±108.8	0.1983	6	{3.64, 9.95, 11.00, 11.65, 12.18, 12.60}	{1,1,1,1,1,1}	13.02
$C_2^H = 0.2$	0.2020±0.0034	692.6±415.7	0.1982	6	{3.62, 9.93, 11.01, 11.79, 12.40, 12.86}	{1,1,1,1,1,1}	13.37
$C_2^H = 0.3$	0.2051±0.0146	896.2±252.5	0.1984	6	{3.64, 9.76, 10.86, 11.50, 11.94, 12.60}	{1,1,1,1,1,1}	13.16
$C_2^H = 0.4$	0.2114±0.0204	896.5±384.5	0.1988	5	{3.61, 9.62, 10.73, 11.43, 12.04}	{1,1,1,1,1}	12.74
$C_1^I = 0.01$	0.1987±0.0032	889.5±254.7	0.1953	5	{3.61, 9.82, 10.94, 11.69, 12.24}	{1,1,1,1,1}	12.86
$C_1^I = 0.03$	0.2033±0.0025	780.4±373.9	0.2000	5	{3.63, 9.91, 10.74, 11.54, 12.11}	{1,2,2,1,2}	12.66
$C_1^I = 0.05$	0.2038±0.0027	849.5±321.8	0.2010	6	{3.64, 10.28, 11.17, 11.93, 12.27, 12.61}	{2,2,2,2,2,2}	13.21
$C_1^I = 0.005$	0.2041±0.0052	853.5±337.0	0.1973	6	{3.62, 9.74, 10.75, 11.30, 11.80, 12.18}	{1,1,2,2,2,2}	12.84
$C_2^I = 0.015$	0.2036±0.0049	726.6±346.6	0.1983	6	{3.63, 9.84, 10.91, 11.94, 12.39, 12.89}	{1,1,1,1,1,1}	13.43
$\lambda = 0.25$	0.1707±0.0023	844.6±292.0	0.1689	5	{4.00, 10.16, 11.45, 12.24, 12.80}	{1,1,1,2,2}	13.44
$\lambda = 0.75$	0.2223±0.0067	697.8±389.3	0.2151	8	{2.92, 10.03, 10.67, 11.12, 11.47, 11.80, 12.05, 12.30}	{1,2,2,2,2,2,2,2}	12.54
p = 0.05	0.1647±0.0008	935.9±79.1	0.1633	4	{4.15, 9.52, 10.76, 11.35}	{2,1,2,1}	12.04
p = 0.15	0.2336±0.0016	928.1±147.2	0.2321	5	{2.82, 4.22, 10.31, 11.46, 12.24}	{1,1,1,1,1}	13.00

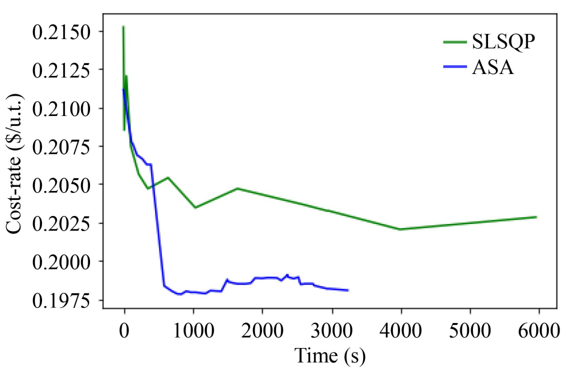

Fig. 8 Candidate solutions founding during the iterations for both algorithms.

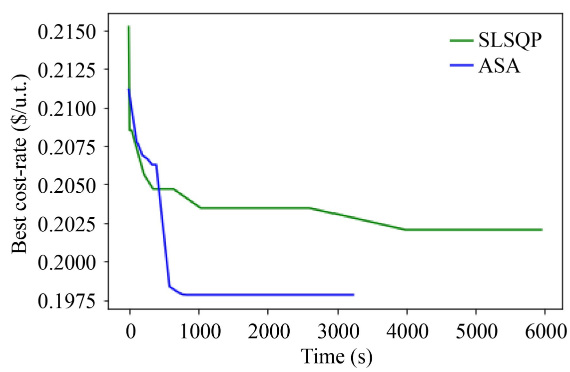

Fig. 9 Best solution found so far for both algorithms.

the freezing mechanism. In Fig. 10, we compare the average performance of the algorithm without the freezing operator to a typical run that includes it. A clear pattern emerges: the freezing mechanism tends to achieve faster convergence, as shown by its curve lying below the average curve without the operator. Beyond the improved convergence speed, the freezing mechanism is expected to offer even greater benefits in more challenging instances. In such cases, the neighbor operators may become increasingly time-consuming, and the freezing mechanism helps avoid unproductive search directions with poor recent performance. Naturally, after some time, reactivating these operators can again be useful for escaping local optima.

4.3 Sensitivity analysis

In this Subsection, we share and explain findings for the maintenance manager introduced by the new problem. We investigated the sensitivity of the model when selecting inspector contracts, analyzing a more real scenario with three different types of inspectors, each varying in cost and inspection quality based on their experience level. The types of inspection teams considered are trainee, regular, and expert, with their respective parameters shown in Table 8. We set up a maximum threshold of execution time in 4000 s for these tests.

The parameters were chosen based on a hypothetical situation where the solutions using exclusively one type of inspection team were nearly identical. Specifically,

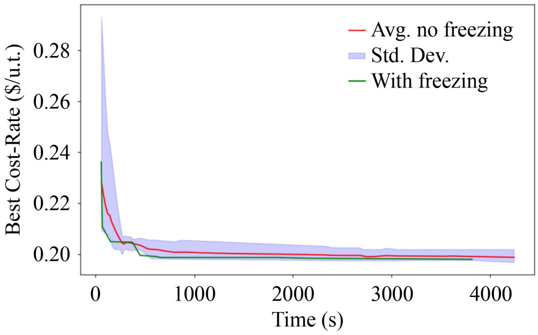

Fig. 10 Efficiency of the freezing mechanism.

Table 8 Parameters of trainee, regular and expert teams

Parameters	Team		
	Trainee (1)	Regular (2)	Expert (3)
$\alpha(\%)$	0.0	0.0	0.0
$\varepsilon(\%)$	15.00	8.55	0.00
$C^I(\$)$	0.0750	0.0850	0.0984
$C^H(\$)$	0.0	0.0	0.0

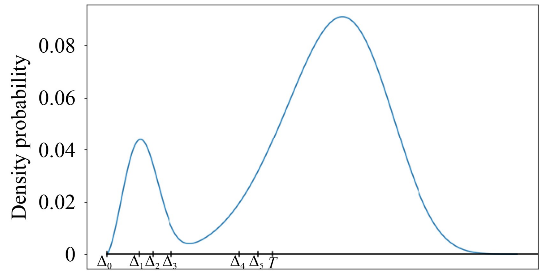

Fig. 11 Solution for the base-case over the distribution for the defect arrival.

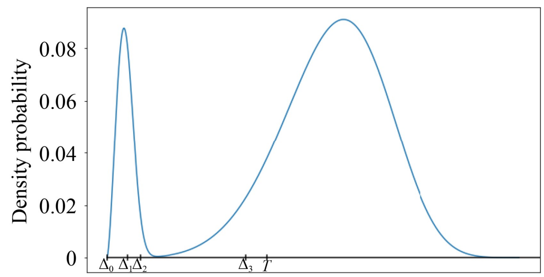

Fig. 12 Solution for $\eta_1 = 1.5$ over the distribution for the defect arrival.

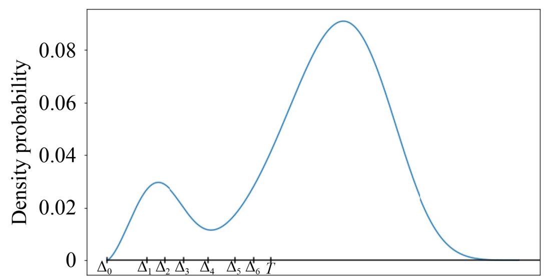

Fig. 13 Solution for $\eta_1 = 4.5$ over the distribution for the defect arrival.

with the parameters in Table 8, the solutions using only trainee, regular, and expert teams were very similar, i.e., $CR_0 = 0.194261$, $CR_1 = 0.194267$ and $CR_2 = 0.194266$ (in \$/t.u.).

The base case involves five inspections distributed across two distinct regions until the programmed action T, as illustrated in Fig. 11. In this figure, we present the timing of the optimal policy against the distribution of the defect arrival to better understand the dependence of the solution on defect arrival. Notably, the region least likely for defect arrival is where no inspection is assigned. Additionally, the age-based action was performed earlier due to the high significance of the corrective cost C^F. In this scenario, the less expensive team is generally used. However, the last two inspections (Δ_2 and Δ_3) before a significant period without inspections are conducted by the expert team. This suggests that, given the nature of the problem involving two types of components, the optimal solution allocates inspections into two regions to prevent failures. Moreover, because there will be a considerable amount of time without planned inspections, the preceding inspections are performed by the best team to ensure reliability.

Concerning the sensitivity of the model, our analysis suggests that it exhibits a higher degree of sensitivity when there is a decrease in the scale parameter of the Weibull distribution for the defect arrival of the "weak" component (η_1) compared to an increase. This lack of sensibility on η_1 is primarily attributed to the relatively low significance of the mixing parameter ($p = 0.13$). Additionally, we uncovered an interesting phenomenon that was not expected: "weaker" components result in lower cost rates, while the opposite holds true for strengthened components. To gain a deeper understanding of this behavior, we have depicted the solutions and defect arrival distributions for both scenarios in Figs. 12 and 13.

Analyzing Fig. 12, we can understand the reason behind the decrease in the cost-rate when η_1 was decreased. In this scenario, although the mean of the distribution for the "weak" component is lower, there is a significant increase in the concentration of this distribution, resulting in a more predictable defect arrival pattern around the mean. Conversely, for $\eta_2 = 4.5$ (Fig. 13), the opposite trend is observed, with defects appearing later but being more widely distributed. This explains the unexpected behavior for the variation in this parameter.

Regarding the shape parameter of this component, β_1, the model is slightly more sensitive compared to η_1. When the defect arrival is not well-behaved around the mean, one more inspection is conducted, and the expert team is not used. This suggests that in situations of unpre-

Table 9 Sensitive analysis

η_1	β_1	η_2	β_2	λ	p	C^R	C^F	K	Δ	Y	T	CR
3.0	2.5	18	5	0.5	0.13	1	10	5	{2.37, 3.35, 4.71, 9.69, 11.04}	{1,3,3,1,1}	12.14	0.1937
1.5	2.5	18	5	0.5	0.13	1	10	3	{1.47, 2.40, 10.11}	{3,3,3}	11.65	0.1839
4.5	2.5	18	5	0.5	0.13	1	10	6	{2.89, 4.19, 5.61, 7.42, 9.36, 10.69}	{1,3,3,1,1,1}	11.96	0.1962
3.0	1.0	18	5	0.5	0.13	1	10	6	{1.10, 2.30, 3.97, 6.58, 9.30, 10.79}	{1,1,2,1,1,1}	11.98	0.2009
3.0	5.0	18	5	0.5	0.13	1	10	2	{2.85, 3.61}	{1,3}	10.90	0.1838
3.0	2.5	9	5	0.5	0.13	1	10	4	{2.82, 3.85, 4.82, 5.65}	{3,1,1,1}	6.37	0.3244
3.0	2.5	27	5	0.5	0.13	1	10	4	{1.92, 2.60, 3.57, 4.98}	{1,1,1,3}	16.21	0.1386
3.0	2.5	18	3	0.5	0.13	1	10	7	{2.31, 3.30, 4.61, 6.60, 8.10, 9.26, 10.25}	{1,3,3,1,1,1,1}	11.21	0.2381
3.0	2.5	18	5	0.5	0.10	1	10	3	{3.02, 4.41, 9.43}	{2,3,1}	11.31	0.1831
3.0	2.5	18	5	0.5	0.15	1	10	5	{2.49, 3.58, 4.88, 9.25, 10.63}	{3,3,3,1,1}	11.97	0.2015
3.0	2.5	18	5	0.5	0.20	1	10	9	{1.94, 2.66, 3.42, 4.16, 5.20, 9.10, 10.73, 11.54, 12.37}	{1,1,1,1,1,1,1,1,1}	13.14	0.2198
3.0	2.5	18	5	0.5	0.13	1	5	2	{2.66, 3.88}	{1, 2}	12.55	0.1491
3.0	2.5	18	5	0.5	0.13	1	50	8	{1.71, 2.39, 2.91, 3.52, 4.28, 5.22, 7.56, 8.75}	{3, 3, 3, 3, 3, 3, 3, 3}	9.74	0.3745
3.0	2.5	18	5	0.5	0.13	1	100	9	{1.68, 2.14, 2.64, 3.09, 3.60, 4.14, 5.13, 6.13, 7.79}	{3, 3, 3, 3, 3, 3, 3, 3, 3}	8.37	0.5405

dictability, low and medium quality teams are used because they are cheaper, and in this particular case, with the last inspection of the first sequence performed by the medium quality inspector. On the other hand, when the predictability is very high, only two inspections are conducted around the η_1 value, with the inspection after η_1 carried out by the high quality team. In this situation, there are two distinct peaks of probability for defect arrival, and the model leverages this by scheduling inspections around the defect arrival of the "weak" component and employing high quality teams after the most likely defect arrival moment.

When the mean defect arrival for the “strong” component is decreased to $\eta_2 = 9$, inspections are well distributed along the expected lifetime of the component. Surprisingly, this solution is very similar to the situation where the defect arrival occurs later ($\eta_2 = 27$), however the last inspection is still conducted by the more reliable team to avoid errors in the final evaluation. We also highlight the dependence of the age-based action on the defect arrival of the “strong” component. When the shape parameter β_2 is decreased, the number of inspections increases, and the two regions of inspections are less distinct. However, the last two inspections related to the “weak” component are still conducted by the high quality team.

When the mixing parameter is decreased p, we observed two regions of inspections with fewer inspections overall, but the last inspection is still conducted by the expert team. When this parameter is increased, inspections are concentrated around the region for the “weak” component. Given the high number of inspections, the cheaper inspector is mostly utilized. Lastly, we observed an interesting behavior regarding the variation in the failure cost C^F. When this parameter is decreased, only two inspections are conducted around the defect arrival for the “weak” component, with the final inspection performed by a higher quality team. Conversely, when this parameter is very high, several inspections are conducted over the entire period until T, with the age-based action planned earlier, and the expert team predominantly used. When the cost is further increased to C^F = 100, the model schedules all maintenance actions earlier and adds one more inspection compared to the scenario with $C^F = 50$. We also observe a tendency to use higher quality inspectors when this cost is excessively high, which is expected since the consequence of an error is substantial, necessitating the use of inspectors who are less prone to errors. However, we do not see an “explosion” in the number of inspections. Instead, inspections are well-distributed up to T, with a recurring pattern of clustering inspections into two distinct regions.

4.4 Practical implications

Based on numerical results and sensitivity analysis, the proposed model provides actionable managerial insights that go beyond scheduling, integrating the optimization of both the timing of maintenance actions and the assignment of the most suitable inspection team.

For example, in a large infrastructure company, such as a water distribution network, a maintenance manager may work with teams of varying experience and cost: Trainee (low cost, higher false-negative risk), Regular (intermediate) and Expert (high cost, minimal errors). Each team differs in performance, experience, available tools and error probabilities. The model guides not only when to inspect, but also who should inspect and why, optimizing overall cost-effectiveness.

For critical components with high failure costs, the model may recommend assigning an Expert inspector before long intervals without preventive inspections, reducing the probability of faults and costly corrective actions. It also identifies clusters of inspections, enabling managers to optimize routes, dispatch teams efficiently and cut logistical costs. This structured allocation of teams supports smarter contract negotiations and justifies investments in training or specialized tools for long-term savings.

Finally, the user-friendly online tool allows managers without advanced mathematical expertise to simulate scenarios, evaluate financial impacts and avoid random assignments that lead to suboptimal solutions. By aligning workforce quality with system needs and costs, the model transforms maintenance planning from reactive to proactive and strategically optimized.

5 Conclusions

As discussed, maintenance policies have long been relied upon to guide maintenance actions, yet many existing models still make strong assumptions. We specifically investigated one such assumption: the presence of only a single inspector. In response, we developed a maintenance model that can deal with scenarios with two or more inspectors, drawing from theory on the renewal-cycle. Additionally, we introduced a Simulated Annealing algorithm with adaptive mechanisms to tackle more complex instances.

Our numerical results underscored the impact of having mixed cohorts and also the effectiveness of our algorithm compared to one of the primary “black-box” algorithms used in maintenance policy optimization. The Simulated Annealing (SA) algorithm yielded high-quality solutions within short time frames, thus demonstrating its ability to avoid local optima. Additionally, we examined instances where changes occurred in the inspector contracts and assessed the sensitivity of the model to the parameters introduced by the novel decision proposed in this paper. Lastly, we explored the significance of incorporating mixed contracts and the potential consequences of overlooking this possibility.

Looking ahead, future lines of research will involve expanding the model to incorporate dependencies between inspectors. Additionally, we highlight the pressing need for further exploration of optimization methods tailored to these maintenance policies, an area that has been largely overlooked in the literature.

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Competing Interests The authors declare that they have no competing interests.

Appendix A

Table A1 Calibrated parameters

Parameters	Description	Value
	InitialSolution()	
$\text{Iterations}_{\text{MS}}$	Number of iterations	8
Refinements	Number of refinements for each trial solution	4
MaxInsp	Maximum number of inspections for generating the random solution	8
AvgTimeInsp	Average time between inspections for generating the random solution	2
Max$T$	Maximum value for T	100
	Adaptive Simulated Annealing()	
$\text{Temperature}_{\text{Init}}$	Initial temperature	0.01
$\text{Temperature}_{\text{End}}$	Final temperature	0.001
$\text{Iterations}_{\text{ASA}}$	Number of candidate solutions generated for each temperature	1
θ	Aspiration level	0.9
t	Number of freezing iterations for operators	5
$o_{\max}$	Maximum number for the historical counters	5

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