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RESEARCH ARTICLE

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Optimal control approach for solving a supply chain problem under variable demand and emissions tax regulation with an unknown production rate

Abstract Supply chains and other complex systems can be effectively managed and optimised with the help of optimal control techniques. Optimal control, as used in supply chain management, is the process of using mathematical optimisation techniques to identify the best course of action for controlling a given objective function over time. Modeling the supply chain's dynamics, which include elements like production rates, inventory levels, demand trends, and transportation constraints, is the best control strategy when applied to a supply chain. In this study, we have considered that production rate is an unknown function of time, which is a controlling function. The demand for the product is taken as a function of price and time. The emission of carbon is taken as a linear function of the production rate of the system. To solve the suggested supply chain system, we have used an optimal control approach for determining the unknown production rate. To find the optimal values of the objective function as well as the decision variables, we have used different meta-heuristic algorithms and compared their results. It is observed that the equilibrium optimizer algorithm performed better than other algorithms used. Finally, a sensitivity analysis is performed, which is presented graphically in order to choose the best course of action.

Keywords two-layer supply chain, control theory, price & time dependent demand, carbon emissions, Meta-heuristic algorithms, equilibrium optimizer algorithm (EOA)

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1 Introduction

The interdependent connection between manufacturers and retailers is a crucial element of effective supply chain management in the current, continuously evolving business environment. Nevertheless, this collaboration encounters exceptional difficulties because of fluctuating demand patterns and strict carbon tax laws. With the continuous evolution of global markets and the increasing importance of environmental issues, it is now more pressing than ever to align operations and strategies between producers and retailers. It is noteworthy that production processes generate substantial amounts of greenhouse gas (GHG) emissions, which play a major role in driving global climate change. One-fifth of worldwide carbon emissions are attributed to the industrial and production sector. Fig. 1 depicts a comprehensive examination of global GHG emissions resulting from production processes in 2019, with a specific emphasis on specific materials. In the year 2019, the primary source of emissions was iron and steel manufacturing, which released 3.70 billion metric tons of carbon dioxide equivalent (CO_{2e}). Cement production closely followed, generating 2.90 billion metric tons of CO_{2e}. The manufacture of plastic and rubber resulted in the emission of 1.40 billion metric tons of CO_{2e}, while wood products and aluminum production released 0.90 and 0.60 billion metric tons of CO_{2e}, respectively. Metals, glass, and minerals other than those mentioned had lesser emissions, contributing between 0.40 and 1.10 billion metric tons of CO_{2e}. Therefore, stakeholders should actively contribute to global endeavors aimed at mitigating climate change and attaining a more sustainable future by giving precedence to sustainable

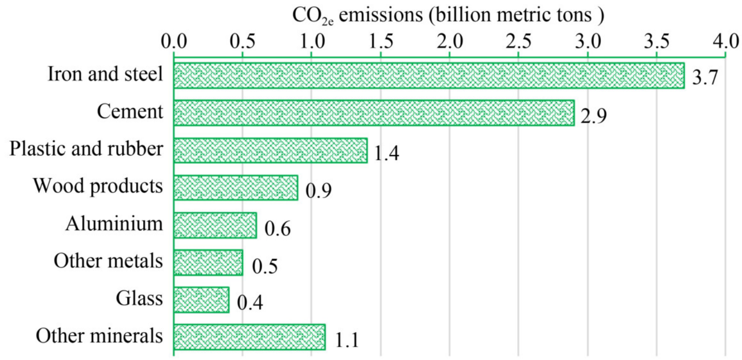

Fig. 1 Global CO_{2e} emissions from production processes in 2019, categorized by specific materials.

practices and fostering innovation in material production processes.

The presence of variable demand, which is impacted by factors such as seasonality, consumer preferences, and market trends, presents a significant challenge to the coordinating efforts of traditional supply chains. The task becomes more difficult when combined with emissions tax requirements, which require firms to adjust their production and distribution systems in order to reduce their environmental impact while still remaining profitable. The government implements several emissions regulatory measures, including cap-and-price, carbon tax, carbon offset, and other legislation, in order to mitigate and manage carbon emissions (Khan et al., 2024a). Out of all these restrictions, the carbon tax legislation stands out as one of the most widely embraced measures for controlling carbon emissions (Zhu and Ma, 2022). Of the European nations, Norway and Sweden employed carbon taxes to regulate carbon emissions as early as 1991 (Khan et al., 2023a), whereas France has only been implementing the rule since 2009 (Bureau, 2011). Therefore, this study examines the complex dynamics of coordination between manufacturers and retailers in the context of fluctuating demand and regulations on carbon tax.

The optimal control theory is widely recognized as a suitable method for modeling complex networks. It has a significant impact on assessing system performance and facilitating optimal decision-making. Optimal control techniques provide an alternative viewpoint compared to mathematical programming methods since they express plans as trajectories (Dolgui et al., 2019). Production systems often include optimal control applications for time management challenges (Giglio, 2015). Thus, this study offers a model that utilizes the optimal control theory for this specific goal. The key objective is to bring together the fields of economics, environmental policy, and supply chain management to provide new and valuable insights. These insights will not only improve operational efficiency but also reduce the negative effects on the environment. More precisely, this research aims to answer the following inquiries:

i) How do manufacturers and retailers adjust their demand forecasting techniques to accurately predict and react to fluctuating demand patterns impacted by factors such as seasonality, selling price, and market trends?

ii) How can manufacturers and merchants align their inventory management procedures to accommodate changing demand, and how do these practices adapt in response to an emissions tax regulation?

iii) How do pricing strategies change when manufacturers and retailers coordinate their efforts in response to fluctuating demand and carbon tax regulations?

iv) What are the economic consequences of carbon tax regulation on manufacturing and retail operations, and how do businesses manage the compromises between environmental compliance, operational expenses, and customer pricing preferences?

The presented study supports the following areas of carbon tax regulation and manufacturer-retailer coordinating systems under fluctuating demand:

• No previous studies in the field of manufacturer-retailer coordination research have taken the combined variable demands of both the retailer and the customers into account while also adhering to an emissions guideline to maximise the profit of the coordination.

• Using optimal control theory, widely acknowledged as an appropriate approach for analyzing complex networks, no researcher examined the performance of the manufacturer-retailer chain system and optimal decision-making under a carbon tax guideline.

• Through an analysis of the interactions among changeable demands, inventory control, pricing tactics, and environmental compliance, this study aims to clarify the complex trade-off between sustainability and economic feasibility for a manufacturer-retailer collaboration.

By combining quantitative models, qualitative evaluations, and scenario analyses, this research aims to provide practical suggestions that enable companies to handle the challenges of modern supply chain management confidently and strategically. Therefore, by highlighting the way towards synergy between profitability and responsibility toward the environment, this study aims to promote revolutionary change within the global supply chain ecosystem.

When it comes to managing and improving sophisticated systems like supply chains, optimal control techniques can be a very effective instrument. The use of mathematical optimisation techniques to identify the optimal control actions over time to minimise or maximise some objective functions, such as cost, inventory levels, or service levels, is known as optimal control in the context of supply chain management. The best control strategy in a supply chain setting entails modeling the chain's dynamics, taking into account variables like production rates, inventory levels, demand trends, and transportation limitations. The goal is to identify control actions that maximise supply chain performance in accordance with the selected objective function. Examples of these actions include production rates, inventory policies, and transportation schedules. Outlining supply chain problems as dynamic optimisation problems, which are essentially a type of optimal control problem, is a popular strategy for applying optimal control to solve supply chain problems. To do this, a mathematical model of the supply chain dynamics must be established, along with the objective function that needs to be optimised. The problem must then be solved, and the best course of action for control must be determined using mathematical optimisation techniques like dynamic programming, Pontryagin's maximum principle, or numerical optimisation algorithms. Now there are some additional questions that arose during the formulation of this supply chain model, which are given below:

➢ How can the production rate of a supply chain system be accurately measured?

➢ Which systems tend to be the most profitable, and what factors contribute to their profitability?

➢ How does one effectively influence production control across the entire supply chain?

➢ What is the approach for solving the suggested model?

The answers to the above-mentioned questions are the main contribution of this research.

1.1 Literature review

This pivotal section delves into the intricate motivations behind the present investigation and conducts a thorough review of the large amount of research that has already been published. The objective is to clearly explain the motivations behind the current research efforts and to clarify the importance and pertinence of this work in the wider academic discussion. In addition, by doing an in-depth examination of the current body of literature, the aim is to identify any gaps, inconsistencies, and opportunities for further investigation.

1.1.1 Literature review on the application of optimal control theory

Optimal control models can handle the complexity of contemporary supply chain management, making them important for manufacturer-retailer supply networks. The literature emphasizes how these models improve supply chain decision-making in production, inventory management, transportation, and pricing strategies. These models allow producers and retailers to quickly adapt to changing market conditions, demand, and regulations by combining dynamic optimisation with real-time data analytics. Ortega and Lin (2004) reviewed significant research on control theoretic approaches to production-inventory operations, and Dolgui et al. (2019) examined the use of optimal control in scheduling across production, supply chain, and Industry 4.0 systems. Wu and Chen (2010) used optimal control theory to study inventory and pricing over time and production stages. Ivanov et al. (2012) examined optimal control theory's potential for flexible supply chain scheduling and planning. Afterwards, using optimal control theory, Vercraene and Gayon (2013) examined the best approach by adding the product return attribute to a production-inventory scheme. Gayon et al. (2017) classified serviceable and disposable items from stochastic product returns in a production-inventory network using optimal control theory. Zhao et al. (2017) examined production control in a partial-flexibility production-inventory system. Gayon et al. (2017) optimally controlled product returns in a production-inventory system with two disposal options. In a failing manufacturing system, Kang and Subramaniam (2018) implemented preventive maintenance and production control. Kang and Subramaniam (2018) found the best control policy for a deteriorating single machine production scheme by minimizing the total manufacturing cost of an integrated control and preventive maintenance scheme. Wu (2019) used optimal control for consignment supply chain advertising. Dolgui et al. (2019) studied optimal control scheduling in production, supply chain, and Industry 4.0 systems: principles, cutting edge, and applications. Dizbin and Tan (2020) used optimal control theory to examine a demand-based inventory management strategy that is correlated with inter-arrival and processing time. Yu et al. (2020) solved a low-carbon supply chain's ideal control model: cooperative emission reduction, price schemes, and new coordination contract design. Zu et al. (2021) examined how consignment contracts and wholesale prices can efficiently reduce supply chain carbon emissions. Papanagnou (2022) solved closed-loop supply chain bullwhip measurement and elimination using control theory. Ali et al. (2023) created a model to optimise an imperfect green product manufacturing process. Das et al. (2023) used interval optimal control to reduce emissions and implement reworking regulations in an imperfect manufacturing scheme. Bao et al. (2023) used epidemic dynamics and production-inventory models to find the best epidemic control methods. Akhtar et al. (2024) apply optimum control theory to reduce emissions for eco-friendly products in a production system. Das et al. (2024a) established pricing and dynamic service policies for a manufacturing process with interval uncertainty using the extended Pontryagin's maximum principle. No studies have developed an optimal control model for coordinating manufacturer-retailer supply, but optimal control models can align supply chain objectives with organizational goals like profitability, operational efficiency, and customer service.

1.1.2 Literature review on coordination contract

Manufacturers and retailers must coordinate their supply chains to maximize efficiency, reduce costs, and improve customer satisfaction. Sarkar (2013) calculated the minimum cost of manufacturer-retailer coordination for a deteriorating item using algebra. Giri and Sharma (2014) examined a producer-two rival store cost-sharing arrangement for advertising that affects market demand. Cárdenas-Barrón and Sana (2014) examined sales team promotions in oligopolistic marketing. These activities increased end-customer demand, which increased demand for manufacturers and retailers in the supply chain. Saha and Goyal (2015) coordinated the supply chain using retail price dependence and inventory. Yan et al. (2016) examined risk-reimbursement trade credit supply chain coordination agreements. Giri and Sarker (2016) examined a supply chain network with two vendors competing on price and service quality and a single manufacturer that could experience production interruptions. A two-tiered payment delay contract for supply chain management was examined by Heydari et al. (2017). Manna et al. (2018) described a two-stage production inventory framework with two storage facilities and reliability. Song and He (2019) proposed a three-layer contract-coordinated fresh vegetable supply chain. Mohammadi et al. (2019) used a revenue-and-preservation-technology investment-sharing agreement to coordinate the fresh food supply chain and reduce waste. Kırcı et al. (2019) studied a two-level supply chain. The retailer buys finished products made from perishable raw materials from the manufacturer during replenishment cycles. This process occurs in an unpredictable demand and product obsolescence environment. Agrawal and Yadav (2020) examined profit-sharing and pricing strategies in a two-stage supply chain with a single producer and multiple vendors, where selling price affects market demand. Shen (2021) examined a two-level supply chain issue where a producer sells eco-friendly products to a retailer, who sells them to consumers. Retail price and greening affect market demand. The revenue sharing contract and its specifications are designed to optimize supply chain model coordination. Under a revenue-sharing-commission coordination contract, Lan and Yu (2022) solved a supply chain problem with marketing consideration. Soleymanfar et al. (2022) also calculated the most efficient economic order quantity (EOQ) and economic production quantity (EPQ) for items in a two-tier supply chain, including a retailer and a supplier. The product return policy influenced these values. Another study, Ghosh et al. (2023), examined a dual channel two-stage coordination model with one producer and one retailer for a single product, accounting for consumer feedback and advertising or sales team promotions. Saha et al. (2023) proposed a profit-sharing method to coordinate producer-retailer supply based on the observation that market demand increases with a product's environmental sustainability. Recently, Zhao et al. (2024) used transparency campaigns to reduce product returns and improve supply chain synchronisation. Saha et al. (2024) created a supply chain coordination challenge with green investment, contract sharing, and advertising. None of the manufacturer-retailer coordination studies mentioned above have included the retailer and customer's changing price and time demands. This research gap must be addressed to improve the resilience, adaptability, and competitiveness of manufacturer-retailer supply chains in the current competitive business environment.

1.1.3 Literature review on various demands

In the context of supply chain coordination between manufacturers and retailers, the demand patterns of both the retailer and customers vary over time and in response to price variations. Customers' buying choices are significantly affected by price elasticity, where changes in price have a direct effect on the quantity demanded. Rising prices may cause customers to see things as less cheap, resulting in a drop in demand. Lower pricing may lead to more demand by making things more appealing and accessible to a wider range of customers. Abdul-Jalbar et al. (2009) is the single study that examines the coordination of the supply chain between a storage facility and several retailers while taking into account time-sensitive market demand. Sicilia et al. (2015) investigated the best manufacturing approach for situations where demand and output rates fluctuate over time. Giri and Roy (2016) solved a supply chain inventory system modeling under price-dependent demand with regulated lead times. Pal et al. (2016) solved a two-tier competitive integrated supply chain model with demand that depends on pricing and credit duration. Pakhira et al. (2017) proposed a two-level supply chain with a fixed temporal horizon and demand that is reliant on time, price, and promotional costs. In the same year, Maiti & Giri (2017) determined pricing and decision-making techniques for two periods in a two-tier supply chain with demand that is reliant on price. Pervin et al. (2018) studied time-sensitive consumer buying preferences in an inventory system. Malik et al. (2018) reviewed a time-varying inventory model for non-instantaneously decaying objects with a maximum life span. Chen et al. (2019) established pricing and replenishment strategies that work best for depreciating inventory when demand is contingent on price, time, and stock level. Ghomi-Avili et al. (2019) proposed a multi-objective model that accounts for price dependence in demand, disruption, and scarcity in a closed-loop supply chain network architecture. San-José et al. (2020) solely took into account time-varying demand while maintaining a constant manufacturing rate. Moreover, San-José et al. (2021) examined a demand that is sensitive to both price and time while making decisions about pricing and inventory planning for a retailer. For instance, Rahman et al. (2021) included the influences of price and stock availability simultaneously on customer demand. In addition, Das et al. (2021) investigated pricing and ordering methods in a manufacturer-retailer supply chain while implementing a price reduction strategy. Later, Barman et al. (2022) expanded the cooperation between manufacturers and retailers for a non-instantaneous degrading item. Duary et al. (2022) adopted a nonlinear influence on market demand in inventory systems. A number of academics additionally examined how pricing and other variables affected demand as a consequence. Das et al. (2024b) delineated the resultant impacts of price and the level of environmental friendliness of items. In addition, fluctuations in demand over time are often influenced by seasonal trends, promotional campaigns, and evolving consumer choices. Retailers usually face greater demand during peak shopping seasons like Christmas or back-to-school periods, resulting in increased order volumes and inventory levels. To coordinate supply between a manufacturer and a retailer, Saha et al. (2023) presented a profit-sharing approach, taking into account the correlation between market demand and a product's price and level of environmental sustainability. Rukonuzzaman et al. (2023) performed a case study on the effects of time on demand for a mango company in Bangladesh. De et al. (2024) explored, to address post-Covid-19 supply chain difficulties, an inventory model for degrading products with stock and price-dependent demand under inflation and partial backlogs. A few noteworthy studies in the area of combined price and time-sensitive demand include Cárdenas-Barrón et al. (2021), San-José et al. (2021), Khan et al. (2023b, 2024b), and others. All of these studies focus on the perspective of a retailer and customer demand rate and do not include supply chain coordination. They examined the best pricing plan and inventory scheduling strategy for the chain.

1.1.4 Literature review on carbon emission tax

Carbon emissions tax regulations have a notable impact on profitability in contemporary manufacturer-retailer supply chain coordination. These tax schemes increase expenses for businesses based on their carbon emissions, motivating corporations to decrease their environmental footprint and shift toward more sustainable methods. Benjaafar et al. (2012) suggested carbon footprints and the management of supply chains. Choi (2013) determined the carbon footprint tax on fashion supply chain systems. Jin et al. (2014) showed the effect of carbon regulations on the logistics and supply chain planning of a large retailer. Fahimnia et al. (2015) studied a case strategic supply chain management under a carbon price policy framework. Yang and Yu (2016) analyzed and optimize the carbonization game in a two-tier supply chain under the carbon tax policy. Wang et al. (2017) proposed government carbon tax choices and supply chain enterprise activities taking carbon emissions into account. Yi and Li (2018) cost-sharing agreements for a supply chain's energy conservation and emissions reduction in exchange for government subsidies and a carbon tax. Using a social cost of carbon for emissions from transportation operations solely, Darom et al. (2018) examined a two-phase supply chain with a producer and a single retailer. Subsequently, Bai et al. (2019) suggested a revenue sharing contract throughout the chain within a carbon cap-and-trade guideline to handle supply coordination for degrading commodities. Wang et al. (2019) studied supply chain emission reduction levels under stochastic demand, using carbon tax and low-carbon preferences of customers. Moreover, Huang et al. (2020) studied how carbon regulations and green equipment impact the overall inventory of a two-level supply coordination, taking carbon emissions from manufacturing, distribution, and warehousing into account and minimized the cost under the considered emissions guidelines. In a stochastic setting with reduced carbon emissions for a mixed production process combining normal and green manufacturing, Jauhari et al. (2021) examined a two-phase inventory framework for closed-loop supply coordination with a producer and a retailer under a carbon tax scheme. Lu et al. (2022) examined the impact that various combinations of emissions laws for various supply chain participants had on the overall profit of the chain collectively in each scenario. In addition, Manna et al. (2022), Das et al. (2022) and Akhtar et al. (2023) investigated the impact of a carbon tax policy with interval uncertainty. Jauhari et al. (2023) introduced a supply chain model involving a single manufacturer and buyer dealing with uncertain demand. Khan et al. (2024c) identified the best investment practices for reducing emissions and ensuring compliance with the carbon tax regulations for livestock production companies. Carbon trading price, carbon tax, and low-carbon product subsidy viewpoints on how carbon policies affect the equilibrium of supply chain networks are proposed by Duan et al. (2024). Eslamipoor and Sepehriyar (2024) encouraged the use of carbon-free supply chains through carbon trading, carbon quotas, and taxes. Song et al. (2024) determined the effects of the carbon tax under port competition on the supply chain for environmentally friendly shipping. The model integrates carbon tax policies to reduce emissions in a hybrid production setup that combines traditional and eco-friendly facilities, aiming to manage increased production expenses while minimizing environmental impacts. One common characteristic of the mentioned studies is that the production rate remains constant, resulting in a consistent level of emissions from manufacturing activities. However, the connection between emissions from a manufacturing system and time-sensitive production rates is complicated. Time-sensitive production rates are the changes in production rates over time, impacted by variables including demand changes, operational limitations, and production scheduling. Increased production rates often result in higher emissions as a result of elevated energy consumption, heightened utilization of raw materials, and increased waste output. On the other hand, decreased production rates might minimize emissions but can cause inefficiencies and higher costs if not controlled correctly. Thus, adjusting production rates to meet demand while reducing emissions is crucial for implementing sustainable and ecologically conscious manufacturing methods. Through manufacturer-retailer supply coordination, this research seeks to promote sustainability and environmental performance by minimizing emissions while meeting demand at a time-sensitive production rate.

1.1.5 Literature review on soft computing

Due to the complex and dynamic nature of inventory control and supply chain problems, soft computing approaches have become more and more important in the investigation of optimum policies for these kinds of problems. While traditional approaches frequently fail to manage the same uncertainties and complexity, these strategies offer durable, adaptable, and adaptive solutions. Using a modified meta-heuristic method to optimize a bi-objective inventory model for a two-tier supply chain was solved by Bakeshlu et al. (2014). Fattahi et al. (2015) solved an inventory control problem for bi-objective continuous review using Pareto-based meta-heuristic algorithms. Sadeghi (2015) introduced a tuned-parameter hybrid meta-heuristic for a multi-item integrated inventory model in two-echelon supply chain management with varying retailer replenishment frequency. Rooeinfar et al. (2016) proposed modeling and optimizing multi-tier supply chain networks using metaheuristic methods and simulation. Kaasgari et al. (2017) established a discount optimization in a vendor managed inventory (VMI) supply chain for perishable goods: two meta-heuristic algorithms that are calibrated. Metaheuristic algorithms are used in a vendor-managed inventory control system for deteriorating products, as solved by Rabbani et al. (2018). Fatemi Ghomi and Asgarian (2019) developed metaheuristic to address the location routing problem of transportation inventory while accounting for missed sales of perishable products. El Raoui et al. (2020) offered a thorough analysis and taxonomy of the benefits of working together when combining simulation, optimization, and soft computing approaches in supply chain systems. Najafnejhad et al. (2021) proposed a VMI policy-based mathematical inventory model for a supply chain with a single vendor and several retailers. Baghizadeh et al. (2022) optimized lowering the risk of supplier interruption by applying four metaheuristic algorithms to a mathematical inventory model for the provision of spare parts. Using meta-heuristic algorithms, a manufacturing inventory model based on warranty policies and carbon emission regulated investments was solved by Manna et al. (2023). Das et al. (2024b) used the TLBO algorithm to construct the best-finding policies of an inventory model for green manufacturing with price and green index dependent demand under various payment schemes. Metaheuristic algorithms are used to price, prepay, and preserve an inventory model with degradation optimized by Jain and Singh (2024).

Within the framework of coordinating the supply chain between manufacturers and retailers, the demand patterns of both the retailer and consumer fluctuate over time and in reaction to price fluctuations. Retailers undergo demand fluctuations throughout time, which are impacted by variables like seasonality, market trends, and promotional activity. Temporal variables like holidays or special events may magnify or diminish these fluctuations, affecting the retailer's purchasing habits and inventory management tactics. On the other hand, the purchasing patterns of customers are significantly influenced by changes in prices. Fluctuations in pricing prompt consumers to reevaluate their inclination to make purchases, since higher prices often result in reduced demand, while lower prices tend to stimulate demand. The interaction between the fluctuating demand from customers over time and the demand sensitivity to prices highlights the complex dynamics involved in coordinating the supply chain between manufacturers and retailers. This calls for sophisticated strategies to optimize inventory levels, pricing choices, and the overall performance of the supply chain. Therefore, the main goal of this study is to provide concrete recommendations for businesses to improve manufacturer-retailer coordination in the face of fluctuating demand and emissions tax regulations, using a combination of quantitative models by using optimal control theory, qualitative assessments, and scenario analyses. This is crucial for achieving sustainable economic growth. This study intends to inspire significant change in the global supply chain ecosystem by showing how profitability and environmental stewardship may work together harmoniously. In Table 1, a comparative analysis of our proposed work is presented.

1.1.6 Research gap and contribution

The comparison table (cf. Table 1) related to the investigated supply chain model reveals several research gaps. It is observed from Table 1 that Manna et al. (2018)

Table 1 Comparative assessment of the current study and related literature

Authors' name	Model type	Demand function depends on	Production rate	Carbon emission	Application of control theory	Decentralize game type	Coordination contract	Solution procedure
Sicilia et al. (2015)	EOQ	Time	×	×	×	×	×	Analytically
Giri & Sarkar (2016)	Supply chain	Service level & price	Constant	×	×	Stackelberg's game approach	×	Analytically
Manna et al. (2018)	Supply chain	Stock	Constant	×	×	×	×	Generalized Reduced Gradient Method
Pervin et al. (2018)	EOQ	Time	×	×	×	×	×	Mathematica software
Bai et al. (2019)	Supply chain	Green level and price	Constant	√	×	Stackelberg's game approach	√	Analytically
Wu (2019)	Supply chain	Platform goodwill, brand goodwill & price	×	×	√	Stackelberg's game approach	√	Analytically
San-José et al. (2020)	EOQ	Time & price	×	×	×	×	×	Analytically
Yu et al. (2020)	Supply chain	Price, promotional effort & emission reduction level	×	√	√	Stackelberg's game approach	√	Analytically
Zu et al. (2021)	Supply chain	Emission reduction level & selling price	×	√	√	Stackelberg's game approach	√	Analytically
Jauhari et al. (2021)	Supply chain	Stochastic	Constant	√	×	Decentralize method	×	Analytically
Das et al. (2021)	Supply chain	Iso-elastic price	×	×	×	Stackelberg's game approach	√	Analytically
Barman et al. (2022)	Supply chain	Price	Constant	×	×	Stackelberg's game approach	×	Analytically
Das et al. (2022)	EPQ	Price & specific absorption rate (SAR)	Unknown function of time	√	√	–	×	*c-r* optimization technique & soft computing
Ghosh et al. (2023)	Supply chain	Stochastic	×	×	×	Decentralize method	×	Analytically
Das et al. (2024a)	EPQ	Price & service level	Unknown function of time	√	√	–	×	*c-r* optimization technique & soft computing
Bera & Giri (2024)	Supply chain	Price, product transparency & traceability level	×	×	√	Nash equilibrium	√	Analytically
This work	**Supply chain**	**Price and time**	**Unknown function of time**	√	√	**Stackelberg's game approach**	√	**Metaheuristic algorithms and Equilibrium Optimizer Algorithm (EOA)**

developed a supply chain model with stock-dependent demand where the production rate is constant. Again, Bai et al. (2019) solved a supply chain problem using Stackelberg's game approach, considering price and green level-dependent demand with a constant production rate. A supply chain considering carbon emissions and stochastic demand was created and solved using the decentralized method by Jauhari et al. (2021). Barman et al. (2022) investigated the optimal policy of a supply chain model under price-dependent demand using Stackelberg's game approach, where the rate of production of the manufacturer is constant. After that, Ghosh et al. (2023) studied a supply chain model with stochastic demand without consideration of carbon emissions or production. Das et al. (2024a) developed a production inventory model considering an unknown function of production rate with price and service level sensitive demand using control theory and metaheuristic algorithms. Bera and Giri (2024) analyzed a supply chain model using control theory and Nash equilibrium with price, product transparency, and traceability level-dependent demand. However, none of these mentioned studies investigates the consequences of an unknown production rate on the coordination of the chain in an economical way. In addition, there are only studies (Das et al., 2022, 2024a) explored the consequences of an unknown production rate only on production systems. There is no study on manufacturer-retailer supply coordination under an unknown production rate to promote both sustainability and environmental performance by minimizing emissions while meeting variable demand.

To address the research gaps identified in the existing literature, a supply chain model is developed that incorporates price- and time-dependent demand under a coordination contract. Moreover, in this study, the production rate of the manufacturer is adopted as an unknown function of time, whereas the per-unit production cost and carbon emission rate of the manufacturing firm are an increasing function of production rate. Due to the unknown functions of production and carbon emission rates, a variational problem arises. To address this issue, control theory is applied. Additionally, in the decentralized scenario, Stackelberg's game approach is utilized. The Equilibrium Optimizer Algorithm (EOA) (Faramarzi et al., 2020) is employed to find the optimal solutions to the formulated optimization problems. To benchmark the results obtained from EOA, we also compare them against five other metaheuristic algorithms, such as,

✶ Artificial electric field algorithm (AEFA) (Yadav, 2019);

✶ Firefly algorithm (FA) (Yang and Yu, 2016);

✶ Grey wolf optimizer algorithm (GWOA) (Mirjalili, 2014);

✶ Sparrow Search Algorithm (SSA) (Xue and Shen, 2020);

✶ The Whale optimizer algorithm (WOA) (Mirjalili and Lewis, 2016);

1.2 Organization of the manuscript

The remaining part of the manuscript is formulated as follows: Section 2 introduces the notation and underlying assumptions. Section 3 provides information on formulating problems and creating mathematical models. In Section 4, an efficient method is described for solving the suggested optimisation problem. Section 5 provides numerical examples, performs statistical analysis, shows the convergence graph of the used algorithms, and shows the concavity of the objective function graphically. In Subsection 5.7, a numerical experiment using a coordination contract is proposed. In Section 6, a sensitivity analysis is performed and shown graphically. In Section 7, the research conclusion is described along with managerial implications for building a robust and successful supply chain.

2 Notations and assumptions

The subsequent notations and assumptions are used to formulate a two-layer supply chain model.

2.1 Notations

The notations employed through the study is summarized in Table 2.

Table 2 List of notations

Varible	Description
$x_m(t)$	Manufacturer's stock level at time t (units)
$x_r(t)$	Retailer's stock level at time t (units)
$D_r(p_m,t)$	Demand of retailer to the manufacturer (unit/time)
$D_c(p_r)$	Demand of customer to the retailer (unit/time)
$P(t)$	Production rate at time t (unit/time)
a_m,b_m,c_m,a_c,b_c	Demand parameters
h_{cm}	Holding cost per unit per unit of time for the manufacturer ($)
h_{cr}	Holding cost per unit per unit of time for the retailer ($)
λ_m	Fixed initial cost of per unit production ($)
μ_m	Per unit production cost parameter
e_m	Per unit tax for carbon emission ($)
a,b	Parameters of carbon emission rate
ϕ_m	Fraction of emission tax paid by retailer
ϕ_r	Fraction of total sales revenue given by manufacturer to the retailer
$C_p(P)$	Per unit production cost ($)
$C_{em}(P)$	Carbon emission rate
A_m	Manufacturer's setup cost/cycle ($)
A_r	Retailer's setup cost/cycle ($)
SR_m	Sales revenue of the manufacturer ($)
HC_m	Total holding cost of manufacturer ($)
PC_m	Total production cost of manufacturer ($)
$ETax_m$	Total carbon emission tax of the manufacturer due to production ($)
SR_r	Sales revenue of the retailer ($)
HC_r	Total holding cost of retailer ($)
PC_r	Total purchasing cost of retailer ($)
T	Business cycle length (in month)
$\pi_m^{co}(t_m)$	Manufacturer's average profit function under coordination contract ($)
$\pi_r^{co}(t_m,p_r)$	Retailer's average profit function under coordination contract ($)
$\pi_{sc}^{co}(t_m,p_r)$	Integrated profit function under coordination contract ($)
$\pi_{sc}(t_m,p_r,T)$	Average profit function of the integrated supply chain system ($)
$\pi_m(t_m)$	Average profit function of the manufacturer's ($)
$\pi_r(p_r,t_m,T)$	Average profit function of the retailer's ($)
p_r	Retailer's selling price/unit ($)
t_m	Manufacturer's production period (in month)

2.2 Assumptions

(i) A supply chain consisting of a single manufacturer and single retailer, where manufacturer produced single type of product with finite time horizon.

(ii) Production rate of the manufacturer $P(t)$ is an unknown function of time.

(iii) Per unit production cost of the produced items is a linear function of production rate $P(t)$, which is denoted as $C_p(P(t)) = \lambda_m + \mu_m P(t)$, where $\lambda_m, \mu_m (>0)$.

(iv) The rate of demand for the retailer (D_r) is a linear function of time (t) and selling price (p_m) of the manufacturer i.e., $D_r = a_m - b_m p_m + c_m t$ (where primary demand $a_m > 0$ and price elasticity $b_m > 0$, $a_m >> b_m$ and $c_m > 0$, $D_r > 0$) are present. The customer demand rate (D_c) to the retailer is only dependent on selling price (p_r) of the retailer i.e., $D_c = a_c - b_c p_r$ (where primary demand $a_c > 0$ and price elasticity $b_c > 0$, $a_c >> b_c$ and $D_c > 0$). When the selling price increases, both the retailer's and the customer's demand rates decline, and vice versa. Additionally, the retailer's demand rate is significantly higher than that of the clients i.e., $D_r > D_c$.

(v) Carbon emission rate is linearly dependent on the production rate $P(t)$ (Das et al., 2023), i.e., $C_{em}(P(t)) = b + aP(t)$, $a > 0$ and $b > 0$.

(vi) Supply chain management takes into account the combined impact of the manufacturer and the retailer.

(vii) Lead time is negligible, and shortages are not allowed in this two-layer supply chain model.

3 Formulating problems and creating mathematical models

In this work, a two-layer supply chain model is established wherein the manufacturer makes the product and then the retailer sells the product to customers. Here, the manufacturer produces an item with a production rate $P(t)$, which is unknown function of time. The production period of the manufacturer is $[0, t_m]$. The manufacturer offers his or her merchants a set price (p_m) for the manufactured goods. Once more, the retailer offers the products to customers at the price (p_r) during $[0, T]$. Additionally, the manufacturer must pay a carbon tax amount that varies based on the firm's production rate $P(t)$. The manufacturer faces a price- and time-dependent demand from the retailer and supplies the completed goods to the retailer. The retailer interacts directly with customers, selling them products based on the price-dependent demand rate seen in Fig. 2. The manufacturer's profit function has been determined after the retailer's profit functions have been developed in this part. Fig. 3 depicts the inventory in visual form.

In Section 3.1, the manufacturer's inventory system is covered. Section 3.2 covered the retailer's inventory system, while Section 3.3 covered the supply chain system's combined profit.

3.1 Model for manufacturer

According to the suggested model, up to production run time t_m, the manufacturer creates the final goods at a rate of $P(t)$. Up to time t_m, the producer supplies the retailer's demand at a rate $D_r = a_m - b_m p_m + c_m t$. The final product inventory accumulates at a rate of $P(t) - D_r$ during the production run period $(0, t_m)$. The following equations depict the manufacturer's inventory system at time t.

$$\frac{\mathrm{d}x_m(t)}{\mathrm{d}t} (= \dot{x}_m(t)) = P(t) - D_r(p_m, t), \tag{1}$$

subject to

$$x_m(0) = 0 \text{ and } x_m(t_m) = 0. \tag{2}$$

The system's various expenses and revenues are calculated in the manner described below:

Fig. 2 Flow of supply chain system.

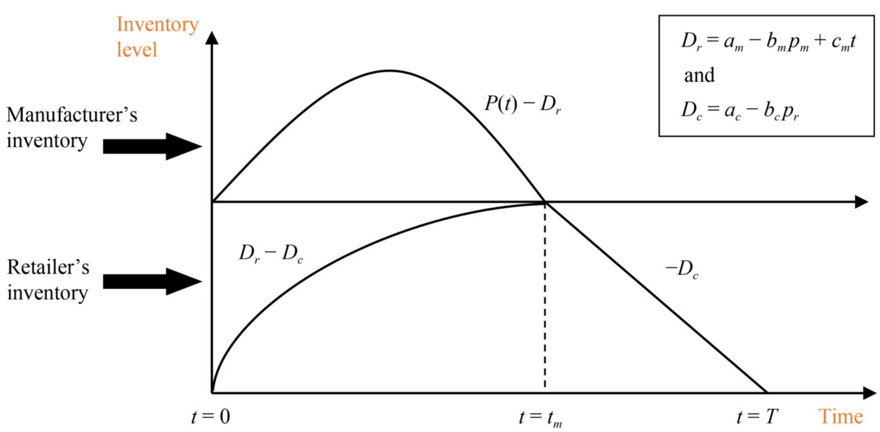

Fig. 3 Logistic diagram of the model.

(i) Total set-up cost for the manufacturer,

$$SCO_m = A_m. \tag{3}$$

(ii) The total cost of production is calculated as:

$$PC_m = \int_0^{t_m} P(t)C_p(P(t))\,\mathrm{d}t = \int_0^{t_m} P(t)\{\lambda_m + \mu_m P(t)\}\,\mathrm{d}t. \tag{4}$$

(iii) Total holding cost is derived as:

$$HC_m = h_{cm}\int_0^{t_m} x_m(t)\,\mathrm{d}t. \tag{5}$$

(iv) Total carbon emission tax by the manufacturer is given by:

$$ETax_m = e_m\int_0^{t_m} C_{em}(P(t))\,\mathrm{d}t = e_m\int_0^{t_m}\{b + aP(t)\}\,\mathrm{d}t. \tag{6}$$

(v) The entire sales revenue is calculated as

$$SR_m = \int_0^{t_m} p_m D_r(p_m, t)\,\mathrm{d}t. \tag{7}$$

Therefore, using Eqs. (3)–(7), the average profit of manufacturer is computed as follows:

$$\pi_m = \frac{1}{t_m}\left(\begin{array}{c} < \text{Sales revenue} > - < \text{Production cost} > - < \text{Set up cost} > \\ - < \text{Carbonemission tax} > - < \text{Holding cost} > \end{array}\right) = \frac{1}{t_m}(SR_m - PC_m - A_m - ETax_m - HC_m). \tag{8}$$

Using Eqs. (3)–(7), Eq. (8) takes the following form:

$$\begin{aligned}\pi_m &= \left(\int_0^{t_m}\frac{1}{t_m}p_m D_r(p_m,t)\,\mathrm{d}t - \int_0^{t_m}\frac{1}{t_m}P(t)\{\lambda_m + \mu_m P(t)\}\,\mathrm{d}t - \frac{1}{t_m}A_m - h_{cm}\int_0^{t_m}\frac{1}{t_m}x_m(t)\,dt - e_m\int_0^{t_m}\frac{1}{t_m}\{b + aP(t)\}\,\mathrm{d}t\right)\\ &= \int_0^{t_m}\frac{1}{t_m}\{p_m(a_m - b_m p_m + c_m t) - P(t)\{\lambda_m + \mu_m P(t)\} - A_m - h_{cm}x_m(t) - e_m\{b + aP(t)\}\}\,\mathrm{d}t.\end{aligned} \tag{9}$$

As a result, the manufacturer's profit-corresponding optimum control problem for Eq. (9) provided by

$$\text{Maximize } \pi_m = \int_0^{t_m}\frac{1}{t_m}\{p_m(a_m - b_m p_m + c_m t) - P(t)\{\lambda_m + \mu_m P(t)\} - A_m - h_{cm}x_m(t) - e_m\{b + aP(t)\}\}\,\mathrm{d}t. \tag{10}$$

subject to the following dynamic relationship between the system's stock level and manufacturing rate:

$$P(t) = \dot{x}_m(t) + D_r(p_m, t), 0 \leqslant t \leqslant t_m. \tag{11}$$

To determine the ideal values for the manufacturer's production time (t_m^*) and maximize the average profit functions (π_m), we must solve the optimum control problems (10) and (11).

3.2 Model for retailer

The retailer receives finished goods from the producer at a demand rate D_r, which persisted until time t_m. Up to time t_m, the retailer's stock builds up with a demand rate $(D_r - D_c)$ as the customers' demand is satisfied at a demand rate of D_c. While the only demand causes the inventory to drop during the $[t_m, T]$ timeframe. Thus, the governed differential equations for the retailer are given by

$$\frac{\mathrm{d}x_r(t)}{\mathrm{d}t}(= \dot{x}_r(t)) = \begin{cases} D_r - D_c, & 0 \leqslant t < t_m, \\ -D_c, & t_m \leqslant t \leqslant T. \end{cases} \tag{12}$$

subject to $x_r(0) = 0, x_r(T) = 0$ and $x_r(t)$ is continuous at

$$t = t_m. \tag{13}$$

Solving Eq. (12) using the boundary Eq. (13), we get the stock level of the retailer given by Eq. (14).

$$x_r(t) = \begin{cases} k_9 t + \frac{1}{2}c_m t_m^2, & 0 \leqslant t < t_m, \\ D_c(T - t), & t_m \leqslant t \leqslant T, \end{cases} \tag{14}$$

where $k_9 = k_1 - a_c + b_c p_r$ and $k_1 = a_m - b_m p_m$.

Also, from the continuity condition at $t = t_m$, we find the closed form of the business total cycle length,

$$T = t_m + \frac{1}{D_c}\left\{k_9 t_m + \frac{1}{2}c_m t_m^2\right\}. \tag{15}$$

Therefore, the system's various expenses and revenues are the follows:

(i) Total set-up cost for the retailer

$$SCO_r = A_r. \tag{16}$$

(ii) The total purchasing cost of the retailer is calculated as follows:

$$\begin{aligned}PC_r &= \int_0^{t_m} p_m D_r(p_m,t)\,\mathrm{d}t = \int_0^{t_m} p_m(a_m - b_m p_m + c_m t)\,\mathrm{d}t\\ &= p_m\left(k_1 t_m + \frac{1}{2}c_m t_m^2\right), \text{ where } k_1 = (a_m - b_m p_m).\end{aligned} \tag{17}$$

(iii) Total holding cost is as follows:

$$HC_r = h_{cr}\int_0^T x_r(t)\,\mathrm{d}t = h_{cr}\left\{\int_0^{t_m} x_r(t)\,dt + \int_{t_m}^T x_r(t)\,\mathrm{d}t\right\}$$
$$= h_{cr}\left\{\frac{1}{2}k_9 t_m^2 + \frac{1}{6}c_m t_m^3 + \frac{1}{2}D_c(T-t_m)^2\right\}. \quad (18)$$

(iv) The entire sales revenue of the retailer is

$$SR_r = \int_0^T p_r D_c(p_r)\,\mathrm{d}t = p_r D_c T. \quad (19)$$

Therefore, using Eqs. (16)–(19), closed form of the average profit of the retailer is computed as follows:

$$\pi_r = \frac{1}{T}\{\text{< Sales revenue >} - \text{< Purchasing cost >} - \text{< Set up cost >} - \text{< Holding cost >}\}$$
$$= \frac{1}{T}(SR_r - PC_r - A_r - HC_r) = \frac{1}{T}\left\{p_r D_c T - p_m\left(k_1 t_m + \frac{1}{2}c_m t_m^2\right) - A_r - h_{cr}\left\{\frac{1}{2}k_9 t_m^2 + \frac{1}{6}c_m t_m^3 + \frac{1}{2}D_c(T-t_m)^2\right\}\right\}, \quad (20)$$

where $k_9 = k_1 - a_c + b_c p_r$ and $k_1 = a_m - b_m p_m$

3.3 Model for centralize system

Now, the integrated supply chain profit using Eqs. (9) and (20) is given by

$$\pi_{sc} = \pi_m + \pi_r = \left[\begin{array}{l} \int_0^{t_m} \frac{1}{t_m}\{p_m(a_m - b_m p_m + c_m t) - P(t)\{\lambda_m + \mu_m P(t)\} - A_m - h_{cm}x_m(t) - e_m\{b + aP(t)\}\}\,\mathrm{d}t + \\ \frac{1}{T}\left\{p_r D_c T - p_m\left(k_1 t_m + \frac{1}{2}c_m t_m^2\right) - A_r - h_{cr}\left\{\frac{1}{2}k_9 t_m^2 + \frac{1}{6}c_m t_m^3 + \frac{1}{2}D_c(T-t_m)^2\right\}\right\} \end{array}\right]. \quad (21)$$

4 Solution methodology

At first, we have to determine the manufacturing rate $P(t)$, and corresponding inventory level $x_m(t)$ at time $t \in [0, t_m]$. Accordingly, in the terms of selling price (p_m), production time (t_m), and cycle length (T), we have to determine the analytic form of average profit (π_m). To serve the purpose, we have to solve the optimal control problem (10).

Now, by introducing a continuous co-state variable $x_m(t)$, the Hamiltonian of the optimal control problems (10) is given by

$$\begin{aligned} H(t, x_m, \dot{x}_m) &= p_m(a_m - b_m p_m + c_m t) - P(t)\{\lambda_m + \mu_m P(t)\} \\ &\quad - A_m - h_{cm}x_m(t) - e_m\{b + aP(t)\} \\ &= p_m(a_m - b_m p_m + c_m t) \\ &\quad - (\lambda_m + ae_m)(\dot{x}_m(t) + D_r(p_m, t)) \\ &\quad - \mu_m(\dot{x}_m(t) + D_r(p_m, t))^2 \\ &\quad - h_{cm}x_m(t) - A_m t_m \end{aligned} \quad (22)$$

Now, in the following theorem is proposed to navigate the production rate $P(t)$ and corresponding inventory level $x_m(t)$ at the time $t \in [0, t_m]$ for the manufacturer.

Theorem 1 The manufacturer production rate $P(t)$ of the system is a strictly increasing function of time.

Proof: Using the Euler-Lagrange's equation in Eq. (22), one finds

$$\frac{\partial H}{\partial x_m} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial H}{\partial \dot{x}_m}\right) = 0 \text{ or, } h_{cm} - 2\mu_m(\ddot{x}_m + c_m) = 0. \quad (23)$$

Using the boundary conditions, $x_m(0) = 0$ and $x_m(t_m) = 0$, the solution of Eq. (23) is

$$x_m(t) = \frac{1}{2}\left(c_m - \frac{h_{cm}}{2\mu_m}\right)t(t_m - t), 0 \leqslant t \leqslant t_m. \quad (24)$$

Therefore, from Eq. (24), the production rate of the manufacturer is given by

$$\begin{aligned} P(t) &= \dot{x}_m(t) + D_r(p_m, t) = \dot{x}_m(t) + a_m - b_m p_m + c_m t \\ &= \left(c_m - \frac{h_{cm}}{2\mu_m}\right)\left(\frac{t_m}{2} - t\right) + a_m - b_m p_m + c_m t \\ &= \frac{h_{cm}}{2\mu_m}t + a_m - b_m p_m + \left(c_m - \frac{h_{cm}}{2\mu_m}\right)\frac{t_m}{2} \\ &= At + B(say), \end{aligned} \quad (25)$$

where

$$A = \frac{h_{cm}}{2\mu_m} \text{ and } B = a_m - b_m p_m + \left(c_m - \frac{h_{cm}}{2\mu_m}\right)\frac{t_m}{2}. \quad (26)$$

It is observed from the Eq. (25), that is $P(t)$ a linearly increasing function of time t (as $\frac{h_{cm}}{2\mu_m} > 0$) and the production rate is going to be constant when $\frac{h_{cm}}{2\mu_m} = 0$.

Now, using Eqs. (24)–(26), the different expenses and revenue of the manufacturer are given in the following:

(i) Manufacturer's sales revenue of the system is calculated as follows:

$$SR_m = \int_0^{t_m} p_m D_r(p_m, t)\,\mathrm{d}t = \int_0^{t_m} p_m (a_m - b_m p_m + c_m t)\,\mathrm{d}t$$
$$= p_m\left(k_1 t_m + \frac{1}{2}c_m t_m^2\right), \text{ where } k_1 = (a_m - b_m p_m). \tag{27}$$

(ii) Total production cost of the manufacturer is given by

$$PC_m = \int_0^{t_m} P(t) C_p(P(t))\,\mathrm{d}t = \int_0^{t_m} P(t)\{\lambda_m + \mu_m P(t)\}\,\mathrm{d}t$$
$$= \lambda_m t_m (B + A\frac{t_m}{2}) + \mu_m \frac{t_m}{6}(2A^2 t_m^3 + 3ABt_m + 6B^2). \tag{28}$$

(iii) Total carbon emission tax for the manufacturer

$$ETax_m = e_m \int_0^{t_m} C_{em}(P(t))\,\mathrm{d}t = e_m \int_0^{t_m} \{b + aP(t)\}\,\mathrm{d}t$$
$$= be_m t_m + ae_m t_m (B + A\frac{t_m}{2}). \tag{29}$$

(iv) Total holding cost of the manufacturer

$$HC_m = h_{cm}\int_0^{t_m} x_m(t)\,\mathrm{d}t = \frac{1}{12}h_{cm}\left(c_m - \frac{h_{cm}}{2\mu_m}\right)t_m^3. \tag{30}$$

(v) Total set-up cost for the manufacturer

$$SCO_m = A_m. \tag{31}$$

Therefore, using Eqs. (27)–(31) and (26), the average profit of the manufacturer is

$$\pi_m = \frac{1}{t_m}\left[\begin{array}{l} p_m\left((a_m - b_m p_m)t_m + \frac{1}{2}c_m t_m^2\right) - \lambda_m t_m\left(B + A\frac{t_m}{2}\right) - \mu_m \frac{t_m}{6}(2A^2 t_m^3 + 3ABt_m + 6B^2) \\ -be_m t_m + ae_m t_m\left(B + A\frac{t_m}{2}\right) - \frac{1}{12}h_{cm}\left(c_m - \frac{h_{cm}}{2\mu_m}\right)t_m^3 - A_m \end{array}\right]. \tag{32}$$

Thus, the integrated supply chain profit is as follows:

$$\pi_{sc} = \pi_m + \pi_r = \left[\begin{array}{l} \frac{1}{t_m}\left\{\begin{array}{l} p_m\left((a_m - b_m p_m)t_m + \frac{1}{2}c_m t_m^2\right) - \lambda_m t_m\left(B + A\frac{t_m}{2}\right) - \mu_m \frac{t_m}{6}(2A^2 t_m^3 + 3ABt_m + 6B^2) \\ -be_m t_m + ae_m t_m\left(B + A\frac{t_m}{2}\right) - \frac{1}{12}h_{cm}\left(c_m - \frac{h_{cm}}{2\mu_m}\right)t_m^3 - A_m \end{array}\right\} \\ +\frac{1}{T}\left\{p_r D_c T - p_m\left(k_1 t_m + \frac{1}{2}c_m t_m^2\right) - A_r - h_{cr}\left\{\frac{1}{2}k_9 t_m^2 + \frac{1}{6}c_m t_m^3 + \frac{1}{2}D_c(T - t_m)^2\right\}\right\} \end{array}\right], \tag{33}$$

where $k_9 = k_1 - a_c + b_c p_r$, $k_1 = a_m - b_m p_m$, $A = \dfrac{h_{cm}}{2\mu_m}$ and $B = a_m - b_m p_m + \left(c_m - \dfrac{h_{cm}}{2\mu_m}\right)\dfrac{t_m}{2}$.

Hence, the corresponding optimization problems by using Eqs. (20), (32) and (33) are, respectively,

Problem 1 The optimization problem corresponding to the manufacturer's average profit takes the form

$$\text{Maximize } \pi_m(t_m) = \frac{1}{t_m}(SR_m - PC_m - A_m - ETax_m - HC_m)$$
$$\text{subject to } t_m > 0. \tag{34}$$

Problem 2 The optimization problem corresponding to the retailer's average profit is given by

$$\text{Maximize } \pi_r(p_r, t_m) = \frac{1}{T}(SR_r - PC_r - A_r - HC_r)$$
$$\text{subject to } p_r > 0,\ t_m > 0,\ T > 0. \tag{35}$$

Problem 3 The optimization problem corresponding to integrated supply chain profit is given by

$$\text{Maximize } \pi_{sc}(p_r, t_m) = \pi_m(t_m) + \pi_r(p_r, t_m),$$
$$\text{subject to } p_r > 0,\ t_m,\ T > 0. \tag{36}$$

4.1 Model for coordination contract

One party between the manufacturer and the retailer will be at a disadvantage in traditional supply chain models for centralized systems. Therefore, in this section, model for coordination contract is presented to obtain a win-win solution for both retailer and manufacturer. Suppose the retailer gives the manufacturer ϕ_m of manufacturer total emission tax ($ETax_m$). As a reward, the manufacturer gives the retailer ϕ_r of its total sales revenue (SR_m). Then, the average profits of the manufacturer and retailer is described, respectively, as follows

$$\pi_m^{co}=\frac{1}{t_m}((1-\phi_r)SR_m-PC_m-A_m-(1-\phi_m)ETax_m-HC_m)$$
$$=\frac{1}{t_m}\left[\begin{array}{l}(1-\phi_r)p_m\left((a_m-b_mp_m)t_m+\frac{1}{2}c_mt_m^2\right)-\lambda_mt_m\left(B+A\frac{t_m}{2}\right)-\mu_m\frac{t_m}{6}(2A^2t_m^3+3ABt_m+6B^2)\\-(1-\phi_m)\left\{be_mt_m+ae_mt_m\left(B+A\frac{t_m}{2}\right)\right\}-\frac{1}{12}h_{cm}\left(c_m-\frac{h_{cm}}{2\mu_m}\right)t_m^3-A_m\end{array}\right], \tag{37}$$

and

$$\pi_r^{co}=\frac{1}{T}(SR_r+\phi_rSR_m-PC_r-A_r-\phi_mETax_m-HC_r)$$
$$=\frac{1}{T}\left\{\begin{array}{l}p_rD_cT+\phi_rp_m\left((a_m-b_mp_m)t_m+\frac{1}{2}c_mt_m^2\right)-p_m\left(k_1t_m+\frac{1}{2}c_mt_m^2\right)-A_r\\-\phi_m\left\{be_mt_m+ae_mt_m\left(B+A\frac{t_m}{2}\right)\right\}-h_{cr}\left\{\frac{1}{2}k_9t_m^2+\frac{1}{6}c_mt_m^3+\frac{1}{2}D_c(T-t_m)^2\right\}\end{array}\right\}. \tag{38}$$

Therefore, the corresponding optimization problems for manufacturer and retailer are given, respectively, as follows:

Problem 4 The optimization problem corresponding to the manufacturer's average profit takes the following form by using the coordination contract:

$$\text{Maximize } \pi_m^{co}(t_m)=\frac{1}{t_m}((1-\phi_r)SR_m-PC_m-A_m-(1-\phi_m)ETax_m-HC_m),\quad \text{subject to } t_m>0. \tag{39}$$

Problem 5 The optimization problem corresponding to the retailer's average profit takes the following form by using the coordination contract:

$$\text{Maximize } \pi_r^{co}(p_r,t_m)=\frac{1}{T}(SR_r+\phi_rSR_m-PC_r-A_r-\phi_mETax_m-HC_r),\quad \text{subject to } p_r>0,\ t_m>0,\ T>0. \tag{40}$$

Problem 6. The optimization problem corresponding to the integrated profit takes the following form by using the coordination contract:

$$\text{Maximize } \pi_{sc}^{co}(p_r,t_m)=\pi_m^{co}(t_m)+\pi_r^{co}(p_r,t_m),\quad \text{subject to } p_r>0,\ t_m>0,\ T>0. \tag{41}$$

4.2 Motivation for solving optimization problems, which correspond to the Centralize and Decentralize scenarios, using a metaheuristic approach

Using the Stackelberg game approach, in our proposed supply chain model, two decentralized scenarios have appeared. In the first scenario, the manufacturer acts as the leader, while the retailer takes on the role of the follower. First, the retailer evaluates his/her profit function (π_r) with respect to the retail price (p_r) and production time (t_m). Then, the manufacturer optimizes his/her profit function (π_m) for any given value of (p_r) and (t_m) by optimizing the profit function of the retailer. In the second scenario, we considered the retailer as the leader and the manufacturer as the follower. In this case, the manufacturer maximizes his/her profit function (π_m) with respect to the decision variable production time (t_m). After that, the retailer maximizes his/her profit function (π_r) with respect to the variable retail price (p_r) for given value of production time (t_m) by optimizing the profit function of the manufacturer. Both decentralized scenarios have optimal results, which are discussed in sub-section 5.2.

Upon careful examination of the optimization problems associated with the decentralize and centralize systems, it becomes evident that the objective functions in all three problems exhibit a significant degree of nonlinearity with respect to the decision variables, namely the retailer's selling price (p_r) and production time (t_m). In this instance, to address these highly nonlinear optimization problems, we have explored the equilibrium optimizer algorithm (EOA) (Faramarzi et al., 2020).

The reasons for choosing EOA to solve the optimization Problems 1-6 corresponding to Centralize and Decentralize scenarios are as follows:

(i) No one has used EOA to address optimal control issues in the supply chain.

(ii) This is a new and highly effective metaheuristic algorithm.

(iii) Because it is predicated on the equilibrium of a forced- orientated system, EOA differs from other metaheuristics.

Note that a brief description of EOA is provided in the Appendix section.

Furthermore, five more well-known and reputable metaheuristics are concurrently used to provide a numerical solution of optimization Problems 1–6 and assess the efficacy of EOA in solving them numerically. The following list of metaheuristics includes them:

☆ Artificial electric field algorithm (AEFA) (Yadav, 2019);

☆ Firefly algorithm (FA);

☆ Grey wolf optimizer algorithm (GWOA) (Mirjalili, 2014);

☆ Sparrow Search Algorithm (SSA) (Xue and Shen, 2020);

☆ The Whale optimizer algorithm (WOA) (Mirjalili and Lewis, 2016);

5 Numerical analyses for the scenario without a coordination contract

To substantiate the model in numerical format, we have utilized the numerical illustration. In the given numerical illustration, the hypothetical input data for different inventory parameters is obtained. To address the given problem, we utilize both the Decentralize and Centralize models. We employ six metaheuristic algorithms, namely EOA, AEFA, FA, GWOA, SSA, and WOA, which were previously mentioned. These algorithms are implemented in MATLAB software on a laptop equipped with an 11th generation, 2.40 GHz Intel Core i-5 processor, running on the Windows 11.1 operating system. After carefully examining the stability of the optimization results for Problems 1-6 using various algorithms (namely EOA, FA, AEFA, GWOA, SSA, and WOA), a population size of 50 (pop_size) and a maximum generation of 500 (Max_gen) were chosen for each algorithm.

Fifty iterations are conducted independently for each algorithm to acquire the optimal and suboptimal solutions for both scenarios. Separate tables showcase the optimal solutions, and the least favorable solutions obtained from various algorithms (namely EOA, FA, AEFA, GWOA, SSA, and WOA) for both the Decentralize and Centralize cases in the given example. Moreover, the efficiency of the aforementioned metaheuristic algorithms (EOA, FA, AEFA, GWOA, SSA, and WOA) in solving Example 1 is assessed through statistical experimentation, analysis of variance (ANOVA), and convergence graphs.

Example 1 The model's parameters are regarded as the values:

a_m=220; b_m=2.5; c_m=30; a_c=190; b_c=1.6; h_{cm}=1.5; h_{cr}=0.4; λ_m=10; μ_m=1.2; a=3.1; b=3.5; e_m=\$1; A_m=\$500; A_r=\$250; p_m=\$45;

Now, it is necessary to ascertain the optimal price (p_r^*), production time (t_m^*), and business period (T^*) to achieve the highest average profit. Additionally, it is crucial to determine the corresponding optimal value of the system.

Solution: Here, Example 1 is considered for Problems 1–3 and is solved for the respective scenarios of centralization and decentralization.

5.1 Centralize scenario

Table 3 displays the best-found solutions for centralised system, corresponding to Example 1 of Problem 3 (i.e., equation no. 36), while Table 4 presents the worst favorable outcomes encountered. Additionally, the outcomes derived from the statistical investigation are showcased in Table 5.

Table 3 Best-found result for centralize system corresponding to Example 1 of Problem 3 obtained from various metaheuristic algorithms

Algorithm	π_{sc}^* (\$/month)	π_m^* (\$/month)	π_r^* (\$/month)	p_r^* (\$/unit)	t_m^* (month)	T^* (month)	Run time (second)
EOA	6572.534995	4495.864647	2076.670349	81.64728	3.820031	10.60473	0.064629
AEFA	6572.534994	4495.865277	2076.669718	81.647573	3.82006	10.604921	0.133425
FA	6572.534995	4495.864866	2076.670130	81.647222	3.820041	10.604751	0.101156
GWOA	6572.534961	4495.87714	2076.657821	81.646862	3.820602	10.606745	0.093878
SSA	6572.534995	4495.864647	2076.670349	81.647281	3.820031	10.604730	0.069970
WOA	6572.534994	4495.865091	2076.669905	81.647173	3.820051	10.604775	0.073121

Table 4 Worst-found results for centralize system corresponding to Example 1 of Problem 3 obtained from various metaheuristic algorithms

Algorithm	π_{sc}^* (\$/month)	π_m^* (\$/month)	π_r^* (\$/month)	p_r^* (\$/unit)	t_m^* (month)	T^* (month)	Run time (second)
EOA	6572.534995	4495.864648	2076.670348	81.64728	3.820031	10.60473	0.086774
AEFA	6572.529314	4496.013549	2076.515766	81.664959	3.827034	10.636009	0.119223
FA	6572.534995	4495.864617	2076.670378	81.647187	3.820030	10.604698	0.113295
GWOA	6572.529409	4495.757733	2076.771676	81.693047	3.815264	10.599976	0.066149
SSA	6572.534995	4495.864645	2076.670350	81.647280	3.820031	10.604729	0.087430
WOA	6572.534468	4495.818202	2076.716266	81.654303	3.817936	10.598897	0.051528

Table 5 Statistical experiment results of total integrated supply chain profit (π_{sc}^*) obtained from various metaheuristic algorithms

Algorithm	Best-found π_{sc}^*	Worst-found π_{sc}^*	Mean of π_{sc}^*	Median of π_{sc}^*	Mode of π_{sc}^*	Standard deviation
EOA	6572.534995	6572.534995	6572.534995	6572.534995	6572.534995	4.59364×10^{-12}
AEFA	6572.534995	6572.529314	6572.533977	6572.534626	6572.534973	0.001415
FA	6572.534995	6572.534995	6572.534995	6572.534995	6572.534995	4.59364×10^{-12}
GWOA	6572.534961	6572.529409	6572.533056	6572.533370	–	0.001458
SSA	6572.534995	6572.534995	6572.534995	6572.534995	6572.534995	4.59364×10^{-12}
WOA	6572.534994	6572.534468	6572.534873	6572.534923	6572.534994	0.000127

5.1.1 Observations and discussions

From Tables 3–5, the following implications are observed:

□ Table 3 shows that the best-found average profit value for Example 1’s centralised situation (i.e., π_{sc}^*) accords with the results of EOA, FA, and SSA, on the other hand, it differs from the results of the AEFA, WOA, and GWOA algorithms.

□ Once more, Tables 3 and 4 show that the best- and worst-found values for the information gleaned from EOA, FA, and SSA correspond. Furthermore, the best-found value of π_{sc}^* is executed with the least amount of processing time required by EOA.

□ Again, it is evident from the statistical experiment (see Table 5) that the EOA, FA, and SSA standard deviations throughout the computation of the solution for the Centralize case of Example 1 are the same and smallest.

□ As a result, when it comes to finding the answer for the Centralize instance of Example 1, EOA, FA, and SSA are all equally efficient.

5.2 Decentralize scenario

In this section, we have discussed the best and worst found results for the decentralized system. For the decentralize process, two cases appear. In the first case, the manufacturer takes the leader role, whereas the retailer plays the follower role. For this case, the best found and worst found results are shown in table form in Section 5.2.1. And another case is when the retailer plays as the leader and the manufacturer as the follower, and the corresponding results are shown in table form in Section 5.2.2.

Finally, we compare the results of centralize profit of the supply chain system for the centralised scenario and the decentralised scenario, which are shown in Table 10 in Section 5.3.

5.2.1 Results when the manufacturer is the leader

Table 6 displays the best-found solutions for the decentralized system (when the manufacturer is the leader), corresponding to Example 1 of Problem 1 (i.e., equation no. 34), while Table 7 presents the worst favorable outcomes encountered.

5.2.2 Results when the retailer is the leader

Table 8 displays the best-found solutions for the decentralized system (when the retailer is the leader), corresponding to Example 1 of Problem 2 (i.e., equation no. 35), while Table 9 presents the worst found results.

5.3 Comparing results

Here, we compare the best-found results of the outcomes from EOA for both centralize and decentralize scenarios. It shows that the integrated profit is higher in centralize system than in both cases of the decentralize system, which is shown in Table 10 as follows:

5.4 Analysis of variance (ANOVA)

It is evident from the statistical experiment for the centralize instance of Example 1 corresponding to Problem 3 that the metaheuristic algorithms EOA, FA, and

Table 6 Best-found of Example 1, when the manufacturer is the leader, obtained from different metaheuristic algorithms

Algorithm	π_m^* ($/month)	π_r^* ($/month)	$\pi_{sc}^* (=\pi_m^*+\pi_r^*)$ ($/month)	p_r^* ($/unit)	t_m^* (month)	T^* (month)	Run time (second)
EOA	4497.086053	2074.160925	6571.246978	81.640488	3.931427	11.022610	0.051953
AEFA	4497.08206	2074.306516	6571.388576	81.594779	3.925050	10.984880	0.101471
FA	4497.086053	2074.151944	6571.237997	81.702223	3.931431	11.041000	0.057066
GWOA	4497.086053	2074.002216	6571.088269	81.942669	3.931384	11.112950	0.048971
SSA	4497.086053	2073.291242	6570.377295	80.889035	3.931427	10.803840	0.055489
WOA	4497.086053	2074.154803	6571.240856	81.689882	3.931426	11.037304	0.046296

Table 7 Worst-found results of Example 1, when the manufacturer is the leader, obtained from different metaheuristic algorithms

Algorithm	π_m^* (\$/month)	π_r^* (\$/month)	$\pi_{sc}^* (= \pi_m^* + \pi_r^*)$ (\$/month)	p_r^* (\$/unit)	t_m^* (month)	T^* (month)	Run time (second)
EOA	4497.086053	1857.883085	6354.969138	70	3.931427	8.390642	0.056892
AEFA	4497.074921	1880.402373	6377.477294	70.616929	3.920780	8.467043	0.100767
FA	4497.086053	2028.069446	6525.155499	76.259185	3.931432	9.626657	0.057284
GWOA	4497.086051	1857.880479	6354.966530	70	3.931588	8.391108	0.050369
SSA	4497.086053	1870.833707	6367.91976	70.353465	3.931427	8.451924	0.061712
WOA	4497.086053	1857.883123	6354.969176	70	3.931424	8.390636	0.041681

Table 8 Best-found of Example 1, when the retailer is the leader, obtained from different metaheuristic algorithms

Algorithm	π_r^* (\$/month)	π_m^* (\$/month)	$\pi_{sc}^* (= \pi_m^* + \pi_r^*)$ (\$/month)	p_r^* (\$/unit)	t_m^* (month)	T^* (month)	Run time (second)
EOA	2094.487039	4286.244141	6380.731180	81.948276	2.5	6.156301	0.045874
AEFA	2094.487038	4286.244237	6380.731275	81.947665	2.5	6.156200	0.078295
FA	2094.487039	4286.244141	6380.731180	81.948236	2.5	6.156295	0.048637
GWOA	2094.487039	4286.244141	6380.73118	81.948375	2.5	6.156318	0.039643
SSA	2094.487039	4286.244141	6380.73118	81.948276	2.5	6.156301	0.043807
WOA	2094.451375	4290.249303	6384.700678	81.925365	2.513193	6.193379	0.035349

Table 9 Worst-found results of Example 1, when the retailer is the leader, obtained from different metaheuristic algorithms

Algorithm	π_r^* (\$/month)	π_m^* (\$/month)	$\pi_{sc}^* (= \pi_m^* + \pi_r^*)$ (\$/month)	p_r^* (\$/unit)	t_m^* (month)	T^* (month)	Run time (second)
EOA	2094.487039	4286.244141	6380.73118	81.948276	2.5	6.156301	0.048059
AEFA	2094.487004	4286.244141	6380.731145	81.943561	2.5	6.155513	0.077540
FA	2094.487039	4286.244141	6380.73118	81.948244	2.5	6.156296	0.101961
GWOA	2094.486861	4286.244141	6380.731002	81.958822	2.5	6.158066	0.039015
SSA	2094.487039	4286.244141	6380.731180	81.948276	2.5	6.156301	0.045309
WOA	2094.485182	4286.244141	6380.729323	81.914204	2.5	6.150607	0.034956

Table 10 Comparing the results from Centralize & Decentralize scenarios

Optimal values	Centralize scenario	Decentralize scenario	
		Manufacturer as leader	Retailer as leader
Integrated total profit (π_{sc}) (\$)	**6572.534995**	6571.246978	6380.73118
Retail price (p_r) (\$)	81.64728	81.640488	81.948276
Production time (t_m) (months)	3.820	3.931427	2.5
Business period (T) (months)	10.60473	11.02261	6.156301

SSA all perform comparably. Nevertheless, the analysis of variance (ANOVA) is carried out for the centralize system of Example 1 in order to evaluate the importance of EOA and the other five metaheuristic algorithms in solving it. EOA is used as the controlling algorithm when doing an ANOVA test, and Table 11 only provides the findings for the centralize case.

It is evident from Table 11 that the F-static values for FA and SSA are lower than the F-critical values. Therefore, the null hypothesis for FA & SSA is accepted. Further, the P-value due to SSA and FA is 1, which is more than 0.05. Therefore, due to the ANOVA test, EOA, SSA and FA perform equally during the computation of the solution of Example 1 for the centralised system.

Once more, F-static values for GWOA, AEFA, and WOA are higher than F-critical values. For AEFA, GWOA, and WOA, the null hypothesis is thus accepted. Further, P-values due to WOA, AEFA, and WOA are less than 0.05. As a result, compared to AEFA, WOA, and GWOA, EOA performs substantially more efficiently in computing Example 1 at the 5% level of significance.

Table 11 ANOVA results for Centralize system of Example 1

EOA vs	Count	Mean of Centralize profit	Variance	Source of Variation						F-static	F-crit	P-value
				Between Groups			Within Group					
				SS	*df*	*MS*	*SS*	*df*	*MS*			
AEFA	50	6572.533977	2.003×10^{-6}	2.59×10^{-5}	1	2.59×10^{-5}	9.81×10^{-5}	98	1.001×10^{-6}	25.88	3.93	1.74×10^{-6}
FA	50	6572.534995	2.11×10^{-23}	0	1	0	2.06×10^{-21}	98	2.11×10^{-23}	0	3.93	1
GWOA	50	6572.533056	2.12×10^{-6}	9.4×10^{-5}	1	9.4×10^{-5}	0.000198	98	1.06×10^{-6}	88.46	3.93	2.36×10^{-15}
SSA	50	6572.534995	2.11×10^{-23}	0	1	0	2.06×10^{-21}	98	2.11×10^{-23}	0	3.93	1
WOA	50	6572.534873	1.62×10^{-8}	3.7×10^{-7}	1	3.7×10^{-7}	7.94×10^{-7}	98	8.1×10^{-9}	45.73	3.93	9.85×10^{-10}

5.5 Convergence graph

Fig. 4 illustrates the convergence of the six metaheuristic algorithms under consideration (EOA, FA, AEFA, SSA, WOA, and GWOA) in solving the centralize system of Example 1.

From Fig. 4, it can be observed that EOA, AEFA, SSA, and FA algorithms converge to the solution 6572.534995 of π_{sc} after a certain iteration, but EOA more quickly converges to the optimal solution.

5.6 Concavity figures

The concavity of π_{sc}, π_m, and π_r with respect to the decision variables p_r and t_m for both centralized and decentralized systems is shown graphically in Figs. 5–8.

5.7 Numerical experiment using coordination contract

To validate the model using a coordination contract, another example is considered, and the best-found solutions for both centralize and decentralize scenarios are computed.

Example 2 The model's parameters are regarded as the values: a_m=220; b_m=3.5; c_m=5; a_c=210; b_c=1.5; h_{cm}=\$1.5; h_{cr}=\$0.8; λ_m=\$5; μ_m=1.2; a=3.1; b=3.5; e_m=\$1; A_m=\$500; A_r=\$250; p_m=\$45;

Solution: Here, Example 2 is considered for Problems 4–6 and is solved for the respective scenarios of centralization (under coordination) and decentralization (without coordination).

The optimal solution to Problems 4–6 (Eqs. (39) and (40)) for Example 2, under decentralize model is displayed in Table 11. Furthermore, for the coordination contract, we have considered the values of two parameters $\phi_m = 0.2$ and $\phi_m = 0.1$ associated with centralize coordination contract. The best-found solution for this scenario is also presented in Table 12.

It is observed from Table 12 that integrated supply chain profit under coordination contract is greater than the centralize profit under decentralized scenario without using a coordination contract. In addition, we have seen that in the coordination contract, a win-win outcome is attained for both the manufacturer and the retailer.

To investigate the impact of coordination parameters ϕ_m and ϕ_r on optimal decisions and the profits of the retailer, manufacturer, and the entire supply chain, a sensitivity analysis is conducted under the coordination

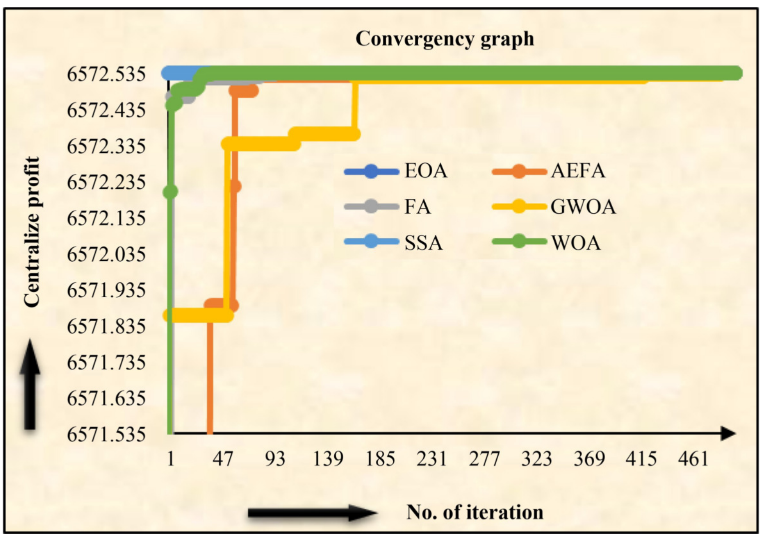

Fig. 4 Convergence graphs of different algorithms for the centralize scenario of Example 1.

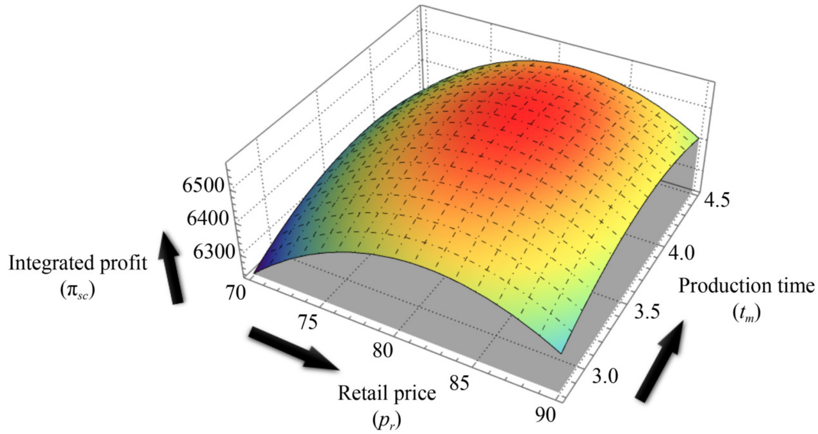

Fig. 5 Concavity of integrated supply chain profit for centralized system.

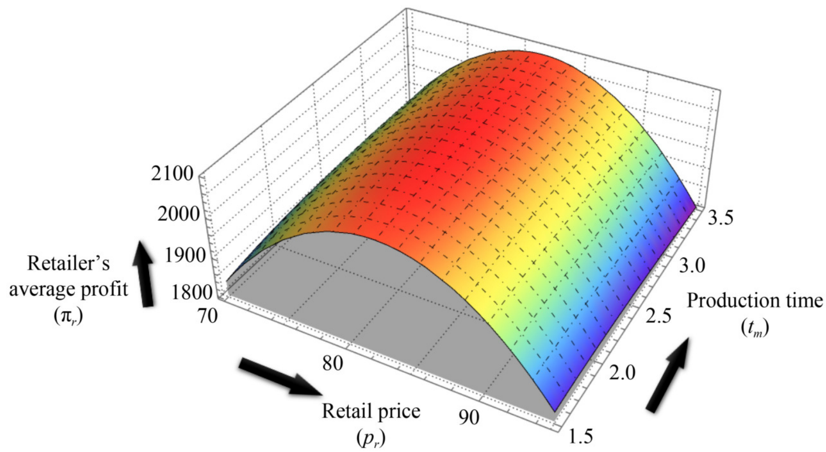

Fig. 6 Concavity of the retailer's average profit for decentralized system (where the retailer is the leader).

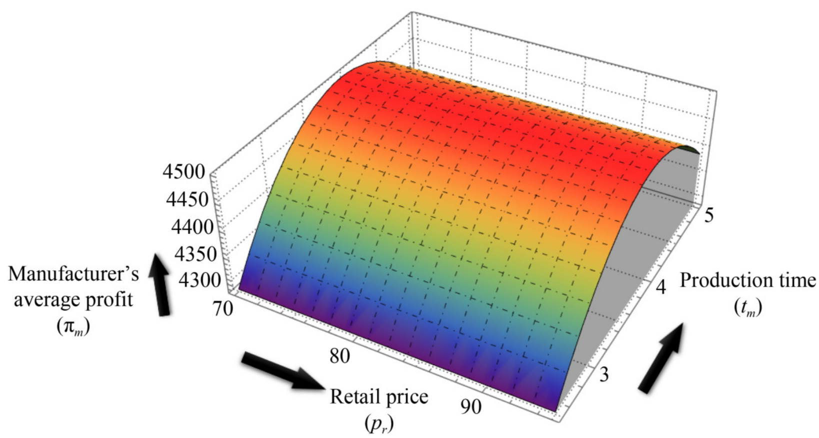

Fig. 7 Concavity of the manufacturer's average profit for decentralized system (where the manufacturer is the leader).

contract, with the numerical results presented in Table 13.

From Table 13, it is straightforwardly observed that, for the value of $\phi_r \leqslant 0.03$ when $\phi_m = 0.2$ and $\phi_r \geqslant 0.15$ when $\phi_m = 0.2$, achieving a win-win solution is not feasible. In all other cases, the coordination contract results in a win-win outcome.

6 Sensitivity experiment

Using the Centralize scenario of the numerical Example 1, a sensitivity experiment is conducted to investigate the effects of different input parameters on the integrated profit (π_{sc}), retail pricing (p_r), production time (t_m), and

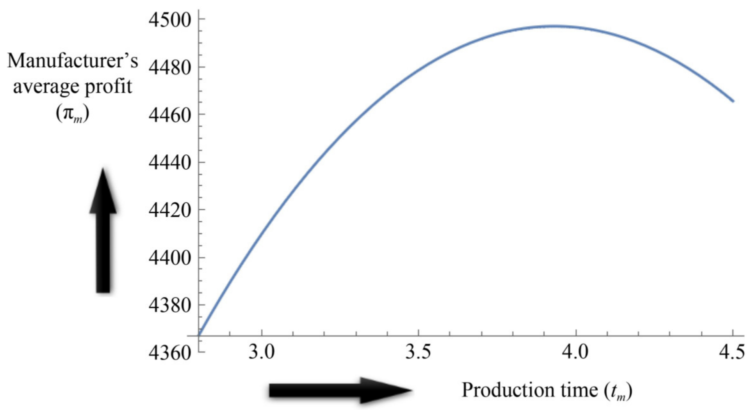

Fig. 8 Concavity of the manufacturer's average profit with respect to the production time (t_m).

Table 12 Comparing the best-found results from Centralize (under coordination) & Decentralize scenarios (without coordination) corresponding to Example 2

Objective functions	Coordination contract	Without coordination
	Centralize scenario	Decentralize scenario
Integrated supply chain profit (π_{sc}^{co}) ($)	7024.368275	6672.157703
Manufacturer's profit(π_m^{co}) ($)	**3442.598395**	3327.081056
Retailer's profit(π_r^{co}) ($)	**3581.769880**	3345.076646

Table 13 Results of sensitivity experiment for centralize (coordination contract) and decentralize (without coordination) scenarios of Example 2

Model	p_r^* ($)	t_m^* (month)	T^* (month)	Manufacturer average profit	Retailer average profit	Supply chain profit
Decentralize (Without coordination)	91.241806	6.922034	7.553104	**3345.076646**	**3327.081056**	6672.157703
Centralize (Coordination)	86.467426	17.048011	22.317718	**3442.598395**	**3581.769880**	7024.368275
Impact of coordination parameters ϕ_m and ϕ_r **on Centralize scenario's optimal results**						
$\phi_m = 0.1$; $\phi_r = 0.1$;	86.459392	17.043976	22.306947	3440.621437	3583.298385	7023.919822
$\phi_m = 0.15$; $\phi_r = 0.1$;	86.46341	17.045994	22.312334	3441.60994	3582.534064	7024.144004
$\phi_m = 0.2$; $\phi_r = 0.1$; **(Main Result)**	**86.467426**	**17.048011**	**22.317718**	**3442.598395**	**3581.769880**	**7024.368275**
$\phi_m = 0.3$; $\phi_r = 0.1$;	86.475457	17.052042	22.328486	3444.575201	3580.241885	7024.817086
$\phi_m = 0.4$; $\phi_r = 0.1$;	86.483487	17.056072	22.339255	3446.551896	3578.714357	7025.266254
$\phi_r = 0.01$; $\phi_m = 0.2$;	88.179148	18.303573	25.491994	3900.953163	3235.490969	7136.444131
$\phi_r = 0.03$; $\phi_m = 0.2$;	87.798986	18.023883	24.758697	3797.579234	3311.961319	7109.540554
$\phi_r = 0.05$; $\phi_m = 0.2$;	87.418703	17.744553	24.041597	3695.073117	3388.706061	7083.779178
$\phi_r = 0.1$; $\phi_m = 0.2$; **(Main result)**	**86.467426**	**17.048011**	**22.317718**	**3442.598395**	**3581.769880**	**7024.368275**
$\phi_r = 0.15$; $\phi_m = 0.2$;	85.515233	16.354464	20.688659	3195.524675	3776.554679	6972.079354
$\phi_r = 0.2$; $\phi_m = 0.2$;	84.561987	15.664577	19.150319	2953.830456	3973.066612	6926.897069
$\phi_r = 0.3$; $\phi_m = 0.2$;	82.651619	14.299198	16.331382	2486.464179	4371.307986	6857.772165

Note: " Green color" results indicates that the coordination contracts' result greater than the decentralize (without coordination) result for the manufacturer and the retailer. "Red color" results indicates that the coordination contracts' result less than the decentralize (without coordination) result for the manufacturer and the retailer.

business length (T). To conduct this experiment, one parameter will be changed at a time, from −20% to 20%, while the remaining parameters will remain at their initial values. Table 14 displays the experiment's collected findings.

Additionally, a sensitivity analysis is conducted for the emission parameters of the supply chain, as presented in Table 15.

6.1 Graphical representation of sensitivity

In this section, we show the impact of inventory parameters

Table 14 Results of sensitivity experiment for centralised system of Example 1

Parameters	Parameters change (%)	Change in decision variables and total profit (%)			
		p_r	t_m	T	π_{sc}
a_m	20	0.051	1.674	−24.796	−29.709
	10	0.022	0.791	−12.346	−14.848
	−10	−0.016	−0.732	12.227	14.841
	−20	−0.029	−1.425	24.326	29.677
b_m	20	−0.016	−0.748	12.503	15.178
	10	−0.009	−0.380	6.268	7.590
	−10	0.010	0.395	−6.299	−7.592
	−20	0.022	0.810	−12.628	−15.186
c_m	20	−0.123	17.800	15.129	7.277
	10	−0.074	10.055	9.514	3.832
	−10	0.064	−8.305	−7.899	−4.179
	−20	0.122	−15.282	−14.564	−8.701
μ_m	20	−0.149	17.800	24.683	8.115
	10	−0.082	9.616	13.075	3.834
	−10	0.072	−7.858	−10.233	−3.484
	−20	0.136	−14.392	−18.433	−6.704
a_c	20	−14.265	−0.926	46.563	−18.018
	10	−7.268	−0.453	18.338	−9.867
	−10	7.268	0.451	−13.277	11.584
	−20	10.230	1.164	−28.217	24.585
h_{cm}	20	0.00012	−0.014	−0.018	−0.003
	10	0	−0.007	−0.009	−0.002
	−10	0	0.008	0.010	0.002
	−20	0.00015	0.017	0.022	0.004
h_{cr}	20	0.099	0.761	1.247	0.231
	10	0.049	0.379	0.620	0.115
	−10	−0.049	−0.376	−0.613	−0.114
	−20	−0.097	−0.749	−1.218	−0.228
b_c	20	10.230	0.731	−19.853	18.221
	10	8.079	0.169	−5.450	8.328
	−10	−6.610	−0.170	6.147	−6.594
	−20	−12.117	−0.340	13.134	−11.888

on the optimal policy of this two-layer supply chain model in visual form in the following Figs. 9–19.

From Fig. 9, it is evident that, increasing the value of the parameter a_m, the integrated profit (π_{sc}) increases highly, whereas the business length (T) significantly reduces. Retail price (p_r) and production time (t_m) are insensitive with respect to this parameter, i.e., very small changes in these optimal values.

Figure 10 exposes that the integrated profit (π_{sc}) and business length (T) are moderately sensitive with respect to the parameter b_m. Here, as the of value b_m increases, decreases the value of (π_{sc}) and (T) moderately. Production time (t_m) and retail price (p_r) are very less sensitive with respect to changes in the parameter b_m.

Figure 11 shows that increasing the value of parameter c_m, the time of production (t_m) and business cycle length (T) decreases moderately, whereas the price (p_r) changes is insensitive and profit of the supply chain (π_{sc}) about the changes of c_m is less sensitive.

It is observed from Fig. 12, the business cycle (T) is highly decreases as the increasing of parameter μ_m. Production time (t_m) is moderately sensitive, integrated profit (π_{sc}) is less sensitive and retail price (p_r) is insensitive with the changes of parameter μ_m.

Table 15 Impact of emission parameters on optimal values of centralize scenario for Example 1 To better illustrate the impact of the supply chain's emission parameters on optimal values, the percentage changes are graphically represented in Figs. 17–19.

Parameter	Parameters change (%)	Optimal values					
		p_r (\$/unit)	t_m (month)	T (month)	π_m (\$/cycle)	π_r (\$/cycle)	π_{sc} (\$/cycle)
a	20	81.647	3.820	10.605	4477.265	2076.670	6553.935
	10	81.647	3.820	10.605	4486.565	2076.670	6563.235
	0	81.647	3.820	10.605	4495.865	2076.670	6572.535
	−10	81.647	3.820	10.605	4505.165	2076.670	6581.835
	−20	81.647	3.820	10.605	4514.465	2076.670	6591.135
b	20	81.647	3.820	10.605	4495.165	2076.670	6571.835
	10	81.647	3.820	10.605	4495.515	2076.670	6572.185
	0	81.647	3.820	10.605	4495.865	2076.670	6572.535
	−10	81.647	3.820	10.605	4496.215	2076.670	6572.885
	−20	81.647	3.820	10.605	4496.565	2076.670	6573.235
e_m	20	81.647	3.820	10.605	4476.565	2076.670	6553.235
	10	81.647	3.820	10.605	4486.215	2076.670	6562.885
	0	81.647	3.820	10.605	4495.865	2076.670	6572.535
	−10	81.647	3.820	10.605	4505.515	2076.670	6582.185
	−20	81.647	3.820	10.605	4515.165	2076.670	6591.835

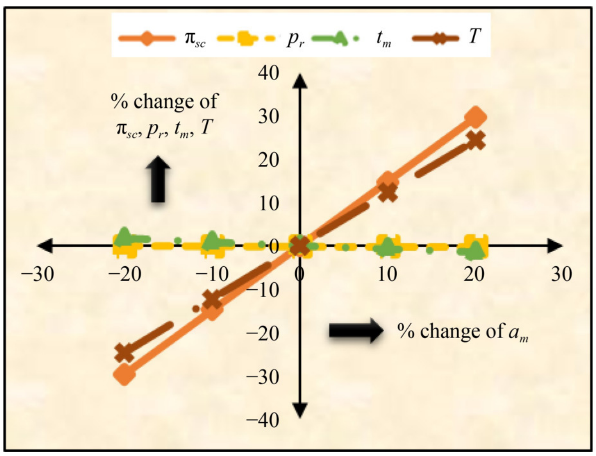

Fig. 9 Effect of a_m on the optimal solutions.

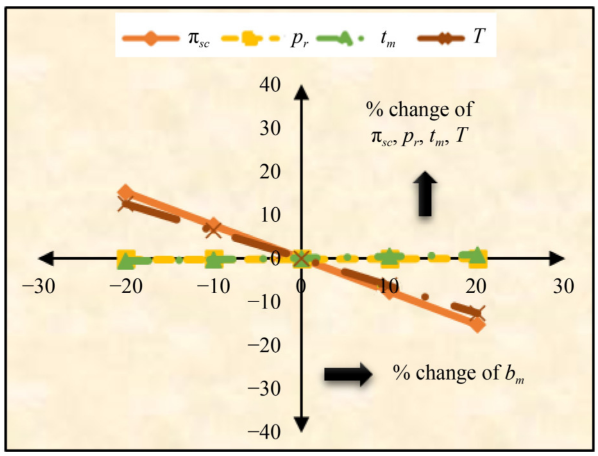

Fig. 10 Effect of b_m on the optimal solutions.

It is clear from Fig. 13 that the increases of a_c, the value of the supply chain profit (π_{sc}) increases and cycle length (T) decreases significantly. The production time (t_m) has very less impact on a_c, where the impact on the retail price (p_r) is moderate of this parameter.

From Figs. 14 and 15, it is seen that all the optimal values are insensitive to changes in h_{cm} and h_{cr} parameters, i.e., there is very less effect of h_{cm} and h_{cr} on the optimal policy.

Figure. 16 exposes that the integrated profit (π_{sc}) and retail price (p_r) are moderately decreases as the increases of b_c, whereas the cycle length (T) is equally sensitive and it's increases as the increasing the value of b_c. Production time (t_m) is insensitive with respect to the changes of parameter b_c.

From Figs. 17–19, it is seen that all the optimal values of decision variables are insensitive with respect to the emission parameters a, b and e_m. However, the sensitivity analysis demonstrates that the emission parameters a and the tax rate e_m have a noticeable negative impact on the profitability of both the entire supply chain and the manufacturer, while the retailer's profit remains unaffected. The emission parameter b, however, shows minimal impact on the supply chain's overall profitability.

6.2 Experimental observations

From Tables 14–15 and Figs. 9–19, the following observations are made:

◆ Integrated profit (π_{sc}) is highly sensitive to changes

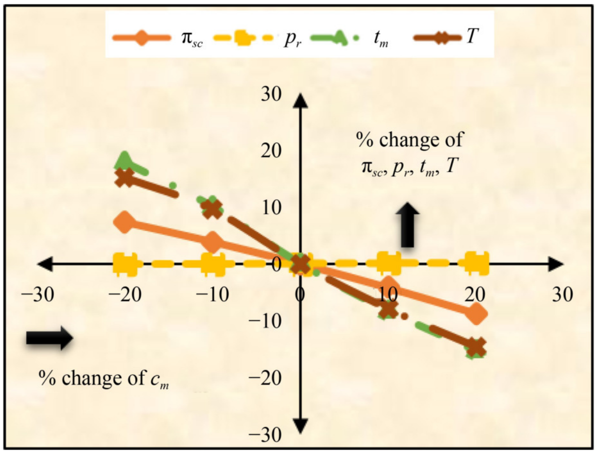

Fig. 11 Effect of c_m on the optimal solutions.

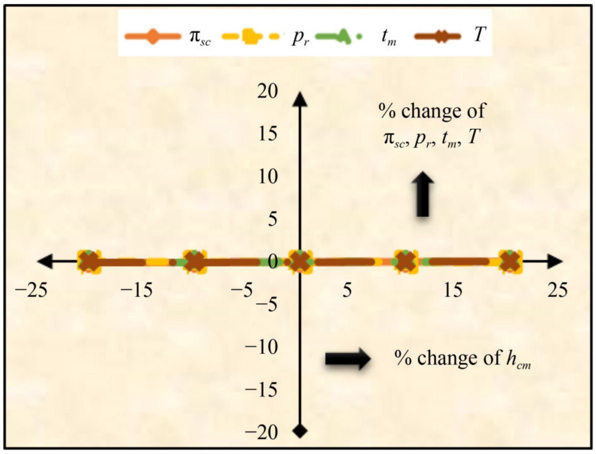

Fig. 14 Effect of h_{cm} on the optimal solutions.

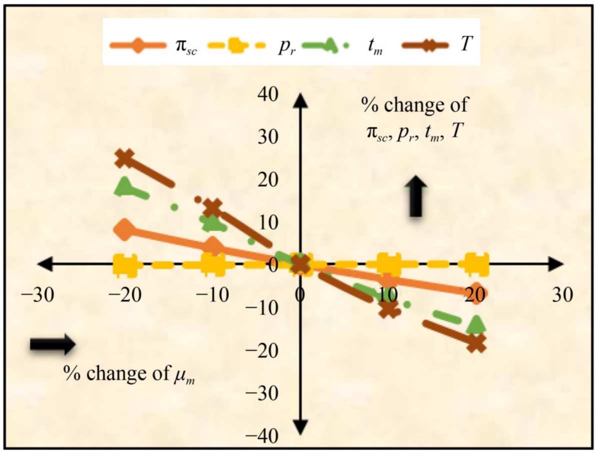

Fig. 12 Effect of μ_m on the optimal solutions.

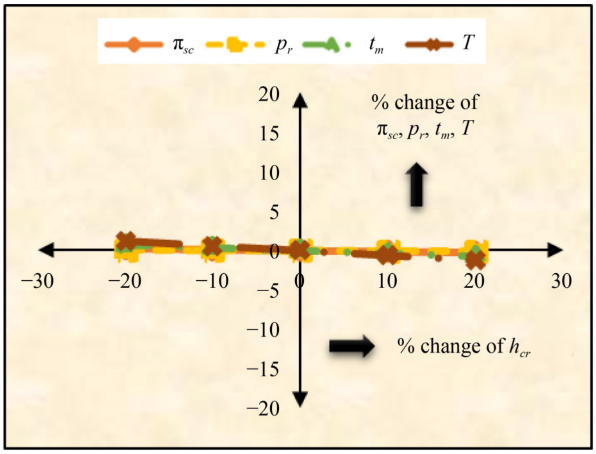

Fig. 15 Effect of h_{cr} on the optimal solutions.

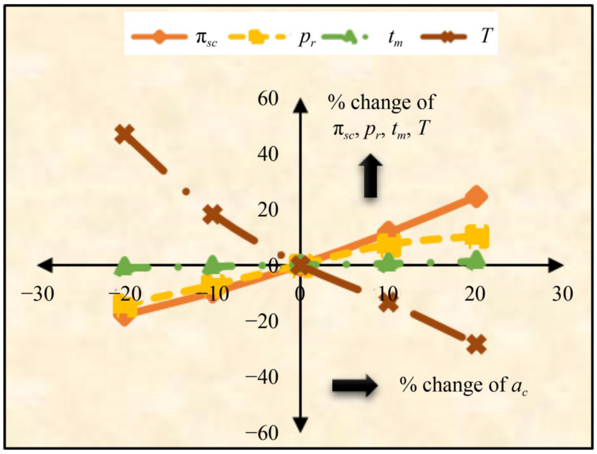

Fig. 13 Effect of a_c on the optimal solutions.

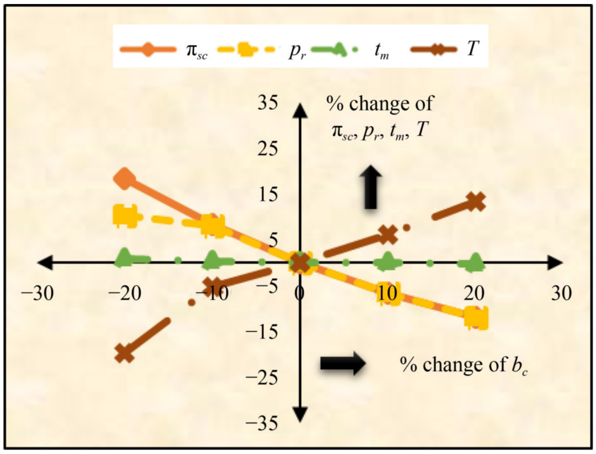

Fig. 16 Effect of b_c on the optimal solutions.

of demand parameters a_m, a_c. Also, it is equally sensitive with respect to the parameter b_c and sensitive moderately with respect to the parameter b_m. Further integrated profit (π_{sc}) is less sensitive against the changes of c_m and μ_m, whereas h_{cm} and h_{cr} have no effect on the integrated profit (π_{sc}).

◆ Retailer's selling price (p_r) is effective moderately with the changes of the parameters a_c and b_c, while it is not sensitive with respect to the other parameters.

◆ Manufacturer's production time (t_m) is sensitive

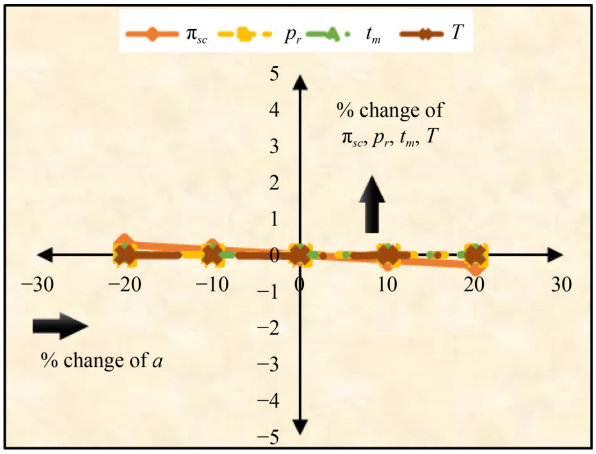

Fig. 17 Effect of a on the optimal solutions.

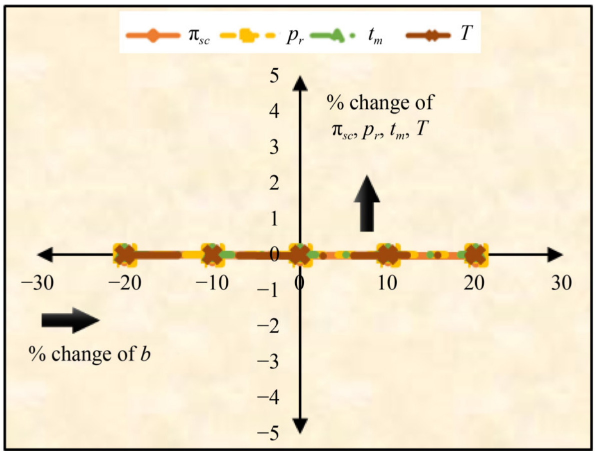

Fig. 18 Effect of b on the optimal solutions.

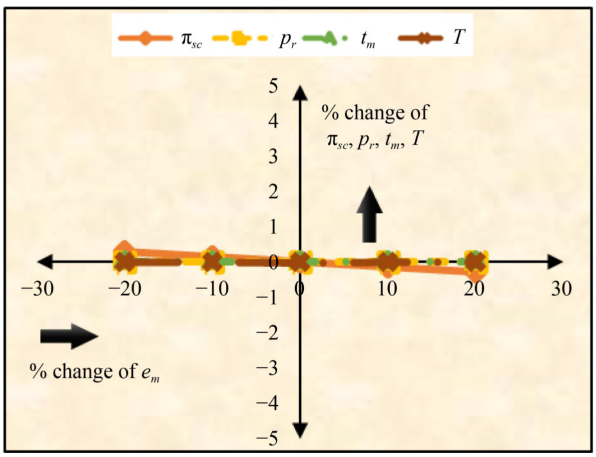

Fig. 19 Effect of e_m on the optimal solutions.

moderately with respect to the parameters c_m and μ_m, whereas the remaining parameters have no effect on this.

◆ Business cycle length (T) is highly sensitive against the changes of the parameters a_m, a_c, and μ_m and equally sensitive with the changes of b_c. Further, effect of b_m and c_m on (T) is moderately whereas it is insensitive with respect to the parameters h_{cm} and h_{cr}.

◆ As the emission parameter a increases, the profit of the entire supply chain and the manufacturer decreases progressively though changes in a do not directly affect the retailer's profitability. This indicates a negative correlation between the emission parameter a and the overall profitability of the supply chain.

◆ The manufacturer's and the supply chain's overall profitability are both noticeably impacted negatively by tax rate e_m, while the retailer's profit is unaltered.

7 Research conclusions and management insights

Optimal control techniques are useful in the effective management and optimisation of complex systems, such as supply chains. In supply chain management, optimal control refers to the process of determining the best course of action for controlling a given objective function over time through the application of mathematical optimisation techniques. The best control strategy for a supply chain is to model its dynamics, which include things like production rates, inventory levels, demand trends, and transportation constraints. We have taken into consideration in this study that the production rate is an unknown function of time, a controlling function. Price and time are taken into account when determining the product's demand. Carbon emissions are assumed to be a linear function of the system's production rate. We have utilized an optimal control approach to solve the proposed supply chain system by figuring out the unknown production rate.

The Stackelberg game theory approach is used for solving decentralised optimization problems. We have employed various meta-heuristic algorithms and compared their outcomes to determine the ideal values of the objective function as well as the decision variables. It is observed that the equilibrium optimizer algorithm outperformed the other ones. It is also noted that a centralized system provides the best solution compared to a decentralized system. Based on the findings, the manufacturer and the entire supply chain experience a considerable decline in profit when the emission rate rises. Consequently, the manufacturer must make investments in more efficient and cleaner production methods to lower the emission rate. A more robust and successful supply chain may result from developing a coordinated strategy for emission reduction, such as exchanging best practices or pooling resources for environmentally friendly technologies. The findings provide the following key managerial implications for building a robust and successful supply chain:

• An organized framework for assessing and improving supply chain operations is given to managers by the optimal control approach, which boosts productivity, lowers expenses, and improves overall performance.

• By determining the best course of action over long-time horizons, optimal control can assist long-term planning. This can assist in adjusting to shifting market conditions and coordinating supply chain decisions with corporate objectives.

• While the retailer's profit is not directly affected by emission parameters, collaboration with the manufacturer to promote and sell environmentally friendly products could enhance the retailer's market position and brand reputation. This could lead to increased demand and potentially higher profits in the long-term. Effective supply and demand matching is facilitated by optimal control, which enables production rates to be changed in response to changing demand. Due to the system's ability to adjust to changing circumstances, this flexibility can also lessen the effects of unknown production rates.

• The profit of both the entire supply chain and the manufacturer decreases significantly as the emission rate increases. It is crucial for the manufacturer to invest in cleaner and more efficient production technologies that reduce the emission rate. Establishing a coordinated approach to emission reduction, such as sharing best practices or co-investing in green technologies, could lead to a more resilient and profitable supply chain. Coordination contracts or incentive schemes that align the interests of both parties in reducing emissions could be particularly effective.

For the future scope of research, different types of facilities may be incorporated for developing a new supply chain model, such as advance payment, trade credit facilities, quantity discounts, nonlinear stock-dependent demand, etc. On the other hand, anyone can develop the model by taking supply chain parameters as interval-valued, fuzzy-valued, or uncertain in nature.

Data availability statement No data were used to support this study.

Competing Interests The authors declare that they have no competing interests

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Appendix

A Brief description about EOA

The origin of the EOA metaheuristic algorithm was a straightforward well-mixed dynamic mass balancing problem across a control volume. The mass balance equation in this problem, along with its various sources and sinks, explains the concentration of non-reactive components in the control volume. The mass balance equation provides the underlying physics for entering the control volume, preserving mass, leaving it, and spawning there. The general mass balance Eq. (A1), which says that the quantity entering the system less the quantity leaving it, plus the quality of the input system equals the change in mass over time. This is represented by the first order ordinary differential equation:

$$\zeta \frac{\mathrm{d}\tau}{\mathrm{d}t} = \nu\tau_{eq} - \nu\tau + \Upsilon, \tag{A1}$$

where τ be the rate in volume is called concentration. ζ, ν be the volumetric flow rate and Υdenotes the rate of mass generation. When $\zeta \frac{\mathrm{d}\tau}{\mathrm{d}t}$ becomes zero, after which a stable equilibrium condition is reached.

Again, fulfils the relationship listed below:

$$\tau = \tau_{eq} + (\tau_0 - \tau_{eq})\kappa + \frac{\Upsilon}{\lambda\zeta}(1-\kappa). \tag{A2}$$

The phrase for the exponential factor is generated by using Eqs. (A1) and (A2) as

$$\kappa = \exp[-\lambda(t-t_0)]. \tag{A3}$$

Here τ_0 and t_0 be the initial mass concentration of the system and initial starting time. Now, Eq. (A2) predicts that the mass volume concentration will evolve at a linear regression of generating rate. EOA is actually built using Eqs. (A2) and (A3). With every particle changing its concentration of mass is based on three different terms, Eq. (A3) now reflects an update rule for particles. One of the top answers chosen at random from an equilibrium pool is what is meant by the first word. The difference in concentration between the equilibrium state and the particle, which is related to the second term, involve for a direct search engine. The phrase stimulates particles to conduct global domain searches and serve as discoverers whereas the third term, which refers to generation rate, primarily serves as a developer or refiner of solutions, particularly for minor steps, though it can also occasionally act as an explorer. Below are the definitions of each phrase and how it influences search trends.

A.1 Initialization

To begin an optimisation process utilizing EOA, a starting population is necessary. According to the following linearized equation, the quantity of particles in a uniform search space created random, is used to initialise the concentration:

$$\tau_i^{\text{ini}} = \tau i(\tau \min_{\max}, i = 1,2,...,n)_{\min}. \tag{A4}$$

where τ_i^{ini} be the concentration as initial and $\tau_{\min}$, $\tau_{\max}$ are minimum and maximum values inside the searching space. Additionally, n represents the population's particles. To find equilibrium particles, fitness values of the particles are assessed here, and their values are saved.

A.2 Equilibrium pool and candidates (τ_{eq})

When there are fewer than four possibilities, multimodal and combinatorial methods are simplified, while work on unimodal functions enhances results. The reverse outcome will occur with more than four candidates. These five particles are chosen as equilibrium candidates and utilized to build the equilibrium pool, a vector:

$$\vec{\tau}_{eq.pool} = (\vec{\tau}_{eq(1)}, \vec{\tau}_{eq(2)}, \vec{\tau}_{eq(3)}, \vec{\tau}_{eq.(ave)}). \quad \text{(A5)}$$

Each particle changes its concentration during an iteration by choosing randomly from candidates who have the same chance of being chosen. For instance, the first particle updates all of its concentrations in accordance with $\vec{\tau}_{eq(1)}$, subsequently, it can adjust its concentrations in accordance with $\vec{\tau}_{eq(ave).}$ At the end of the optimisation process, which entails updating each particle individually, all candidate solutions receive almost the same number of updates for each particle.

A.3 Generation rate and exponential term (E)

In this scenario, the exponential term κ helps maintain a balance between exploitation and exploration while also updating the rules in the EOA. The exponential term's vector can now be found by:

$$\vec{\kappa} = \exp\left[-\vec{\lambda}(t - t_0)\right]. \quad \text{(A6)}$$

If time t is given in the following equation, where iteration of EOA is defined as a function of time

$$u : t = \left(1 - \frac{it_{er}}{\max\_it_{er}}\right)^{\left(r\frac{it_{er}}{\max\_it_{er}}\right)}, \quad \text{(A7)}$$

where r is an appropriate constant to use in controlling the system's rate of exploration. Additionally, it is suggested to use the generation rate to develop the precise answer by enhancing the system's phase. The exponential term is used to define the rate of generation of EOA as follows:

$$\vec{\Upsilon} = \vec{\Upsilon}_0 \exp\left[-\vec{\lambda}(t - t_0)\right] = \vec{\Upsilon}_0 \vec{\kappa}. \quad \text{(A8)}$$

Here $\vec{\lambda}$ is decaying vector and $\vec{\Upsilon}_0$ be the initial generation rate. Hence, the EOA generation rate parameter is defined by

$$\overrightarrow{grp} = \begin{cases} 0.5 m_1 m_2 \geqslant gp, \\ 0 m_2 < gp. \end{cases} \quad \text{(A9)}$$

By taking into account two random numbers $m_1, m_2 \in [0, 1]$. It is believed that the controlling parameter of generation rate of EOA will result in an ideal equilibrium among the exploration rate and exploitation rate i.e., $gp = 0.5$ and in this instance, the EOA updating formula is provided by

$$\vec{\tau} = \vec{\tau}_{eq} + (\vec{\tau} - \vec{\tau}_{eq}).\vec{\kappa} + \frac{\vec{\Upsilon}}{\vec{\lambda}\zeta}(1 - \vec{\kappa}). \quad \text{(A10)}$$

A.4 Saving of memory

Since each particle keeps track of its spatial coordinates, which influences the fitness value, the addition of memory-saving techniques benefits every particle. The PSO p-best idea and this procedure are similar. The current iteration's fitness value for each particle should be compared to the previous iteration's value; if a better fit is found, the new value should be used. This method facilitates the use of abilities, but if global exploration is not used, it increases the chance that the approach will become stuck in local minima.

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