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RESEARCH ARTICLE

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Joint optimization of production, maintenance, and quality control considering the product quality variance of a degraded system

Abstract The joint optimization of production, maintenance, and quality control has shown effectiveness in reducing long-term operational costs in production systems. However, existing studies often assume that changes in the mean value of product quality characteristics in a deteriorating system follow a specific distribution while keeping variance constant. To address this limitation, we propose an innovative method based on the continuous ranking probability score (CRPS). This method enables the simultaneous detection of changes in mean and variance in nonconformities, thus removing the assumption of a specific distribution for quality characteristics. Our approach focuses on developing optimal strategies for production, maintenance, and quality control to minimize cost per unit of time. Additionally, we employ a stochastic model to optimize the production time allocated to the inventory buffer, resulting in significant cost reductions. The effectiveness of our proposed joint optimization method is demonstrated through comprehensive numerical experiments, sensitivity analysis, and a comparative study. The results show that our method can achieve cost reductions compared to several other related methods, highlighting its practical applicability for manufacturing companies aiming to reduce costs.

Keywords joint optimization, degraded system, CRPS control chart, uncertain buffer stocking time

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1 Introduction

In the era of smart manufacturing, traditional standalone maintenance strategies cannot meet the manufacturing industry's goals of cost reduction and productivity improvement. Implementing smart maintenance could provide a solution to this problem (Tambe and Kulkarni, 2022), and the internal integration strategy is one of the four aspects of smart maintenance (Bokrantz et al., 2020). This strategy involves integrating the organization's functions (e.g., production, quality, and maintenance) into a unified management and control system.

From a company's perspective, enhancing overall revenue is the primary objective, and the overall performance is more crucial than the individual functions' performances (Tambe and Kulkarni, 2022). Panagiotidou and Tagaras (2010) have demonstrated that a joint strategy can increase profits by approximately 8% compared to the condition-based maintenance strategy. Extensive studies have shown that the joint control of production, maintenance, and quality can significantly enhance system efficiency while reducing the overall operational cost (An et al., 2022; Fitouhi and Nourelfath, 2012; Kumar et al., 2018; Liu et al., 2021a). Thus, achieving overall optimal performance requires integrating these three functions (Ben-Daya and Rahim, 2000; Hajji et al., 2012), which has been a significant challenge in both industry and theory (Azimpoor and Taghipour, 2021; Farahani and Tohidi, 2021; Hadian et al., 2021; Obaidat and Liao, 2021; Tambe and Kulkarni, 2022). Production, maintenance, and quality control are the most three critical functions in the manufacturing industry. The close relationship between these functions has attracted significant attention from both managers and scholars (Chakraborty et al. 2009). Over time, equipment can deteriorate due to environmental and human factors, which can impact production plans, reduce product quality, and lead to failing customer demands. Therefore, maintenance activities should prioritize equipment reliability and product quality. However, if equipment degradation cannot be detected in real-time, product quality feedback can be utilized for decision-making regarding equipment maintenance (Hadian et al. 2021). Consequently, the coordination of these three functions has become increasingly important for managers in recent years. Specifically, the grinder used for producing finishes for excavation tools has received particular attention (Panagiotidou and Tagaras, 2010). Within this grinder, the wheels rotate at a constant rate, and the power source includes a three-belt transmission. Should a belt break, the machining of the grinder will remain unaffected, but the grinding of the machined part may be less effective due to decreased power transmission. Unfortunately, a broken belt cannot be detected from the outside of the grinder, and it is only through the processed parts that any faults in the grinder and their causes can be inferred.

An important aspect of joint optimization is the quality control strategy. Control charts are widely used in quality control to identify changes in the mean and variance of product quality (de Vargas et al. 2004). Previous research has successfully achieved joint optimization of production planning, quality control, and maintenance by incorporating quality factors into the economic production quantity (EPQ) model (Pan et al., 2012; Zhou et al., 2015), as well as through the integration of statistical process monitoring (SPM) with condition-based maintenance (Panagiotidou and Tagaras, 2010, 2007; Xiang, 2013; Le and Tan, 2013; He et al., 2020). Furthermore, Peng et al. (2022) proposed an innovative preventive maintenance strategy for heterogeneous parallel systems, while Liu et al. (2021b) used a partially observable Markov decision process framework to determine the health status. However, it is worth noting that the aforementioned joint control strategies assume that the scale shift is zero and that the control chart can only detect a location shift. Huang et al. (2020) addressed this limitation by proposing a dual-sampling SPM scheme that can simultaneously monitor both location and scale shifts within a joint control model. Nevertheless, this scheme monitors the location and scale shifts separately rather than jointly. Farahani and Tohidi (2021) conducted a comprehensive review of the existing literature on the joint optimization of quality control and maintenance.

To achieve a company's objectives of meeting customer demands and minimizing costs, managers require a comprehensive understanding of a production system, which involves designing an integrated strategy for production, maintenance, and quality control (Megoze Pongha et al., 2022). However, developing an optimal strategy that includes these three aspects is challenging due to the stochastic nature of production systems, including component degradation, equipment failure, maintenance downtime, and product quality (Ait-El-Cadi et al., 2021a; Gan S et al. 2021; Megoze Pongha et al., 2022).

The seamless coordination of these manufacturing elements has the potential to reduce costs and ensure the reliability of a production system (Hajej et al., 2021). Recently, there has been significant interest in integrating quality aspects into combined production and preventive maintenance models, driven by the adoption of advanced inspection strategies. Nourelfath et al. (2016) pioneered a comprehensive joint control framework that incorporates quality, production, and maintenance for a multi-cycle, multi-product production process, taking into account allowable lot sizes. Similarly, Fakher et al. (2018) investigated the inherent connections between these three elements, analyzing trade-offs through computational experiments. Rivera-Gómez et al. (2020) proposed an advanced stochastic model for integrated deterioration processes, highlighting the impact of control parameters in the model. Bahria et al. (2019), Bouslah et al. (2018), Duffuaa et al. (2020), and Salmasnia et al. (2018) examined an integrated approach for joint control across batch manufacturing systems. Furthermore, Cheng and Li (2020) expanded the scope of joint optimization from single-machine scenarios to serial-parallel multi-stage production systems. On a different note, Tasias (2022) developed a Bayesian model that optimizes the interrelated aspects of production, maintenance, and quality in modern manufacturing environments. This model enables real-time decision-making based on a cost optimization criterion. However, many existing studies on joint optimization using control charts have assumed constant scales and monitored only location shifts (Ait-El-Cadi et al. 2021b; Azimpoor and Taghipour, 2021; Hadian et al. 2021; Salmasnia et al. 2017). Additionally, it has been assumed that quality characteristics follow a normal distribution.

Performing maintenance on a production system can disrupt production and disrupt production planning, without guaranteeing continuous operation. As a proactive approach to ensure seamless and uninterrupted production, the introduction of a buffer zone between upstream and downstream equipment in the production system has gained popularity in recent years in the field of joint optimization research (Radhoui et al. 2009; Sett et al. 2017; Zequeira et al. 2008, 2004). Previous studies have commonly scheduled the restocking of buffer stocks at the beginning of each cycle to simplify calculations and improve model usability (Ait-El-Cadi et al. 2021b; Hadian et al. 2021; Rivera-Gómez et al. 2021; Salmasnia et al. 2017). However, this approach leads to higher overall costs due to increased inventory holding costs.

This study proposes an innovative joint control strategy that uses a continuously ranked probability score (CRPS) control chart to efficiently track changes in the mean and variance of product quality without relying on distributional assumptions. The proposed strategy enables timely replenishment, thus reducing total costs. Additionally, the joint control strategy aims to optimize multiple key aspects, including production cycle, maintenance frequency, and sampling/testing strategies, with the overall objective of minimizing the total cost incurred by the system.

The main contributions of this work are as follows:

(1) By integrating CRPS control charts into the joint control strategy, this study provides better monitoring of product quality degradation, allowing for the detection of changes in both quality mean and variance. This enables faster decision-making in response to product quality changes.

(2) To the author's knowledge, this is the first study to apply the CRPS control chart to the joint optimization of production, maintenance, and quality control. The proposed joint control strategy relaxes the assumption that quality characteristics follow a specific distribution, making it possible to monitor quality without considering the data distribution. The feasibility of the strategy is demonstrated through numerical case studies considering three different distributions.

(3) In previous studies, stocking inventory in the buffer zone from the beginning of the production cycle led to significant increases in inventory holding costs. To address real-world production scenarios, this study refines the replenishment timing in the joint control strategy, resulting in cost reduction. The effectiveness of the proposed approach is validated through comparative analysis, showing improvements of 4.731% and 1.084% in cost-effectiveness compared to the methods of Hadian et al. (2021) and Shi et al. (2023), respectively.

The remaining portion of this paper is structured as follows. Section 2 provides a definition of the notation and research inquiries guiding the model. Section 3 outlines the methodologies employed in this study. Section 4 presents the mathematical formulation of a stochastic model that includes joint quality, production, and maintenance. Section 5 presents numerical results and comparisons, including three numerical examples, sensitivity analysis, and a comparative examination. Finally, Section 6 concludes the main findings of this work and suggests future research directions.

2 Problem statement and notations

2.1 Problem statement

This study focuses on a stand-alone machine manufacturing system that operates consistently at a certain level of productivity but experiences degradation throughout the production process. The production system initially operates in a controlled state, but transitions into a runaway state as degradation occurs, resulting in substandard product quality. During the production cycle, the product's quality is assessed through intermittent sampling at h time intervals, with each sampling operation consisting of a sample size of n. The quality characteristics of the product, such as its diameter, thickness, and width, are evaluated using a CRPS chart. At the start of each production cycle, the system operates in a controlled state. However, as the machines and equipment are utilized, the system gradually deteriorates, eventually transitioning into a runaway state at a random point denoted as X. This deterioration, known as an assignable cause, can be attributed to various factors, including human behavior, environmental influences, changes in raw materials, and mechanical failures.

If the control chart does not indicate an alarm after the kth test (i.e., within control limits), the system is considered to be under control, and preventive maintenance (PM) is performed. An alarm occurring before the kth test signifies an out-of-control system, triggering corrective maintenance (CM). Both PM and CM actions are strategically designed to restore the system to its desired state (Hadian et al., 2021; Salmasnia et al., 2017). Type I errors occur when an alarm is raised by an in-control system, necessitating an investigation of the time and resources expended. Conversely, Type II errors arise when an out-of-control system goes unnoticed by the control chart, leading to CM instead of the intended PM after the kth test.

PM and CM durations represent random variables with a general distribution. During the maintenance process, production halts, and an inventory buffer is used to minimize shortage costs. The proposed method replenishes the buffer before maintenance, unlike traditional methods that refill the buffer at the cycle's start and increase inventory expenses. The system starts production at the normal production rate q_1, whereas buffer replenishment operates at higher productivity q_2 (i.e., $q_2 > q_1$). When the replenishment reaches S, buffer replenishment stops. During preventive and corrective maintenance, the buffer inventory decreases at a demand rate q_1. When the maintenance time is longer than S/q_1, a shortage occurs, and a shortage cost is incurred. However, when the maintenance time is shorter than S/q_1, there is no shortage, and the buffer is consumed preferentially after the maintenance activity. After the buffer has been depleted, a new round of production starts. The specific integration process of production, maintenance, and quality control is shown in Fig. 1.

When maintenance activities are performed on a production system, there is a risk of the system failing to meet downstream production or customer product requirements. The buffer inventory helps mitigate this

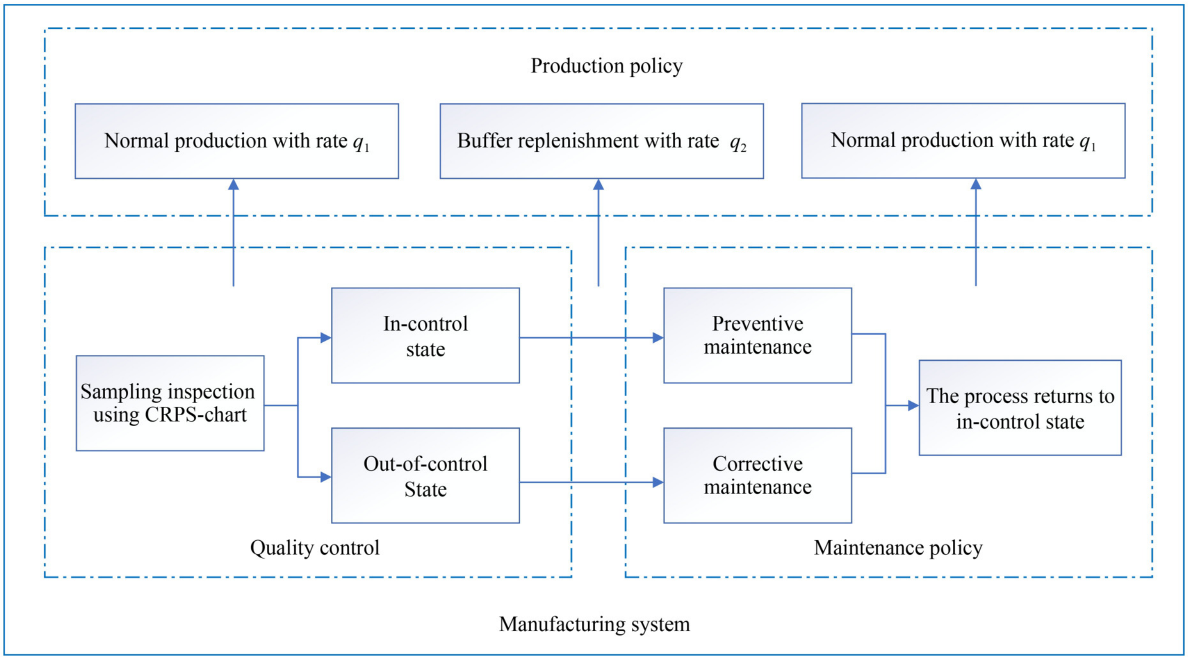

Fig. 1 The process of joint production, maintenance, and quality control.

risk. If there is excessive inventory in the buffer, inventory holding costs will increase. Conversely, if there is insufficient inventory and it runs out before the maintenance activity is completed, there will be out-of-stock costs. Additionally, system degradation can result in defective goods, so timely quality monitoring is necessary. However, if monitoring is performed too frequently, there will be additional quality inspection costs. Therefore, the inspection frequency represents an important decision variable. Moreover, the choice of control limits for the control chart affects the stringency level of product quality and the selection of the sample number (n). In summary, production, maintenance, and quality are closely interconnected and mutually influence each other. Therefore, only the decision that minimizes the overall cost can be considered the optimal solution.

2.2 Notations

n	(Decision variable) Sample number
k	(Decision variable) Inspection frequency
h	(Decision variable) Time interval between successive samples
UCL	(Decision variable) Control chart's limits
S	(Decision variable) Safety stock levels in the buffer zone
X	(Random variable) Transition time from the in-control state to the out-of-control state
$f(x)$	Probability density function (PDF) of x
$F(x)$	Cumulative distribution function (CDF) of x
$\overline{F}(x)$	$1 - F(x)$
Y_1	(Random variable) Time required for preventive maintenance
$g_1(y_1)$	PDF of Y_1
$G_1(y_1)$	CDF of Y_1
Y_2	(Random variable) Time required for corrective maintenance
$g_2(y_2)$	PDF of Y_2
$G_2(y_2)$	CDF of Y_2
γ	Set to zero when production halts during the false alarm analysis; otherwise, set to one
t_f	Analyze and rectify false alarm durations
t_s	Duration of time per sample inspection
t_a	Assignable cause analysis duration
q_1	Productivity of normal production
q_2	Replenishment rate of buffer zone inventory ($q_2 > q_1$)
C_f	Fixed cost for sample inspection
C_v	Variable cost for sample inspection
C_{in}	Operational cost per time unit in the in-control state
C_{out}	Operational cost per time unit in the out-of-control state
C_I	Cost of the false alarm analysis
C_h	Inventory holding cost per unit item per time unit
C_s	Cost of stockouts per time unit
C_{CM}	Corrective maintenance cost
C_{PM}	Preventive maintenance cost ($C_{CM} > C_{PM}$)

Q_{in} Cost per unit of the quality lost when the production process is in the in-control state

Q_{out} Cost per unit of the quality lost when the production process is in the out-of-control state

ARL_0 Average run length when the production process is in the in-control state

ARL_1 Average run length when the production process is in the out-of-control state

α Probability of occurrence of Type I errors

β Probability of occurrence of Type II errors

3 CRPS and renewal process

3.1 CRPS

Originally developed for probabilistic forecasting systems, the CRPS has emerged as a valuable tool for the evaluation of ensembles. The CRPS measures the divergence between the CDFs of predicted and observed values, providing a comprehensive assessment of forecast performance (Hersbach, 2000; Pinson et al., 2012; Thorarinsdottir and Gneiting, 2010; Zhou et al., 2023). Beyond forecasting, CRPS-based methods have found applications in various domains. Shi et al. (2018, 2016) proposed a CRPS-based approach for analyzing process capabilities, along with a novel control chart-based CRPS designed for data-rich environments. Additionally, Harrou et al. (2018) proposed an enhanced sensitivity alternative to conventional partial least squares (PLS) methods, utilizing a CRPS chart for fault detection. This approach yielded improved results through the use of the PLS-based CRPS-exponentially weighted moving-average (CRPS-EWMA) scheme. The CRPS statistic is defined as

$$CRPS = v(F,z) = \int_{-\infty}^{+\infty}(F(t)-H(t-z))^2\mathrm{d}t, \tag{1}$$

where t represents the observed value, and z is the target value; $v(F,z)$ is the CRPS value, which is derived by integrating the squared difference between the CDFs of F and H; F corresponds to the CDF of the process, and H denotes the Heaviside function.

In essence, the relationship is expressed as follows:

$$H(t-z)=\begin{cases}0 & t<z\\ 1 & t\geqslant z\end{cases}. \tag{2}$$

When designing control charts, the most critical task is setting their upper and lower control limits; that is, a reasonable range of CRPS values needs to be found. The upper control limit (UCL) of the CRPS chart can be obtained by Eqs. (3) and (4), where t_1 and t_2 are the CRPS values in the in-control and out-of-control states, respectively. Let Z be the distribution of the CRPS value and z_1, z_2 be the probability density functions of CRPS.

$$\frac{1}{ARL_0}=\int_{UCL}^{\infty}z_1(t_1)\,\mathrm{d}t_1, \tag{3}$$

$$1-\frac{1}{ARL_1}=\int_{-\infty}^{UCL}z_2(t_2)\,\mathrm{d}t_2. \tag{4}$$

A reduced CRPS value indicates a closer alignment between desired and observed values, signifying a proficient process capable of assessing the variability of continuous processes. Conversely, higher CRPS values suggest potential deviations and highlight issues in control. Notably, the CRPS value is always nonnegative, hence this study employs a lower control limit (LCL) of zero for the CRPS chart. One limitation of using the CRPS control chart is the requirement for a substantial amount of data, which this study addresses by considering a data-rich environment.

3.2 Renewal process

Each production cycle operates independently, beginning in the in-control state, undergoing maintenance activities, and ultimately returning to the controlled state. This sequential progression characterizes the renewal process. The update theorem, as described by Christer (1978), is employed to compute the expected unit cost (ECT) over an infinite timeframe, emphasizing the perpetual nature of this evaluation:

$$ECT=\frac{E(C)}{E(T)}, \tag{5}$$

where $E(C)$ is the total cost over a cycle, and $E(T)$ represents the cycle length.

4 Model design

4.1 Various possible scenarios

This study analyzes a manufacturing process including assignable causes, including four distinct scenarios, as shown in Fig. 2. The four scenarios differ in descriptive factors, such as production process timing, assignable cause occurrence, control chart alarm activation, and maintenance mode.

Scenario 1: In this scenario, the production system consistently operates at a controlled productivity level of q_1; k quality samplings are conducted during the production process. Following the kth inspection, buffer inventory replenishment is performed, enhancing the production rate by (q_2-q_1). Once the buffer inventory attains the predefined level of S, the PM promptly ensues. The cycle draws to a close upon completion of the maintenance

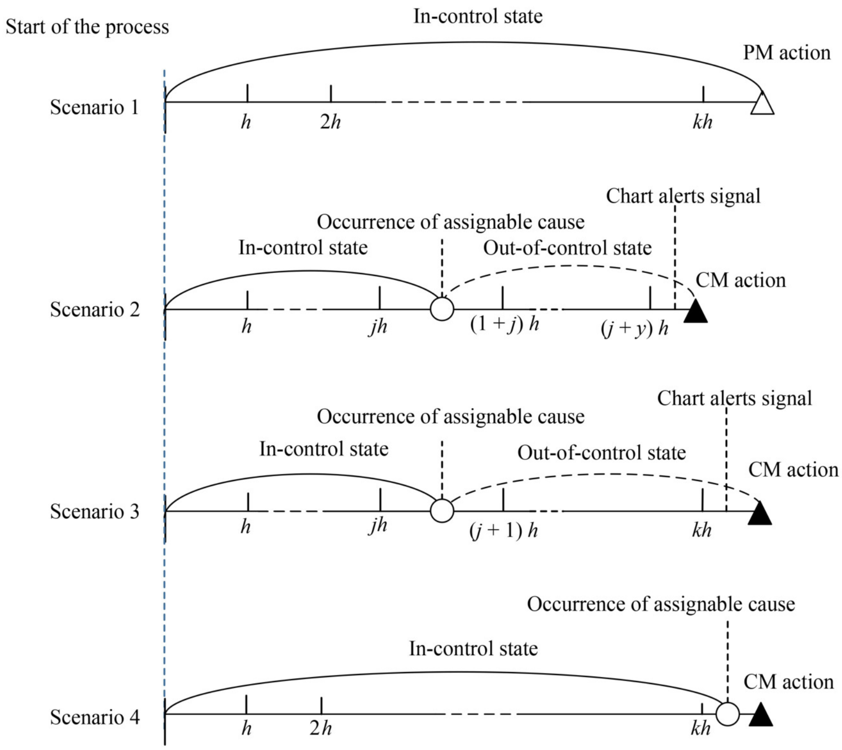

Fig. 2 Graphical representation of the four scenarios.

activity and depletion of the buffer inventory. The probability in Scenario 1 is calculated by

$$P(S_1) = \overline{F}\left(kh + nt_s + \frac{S}{q_2 - q_1}\right). \tag{6}$$

Scenario 2: The production process commences in the control state. A random occurrence marked by X leads to a change into an out-of-control state ($jh \leqslant X \leqslant (j+1)h$). Upon completion of the $(j+y)$th sample inspection, $j+1 \leqslant j+y \leqslant k$, the control chart signals, triggering an immediate analysis of the out-of-control process and prompting the CM measures. Notably, this scenario does not allow the production process to restock the buffer, resulting in substantial shortage costs during maintenance activities. The probability in Scenario 2 is obtained by

$$P(S_2) = F(kh)P(\text{control chart signals} \,|\, \text{out of control}). \tag{7}$$

Scenario 3: The production process initiates in the control state. At a stochastic time denoted as X, the process transitions to the out-of-control state ($jh \leqslant X \leqslant (j+1)h$). It should be noted that the control chart remains inert until the cycle reaches its culmination. After the kth inspection, the buffer inventory is replenished, which is followed by the execution of PM action. The point of transition hinges on the implementation of the PM action, further initiating the CM action. The fault analysis process has a duration of t_a units of time. Then, the probability characterizing Scenario 3 is given by:

$$P(S_3) = F(kh) - P(S_2). \tag{8}$$

Scenario 4: Commencing in the control state, the production process undergoes distinctive dynamics. An assignable cause emerges after the kth test but before the completion of buffer stock replenishment. In this scenario, the execution of CM actions is mandated, irrespective of whether the control chart indicates an anomaly. Upon replenishing the buffer inventory, a period of t_a units of time is allocated for cause analysis. Finally, the probability delineating Scenario 4 is calculated by

$$P(S_4) = F\left(kh + nt_s + \frac{S}{q_2 - q_1}\right) - F(kh). \tag{9}$$

4.2 Quality control strategy

Product quality is an effective indicator of a production system's conditions (Hadian et al. 2021). Therefore, regular quality checks of products can effectively predict equipment failure, especially when it is impossible to check the equipment for faults in a timely manner. In the proposed model, the quality inspection tool used is the CRPS control chart. The production system runs for h units of time to realize the product inspection, analyzing n products at a time, as shown in Fig. 1. Inspection is terminated either after the activation of a control chart alarm or after the completion of the kth inspection, and at this point, as appropriate maintenance action is executed.

The specific quality testing strategy is shown in Fig. 3.

Application of the CRPS chart to sample inspection is a key feature of the proposed model. The UCL value for the CRPS chart is obtained by Eqs. (3) and (4). Moreover, it should be noted that the LCL for the CRPS chart is set to zero, and α and β in the control chart are respectively obtained by

$$\alpha = \int_{UCL}^{\infty} z_1(t_1)\,\mathrm{d}t_1, \tag{10}$$

$$\beta = \int_{-\infty}^{UCL} z_2(t_2)\,\mathrm{d}t_2. \tag{11}$$

Further, ARL_0 and ARL_1 can be obtained by

$$ARL_0 = \frac{1}{\int_{UCL}^{\infty} z_1(t_1)\,\mathrm{d}t_1}, \tag{12}$$

$$ARL_1 = \frac{1}{1-\int_{-\infty}^{UCL} z_2(t_2)\,\mathrm{d}t_2}. \tag{13}$$

Costs associated with the quality control strategy include sample testing costs and false alarm costs.

4.2.1 Excepted sampling cost per cycle

In this study, $E(C_{\text{ins}})$ represents the excepted sampling cost per cycle, and it can be calculated by

$$E(C_{\text{ins}}) = P(S_1)E(C_{\text{ins}}|S_1) + P(S_2)E(C_{\text{ins}}|S_2) + P(S_3)E(C_{\text{ins}}|S_3) + P(S_4)E(C_{\text{ins}}|S_4), \tag{14}$$

where $E(C_{\text{ins}}|S_i)$ can be obtained as follows:

$$E(C_{\text{ins}}|S_1) = (C_f + nC_v)k, \tag{15}$$

$$E(C_{\text{ins}}|S_2) = (C_f + nC_v)(E(j+y)), \tag{16}$$

$$E(C_{\text{ins}}|S_3) = (C_f + nC_v)k, \tag{17}$$

$$E(C_{\text{ins}}|S_4) = (C_f + nC_v)k. \tag{18}$$

4.2.2 Excepted cost of false alarm analysis

Similarly, EC_{FA} represents the mean cost attributed to false alarms, and it is calculated by

$$E(C_{\text{FA}}) = P(S_1)E(C_{\text{FA}}|S_1) + P(S_2)E(C_{\text{FA}}|S_2) + P(S_3)E(C_{\text{FA}}|S_3) + P(S_4)E(C_{\text{FA}}|S_4), \tag{19}$$

where $E(C_{\text{FA}} \mid S_i)$ stands for the cost associated with the false alarms in scenario i.

$$E(C_{\text{FA}}|S_1) = C_I\frac{k}{ARL_0}, \tag{20}$$

$$E(C_{\text{FA}}|S_2) = C_I\frac{E(j)}{ARL_0}, \tag{21}$$

$$E(C_{\text{FA}}|S_3) = C_I\frac{E(j)}{ARL_0}, \tag{22}$$

$$E(C_{\text{FA}}|S_4) = C_I\frac{k}{ARL_0}. \tag{23}$$

4.3 Maintenance strategy

As equipment is used over time, the production system can deteriorate due to assignable causes, resulting in poor-quality products. However, implementing a well-designed and effective maintenance strategy can restore the system to optimal condition, thereby reducing costs. The maintenance strategy in the proposed model is

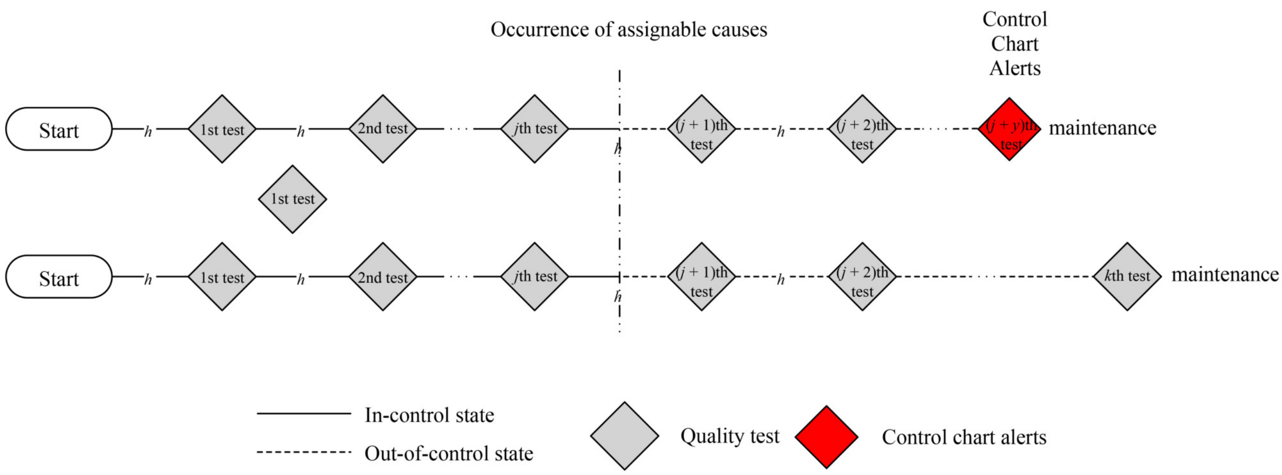

Fig. 3 The quality control strategy.

defined by Eq. (24) and illustrated in Fig. 4.

$$\begin{cases} PM \text{ process is under control} \\ CM \text{ process is out of control and alert is issued} \\ PM \rightarrow CM \text{ process is out of control but no alarm} \\ \quad \text{is issued} \end{cases}. \quad (24)$$

4.3.1 Excepted maintenance cost in each cycle

The excepted maintenance cost in each cycle $E(C_{\mathrm{M}})$ can be obtained by

$$E(C_{\mathrm{M}}) = E(C_{\mathrm{CM}}) + E(C_{\mathrm{PM}}). \quad (25)$$

The PM action is performed in Scenario 1, and the CM action is realized in Scenarios 2–4. The $E(C_{\mathrm{PM}})$ and $E(C_{\mathrm{CM}})$ values can be obtained by

$$E(C_{\mathrm{PM}}) = P(S_1) \times C_{\mathrm{PM}}, \quad (26)$$

$$E(C_{\mathrm{CM}}) = (P(S_2) + P(S_3) + P(S_4)) \times C_{\mathrm{CM}}. \quad (27)$$

4.4 Production strategy

The production system produces at a rate of q_1 to meet the constant demand. After conducting the kth quality inspection, the production system initiates the process of replenishing the buffer, during which the production rate becomes q_2. When preventive or corrective maintenance activities are carried out, production is halted, and the production rate becomes zero, as shown in Fig. 5. The cost of the production strategy includes system operating costs, inventory holding costs, and out-of-stock costs.

4.4.1 Process operational cost

Operating the system in an out-of-control state, especially when control chart alarms are not timely or accurate, leads to higher operational costs. The expected operational cost of the process in each cycle ($E(C_{\mathrm{OC}})$) is calculated using the following formula:

$$E(C_{\mathrm{OC}}) = P(S_1)E(C_{\mathrm{OC}}|S_1) + P(S_2)E(C_{\mathrm{OC}}|S_2) + P(S_3)E(C_{\mathrm{OC}}|S_3) + P(S_4)E(C_{\mathrm{OC}}|S_4), \quad (28)$$

where $E(C_{\mathrm{OC}}|S_i)$ is the projected operational cost of the process within each cycle for scenario i.

This value can be obtained as follows:

$$E(C_{\mathrm{OC}}|S_1) = C_{\mathrm{in}}(kh + nt_{\mathrm{s}}) + Q_{\mathrm{in}}q_1(kh + nt_{\mathrm{s}}) + Q_{\mathrm{in}}q_2\frac{S}{q_2 - q_1}, \quad (29)$$

$$E(C_{\mathrm{OC}}|S_2) = (C_{\mathrm{in}} + Q_{\mathrm{in}}q_1)E(X) + (C_{\mathrm{out}} + Q_{\mathrm{out}}q_1)(h - (E(X) - hE(j)) + hE(y) + nt_{\mathrm{s}} + t_{\mathrm{a}}), \quad (30)$$

$$E(C_{\mathrm{OC}}|S_3) = (C_{\mathrm{in}} + Q_{\mathrm{in}}q_1)E(X) + (C_{\mathrm{out}} + Q_{\mathrm{out}}q_1)(kh + nt_{\mathrm{s}} - E(X)) + (C_{\mathrm{out}} + Q_{\mathrm{out}}q_2)\frac{S}{q_2 - q_1}, \quad (31)$$

$$\begin{aligned} E(C_{\mathrm{OC}}|S_4) = {} & \left(F\left(kh + nt_{\mathrm{s}} + \frac{S}{q_2 - q_1}\right) - F(kh + nt_{\mathrm{s}})\right) \\ & \Bigg((C_{\mathrm{in}} + Q_{\mathrm{in}}q_1)(kh + nt_s) \\ & + (C_{\mathrm{in}} + Q_{\mathrm{in}}q_2)(E(X) - kh - nt_s) \\ & + (C_{\mathrm{out}} + Q_{\mathrm{out}}q_2)\left(\frac{S}{q_2 - q_1} - (E(X) - kh - nt_{\mathrm{s}})\right)\Bigg) \\ & + (F(kh + nt_{\mathrm{s}}) - F(kh))\Bigg((C_{\mathrm{in}} + Q_{\mathrm{in}}q_1)E(X) \\ & + (C_{\mathrm{out}} + Q_{\mathrm{out}}q_1)(nt_{\mathrm{s}} - (E(X) - kh)) \\ & + (C_{\mathrm{out}} + Q_{\mathrm{out}}q_2)\frac{S}{q_2 - q_1}\Bigg). \end{aligned} \quad (32)$$

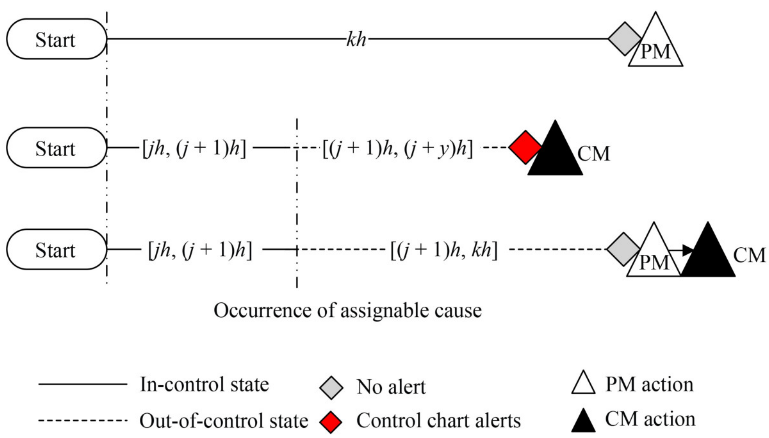

Fig. 4 The maintenance strategy.

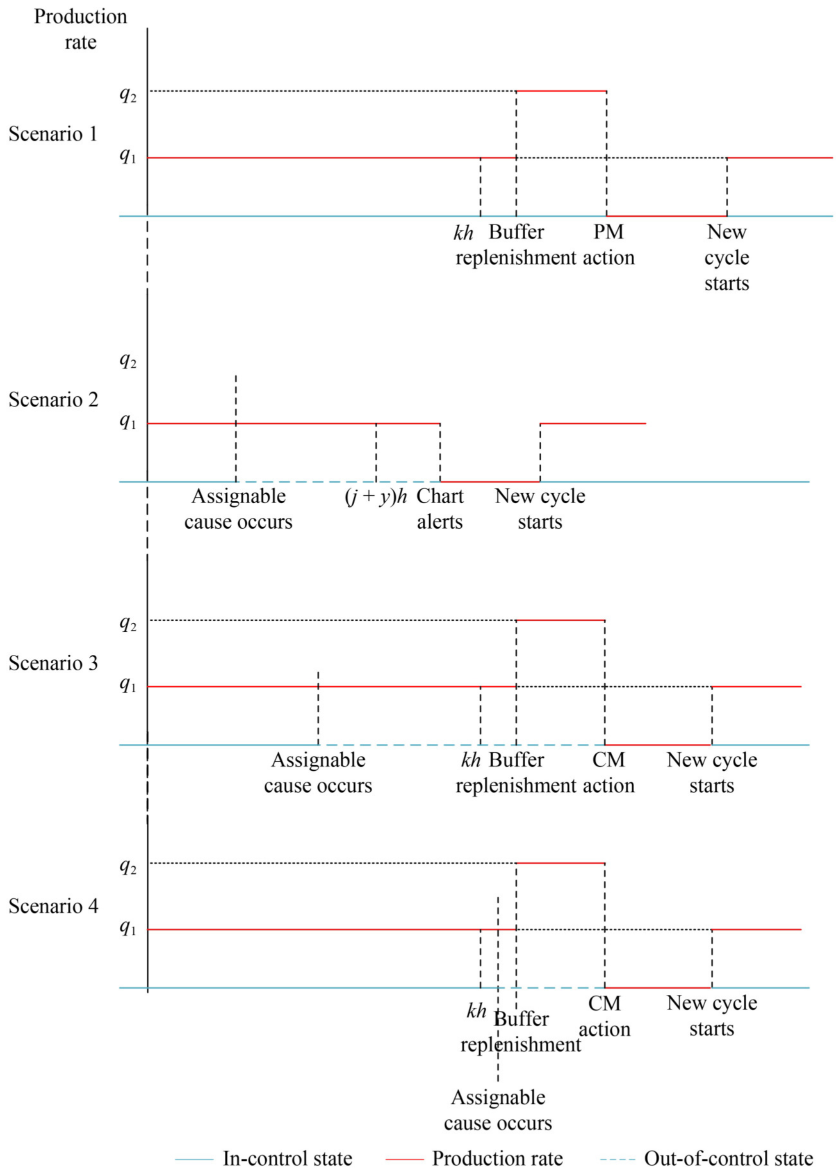

Fig. 5 The production strategy.

4.4.2 Excepted cost of inventory holding in each cycle

Introducing an inventory buffer inevitably incurs inventory holding costs. Previous studies have assumed that the inventory is held in the buffer from the beginning to the end of the cycle, which can significantly increase the cost. In this study, the stocking time of the buffer is set before the maintenance activity. The expected inventory holding cost in each cycle ($E(C_{\mathrm{H}})$) can be calculated using the following formula:

$$E(C_{\mathrm{H}}) = P(S_1)E(C_{\mathrm{H}}|S_1) + P(S_2)E(C_{\mathrm{H}}|S_2) + P(S_3)E(C_{\mathrm{H}}|S_3) + P(S_4)E(C_{\mathrm{H}}|S_4). \quad (33)$$

As shown in Fig. 6, the buffer inventory holdings and holding times are the same for Scenarios 1, 3, and 4. Additionally, in Scenario 2, the cost is zero because there is no time to restock. Then, $E(C_{\mathrm{H}}|S_i)$ can be obtained as follows:

$$E(C_{\mathrm{H}}|S_1) = C_{\mathrm{h}}\left[\frac{S^2}{2(q_2-q_1)} + \frac{S^2}{2q_1}\right], \quad (34)$$

$$E(C_{\mathrm{H}}|S_2) = 0, \quad (35)$$

$$E(C_{\mathrm{H}}|S_3) = C_{\mathrm{h}}\left[\frac{S^2}{2(q_2-q_1)} + \frac{S^2}{2q_1}\right], \quad (36)$$

$$E(C_{\mathrm{H}}|S_4) = C_{\mathrm{h}}\left[\frac{S^2}{2(q_2-q_1)} + \frac{S^2}{2q_1}\right]. \quad (37)$$

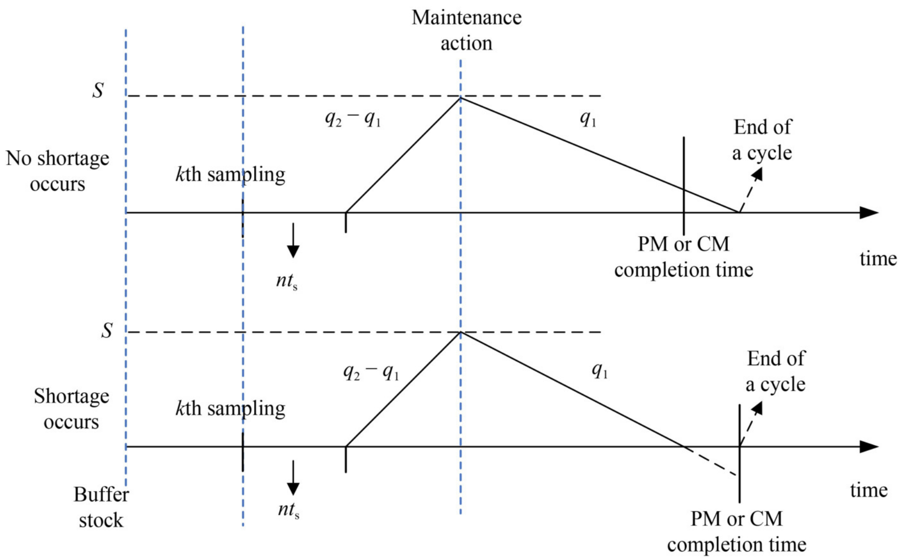

Fig. 6 The buffer stock holding state.

4.4.3 Excepted shortage cost in each cycle

Shortages occur when the buffer's stock is depleted before the end of the maintenance activity, as illustrated in Fig. 6. The projected cost of shortages (ECSH) can be calculated using the following formula:

$$E(C_{\mathrm{SH}})=P(S_1)E(C_{\mathrm{SH}}|S_1)+P(S_2)E(C_{\mathrm{SH}}|S_2)+P(S_3)E(C_{\mathrm{SH}}|S_3)+P(S_4)E(C_{\mathrm{SH}}|S_4), \tag{38}$$

where $E(C_{\mathrm{SH}}|S_i)$ represents the estimated shortage cost in each cycle for scenario i, and it can be determined by

$$E(C_{\mathrm{SH}}|S_1)=C_{\mathrm{S}}\int_{\frac{S}{q_1}}^{\infty}(q_1y_1-S)g_1(y_1)\mathrm{d}y_1, \tag{39}$$

$$E(C_{\mathrm{SH}}|S_2)=C_{\mathrm{S}}\int_0^{\infty}q_1y_2g_2(y_2)\mathrm{d}y_2, \tag{40}$$

$$E(C_{\mathrm{SH}}|S_3)=C_{\mathrm{S}}\int_{\frac{S}{q_1}}^{\infty}(q_1y_2-S)g_2(y_2)\mathrm{d}y_2, \tag{41}$$

$$E(C_{\mathrm{SH}}|S_4)=C_{\mathrm{S}}\int_{\frac{S}{q_1}}^{\infty}(q_1y_2-S)g_2(y_2)\mathrm{d}y_2. \tag{42}$$

Equations (41) and (42) calculate the out-of-stock cost for Scenarios 3 and 4, respectively. The calculation of the shortage cost is identical for these two scenarios since the buffer has already been replenished, and corrective maintenance is performed afterward. Therefore, the formula for shortage cost remains consistent for each scenario.

4.5 Excepted cycle duration

Following the comprehensive probability theorem, $E(T)$ can be defined by:

$$E(T)=P(S_1)E(T|S_1)+P(S_2)E(T|S_2)+P(S_3)E(T|S_3)+P(S_4)E(T|S_4), \tag{43}$$

where $E(T|S_i)$ is the projected duration of a production cycle in scenario i; $P(S_i)(i=1,2,3,4)$ represents the occurring probability of scenario i.

The total time for each scenario includes the equipment production time, buffer stocking time, production system maintenance time, quality inspection time, and control chart alarm cause analysis time. In Scenario 2, the replenishment time is zero because there is no time for buffer replenishment. Equations (44)–(47) define the estimated cycle durations of Scenarios 1–4, respectively.

$$E(T|S_1)=kh+(1-\gamma)\frac{kt_f}{ARL_0}+nt_s+\frac{S}{q_2-q_1}+\max\left(E(Y_1),\frac{S}{q_1}\right), \tag{44}$$

$$E(T|S_2)=E(X)+(1-\gamma)\frac{E(j)t_f}{ARL_0}+h-[E(X)-hE(j)]+hE(y)+nt_s+t_a+E(Y_2), \tag{45}$$

$$E(T|S_3)=kh+(1-\gamma)\frac{E(j)t_f}{ARL_0}+nt_s+\frac{S}{q_2-q_1}+t_a+\max\left(E(Y_2),\frac{S}{q_1}\right), \tag{46}$$

$$E(T|S_4)=kh+(1-\gamma)\frac{E(j)t_f}{ARL_0}+nt_s+\frac{S}{q_2-q_1}+t_a+\max\left(E(Y_2),\frac{S}{q_1}\right). \tag{47}$$

Equations (46) and (47) can be used to compute the desired cycle length for Scenarios 3 and 4, respectively. The computation of the desired cycle length is identical for both scenarios. Although the system has already failed in Scenario 3, the alarm does not occur until after the kth detection, after which corrective maintenance is performed. Conversely, in Scenario 4, the fault occurs after the kth detection, and then corrective maintenance is immediately performed. Although the events occurring in the two scenarios are different, the timing remains consistent. Therefore, the formula for the expected cycle length remains the same for both scenarios.

In Eqs. (45)–(47), $E(j)$ denotes the number of sample inspections conducted before the state transition process, and it is obtained by

$$E(j) = \sum_0^{\infty} j \times P[jh \leqslant X < (j+1)h] = \sum_{j=1}^{k} j[F((j+1)h) - F(jh)]. \quad (48)$$

In Eq. (14), $E(y)$ denotes the average run length when the process is in the out-of-control state (ARL_1). This metric reflects the number of sampling inspections performed starting from the instance the process transitions into an out-of-control state until the point when the control chart triggers an alarm.

4.6 Joint optimization equation

Based on the renewal process theory, the cost per unit of time (ECT) is calculated by

$$ECT = \frac{E(C)}{E(T)}, \quad (49)$$

where $E(C)$ includes the operational cost ($E(C_{\mathrm{OC}})$), inventory holding cost ($E(C_{\mathrm{H}})$), out-of-stock cost ($E(C_{\mathrm{SH}})$), cost of maintenance ($E(C_{\mathrm{m}})$), sample inspection cost ($E(C_{\mathrm{ins}})$), and the cost of analyzing false alarms ($E(C_{\mathrm{FA}})$). Therefore, $E(C)$ is obtained by

$$E(C) = E(C_{\mathrm{OC}}) + E(C_{\mathrm{H}}) + E(C_{\mathrm{SH}}) + E(C_{\mathrm{m}}) + E(C_{\mathrm{ins}}) + E(C_{\mathrm{FA}}). \quad (50)$$

The proposed model's objective is to minimize ECT, and it includes five decision variables: the number of samples (n), the inspection number in a complete cycle (k), the interval between successive samples (h), the control limits of the control chart (UCL), and the buffer inventory (S).

5 Numerical results

The effectiveness and feasibility of the proposed model were demonstrated through practical demonstrations, including a numerical example, sensitivity analysis, and comparative study. Section 5.1 presents several numerical examples assuming the production process follows a normal, uniform, and gamma distribution. Both mean and variance deviations are considered. Section 5.2 presents the sensitivity analysis of parameters and their impact on results. Finally, Section 5.3 provides a comprehensive comparison of the proposed model with existing studies.

5.1 Numerical example

The Weibull distribution was used to characterize various model parameters, including the transition time from the in-control state to the out-of-control state (X), the PM action time (y_1), and the CM action time (y_2). The Weibull distribution has been commonly employed in numerous studies to analyze equipment life and assess reliability (Mahmoudi et al., 2017; Mohtashami, 2014; Wang, 2013). The PDFs of random variable X, y_1, and y_2 were respectively obtained by

$$f(x) = \lambda v(\lambda x)^{v-1} \mathrm{e}^{-(\lambda x)^{v}}; \ x \geqslant 0, \lambda \geqslant 0, v \geqslant 1, \quad (51)$$

$$g_1(y_1) = \lambda_1 v_1 (\lambda_1 y_1)^{v_1 - 1} \mathrm{e}^{-(\lambda_1 y_1)^{v_1}}; \ y_1 \geqslant 0, \lambda_1 \geqslant 0, v_1 \geqslant 1, \quad (52)$$

$$g_2(y_2) = \lambda_2 v_2 (\lambda_2 y_2)^{v_2 - 1} \mathrm{e}^{-(\lambda_2 y_2)^{v_2}}; \ y_2 \geqslant 0, \lambda_2 \geqslant 0, v_2 \geqslant 1. \quad (53)$$

In the proposed model, the CRPS chart is used for sample inspection. In the CRPS chart, the UCL was obtained from Eqs. (8) and (9). The LCL of the CRPS chart was zero; α and β in the control chart were obtained from Eqs. (10) and (11), respectively; ARL_0 and ARL_1 were obtained using Eqs. (12) and (13), respectively.

Further, P(control chart signals | out-of-control) was calculated by:

$$\mathrm{P(control\ chart\ signals \mid out\text{-}of\text{-}control)} = 1 - \prod_{i=1}^{k-j} \beta. \quad (54)$$

Similarly, P(in-control signals | out-of-control) was calculated by:

$$\mathrm{P(in\text{-}control\ signals \mid out\text{-}of\text{-}control)} = \prod_{i=1}^{k-j} \beta. \quad (55)$$

The industrial case study conducted by Hadian et al. (2021) was used to assess the practical applicability of the proposed model. The key parameters of the model and the entire production process were set as shown in Table 1. For the sake of generality, similar constraints on sample size (n) were considered; ARL_0, ARL_1, the number of sampling intervals (k), and other pertinent factors were defined following the work of Hadian et al. (2021). However, their study assumed that the product quality characteristic followed a normal distribution and only focused on changes in the mean value while keeping the variance constant, which is overly stringent and not reflective of real-world scenarios. Hence, this study considered the product quality characteristic to follow a

Table 1 The parameters' values.

γ	t_f	t_s	t_a	C_{CM}	C_{PM}	C_h	C_S	C_{in}	C_{out}	C_f	
1	1	0.01	1	5000	2400	0.5	3	100	300	1	
C_v	C_I	Q_{in}	Q_{out}	q_1	q_2	λ_1	λ_2	λ	v_1	v_2	v
0.2	200	0	0	90	160	0.4	0.4	0.3	2	2	2

normal distribution, uniform distribution, and gamma distribution, with the ability to allow variations in both the mean and standard deviation of the process.

5.1.1 Product quality characteristic following normal distribution

In Case 1, the quality characteristic of the product obeyed a normal distribution, characterized by the mean (μ_0) value and standard deviation (σ_0) of one. Also, when an assignable cause emerged, a shift in both the mean and the standard deviation occurred; particularly, the mean shifted to $\mu_1 = 1.5$, followed by a change in the standard deviation to a value of $\sigma_1 = 2$.

$$s.t.\begin{cases} 5 \leqslant n \leqslant 20, \\ 1 \leqslant k \leqslant 100, \\ 1 \leqslant h \leqslant 50, \\ 0.1 \leqslant UCL \leqslant 0.5, \\ 1 \leqslant S \leqslant 400. \end{cases} \tag{56}$$

The results of Case 1 are presented in Table 2. For the sake of generality, the range of CRPS values was calculated, as illustrated in Fig. 7. After removing outliers, the range of *UCL* and LCL values for the CRPS control chart was between 0.1 and 0.5, and the value of *S* ranged from one to 400 (Zequeira et al., 2008, 2004). Based on Table 2, from an industrial perspective, a quality test was conducted every 50 h with a sample size of five. The control limit of the CRPS control chart was set at 0.5. Preventive maintenance of the manufacturing system was scheduled after the completion of the 4.6518th sample test. Buffer replenishment was implemented with a safety stock *S* set to 347.4615, and the *ECT* was calculated to be 328.5618, reflecting the state with the least cost.

5.1.2 Product quality characteristic following uniform distribution

In Case 2, the product quality characteristic was assumed to follow a uniform distribution with a range of $U(0.065, 7.535)$ based on the work by Shi et al. (2016). When an assignable cause occurred, both the mean and standard deviation of the distribution changed, with a range of $U(1.065, 8.535)$.

Table 2 The results of Case 1

n	h	k	UCL	S	ECT
5	50	4.6518	0.5	347.4615	328.5618

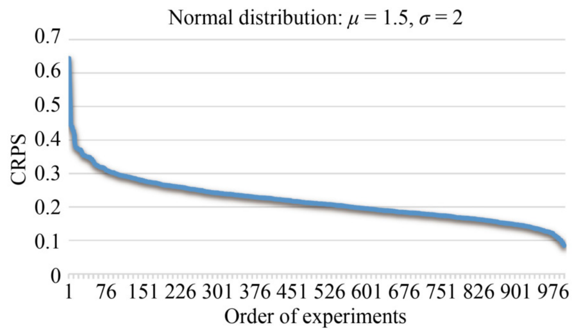

Fig. 7 The CRPS value range in Case 1.

$$\text{Min } ETC$$
$$s.t.\begin{cases} 5 \leqslant n \leqslant 20, \\ 1 \leqslant k \leqslant 100, \\ 1 \leqslant h \leqslant 50, \\ 0.2 \leqslant UCL \leqslant 1.2, \\ 1 \leqslant S \leqslant 400. \end{cases} \tag{57}$$

The CRPS values in Case 2 are depicted in Fig. 8. After removing outliers at both ends, the control chart limited the *UCL* of the CRPS control chart to a range between 0.2 and 1.2. Case 2 results are presented in Table 3. From an industrial standpoint, a quality test was conducted every 50 h, with a sample size of five. The CRPS control chart had a control limit of 1.2. Preventive maintenance of the manufacturing system was performed after the completion of the 49.7103rd sample test. Buffer replenishment was carried out with a safety stock (*S*) set to 309.9749, and the *ECT* was found to be 303.2305.

5.1.3 Product quality characteristic following gamma distribution

In Case 3, the product quality characteristic followed a gamma distribution with *Gamma*(2,1) (Shi et al., 2016). When an assignable cause occurred, both the mean and standard deviation changed, resulting in a distribution of *Gamma* (1,2).

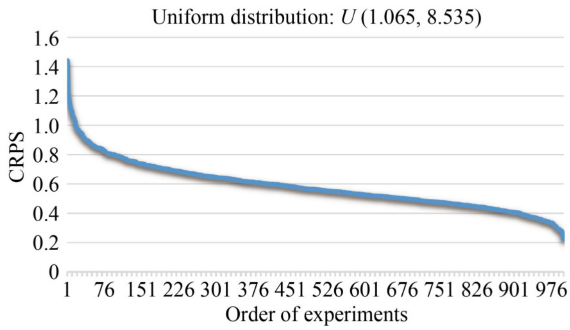

Fig. 8 The CRPS values in Case 2.

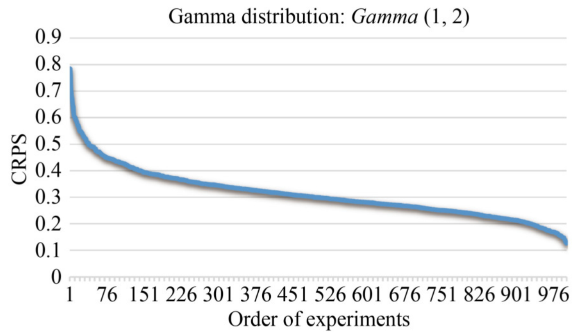

Fig. 9 The CRPS values in Case 3.

Table 3 The results of Case 2

n	*h*	*k*	*UCL*	*S*	*ECT*
5	50	49.7103	1.2	309.9749	303.2305

Table 4 The results of Case 3

n	*h*	*k*	*UCL*	*S*	*ECT*
5	50	77.6591	0.7	1	302.5985

$$\text{Min } ETC$$
$$s.t. \begin{cases} 5 \leqslant n \leqslant 20, \\ 1 \leqslant k \leqslant 100, \\ 1 \leqslant h \leqslant 50, \\ 0.1 \leqslant UCL \leqslant 0.7, \\ 1 \leqslant S \leqslant 400. \end{cases} \quad . \tag{58}$$

The CRPS values in Case 3 are displayed in Fig. 9. After removing outliers at both ends, the *UCL* value of the CRPS control chart ranged from 0.1 to 0.7. Case 3 results are provided in Table 4. From an industrial perspective, a quality test was conducted every 50 h, with a sample size of five. The CRPS control chart had a control limit of 0.7. Preventive maintenance of the manufacturing system was performed after the completion of the 77.6591st sample test. Buffer replenishment was carried out with a safety stock (*S*) set to one. The minimum *ECT* value was calculated to be 302.5985.

5.2 Sensitivity analysis

Next, it was assumed that the product feature masses followed the normal distribution, and the effects of $\lambda, \lambda_1, \lambda_2$ (i.e., the scale parameters of the Weibull distribution) and v, v_1, v_2 (i.e., the shape parameters of the Weibull distribution) on the final result were studied. The values of the main parameters related to the manufacturing process are presented in Table 5. The product quality characteristic followed the normal distribution, with a mean of $\mu_0 = 1$ and a standard deviation of $\sigma_0 = 1$. When the assignable cause occurred, the mean and standard deviation both changed, resulting in a mean of $\mu_1 = 1.5$ and a standard deviation of $\sigma_1 = 2$.

The sensitivity analysis was performed by varying one of the analyzed parameters, including $\lambda, \lambda_1, \lambda_2$ and v, v_1, v_2, while keeping the other parameters constant to examine the effect on the *ECT* and process variables. The results of the sensitivity analysis are presented in Tables 6–11.

The results in the above tables indicate that the considered parameters had a significant effect on *n*, *h*, and *UCL*. These three key decision variables consistently approached their respective boundary values. In practical situations, the range of decision variables can be adjusted judiciously, either narrowed or expanded, based on the unique context and requirements of a specific company.

The number of sampling intervals (*k*) varied as the parameters changed, ranging between 4.5 and 4.7. This can be considered a small change. Furthermore, both parameters had a significant influence on buffer *S*. This suggests that when the CRPS control chart is used in the joint control strategy, the detection buffer inventory *S* is strongly affected by the model parameters.

5.3 Comparative study

The proposed integrated model was compared with the models proposed by Hadian et al. (2021) and Shi et al. (2023). In these two studies, it was assumed that the quality characteristic followed a normal distribution with a mean μ_0 and a variance σ; also, when the assignable cause occurred, the mean of the quality characteristic changed from μ_0 to $\mu_0 + \delta\sigma$, where δ had a value of one. Therefore, in this study, the mean was changed, whereas the variance remained unchanged. The product quality characteristic followed the normal distribution, with a mean of $\mu_0 = 1$ and a standard deviation of $\sigma_0 = 1$. When the assignable cause occurred, the mean changed to $\mu_1 = 2$, and the standard deviation was $\sigma_1 = 1$. For a fair comparison of the two models, the parameter data used in this study were derived from the work of Chakraborty and Giri (2012), Hadian et al. (2021), and Shi et al. (2023), as presented in Table 12.

Table 5 Parameter values

γ	t_f	t_s	t_a	C_{CM}	C_{PM}	C_h	C_S	C_{in}	C_{out}	C_f	
1	1	0.01	1	5000	2400	0.5	3	100	300	1	
C_v	C_I	Q_{in}	Q_{out}	q_1	q_2	λ_1	λ_2	λ	v_1	v_2	v
0.2	200	0	0	90	160	0.4	0.4	0.3	2	2	2

Table 6 Sensitivity analysis results for λ

λ	n	h	k	UCL	S	ECT
0.2	5	50	4.6374	0.5	348.7862	328.6117
0.3	5	50	4.6518	0.5	347.4615	328.5618
0.4	5	50	4.6637	0.5	316.9654	328.4249

Table 7 Sensitivity analysis results for v

v	n	h	k	UCL	S	ECT
1	5	50	4.6170	0.5	339.0196	328.4801
2	5	50	4.6518	0.5	347.4615	328.5618
3	5	50	4.6223	0.5	342.2853	328.5144

Table 8 Sensitivity analysis results for λ_1

λ_1	n	h	k	UCL	S	ECT
0.3	5	50	4.6350	0.5	342.2019	328.5166
0.4	5	50	4.6518	0.5	347.4615	328.5618
0.5	5	50	4.6170	0.5	344.1313	328.5166

Table 9 Sensitivity analysis results for v_1

v_1	n	h	k	UCL	S	ECT
1	5	50	4.6163	0.5	349.7860	328.5169
2	5	50	4.6518	0.5	347.4615	328.5618
3	5	50	5.6577	0.5	341.5521	328.5167

Table 10 Sensitivity analysis results for λ_2

λ_2	n	h	k	UCL	S	ECT
0.3	5	50	4.5604	0.5	372.0660	329.6455
0.4	5	50	4.6518	0.5	347.4615	328.5618
0.5	5	50	4.6858	0.5	320.0389	327.7921

Table 11 Sensitivity analysis results for v_2

v_2	n	h	k	UCL	S	ECT
1	5	50	4.6930	0.5	366.1123	328.8817
2	5	50	4.6518	0.5	347.4615	328.5618
3	5	50	4.6149	0.5	323.6984	328.5246

Table 12 Data used in the comparative study

γ	t_f	t_s	t_a	C_{CM}	C_{PM}	C_h	C_S	C_{in}	C_{out}	C_f	δ
1	1	0.01	1	5000	2400	0.5	3	100	300	1	1
C_v	C_I	Q_{in}	Q_{out}	q_1	q_2	λ_1	λ_2	λ	v_1	v_2	v
0.2	200	0	0	90	160	0.4	0.4	0.3	2	2	2

The comparison results of the three models are presented in Table 13. To maintain generality, the UCL values ranged from 0.1 to 1, while the buffer stock level (S) ranged from 1 to 400, following the approach of previous studies (Zequeira et al., 2008, 2004). As shown in Table 13, the optimal solution was achieved when ECT materialized. Notably, the parameters converged at a sample size of n = 5, a sampling interval of h = 50, and k = 100. The UCL had a value of one, and the safety buffer stock level S was 303.1824. The peak performance was achieved with an optimal ECT value of 301.0150. As presented in Table 13, the proposed strategy resulted in improvements of 4.731% and 1.084% compared to the strategies of Hadian et al. (2021) and Shi et al. (2023), respectively.

Table 13 Comparison results of the three models

	n	h	k	l (UCL)	S	ECT
Hadian et al. (2021)	26	2.2604	27	3.1616	101.0751	315.9644
Shi et al. (2023)	1	10	100	3.5	205.8511	304.3147
This study	5	50	100	1	303.1824	301.0150

6 Conclusions

In pursuit of practical solutions to real-world challenges and with the aim of empowering manufacturing enterprises in their cost-reduction efforts, this study presents a pioneering stochastic model that effectively integrates production, maintenance, and quality aspects in an imperfect production process. Specifically, the model deals with an unreliable production process where n samples are tested every h units of time, and maintenance activities are carried out after k tests. To monitor both mean and variance variation, CRPS control charts are employed to detect samples, and the restriction of a specific distribution is relaxed. PM activity is performed when there is no alarm signal on the control chart after k inspections, while CM activity is conducted otherwise. Notably, the proposed model differs from traditional joint models in that it replenishes the buffer inventory after $(k-1)$ tests, resulting in significant reductions in total unit cost. Additionally, buffer stock is employed to prevent shortages during activities.

The proposed model is validated through numerical experiments that consider various distributions for the product quality characteristics, specifically the normal, uniform, and gamma distributions. The experiments assume varying mean and standard deviation values for the process. The obtained values of variables $(n, h, k, \mathrm{UCL}, \mathrm{S})$ for the three distributions are as follows: (5, 50, 5.6212, 0.5, 400), (5, 50, 50, 1.2, 1), and (5, 50, 9.6593, 0.7, 400), with corresponding *ECT* values of 328.51969, 302.7764, and 333.3183, respectively. Furthermore, a sensitivity analysis is conducted for three scale parameters $(\lambda, \lambda_1, \lambda_2)$ and three shape parameters (v, v_1, v_2). Results demonstrate that both the scale parameters and certain shape parameters significantly influence the testing sample size n. Hence, in practical applications, sufficient historical data are crucial for accurately fitting the maintenance and failure time parameters. The accuracy of these parameters directly impacts the quality of results obtained.

Comparison with two existing models reveals that the proposed model can achieve cost reductions of 4.731% and 1.084%, respectively.

However, there are certain limitations associated with this research. First, in real-world manufacturing systems, maintenance activities include more than just preventive and corrective maintenance; they can be further classified as minor and major repairs. Consequently, future research should consider developing more detailed joint control strategies tailored to specific manufacturing systems. Secondly, this study assumes that the degradation process adheres to the Weibull distribution. However, for a more precise degradation process, it is essential to fit the data to the most accurate distribution based on historical degradation data from a particular implementation. Furthermore, in actual manufacturing systems, quality monitoring using CRPS control charts within a joint control strategy can be conducted regardless of the distribution that governs the quality characteristics. However, this approach necessitates a rich-data environment, making it unfeasible for customized production or multi-species small lot production. Therefore, the future research agenda should focus on developing new tools for monitoring under small sample data within a joint control strategy, representing a potential research direction. Another crucial limitation of the proposed method lies in assuming that the production process of a manufacturing system can be categorized into controlled and uncontrolled states, which falls short in capturing the complexity of real production situations. In reality, the system may need to stop production, rendering the aforementioned assumption unrealistic. Consequently, future research must employ a more detailed classification scheme for the states of a manufacturing system to ensure compatibility with real-world production. Lastly, this study utilizes CRPS control charts for quality monitoring, albeit limited to a single quality feature. To address this limitation, future research could integrate multivariate control charts into the model, enabling the monitoring of multiple quality characteristics within a joint model.

Competing Interests The authors declare that they have no competing interests.

References

Ait-El-Cadi A, Gharbi A, Dhouib K, Artiba A (2021b). Integrated production, maintenance and quality control policy for unreliable manufacturing systems under dynamic inspection. International Journal of Production Economics, 236: 108140

Ait El Cadi A, Gharbi A, Dhouib K, Artiba A (2021a). Joint production and preventive maintenance controls for unreliable and imperfect manufacturing systems. Journal of Manufacturing Systems, 58: 263–279

An Y, Chen X, Hu J, Zhang L, Li Y, Jiang J (2022). Joint optimization of preventive maintenance and production rescheduling with new machine insertion and processing speed selection. Reliability Engineering & System Safety, 220: 108269

Azimpoor S, Taghipour S (2021). Joint inspection and product quality optimization for a system with delayed failure. Reliability Engineering & System Safety, 215: 107793

Bahria N, Chelbi A, Bouchriha H, Dridi I H (2019). Integrated production, statistical process control, and maintenance policy for unreliable manufacturing systems. International Journal of Production Research, 57(8): 2548–2570

Ben-Daya M, Rahim M A (2000). Effect of maintenance on the economic design of $\bar{x}$-control chart. European Journal of Operational Research, 120(1): 131–143

Bokrantz J, Skoogh A, Berlin C, Stahre J (2020). Smart maintenance: instrument development, content validation and an empirical pilot. International Journal of Operations & Production Management, 40(4): 481–506

Bouslah B, Gharbi A, Pellerin R (2018). Joint production, quality and maintenance control of a two-machine line subject to operation-dependent and quality-dependent failures. International Journal of Production Economics, 195: 210–226

Chakraborty T, Giri B C (2012). Joint determination of optimal safety stocks and production policy for an imperfect production system.

Applied Mathematical Modelling, 36(2): 712–722

Chakraborty T, Giri B C, Chaudhuri K S (2009). Production lot sizing with process deterioration and machine breakdown under inspection schedule. Omega, 37(2): 257–271

Cheng G, Li L (2020). Joint optimization of production, quality control and maintenance for serial-parallel multistage production systems. Reliability Engineering & System Safety, 204: 107146

Christer A H (1978). Refined asymptotic costs for renewal reward processes. Journal of the Operational Research Society, 29(6): 577–583

de Vargas V C C, Dias Lopes L F, Mendonça Souza A (2004). Comparative study of the performance of the Cusum and EWMA control charts. Computers & Industrial Engineering, 46(4): 707–724

Duffuaa S, Kolus A, Al-Turki U, El-Khalifa A (2020). An integrated model of production scheduling, maintenance and quality for a single machine. Computers & Industrial Engineering, 142: 106239

Fakher H B, Nourelfath M, Gendreau M (2018). Integrating production, maintenance and quality: A multi-period multi-product profit-maximization model. Reliability Engineering & System Safety, 170: 191–201

Farahani A, Tohidi H (2021). Integrated optimization of quality and maintenance: A literature review. Computers & Industrial Engineering, 151: 106924

Fitouhi M C, Nourelfath M (2012). Integrating noncyclical preventive maintenance scheduling and production planning for a single machine. International Journal of Production Economics, 136(2): 344–351

Hadian S M, Farughi H, Rasay H (2021). Joint planning of maintenance, buffer stock and quality control for unreliable, imperfect manufacturing systems. Computers & Industrial Engineering, 157: 107304

Hajej Z, Nyoungue A C, Abubakar A S, Mohamed Ali K (2021). An integrated model of production, maintenance, and quality control with statistical process control chart of a supply chain. Applied Sciences (Basel, Switzerland), 11(9): 4192

Hajji A, Gharbi A, Pellerin R (2012). Joint production control and product quality decision making in a failure prone multiple-product manufacturing system. International Journal of Production Research, 50(13): 3661–3672

Harrou F, Sun Y, Madakyaru M, Bouyedou B (2018). An improved multivariate chart using partial least squares with continuous ranked probability score. IEEE Sensors Journal, 18(16): 6715–6726

He Y, Zhao Y, Han X, Zhou D, Wang W (2020). Functional risk-oriented health prognosis approach for intelligent manufacturing systems. Reliability Engineering & System Safety, 203: 107090

Hersbach H (2000). Decomposition of the continuous ranked probability score for ensemble prediction systems. Weather and Forecasting, 15(5): 559–570

Huang S, Yang J, Xie M (2020). A double-sampling SPM scheme for simultaneously monitoring of location and scale shifts and its joint design with maintenance strategies. Journal of Manufacturing Systems, 54: 94–102

Kumar S, Purohit B S, Manjrekar V, Singh V, Lad B K (2018). Investigating the value of integrated operations planning: A case-based approach from automotive industry. International Journal of Production Research, 56(22): 6971–6992

Le M D, Tan C M (2013). Optimal maintenance strategy of deteriorating system under imperfect maintenance and inspection using mixed inspection scheduling. Reliability Engineering & System Safety, 113: 21–29

Liu X, Sun Q, Ye Z S, Yildirim M (2021b). Optimal multi-type inspection policy for systems with imperfect online monitoring. Reliability Engineering & System Safety, 207: 107335

Liu Y, Zhang Q, Ouyang Z, Huang H Z (2021a). Integrated production planning and preventive maintenance scheduling for synchronized parallel machines. Reliability Engineering & System Safety, 215: 107869

Mahmoudi M, Elwany A, Shahanaghi K, Gholamian M R (2017). A delay time model with multiple defect types and multiple inspection methods. IEEE Transactions on Reliability, 66(4): 1073–1084

Megoze Pongha P, Kibouka G R, Kenné J P, Hof L A (2022). Production, maintenance and quality inspection planning of a hybrid manufacturing/remanufacturing system under production rate-dependent deterioration. International Journal of Advanced Manufacturing Technology, 121(1-2): 1289–1314

Mohtashami A (2014). A new hybrid method for buffer sizing and machine allocation in unreliable production and assembly lines with general distribution time-dependent parameters. International Journal of Advanced Manufacturing Technology, 74(9-12): 1577–1593

Nourelfath M, Nahas N, Ben-Daya M (2016). Integrated preventive maintenance and production decisions for imperfect processes. Reliability Engineering & System Safety, 148: 21–31

Obaidat S, Liao H (2021). Integrated decision making for attributes sampling and proactive maintenance in a discrete manufacturing system. International Journal of Production Research, 59(18): 5454–5476

Pan E, Jin Y, Wang S, Cang T (2012). An integrated EPQ model based on a control chart for an imperfect production process. International Journal of Production Research, 50(23): 6999–7011

Panagiotidou S, Tagaras G (2007). Optimal preventive maintenance for equipment with two quality states and general failure time distributions. European Journal of Operational Research, 180(1): 329–353

Panagiotidou S, Tagaras G (2010). Statistical process control and condition-based maintenance: A meaningful relationship through data sharing. Production and Operations Management, 19(2): 156–171

Peng R, He X, Zhong C, Kou G, Xiao H (2022). Preventive maintenance for heterogeneous parallel systems with two failure modes. Reliability Engineering & System Safety, 220: 108310

Pinson P, Reikard G, Bidlot J R (2012). Probabilistic forecasting of the wave energy flux. Applied Energy, 93: 364–370

Radhoui M, Rezg N, Chelbi A (2009). Integrated model of preventive maintenance, quality control and buffer sizing for unreliable and imperfect production systems. International Journal of Production Research, 47(2): 389–402

Rivera-Gómez H, Gharbi A, Kenné J P, Montaño-Arango O, Corona-Armenta J R (2020). Joint optimization of production and maintenance strategies considering a dynamic sampling strategy for a deteriorating system. Computers & Industrial Engineering, 140: 106273

Rivera-Gómez H, Gharbi A, Kenné J P, Ortiz-Zarco R, Corona-Armenta J R (2021). Joint production, inspection and maintenance control policies for deteriorating system under quality constraint. Journal of Manufacturing Systems, 60: 585–607

Salmasnia A, Abdzadeh B, Namdar M (2017). A joint design of production run length, maintenance policy and control chart with multiple assignable causes. Journal of Manufacturing Systems, 42: 44–56

Salmasnia A, Kaveie M, Namdar M (2018). An integrated production and maintenance planning model under VP-T2 Hotelling chart. Computers & Industrial Engineering, 118: 89–103

Sett B K, Sarkar S, Sarkar B (2017). Optimal buffer inventory and inspection errors in an imperfect production system with preventive maintenance. International Journal of Advanced Manufacturing Technology, 90(1–4): 545–560

Shi L, Gong L, Lin D K J (2018). CRPS chart: Simultaneously monitoring location and scale under data-rich environment. Quality and Reliability Engineering International, 34(4): 681–697

Shi L, Lv X, He Y, He Z (2024). Optimizing production, maintenance, and quality control for imperfect manufacturing systems considering timely replenishment. International Journal of Production Research, 62(10): 3504–3525

Shi L, Ma H, Lin D K J (2016). Process capability analysis via continuous ranked probability score. Quality and Reliability Engineering International, 32(8): 2823–2834

Tambe P P, Kulkarni M S (2022). A reliability based integrated model of maintenance planning with quality control and production decision for improving operational performance. Reliability Engineering & System Safety, 226: 108681

Tasias K A (2022). Integrated quality, maintenance and production model for multivariate processes: A Bayesian approach. Journal of Manufacturing Systems, 63: 35–51

Thorarinsdottir T L, Gneiting T (2010). Probabilistic forecasts of wind speed: Ensemble model output statistics by using heteroscedastic censored regression. Journal of the Royal Statistical Society. Series A, (Statistics in Society), 173(2): 371–388

Wang W (2013). Models of inspection, routine service, and replacement for a serviceable one-component system. Reliability Engineering & System Safety, 116: 57–63

Xiang Y (2013). Joint optimization of X¯ control chart and preventive maintenance policies: A discrete-time Markov chain approach. European Journal of Operational Research, 229(2): 382–390

Zequeira R I, Prida B, Valdés J E (2004). Optimal buffer inventory and preventive maintenance for an imperfect production process. International Journal of Production Research, 42(5): 959–974

Zequeira R I, Valdes J E, Berenguer C (2008). Optimal buffer inventory and opportunistic preventive maintenance under random production capacity availability. International Journal of Production Economics, 111(2): 686–696

Zhou P P, Wang W, Lin Dennis K J, Liu Y (2023). Another look at run length distribution. Quality and Reliability Engineering International, (forthcoming)

Zhou Y W, Chen J, Wu Y, Zhou W (2015). EPQ models for items with imperfect quality and one-time-only discount. Applied Mathematical Modelling, 39(3-4): 1000–1018