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# Joint optimization of electric bus charging and energy storage system scheduling

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**Abstract** The widespread use of energy storage systems in electric bus transit centers presents new opportunities and challenges for bus charging and transit center energy management. A unified optimization model is proposed to jointly optimize the bus charging plan and energy storage system power profile. The model optimizes overall costs by considering battery aging, time-of-use tariffs, and capacity service charges. The model is linearized by a series of relaxations of the nonlinear constraints. This means that we can obtain the exact solution of the model quickly with a commercial solver that is fully adapted to the time scale of day-ahead scheduling. The numerical simulations demonstrate that the proposed method can optimize the bus charging time, charging power, and power profile of energy storage systems in seconds. Monte Carlo simulations reveal that the proposed method significantly reduces the cost and has sufficient robustness to uncertain fluctuations in photovoltaics and office loads.

**Keywords** electric vehicle, energy storage, mixed integer nonlinear programming, Monte Carlo simulations, public transit.

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## 1 Introduction

The electrification of public transportation has significantly accelerated in recent years as a practical means of achieving carbon neutrality (Attard, 2022; Manzolli et al., 2022; Qu et al., 2022a). Global electric bus sales increased by 40% compared to those in the previous year (IEA, 2022). However, the mass deployment of electric buses has led to a sudden increase in charging demand, which poses a threat to the power quality of the distribution network (Rodrigues and Seixas, 2022). To address this challenge, distributed power sources, such as photovoltaics (PVs) and energy storage systems (ESSs), have been introduced into electric bus transit centers (EBTCs) to achieve local power balance and mitigate grid impacts (Liu and Liang, 2021; Ansariyar and Tahmasebi, 2022; Zhang et al., 2022). This change introduces greater complexity and uncertainty in the operation of transit systems, as it combines the bus scheduling problem with the energy storage system scheduling problem (Li et al., 2022). Therefore, there is an urgent need to study the joint scheduling of buses and energy storage systems to optimize the adjustable capabilities of these systems, reduce operating costs, and address the uncertainty caused by PVs and load fluctuations.

Charging scheduling is a crucial aspect that determines the timing, location, and power levels at which buses are charged (Zhang et al., 2023). The primary objective of charging scheduling is to ensure that electric buses have sufficient battery energy to complete their trips while minimizing electricity costs and battery degradation. Charging scheduling algorithms typically focus on three key optimization objectives: 1) reducing the peak load to decrease capacity service charges (Qin et al., 2016; Jahic et al., 2019); 2) minimizing electricity costs by avoiding high-priced charging periods (Leou and Hung, 2017; Wang et al., 2019; He et al., 2022; Liu et al., 2022; Ji et al., 2022); and 3) mitigating battery aging through specific charging profiles (Schoch et al., 2018; Houbbadi

et al., 2019; Zhou et al., 2022a; Zeng et al., 2022; Wei et al., 2018).

In terms of decision variables, previous charging scheduling algorithms only optimized the start and stop times for bus charging. However, with the advancement of charger intelligence, current algorithms can precisely optimize the charging power at each moment (He et al., 2020) and even enable buses to inject power back into the grid (Lin et al., 2021). Furthermore, charging scheduling is often integrated into bus scheduling (Wang et al., 2017; Messaoudi and Oulamara, 2019; Li et al., 2020; Rinaldi et al., 2020; Abdelwahed et al., 2020; Zhou et al., 2022b; Qu et al., 2022b) and bus route planning (Schoenberg and Dressler, 2023) as a subproblem. This means that the decision variables can include determining which trips each bus services and which stops are included in each trip to optimize the overall charging schedule. Additionally, charging scheduling often considers the siting and capacity of charging facilities (Schoenberg et al., 2023).

At a technical level, the size of the bus charging scheduling model is often constrained to ensure solvability within a reasonable amount of time. This is achieved by discretizing time intervals (Abdelwahed et al., 2020; Fang et al., 2022; Huang et al., 2022). Additionally, due to the high uncertainty of passenger travel demands and random bus delays, stochastic optimization algorithms are commonly employed. In terms of engineering applications, specialized toolkits for optimizing bus scheduling have been developed.

As the adoption of electric vehicles (EVs) continues to increase, managing the impact of bus charging on distribution systems presents challenges that cannot be addressed solely by optimizing charging time and power. In such cases, the installation of distributed power sources, such as energy storage systems, at charging stations becomes necessary to directly mitigate the burden on the system by controlling the power. Ding et al. (2015) utilized a linearized mixed-integer programming model to optimize the capacity and control strategy of energy storage systems, aiming to minimize construction and operating costs. Building on this work, He et al. (2019) formulated a mixed-integer programming problem to minimize the acquisition cost of bus batteries, charging stations, energy storage systems, and electricity demand. Wang et al. (2017) conducted a comparative analysis between deploying individual energy storage systems for each charging post and deploying a centralized energy storage system for the entire charging station, concluding that the centrally configured solution yielded superior results.

To further reduce the overall carbon footprint of transit systems, many charging stations are now equipped with PVs (Torreglosa et al., 2016; Fathabadi, 2017; Cheng and Tao, 2018; Tran et al., 2019). However, this integration introduces new complexities and uncertainties to the transit system. To address these challenges, several energy

management techniques have been developed for charging stations equipped with PV and energy storage systems. Yan et al. (2019) proposed a four-stage intelligent optimal control algorithm to minimize operating costs while ensuring robustness against uncertainties. Shin et al. (2020) adopted a novel multiagent deep reinforcement learning approach to solve charging scheduling problems by considering energy storage systems and PVs. Although these studies focused primarily on charging scheduling for private vehicles, their insights have inspired subsequent research focused on bus systems. Drawing upon these advancements, Liu and Liang (2021) proposed a three-layer optimization framework to address the uncertainty arising from office loads and PVs in transit centers. Their framework optimized the bus charging power and energy storage system charge/discharge power to minimize capacity service charges and battery aging costs.

However, accounting for PV and load fluctuations in optimization problems often entails increased computational resource consumption. For example, Liu and Liang (2021) and Sun et al. (2022) employed a robust optimization over time approach to address uncertainty, which necessitated rolling optimization at each time slot, resulting in an average computation time of 29.84 s per time slot. Furthermore, the prevalent time-sharing tariff is not taken into account when calculating electricity costs.

To optimize the scheduling of bus charging and the charging/discharging schedule of energy storage systems in a faster and more efficient manner, this study presents a comprehensive unified optimization model with a single layer. The proposed model aims to minimize multiple costs, including electricity costs, battery aging, and capacity service charges. By applying a series of relaxations to the nonlinear constraints, the model is linearized, allowing for the swift determination of an exact solution using a commercial solver.

A case study is conducted to validate the effectiveness and computational efficiency of the proposed method. The case study focuses on a bus system consisting of 49 vehicles, six routes, and 275 trips. Actual data for PV generation and office loads (which were first proposed by Liu and Liang (2021) and can be regarded as all the other electricity consumption at transit centers except for charging buses) are derived from actual data. The numerical simulations demonstrate that the proposed method can optimize the bus charging time, charging power, and power profile of energy storage systems in less than one minute. Furthermore, Monte Carlo simulations reveal that the proposed method significantly reduces costs, with the total cost of the optimized case being \$368.4, reflecting a 27.7% decrease compared to the benchmark. Additionally, the proposed method proves to be robust against uncertain fluctuations in PV generation and office loads.

The main contributions of this study to the literature are as follows:

(1) We design a novel unified mathematical model that simultaneously optimizes the bus charging timetable, charging power, and charging/discharging power of energy storage systems for electric bus transit centers. The model incorporates various practical requirements and factors, including partial charging, the number of chargers, the charging capacity, the efficiency of the energy storage system, the office load and PV generation at the transit center, time-sharing tariffs, capacity service charges, and battery aging of the energy storage system.

(2) We also account for the uncertainties of the office load and PV generation at the transit center. The energy storage system is designed to provide sufficient spare capacity to handle such uncertainties. In cases where power fluctuations exceed the reserved spare storage, the power purchased from the grid is adjusted to maintain power balance. By appropriately relaxing constraints or applying equivalent mathematical transformations, all nonlinear constraints in the proposed model can be converted into linear constraints. This allows for the efficient solution of the model using commercial solvers such as Gurobi.

(3) We implement our approach through numerical experiments, including Monte Carlo simulation and parameter sensitivity analysis. The results demonstrate the effectiveness and computational efficiency of the proposed model, even when parameters change or there are uncertain fluctuations in PV and office loads.

The remaining sections of this paper are organized as follows: In Section 2, we describe the proposed unified model that explains the complete problem from bus charging scheduling to energy storage system control. In Section 3, we relax and linearize the model to enhance its solvability. In Section 4, we perform a simulation analysis using actual transit system data. In Section 5, we summarize the contents of this paper and provide an outlook for future work.

## 2 Model formulation

We consider a single-terminal electric bus system with a transit center and multiple buses, where all buses depart from the transit center and follow a predetermined schedule. The time periods during which each bus can be charged (i.e., the time periods during which it stays at the transit center) are predetermined. The trips served by each bus and the corresponding energy consumption during the non-rechargeable time period (i.e., the time period on the road) are deterministic. Single-terminal transit networks, such as those in Shenzhen city and Luxembourg city, are commonly used and have been the subject of extensive research in bus charging scheduling studies (Liu and Liang, 2021; He et al., 2022; Zhou et al., 2022a).

The power consumption of the considered system is divided into two parts: the charging loads of the buses and the office load of the transit center. This power is supplied through rooftop PVs, with any shortfall being provided by the grid or the energy storage system. The energy storage system is also used to absorb any excess energy from the PVs and balance the uncertain power fluctuations of the PVs and the office loads.

The day-ahead charging scheduling problem for such a system can be described by the model shown in Fig. 1. This model consists of several submodels, which are described in the following sections.

### 2.1 Notation and assumptions

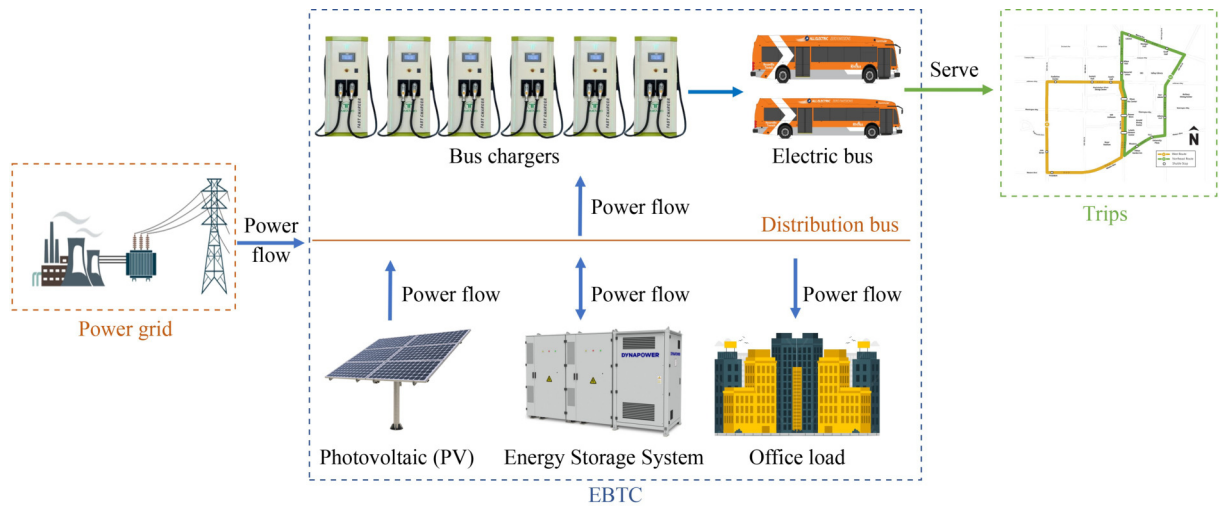
For convenience, Table 1 provides a summary of the critical notations used throughout this study.

The operating time of the bus system is divided into  $T$  time slots indexed from 1 to  $T$ . The bus system is considered with  $N$  buses indexed 1 to  $N_{ch}$ . The trips served by the buses are indexed 1 to  $I$ . For each trip  $i$ , the start time is defined as  $s_i$ , the end time is defined as  $e_i$ , and the power consumed is defined as  $m_i$ . An auxiliary parameter matrix  $\iota$  is introduced to describe the time occupied by each trip, where  $\iota_{i,t} = 1$  indicates that time slot  $t$  is between the starting time and ending time of trip  $i$ .

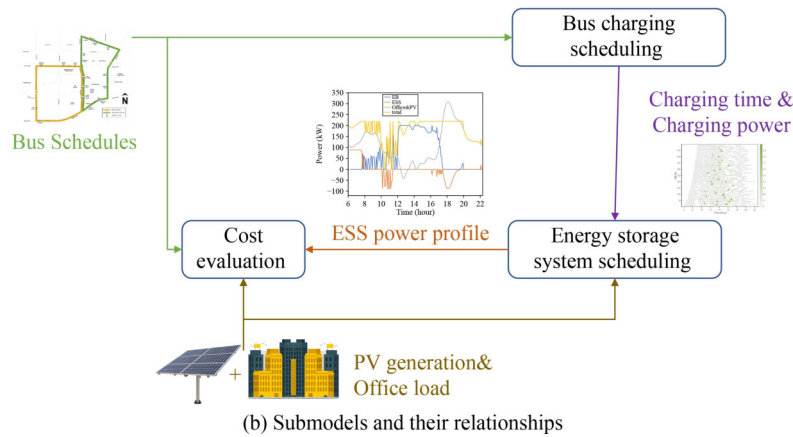
The number of buses required to meet the regular operation requirements of a transit system is often much smaller than the number of trips that need to be served in that system, which indicates that each bus may need to service multiple trips. Bus scheduling assigns each trip to a unique bus, and the trips served by each bus should not overlap. In this case, binary vector  $x$  is introduced to formulate the given bus schedule, where  $x_{n,i} = 1$  indicates that bus  $n$  serves trip  $i$ . We assume that the number of buses and their schedules are predetermined. This assumption is reasonable, as fleet sizes and bus timetables typically remain constant on a daily basis.

The charging and discharging power of the energy storage system and buses is continuously adjustable without exceeding the battery power limit. In this case, continuous decision variables  $y_t^{ch}$  and  $y_t^{dis}$  are introduced to denote the charging and discharging power of the energy storage system at time slot  $t$ , respectively, while  $z_{n,t}$  denotes the charging power of bus  $n$  at time slot  $t$ . The binary variable  $q_{n,t}$  is used to describe the charging schedule of the vehicle, where  $q_{n,t} = 1$  if bus  $n$  is charged at time slot  $t$ . We assume that there are  $N_{ch}$  chargers in the transit center and that the total charging power of these chargers cannot exceed  $z_{ub}^{all}$ . We also assume the omission of battery capacity degradation to concentrate the current model on the operational optimization of charging schedules and energy storage dispatch. This assumption simplifies the model without significantly impacting the short-term operational decisions that our study aims to address.

After finishing all trips for the day, overnight charging



(a) The components of the proposed system and the power flows between them



(b) Submodels and their relationships

Fig. 1 System model.

Table 1 Nomenclature

Sets	
$\mathcal{T}$	Set of time slots
$\mathcal{I}$	Set of trips
$\mathcal{N}$	Set of buses
$\mathcal{T}^s$	Set of trip starting times
$\mathcal{T}^e$	Set of trip ending times
Indices	
$t$	Index of a time slot
$i$	Index of a trip
$n$	Index of a bus
Parameters	
$\Delta t$	Length of a time slot
$s_i$	The starting time of trip $i$
$e_i$	The ending time of trip $i$
$m_i$	The energy consumption during trip $i$
$l_{i,t}$	Binary parameter equal to 1, if time slot $t$ is between the starting time and ending time of trip $i$

(Continued)

Sets	
$P_t^{OV}$	Office load after deducting PV generation at time slot $t$
$err^{OV}$	Maximum prediction error of $P_t^{OV}$
$z_{ub}$	Maximum charging power of a charger
$z_{ub}^{all}$	Maximum total charging power of all the chargers
$N_{ch}$	Number of chargers in the transit center
$y_{ub}^{ch}$	Maximum charging power of the energy storage system
$y_{ub}^{dis}$	Maximum discharging power of the energy storage system
$E^B$	Bus battery capacity
$E^S$	Energy storage system battery capacity
$SoC_{init}^E$	Bus SoC at the beginning of the day
$SoC_{lb}^E$	Lower bound of bus battery SoC
$SoC_{ub}^E$	Upper bound of bus battery SoC
$SoC_{init}^S$	Energy storage system SoC at the beginning of the day
$SoC_{lb}^S$	Lower bound of energy storage system battery SoC
$SoC_{ub}^S$	Upper bound of energy storage system battery SoC
$P_t^e$	Unit electricity price at time slot $t$
$P_{on}^e$	Unit electricity price of overnight charging
$p^c$	Capacity service rate
$p^{bc}$	Battery cost per kWh
$cycle^B$	Battery total life cycle of bus
$cycle^S$	Battery total life cycle of energy storage system
$eff^S$	Charging efficiency of the energy storage system
<b>Variables</b>	
$x_{n,i}$	Binary variable that is equal to 1 if bus $n$ serves trip $i$
$y_t^{ch}$	Decision variable indicating the charging power of energy storage system at time slot $t$
$y_t^{dis}$	Decision variable indicating the discharging power of energy storage system at time slot $t$
$q_{n,t}$	Binary decision variable that is equal to 1 if bus $n$ is charged at time slot $t$
$z_{n,t}$	Decision variable indicating the charging power of bus $n$ at time slot $t$
$v_n$	Decision variable indicating the overnight charging energy of bus $n$
$v_{ch}^S$	Decision variable indicating the overnight charging energy of the ESS
$v_{dis}^S$	Decision variable indicating the overnight discharging energy of the ESS
$w$	Decision variable indicating the maximum transit center total power
$SoC_{n,t}^E$	SoC of bus $n$ at time slot $t$
$SoC_t^S$	SoC of the ESS at time slot $t$
$P_t^{EBTC}$	The power consumption of the entire transit center at time slot $t$
$c$	Total cost of the transit center
$c'$	Total optimizable cost of the transit center
$c^e$	Cost of electricity
$c^{e'}$	Electricity costs incurred for buses charging and the energy storage system charging and discharging
$c^c$	Capacity service charge
$c^{bc}$	Battery aging cost
$c^{bc'}$	Battery aging cost of the energy storage system

will charge the batteries of the buses and energy storage system to their initial SoCs. The decision variables  $y_{\text{on}}^{\text{ch}}$ ,  $y_{\text{on}}^{\text{dis}}$  and  $v_n$  are introduced to denote the overnight charging and discharging energy of the energy storage systems and buses, respectively. To calculate the service charges for capacity costs, the decision variable  $w$  is introduced to denote the peak loads of transit centers during the day.

## 2.2 Bus scheduling model

The proposed joint optimization model requires the assignment of bus trips to individual buses, which can be optimized by a bus scheduling model. This model must ensure that each trip is served by a unique bus and that no bus serves two trips simultaneously to be acceptable. These requirements can be formalized as two constraints as follows:

$$\sum_{n \in \mathcal{N}} x_{n,i} = 1, i \in \mathcal{I}, \quad (1)$$

$$\sum_{i \in \mathcal{I}} x_{n,i} \iota_{i,t} \leq 1, n \in \mathcal{N}, t \in \mathcal{T}. \quad (2)$$

The given constraints involve a variable  $x_{n,i}$ , which denotes whether the  $n$  th bus is assigned to serve the  $i$  th trip, and a constant  $\iota_{i,t}$ , which indicates whether the  $i$  th trip covers time  $t$ . Equation (1) guarantees that each trip is serviced by exactly one bus. Equation (2) ensures that a bus does not serve two different trips simultaneously.

The electric bus scheduling model must allocate sufficient charging time for each bus between their assigned trips while satisfying the constraints above. If a bus is immediately assigned to serve the next trip after completing each trip without leaving any charging time, no matter how effective the charging model is, the bus cannot complete the entire route.

In practice,  $x_{n,i}$  can be obtained through manual scheduling or by utilizing any existing optimization model. As an example, Section 4.2 presents a rule based simple heuristic algorithm that generates  $x_{n,i}$  quickly to provide a general framework. Considering the low computational complexity of the proposed method, embedding it as a lower-level model into any bus scheduling model using a bilevel approach for joint scheduling is also a good idea.

## 2.3 Charging scheduling model

The battery capacities are generally not sufficient to run the buses through all the trips in a day. In this case, buses need to be charged at transit centers between trips. For a given bus, charging scheduling schedules the charging time and the charging power at each time slot for each bus. The charging plan needs to be developed with the following constraints.

**Constraint I:** A bus can only be charged when it stays at the transit center. In other words, the bus charging

must be staggered with the time when the bus is serving a particular trip. This constraint can be expressed as Eq. (3):

$$z_{n,t} \sum_{i \in \mathcal{I}} x_{n,i} \iota_{i,t} = 0, n \in \mathcal{N}, t \in \mathcal{T}. \quad (3)$$

Here,  $x_{n,i}$  represents whether the  $n$  th bus serves the  $i$  th trip, and  $\iota_{i,t}$  represents whether the  $i$  th trip occupies time slot  $t$ . The product of the two,  $\sum_{i \in \mathcal{I}} x_{n,i} \iota_{i,t}$ , indicates whether bus  $n$  is serving any trip at time slot  $t$ . Meanwhile,  $z_{n,t}$  represents the charging power of bus  $n$  at time  $t$ , where 0 denotes no charging. Equation (3) ensures that the bus can only charge when it is not serving any trip.

**Constraint II:** For each time slot, the number of buses charged cannot exceed the total number of chargers in the transit center. This constraint can be mathematically represented by Eqs. (4) and (5):

$$\sum_{n \in \mathcal{N}} q_{n,t} \leq N_{\text{ch}}, t \in \mathcal{T}, \quad (4)$$

with

$$q_{n,t} = \begin{cases} 0, & \text{if } z_{n,t} = 0 \\ 1, & \text{if } z_{n,t} > 0 \end{cases}, n \in \mathcal{N}, t \in \mathcal{T}, \quad (5)$$

where  $q_{n,t}$  represents whether bus  $n$  is charged at time slot  $t$ , and  $N_{\text{ch}}$  denotes the number of chargers in the transit center. Equations (4) and (5) guarantee that the quantity of buses charged at the same time does not surpass the number of chargers accessible.

**Constraint III:** For each time slot, the charging power of any bus cannot exceed the capacity of the charger.

$$0 \leq z_{n,t} \leq z_{\text{ub}}, n \in \mathcal{N}, t \in \mathcal{T}, \quad (6)$$

where  $z_{\text{ub}}$  is the maximum charging power of a charger.

**Constraint IV:** Considering the capacity limitations of transformers and distribution lines, the transit center may not support all chargers in operating at the maximum load. In this case, the total charging power of all chargers should also satisfy a capacity constraint.

$$\sum_{n \in \mathcal{N}} z_{n,t} \leq z_{\text{ub}}^{\text{all}}, t \in \mathcal{T}, \quad (7)$$

where  $z_{\text{ub}}^{\text{all}}$  is the maximum total charging power of the transit center.

**Constraint V:** The SoC of each bus should always be within the safe range. In this case, the SoC of a bus at a certain time slot can be calculated from the amount of power that was previously charged and consumed by this bus.

$$\text{SoC}_{\text{lb}}^{\text{E}} \leq \text{SoC}_{n,t}^{\text{E}} \leq \text{SoC}_{\text{ub}}^{\text{E}}, n \in \mathcal{N}, t \in \mathcal{T}, \quad (8)$$

$$\text{SoC}_{n,t}^{\text{E}} = \text{SoC}_{\text{init}}^{\text{E}} + \frac{\sum_{\tau=1}^t z_{n,\tau} \Delta t - \sum_{s_i \leq t} m_i x_{n,i}}{E^{\text{B}}}, n \in \mathcal{N}, t \in \mathcal{T}, \quad (9)$$

where  $\text{SoC}_{\text{init}}^{\text{E}}$  is the SoC of buses at the beginning of the

day;  $SoC_{lb}^E$  and  $SoC_{ub}^E$  are the lower and upper bounds of the bus SoC, respectively;  $m_i$  is the amount of energy consumed by trip  $i$ ; and  $E^B$  is the bus capacity.  $SoC_{n,t}^E$  is an intermediate variable that represents the SoC of bus  $n$  at time slot  $t$ . Equations (8) and (9) ensure that the SoC of all buses stays within the upper and lower limits during all time slots.

**Constraint VI:** After completing their services for the day, all buses return to the transit center and restore their battery SoC to the initial values through overnight charging.

$$\sum_{i \in \mathcal{I}} z_{n,i} \Delta t - \sum_{i \in \mathcal{I}} m_i x_{n,i} + v_n = 0, \quad n \in \mathcal{N}. \quad (10)$$

Here,  $v_n$  represents the amount of energy obtained by bus  $n$  through overnight charging. Equation (10) ensures that the total energy consumed by each bus within a day is equal to the total energy charged into it.

#### 2.4 Energy storage system scheduling model

The energy storage system installed in the transit center can mitigate the impact of bus charging and PV generation on the distribution system through proper charging and discharging profiles, reducing the peak load of the transit center on the one hand and avoiding reverse flow at the valley on the other hand. Considering the prediction errors in PV generation and office loads, the energy storage system also accepts the heavy responsibility of reducing fluctuations in power uncertainty. The energy storage system can be scheduled considering the following constraints.

**Constraint VII:** The energy storage system should prevent the transit center from injecting reverse power into the grid.

$$P_i^{EBTC} \geq 0, \quad t \in \mathcal{T}, \quad (11)$$

$$P_i^{EBTC} = P_i^{OV} + y_i^{sh} - y_i^{dis} + \sum_{n \in \mathcal{N}} z_{n,t}, \quad t \in \mathcal{T}, \quad (12)$$

where  $P_i^{OV}$  is the office load deducting PV generation at time slot  $t$ . and  $P_i^{EBTC}$  is the total power consumption of the transit center, which can be obtained by summing the power of the energy storage system, photovoltaic generation, office load, and electric vehicle charging.

**Constraint VIII:** Considering the inevitable forecast errors of PV generation and office load, the scheduling curve of the energy storage system must leave enough margin to smooth out the uncertain fluctuations. Here, we assume that the prediction error of  $P_i^{OV}$  does not exceed  $\pm err^{OV}$  and that the energy storage system fills the difference between the actual and predicted power. Even in the worst-case scenario, the energy storage system charging or discharging power must be within limits.

$$0 \leq y_i^{ch} \leq y_{ub}^{ch} - err^{OV}, \quad t \in \mathcal{T}, \quad (13)$$

$$0 \leq y_i^{dis} \leq y_{ub}^{dis} - err^{OV}, \quad t \in \mathcal{T}, \quad (14)$$

$$y_i^{ch} y_i^{dis} = 0, \quad t \in \mathcal{T}, \quad (15)$$

where  $y_{ub}^{ch}/y_{ub}^{dis}$  denotes the maximum charging/discharging power of the energy storage system. It is not feasible to discharge a storage system while simultaneously charging it. Considering that the product of two consecutive variables introduces nonlinearity, Eq. (15) will be relaxed in Section 3 to improve the computational performance.

**Constraint IX:** The energy storage system SoC must be in the safe range in the worst-case scenario.

$$SoC_i^S \leq SoC_{max}^S - \frac{e^{OV}}{E^S} t, \quad t \in \mathcal{T}, \quad (16)$$

$$SoC_i^S \geq SoC_{min}^S + \frac{e^{OV}}{E^S} t, \quad t \in \mathcal{T}, \quad (17)$$

$$SoC_i^S = SoC_{init}^S + \frac{\Delta t}{E^S} \sum_{\tau \leq t} (y_{\tau}^{ch} eff^S - y_{\tau}^{dis}), \quad (18)$$

where  $SoC_{init}^S$  is the SoC of the energy storage system at the beginning of the day;  $SoC_{lb}^S$  and  $SoC_{ub}^S$  are the lower and upper bounds of the energy storage system SoC, respectively;  $eff^S$  is the charging efficiency of the energy storage system; and  $E^S$  is the energy storage system capacity.  $SoC_i^S$  represents the SoC of the energy storage system at time slot  $t$ . It can be calculated by integrating the power flow into and out of the storage system up to time  $t$ .

**Constraint X:** The energy storage system will restore its battery SoC to its initial value by overnight charging or discharging

$$E^S SoC_T^S + v_{ch}^S eff^S - v_{dis}^S = E^S SoC_{init}^S, \quad (19)$$

$$v_{ch}^S, v_{dis}^S \geq 0. \quad (20)$$

Smoothing out all uncertainties by the energy storage system is a rather demanding requirement. If the uncertainty fluctuations in PVs and office loads do not exceed  $e^{OV}$ , such changes will not be transmitted to the grid. In the grid view, the transit center will always follow the predicted power curve.

#### 2.5 Transit center operation cost model

The final submodel of the system model is the cost model. The total cost  $c$  consists of three components: electricity  $c^e$ , capacity service charge  $c^c$ , and battery aging  $c^{bc}$ .

$$c = c^e + c^c + c^{bc}. \quad (21)$$

We can obtain the cost of electricity by multiplying the total electricity consumption of the transit center, including the power consumed by the buses during their trips, the

office loads, and PV power generation, by the unit electricity price  $p_t^e$  and the overnight charging price.

$$c^e = \sum_{t \in \mathcal{T}} P_t^{\text{EBTC}} p_t^e \Delta t + (\sum_{n \in \mathcal{N}} v_n + v_{\text{ch}}^{\text{S}} - v_{\text{dis}}^{\text{S}}) p_{\text{on}}^e. \quad (22)$$

In contrast to the cost of electricity, the capacity service charge, which is positively correlated with the peak load, is a market-based instrument for distribution system operators to limit the peak load. In this study, we obtain the capacity service charge by multiplying the peak load of the transit center by the capacity service rate  $p^c$  (Liu and Liang, 2021).

$$c^c = p^c w, \quad (23)$$

where  $w$  is the maximum total power of the transit center.

$$w = \max_{t \in \mathcal{T}} P_t^{\text{EBTC}}. \quad (24)$$

We use the following linear model (Wirasanti et al., 2018) calibrated by a field test to represent the battery aging cost.

$$c^{\text{bc}} = \frac{p^{\text{bc}}}{\text{cyc}^{\text{S}}} (v_{\text{dis}}^{\text{S}} + \Delta t \sum_{t \in \mathcal{T}} y_t^{\text{dis}}) + \frac{p^{\text{bc}}}{\text{cyc}^{\text{B}}} \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{I}} m_i, \quad (25)$$

where  $p^{\text{bc}}$  is the battery cost per kWh and  $\text{cyc}^{\text{S}}$  and  $\text{cyc}^{\text{B}}$  are the total life cycles of the batteries of the energy storage system and buses, respectively. Note that the total life cycle of a battery varies depending on the operational strategy and the battery profile. Therefore, when calibrating the parameters using a field test, it is necessary for the charging and discharging behaviors of the battery to be as close as possible to the actual situation.

With the optimization objective of minimizing the overall operating cost of the transit center and considering various constraints in charging scheduling and energy storage system scheduling, we develop a joint optimization model to optimize both charging scheduling and transit center energy management.

$$\begin{aligned} & \min_{w, x_{n,t}, y_t^{\text{ch}}, y_t^{\text{dis}}, z_{n,t}} c, \\ & \text{s.t. Eqs. (3)–(25)}. \end{aligned} \quad (26)$$

### 3 Relaxation and rewriting

In the above model, Eqs. (5), (15), and (24) introduce nonlinear terms that create difficulties for the solution. To obtain fast solutions, the equations need to be rewritten as linear formulations.

First, Eqs. (5) and (6) are combined and rewritten as Eq. (27):

$$0 \leq z_{n,t} \leq z_{\text{ub}} q_{n,t}, \quad n \in \mathcal{N}, t \in \mathcal{T}. \quad (27)$$

Compared to Eq. (5), Eq. (27) may overestimate the number of occupied chargers. For example, when  $z_{n,t} = 0$ ,

$q_{n,t}$  can still equal 1. However, a solution with an overestimated number of occupied chargers does not result in a smaller objective function value. The objective function in our model is designed to minimize the total cost, which includes the electricity cost  $c_e$ . If the number of occupied chargers is overestimated, this implies that there are more chargers in use than actually needed. In such a scenario, the calculated costs would be greater because the model assumes that all these chargers incur costs. This indicates that relaxation in this context is acceptable since it does not compromise the effectiveness of the objective function. Therefore, this relaxation is acceptable.

Similarly, the nonlinear equation in Constraint (24) can be relaxed to the linear inequality in Constraint (28):

$$w \geq P_t^{\text{EBTC}}, \quad t \in \mathcal{T}. \quad (28)$$

Considering that  $w$  is proportional to the value of the objective function, Constraint (28) is equivalent to Constraint (24) when the objective function obtains the minimum value.

The rationale for revising Eq. (15) is grounded in considerations of charging efficiency. In an optimal scenario, allowing for simultaneous charging and discharging would not be viable due to the resulting inefficiencies, which unnecessarily escalate costs. Given our aim to minimize expenses, the optimal solution naturally prohibits such concurrent actions. This understanding ensures that Eq. (15) remains valid in the optimal scenario, effectively preventing charging and discharging from occurring simultaneously for any bus. Despite the relaxation in the linearized model, the optimal solution mirrors realistic operational behavior. Therefore, any approximation error arising from this revision is inconsequential to the model's results. Consequently, the revision of Eq. (15) does not affect the objective of cost minimization, and the presence of this constraint is implicitly met in the optimal solution provided by the model. In the objective function,  $P_t^{\text{OV}}$  and  $m_i$  are uncontrollable constants. These constant terms are removed from the objective function because their presence in the objective function does not affect the optimization process. In this case, Eqs. (21), (22), and (25) can be rewritten as Eqs. (29)–(31), respectively:

$$c^e = \sum_{t \in \mathcal{T}} (y_t^{\text{ch}} - y_t^{\text{dis}} + \sum_{n \in \mathcal{N}} z_{n,t}) p_t^e \Delta t + (\sum_{n \in \mathcal{N}} v_n + v_{\text{ch}}^{\text{S}} - v_{\text{dis}}^{\text{S}}) p_{\text{on}}^e, \quad (29)$$

$$c^{\text{bc}'} = \frac{p^{\text{bc}}}{\text{cyc}^{\text{S}}} (v_{\text{dis}}^{\text{S}} + \Delta t \sum_{t \in \mathcal{T}} y_t^{\text{dis}}), \quad (30)$$

$$c' = c^{e'} + c^c + c^{\text{bc}'}, \quad (31)$$

where  $c^{e'}$  is the electricity cost incurred for bus charging and energy storage system charging and discharging,  $c^{\text{bc}'}$  is the battery aging cost of the energy storage system, and

$c'$  is the total optimizable cost of the transit center. For convenience, all total costs in the following refer to the total optimizable cost.

In this case, objective Eq. (26) can be rewritten as Eq. (32), and this mixed integer linear programming model can be solved by a commercial solver such as Gurobi.

$$\min_{w, x_{n,t}, y_{t'}^{\text{ch}}, y_{t'}^{\text{dis}}, z_{n,t}} c', \quad (32)$$

s.t. Eqs. (3), (4), (7)–(14), (16)–(20), (23), (27)–(31).

## 4 Case study

### 4.1 Data and parameters

We verify the proposed model and approach using a real electric bus system, as detailed by Zhang et al. (2021). This system comprises six routes originating from its transit center, accommodating a total of 275 trips. To ensure smooth operation, a minimum of 49 buses must be deployed, considering the temporal overlap of these trips. The original study provides comprehensive information regarding the length and schedule of each bus line, along with the energy consumption for each trip.

The bus parameters, based on the BYD K9 model, are derived from the work of Liu and Liang (2021). Notably, the battery size is standardized at 100 kWh for buses and 500 kWh for energy storage systems.

By incorporating the influence of time-of-use electricity tariffs, we assume the presence of three different pricing tiers throughout the day, as illustrated in Fig. 2. Notably, overnight charging prices align with daytime rates until 8 am. These pricing structures are based on electricity rates observed in Zhengzhou, China, as referenced by Liu et al. (2021). Additionally, a capacity service charge of \$0.39/kW is accounted for.

The office load and PV generation data are taken from actual data. The load and PV uncertainty fluctuation ranges are set to 10% of the average daily power. Table 2 provides the other parameter values.

### 4.2 A rule-based heuristic algorithm for bus scheduling

The proposed joint optimization model assumes that the scheduling of buses is preestablished. In practical applications, a specialized bus scheduling model can be used to determine the bus schedule as input for the proposed model, or manual scheduling can be performed. In this case study, we employ a rule-based heuristic algorithm that mimics the decision-making processes of human schedulers to generate a bus schedule. This algorithm satisfies the constraints specified by Eqs. (1) and (2) while striving to distribute the required trips to each bus in an equitable manner. Additionally, the proposed algorithm provides a straightforward bus charging scheduling

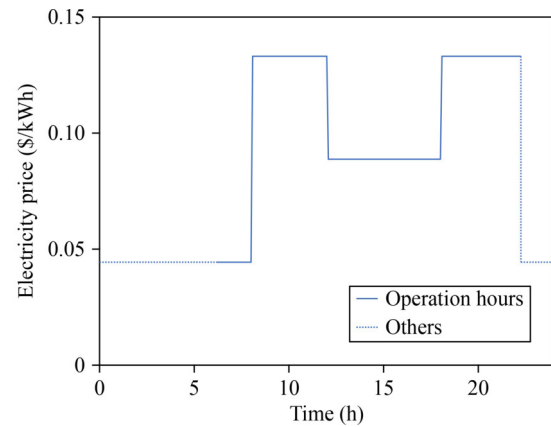


Fig. 2 The electricity price.

Table 2 Constant parameters

Parameter	Value	Unit
$T$	194	–
$I$	275	–
$N_{\text{bus}}$	49	–
$\Delta t$	5	min
$e^{\text{OV}}$	11.2	kW
$z_{\text{ub}}$	80	kW
$y_{\text{ub}}^{\text{ch}}$	100	kW
$y_{\text{ub}}^{\text{dis}}$	100	kW
$E_B$	270	kWh
$E_s$	500	kWh
$SoC_{\text{init}}^E$	100	%
$SoC_{\text{lb}}^E$	20	%
$SoC_{\text{ub}}^E$	100	%
$SoC_{\text{init}}^S$	60	%
$SoC_{\text{lb}}^S$	20	%
$SoC_{\text{ub}}^S$	100	%
$p_c$	0.39	\$/kW
$p_{bc}$	132	\$/kWh
$cyc^S$	2000	–

strategy and energy storage charging and discharging strategy while computing the bus schedule.

The proposed heuristic algorithm consists of two stages. In the first stage, a comprehensive bus schedule, as well as the charging and discharging curve of the energy storage system, is generated. Additionally, an initial bus charging schedule is created. In the second stage, the bus charging schedule obtained in the first stage is refined by shifting the charging period to the nighttime when electricity prices are at their lowest.

During the first stage of the proposed heuristic algorithm, buses, chargers, and energy storage systems in the transit center are scheduled for each time slot using a seven-step process outlined as follows:

**Step 1:** Check whether one or more trips start at the current time slot. If yes, proceed to Step 2; otherwise, skip to Step 3.

**Step 2:** Select the trip with the highest energy consumption among all the trips starting at the current time slot. Simultaneously, the bus with the highest remaining SoC from the buses that stopped at the transit center is selected for the selected trip. This step is repeated until all trips starting from the current time slot are serviced.

**Step 3:** Check whether any trips need to be served after the current time slot. If yes, proceed to Step 4; otherwise, skip to Step 5.

**Step 4:** Examine the SoC of all buses stopped at the transit center to identify buses that require charging. The bus with the lowest SoC is added to the to-be-charged list, followed by the bus with the second lowest SoC. This process is repeated until the number of buses in the to-be-charged list equals the number of chargers or until all buses that are not on the list have sufficient power. In this paper, the criterion for sufficient power is an SoC greater than 80%.

**Step 5:** If charging all buses in the to-be-charged list at maximum power does not exceed the charging station's capacity constraint, then all the buses charge at maximum power. Otherwise, the charging power of these buses is adjusted to meet the capacity constraint of the charging station.

**Step 6:** Check all trips ending in the current time slot. The buses used for these trips are registered as in-station buses, which can be charged or used for other trips in the next time slot.

**Step 7:** Select the charging and discharging behaviors of the energy storage systems based on the current electricity price of the time slot. If the electricity price is high, discharge the energy storage systems as much as possible until all power consumption at the transit center is neutralized. If the price is low, the energy storage system should be charged as much as possible until it is fully charged.

While the first stage of the algorithm may cause overcharging of buses during operational hours, this issue is addressed in the second stage of the algorithm, which includes the following steps.

**Step 1:** The algorithm initiates by systematically checking each time slot, starting from the final time slot of operational hours, to identify any buses whose SoC exceeds the lower limit.

**Step 2:** For a bus in the current time slot that surpasses the lower limit of the SoC, the algorithm assesses whether the bus is being charged during that particular time slot. If the bus is not charged, Step 1 is repeated to check the previous time slot. Conversely, if the bus is being charged, the algorithm proceeds to Step 3.

**Step 3:** The algorithm endeavors to cancel the charging plan assigned to the bus in the current time slot. Following

this, it examines whether the SoC of the bus in every subsequent time slot remains above the lower limit. If this condition is met, the charging plan for the current time slot is canceled, and Step 1 is repeated to investigate the previous time slot. However, if the SoC falls below the lower limit in any subsequent time slot, the charging plan for the current time slot is retained, and further scrutiny of previous time slots through Step 1 is unnecessary.

By implementing the second stage of the algorithm, the charging duration for buses during operational hours is minimized, while the low-price period at night is optimally utilized.

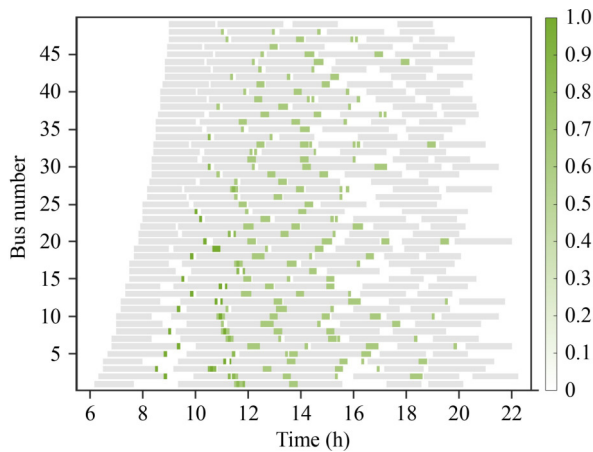
This heuristic method solely bases decisions on data collected directly at each time slot, thus evenly distributing energy consumption and charging opportunities among buses while employing energy storage systems for peak and off-peak power usage. This scheduling strategy aligns with common intuition: 1) priority is given to buses with higher charge levels; 2) buses with low SoCs are charged; 3) electricity is procured when prices are low and sold when they are high; and 4) buses only receive the minimum charging necessary during operational hours, following the simple heuristic algorithm employed as the baseline.

Consequently, we firmly believe that this heuristic algorithm provides a realistic reflection of bus scheduling characteristics to a certain extent. In subsequent case studies, we will utilize the bus schedule generated by this algorithm as input for the joint optimization model and compare the results with those from the proposed model, as well as the heuristic algorithm utilized as a benchmark.

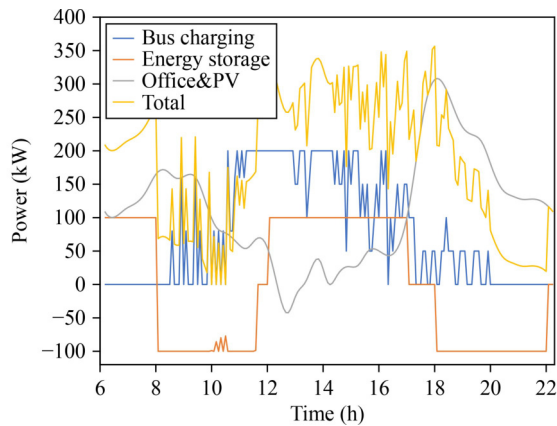
### 4.3 The results of the benchmark

Based on the aforementioned heuristic algorithm, we can establish a benchmark for bus scheduling and charging scheduling, as depicted in Fig. 3. Figure 4 illustrates the aggregated charging power of all buses, the power generated by photovoltaic systems, the power consumed by office loads, the charging and discharging power of energy storage systems, and the total power utilized at the transit center. The SoCs of all buses and the energy storage system are displayed in Fig. 5. The total cost of the benchmark is \$509.2, with electricity costs accounting for \$315.6 or 51.4% of the total cost, battery aging costs accounting for \$49.7 or 9.8% of the total cost, and capacity service charges accounting for \$197.5 or 39.1% of the total cost.

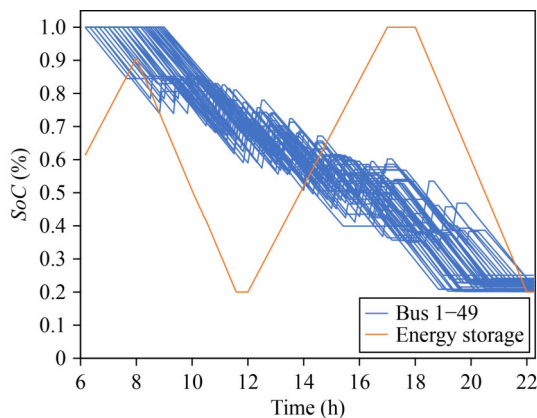
From 8:30 a.m. to 11:30 a.m., the earliest departing buses return to the transit center in a sequential manner for the purpose of charging. At this time, the demand for charging has not yet peaked, and thus, the capacity of the charging station and the number of chargers adequately meet the bus charging needs. After charging, all the bus SoCs are replenished to a level exceeding 80%.



**Fig. 3** Scheduling and charging plan for buses in the benchmark, where the gray blocks indicate the periods when buses are on the road and the green blocks indicate the periods when buses are charging. The green depth indicates the ratio of the charging power to the maximum charging power.



**Fig. 4** Power profiles of the benchmark.



**Fig. 5** SoC profiles of the benchmark.

Although the benchmark unintentionally restricts charging during the morning peak period, there is still a certain level of power consumption from the office loads during this time. Most of this power consumption is offset through PV generation and discharge from the

energy storage system. Comparatively, during the early morning and afternoon periods when electricity prices are lower, the energy storage systems are charged to a sufficiently high SoC, enabling them to handle various scenarios.

Subsequently, between 11:30 a.m. and 6:30 p.m., the transit center operates at full capacity as the number of dispatched buses increases. Consequently, the SoC of the buses begins to gradually decrease. However, by 6:30 p.m., the remaining SoC of most buses is adequate to complete the remaining trips, resulting in a continuous decrease in charging demand. At the end of the operational hours, the SoC of all buses approaches a safety threshold of 20%. In summary, the transit center in the benchmark can meet the charging requirements of the buses.

Significantly, the benchmark highlights the unavoidable need for charging during the evening peak period when electricity prices are at their highest. Additionally, this evening period coincides with the peak consumption hours for office loads. The combination of these factors results in a spike in electricity consumption that lasts for several hours during the high tariff windows, with a maximum power exceeding 350 kW. Consequently, this leads to higher capacity service charges. To mitigate this, the energy storage system is charged and discharged at its maximum power to reduce the difference between the peak and valley loads. However, due to the limited power capacity of energy storage systems, it is challenging to completely suppress the peak load caused by the concentrated charging of buses. Moreover, the repetitive charging and discharging of energy storage system batteries accelerates their aging process, resulting in a daily cost of approximately \$50. This result reflects a portion of the predicament that electric bus charging encounters. In theory, the optimal charging strategy involves minimal charging during operating hours to minimize the electricity costs at night. Simultaneously, the charging demand during operational hours should also consider electricity costs and capacity service charges and should be scheduled during periods of low electricity prices and office loads as much as possible. However, decisions based solely on information from a single moment in time cannot be precise enough to maintain charging at a critical level to minimize costs. Likewise, a simple control strategy allows energy storage systems to achieve lower purchases and higher sales, but effectively controlling charging and discharging power to reduce battery losses is challenging. In general, the benchmark reflects a typical scenario for transit center scheduling. By adhering to simple rules of control, transit centers can streamline resources to maintain daily operations. However, guaranteeing that all the details are optimal is not a simple task.

#### 4.4 Optimization results

In this paper, MATLAB is utilized to invoke Gurobi to

solve the optimization model proposed herein, which aims to reformulate the bus charging schedule as well as the energy storage system charging and discharging schedule. To validate the improvement brought about by the proposed method, the bus scheduling and other parameters are kept identical to those in the benchmark.

The optimization problem involves 9,946 continuous variables and 9,506 binary variables. We set the optimization parameter `mip_gap` to 0 as the condition for Gurobi to conclude the optimization. The total calculation time is 1.5 s, which satisfies the day-ahead scheduling requirements. Furthermore, this computational speed enables us to swiftly adjust the solution in the face of temporary changes in external conditions. In terms of the solving speed, we also conducted tests using a nonlinear solver, which required 466.280662 s to obtain a solution consistent with that derived from our linearized model. This indicates that the mathematical transformations and linear relaxation techniques employed in our paper considerably enhance the efficiency of the solution process while securing a globally optimized, exact solution.

The total cost of the optimized case is \$368.4, which is a 27.7% decrease compared to the benchmark. The battery aging cost is \$10.92, constituting a 78.0% decrease compared to the benchmark. Although the electricity cost (\$271.9) increases by 3% compared to the benchmark, the capacity service charge is \$85.56, signifying a 56.7% decrease compared to the benchmark. Figure 6 illustrates the cost of the optimized components in comparison to the benchmark, while Figs. 7 to 9 present the charging plan, power profiles, and SoC profiles.

The results of the proposed approach are evident, as the total cost is reduced by almost a third. However, it is important to note that the reduction in the cost of each component varies.

The most notable decrease in the optimized cost is observed in the cost of battery aging, which decreases by approximately 80%. This reduction is mainly attributed to the elimination of unnecessary actions in the energy storage system. Figure 9 illustrates that the optimized energy storage system's SoC remains consistently above the initial value, in contrast to the significant fluctuations seen in Fig. 5. This controlled charging and discharging strategy significantly extends the battery's lifespan. Nevertheless, considering the declining average price of lithium batteries to approximately \$100 per kWh (Nykvist and Nilsson, 2015; Kersey et al., 2022), the cost of battery aging for a 500 kWh battery constitutes a marginal proportion of the total cost. The affordability of lithium batteries could have a profound impact on the field of transportation. Specifically, for transit systems, as battery prices continue to decrease, the cost attributed to aging batteries as a percentage of the total transit system costs also diminishes. Consequently, transit companies

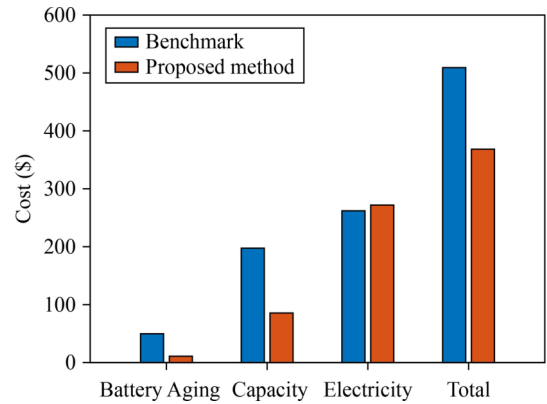


Fig. 6 Cost comparisons.

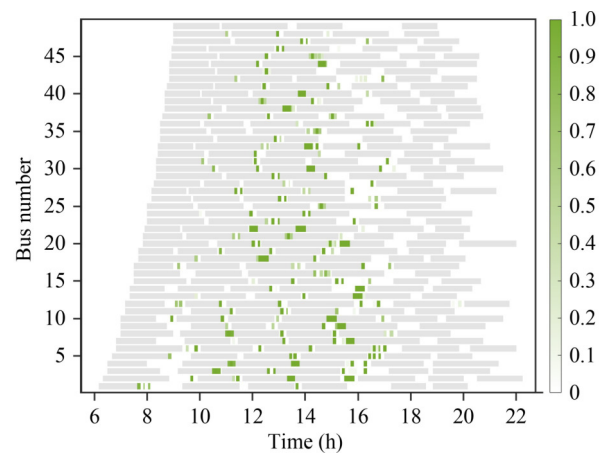


Fig. 7 Scheduling and charging plans for buses, where the gray blocks indicate the periods when buses are on the road and the green blocks indicate the periods when buses are charging. The green depth indicates the ratio of the charging power to the maximum charging power.

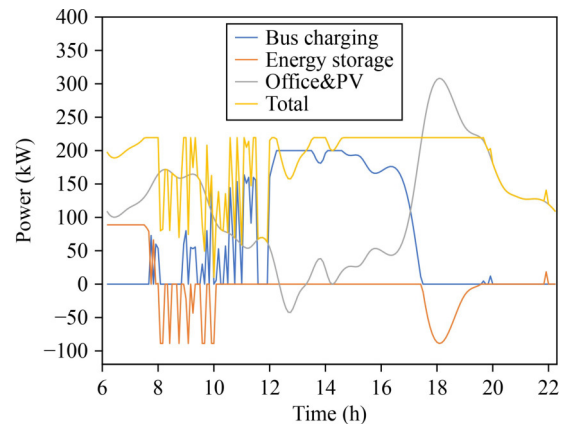
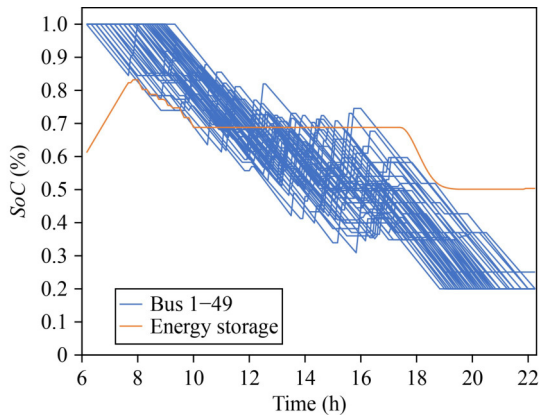


Fig. 8 Optimized power profiles.

may lean toward adopting larger batteries and reducing the number of charging stations.

The electricity cost and capacity service charges account for a comparable percentage of the benchmark and together make up nearly 90% of the total cost. Load



**Fig. 9** Optimized SoC profiles.

shifting is primarily responsible for saving these two cost components. The benchmark successfully shifts most of the charging demand to nonoperational hours to take advantage of lower electricity prices during nighttime charging. Simultaneously, the benchmark's energy storage system maximizes discharging during periods of high electricity prices. It is evident that the benchmark's electricity costs are even lower than those of the optimized results. However, simple scheduling rules cannot precisely align with the peaks and valleys of office loads. Considering the capacity service fees, the optimized results significantly yield cost savings compared to the benchmark. During operating hours, the optimized charging strategy effectively avoids charging during peak office load periods, thereby reducing the capacity service fee. A comparative analysis of Fig. 3 and Fig. 7 reveals that the optimized charging time is shorter, with no charging scheduled between 5:30 and 7:30 pm, the peak office load period. As depicted in Fig. 9, many buses choose to raise their SoCs to a relatively higher level in the afternoon when electricity prices are not as high and then conclude the day's operations with SoCs that are closer to the lower bound. This charging schedule avoids a scenario where a significant number of buses endure the highest electricity prices for continuous charging after 18:00. In addition to the charging time, the optimization of the charging power is also crucial. As observed in Fig. 8, the charging curves of the buses and building load curves exhibit almost axisymmetric patterns in the afternoon. A carefully designed charging schedule mitigates fluctuations in office loads, ensuring a steady total transit center load. During peak load periods, all buses cease charging while the energy storage system commences discharging, thereby maintaining the total load and preventing it from exceeding the previous maximum. Through the joint scheduling of buses and the energy storage system, the capacity service charge was reduced by more than 56%.

#### 4.5 Monte Carlo analysis

The proposed model operates on a day-ahead scheduling

time scale, necessitating the use of forecasted values for office load and PV generation. Due to the inherent uncertainty of PV generation and user behavior, discrepancies may arise between actual and predicted values. The forecast errors for loads and PV generation typically follow a normal distribution with a mean of zero. In particular, the standard deviation of the load forecast error distribution can reach 2%, while that of PVs can reach 20%. Consequently, transit centers may experience varying levels of uncertainty, with standard deviations ranging from 2% to 20%, contingent on the level of PV penetration.

The proposed model demonstrates robustness against office loads and PV fluctuations. When the prediction errors of the office loads and PV generation are less than  $err^{OV}$ , the total power of the transit center can be maintained at the reference value by adjusting the charging and discharging power of the energy storage system alone. Consequently, power prediction errors have no impact on electricity or capacity service fees; only the battery aging cost of energy storage systems may be affected. However, it is worth noting that the impact of battery aging costs on the total cost is minimal, as they constitute a negligible percentage of the overall expenses. On the other hand, if the forecast errors are greater than  $err^{OV}$ , the energy storage system control alone cannot offset them, necessitating an adjustment in the amount of energy purchased from the grid. In such cases, both capacity service charges and electricity costs are subject to change.

To evaluate the performance of the proposed model under different uncertainty levels, four sets of Monte Carlo simulations were conducted. These simulations  $P_t^{OV}$  assumed that the total power of office loads and PV generation had a prediction error with a mean value of 0 and standard deviations of 5%, 10%, 15%, and 20%, respectively. The proposed method optimized the charging strategy of the buses, which remained constant throughout the Monte Carlo simulation. The charging and discharging power of the energy storage system were adjusted based on the optimization results to offset the prediction error of  $P_t^{OV}$  as effectively as possible.

Each set of Monte Carlo simulations was repeated 100,000 times to compare the total costs of the benchmarks and those of the proposed model. The results, shown in Fig. 10, revealed that as uncertainty increases, both the proposed method and the benchmark experience higher total energy consumption, indicating that higher uncertainty leads to higher operating costs for transit centers. Furthermore, regardless of the level of uncertainty, the proposed model consistently outperformed the benchmark, highlighting its superiority. Compared to the benchmark, the proposed method was less likely to result in scenarios that were less costly when prediction errors were not considered. This is because in cases where the power demand is lower than the predicted value, the proposed model reduces the discharge of energy storage

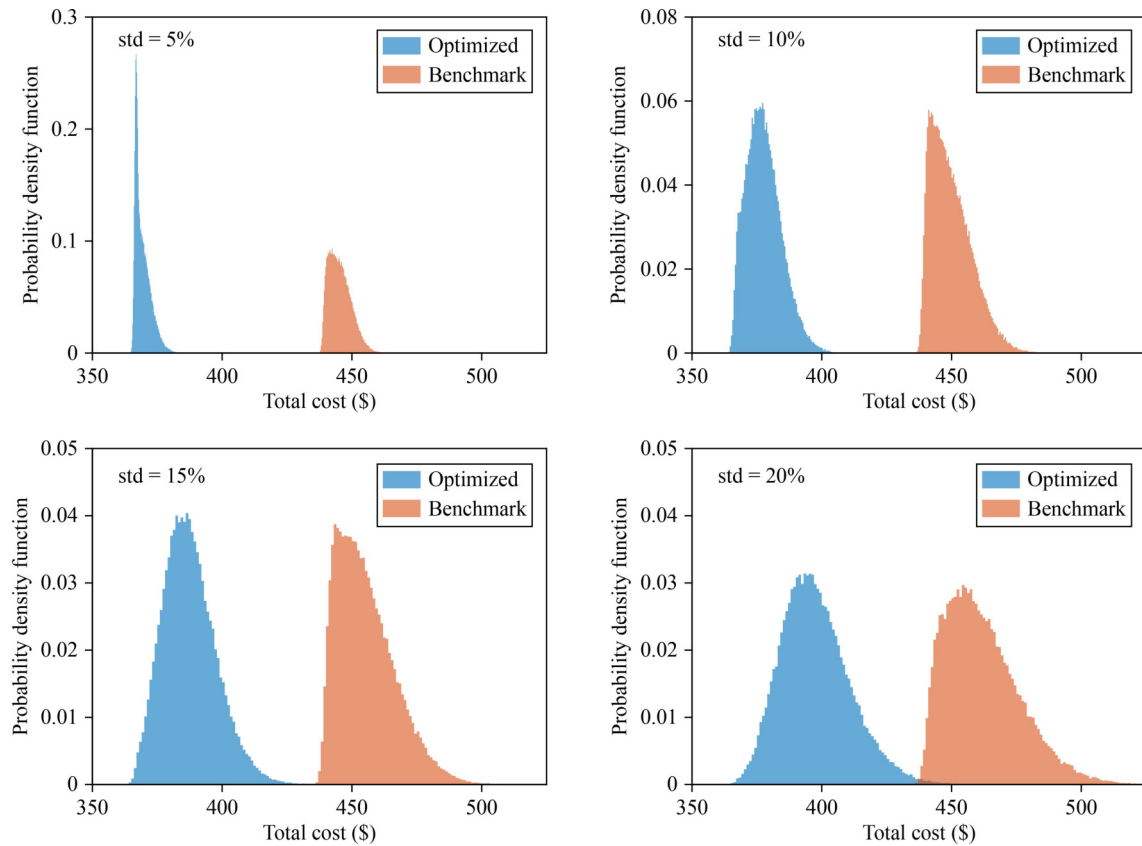


Fig. 10 Monte Carlo analysis under different uncertainty levels.

to ensure that the total power curve of the transit centers aligns more closely with the optimized reference value. Although this setting does not significantly affect the economics of the proposed model, it effectively prevents uncertainty fluctuations from being transmitted to the grid. In practice, the interests of transit centers and grids can be balanced by selecting different real-time control strategies. However, regardless of the chosen underlying control strategy, the proposed approach proves to be highly beneficial.

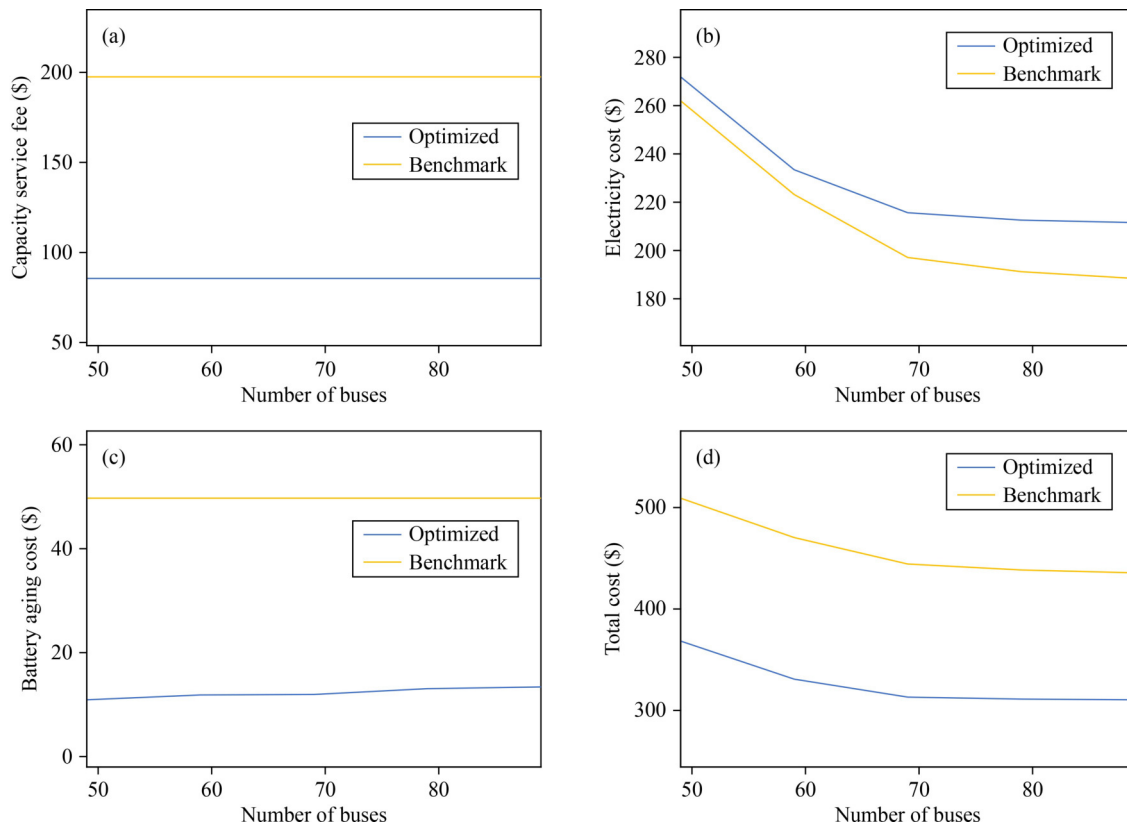
#### 4.6 Sensitivity analysis

Numerous parameters within the proposed model are closely tied to the hardware facilities of the transit centers. It is crucial to analyze the sensitivity of the optimization results in relation to these parameters. Specifically, the previous simulation assumes that the minimum number of buses required to serve all trips is sufficient. However, in reality, the number of buses may exceed this minimum. Therefore, it is necessary to examine the cost implications of increasing the number of buses. Additionally, in some cases, there may be a need to increase the battery capacity to reduce charges. Consequently, analyzing the cost impact of increasing the bus battery capacity is crucial.

Moreover, the previous section assumes the existence of only one 500 kWh energy storage system in the transit

center, with this capacity assumed to meet the power requirements adequately. However, with the continuous decline in battery prices, future transit centers may acquire additional energy storage systems to enhance power regulation. Hence, it is essential to consider the scenario of adding more energy storage systems. Furthermore, the deployment of additional charging infrastructure and increased charging power can have various consequences. On the one hand, the increase in charging power and the number of chargers may result in a higher concentration of charging demand, leading to a more substantial instantaneous load. On the other hand, it is possible that the peak-shaving potential of transit centers could be significantly enhanced, allowing more charging demand to be concentrated during periods of low electricity demand. Thus, a study on the sensitivity of the proposed method to the number of charging stations and charging power is necessary.

First, let us consider the number of buses. The sensitivity analysis results are depicted in Fig. 11. As the number of buses increases, the number of trips required by each bus to serve decreases, consequently reducing the charging demand per bus. This change enables buses that traditionally need to be charged during the day to shift to overnight charging to maintain a safe SoC, thus reducing electricity costs. As a trade-off, fewer buses are charged during the day, resulting in a diminished ability to avoid



**Fig. 11** Sensitivity analysis of each cost to the number of buses: (a) capacity service fee; (b) electricity cost; (c) battery aging; (d) total cost.

reverse power flows from PV systems through bus charging. As a result, a portion of the power regulation task originally performed by buses is shifted to energy storage systems, leading to an increase in battery aging costs. However, the battery aging costs of energy storage systems constitute a small percentage of the total cost, resulting in an overall decrease in the total cost as the number of buses increases.

The simulation demonstrates that increasing the number of buses from 49 to 59 results in an approximately \$50 decrease in the electricity bill. However, as the number of buses continues to increase, the reduction in electricity costs diminishes significantly. When the number of buses increases from 79 to 89, the decrease in total cost becomes almost negligible and is almost parallel to the  $x$ -axis. Furthermore, the total cost barely decreases once the bus count exceeds 69. Therefore, while a slightly greater number of buses may contribute to reducing operating costs, it is not logical to continue increasing the bus count.

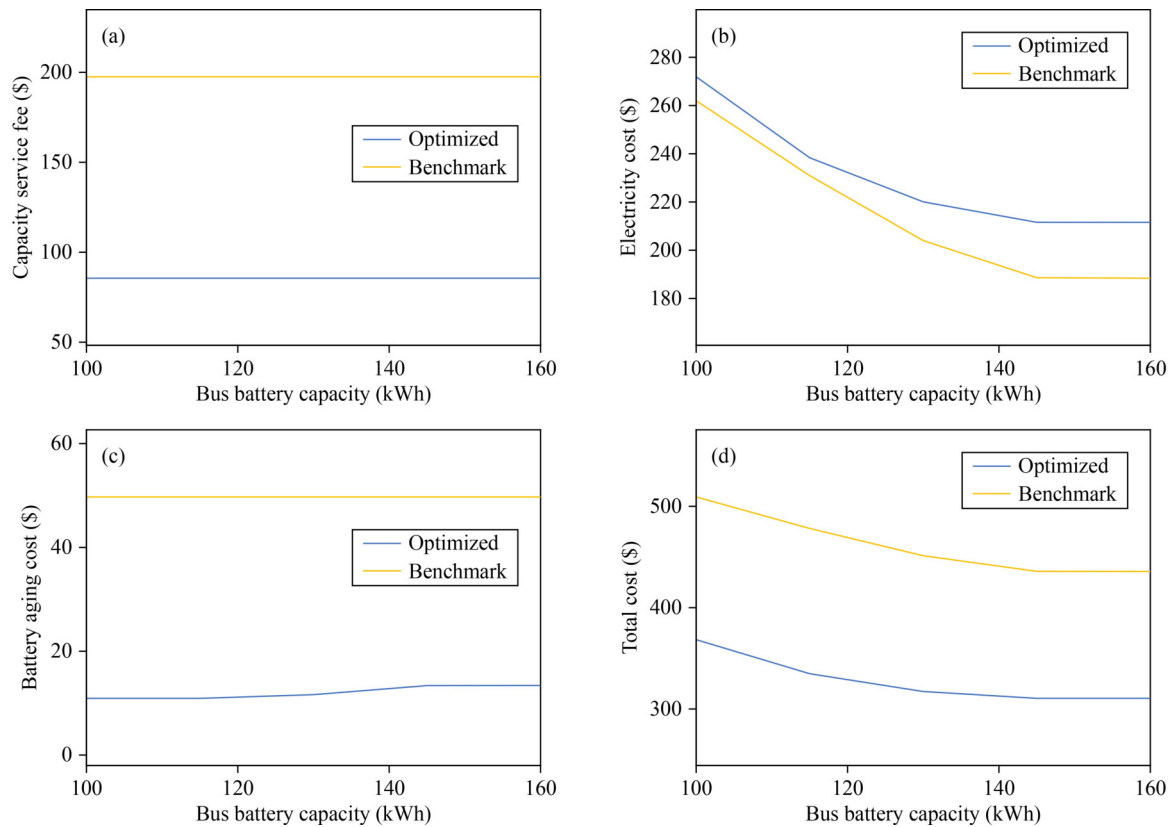
It should be noted that the capacity service fee does not decrease as the number of buses increases. This is because the capacity service charge is dependent on the peak load. Even if all buses are not charged and the energy storage systems are discharged at maximum power, the peak load can only be reduced to the maximum value of PVs and office loads minus the maximum value

of the energy storage system discharge power. As shown in Fig. 8, the peak loads are already optimized to the minimum when there are 49 buses, so increasing the number of vehicles does not lead to a reduction in the capacity service charge.

The cost of the benchmark also decreases with an increase in the number of buses, but the gap between the benchmark and optimized cases does not narrow.

The sensitivity of the cost to the bus cell capacity is depicted in Fig. 12. An increase in battery capacity has a similar impact on operating costs as an increase in the number of buses, as it reduces the need to charge buses during operating hours. When the battery capacities of buses reach 160 kWh, there is no need to charge buses at all during operating hours, and all charging requirements can be met at night.

For commercially available buses with the ability to accommodate 270 kWh or even larger batteries, using a larger battery to reduce daily operating costs is a viable option when the number of trips to be serviced is relatively small compared to the number of buses. However, using a larger battery increases the acquisition cost compared to the option of having more buses. Considering that the value of batteries is declining rapidly, even without factoring in the cost of battery aging, the economic value of batteries will decrease as battery technology advances. Therefore, determining whether it is cost-effective to use



**Fig. 12** Sensitivity analysis of each cost to the bus battery capacity: (a) capacity service fee; (b) electricity cost; (c) battery aging; (d) total cost.

larger batteries to reduce daytime charging requires a comprehensive evaluation of total life-cycle costs. In this paper, the main focus is to validate the effectiveness of the proposed scheduling model; thus, trade-offs at the facility planning level are not further analyzed.

Similar to increasing the number of buses, increasing the battery size also reduces the cost of the benchmark to some extent.

As depicted in Fig. 13, the analysis examines the impact of the number of energy storage system batteries on cost. Unlike the number of buses and bus battery capacities, increasing the number of cells in the energy storage system decreases the capacity service charge. However, using an energy storage system more frequently to reduce peak loads increases battery aging costs. The decrease in capacity service charges is even offset by the increase in battery aging costs. Simulation experiments reveal that even with five energy storage system batteries installed, the total cost only drops by approximately \$20. Therefore, installing more energy storage system batteries than necessary is not a reasonable option for transit centers.

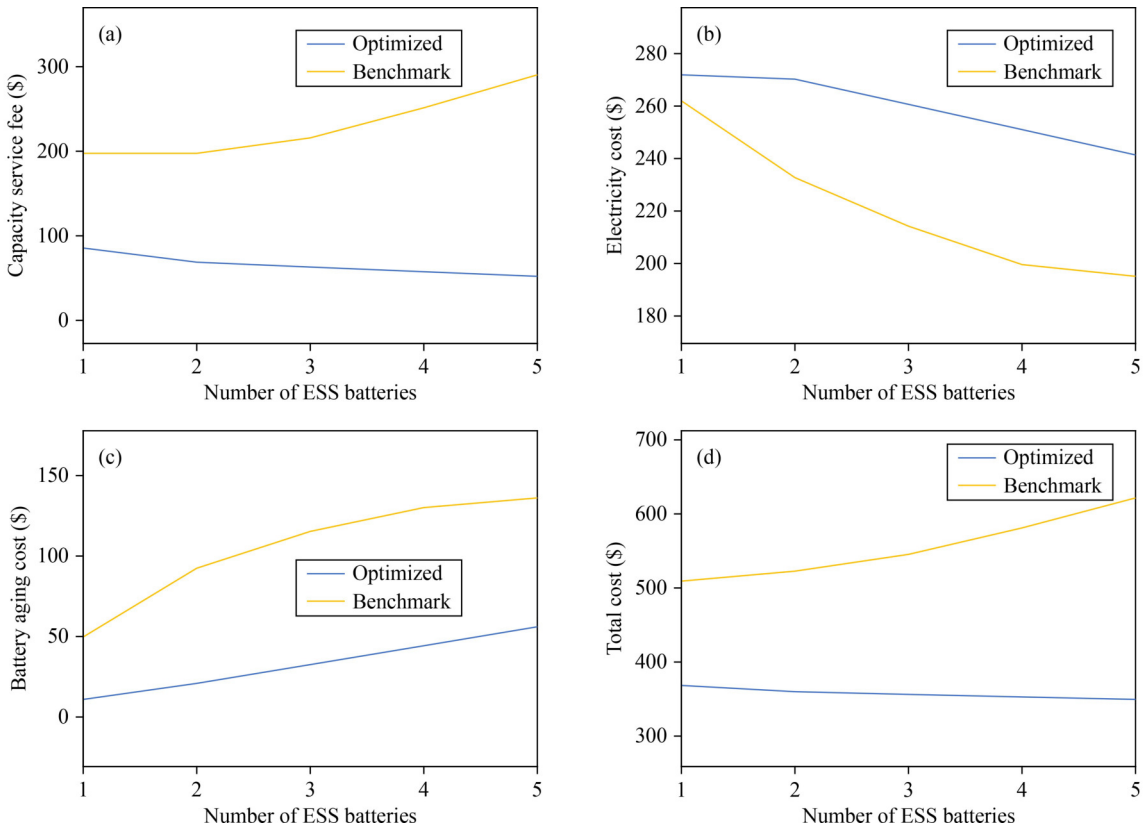
It is noteworthy that the low-buy-high-sell strategy employed in the benchmark enables a rapid decrease in electricity cost as the number of energy storage system batteries increases. However, the higher charging and discharging power of the energy storage system also

leads to an increase in the benchmark's capacity service fee, ultimately increasing the total cost.

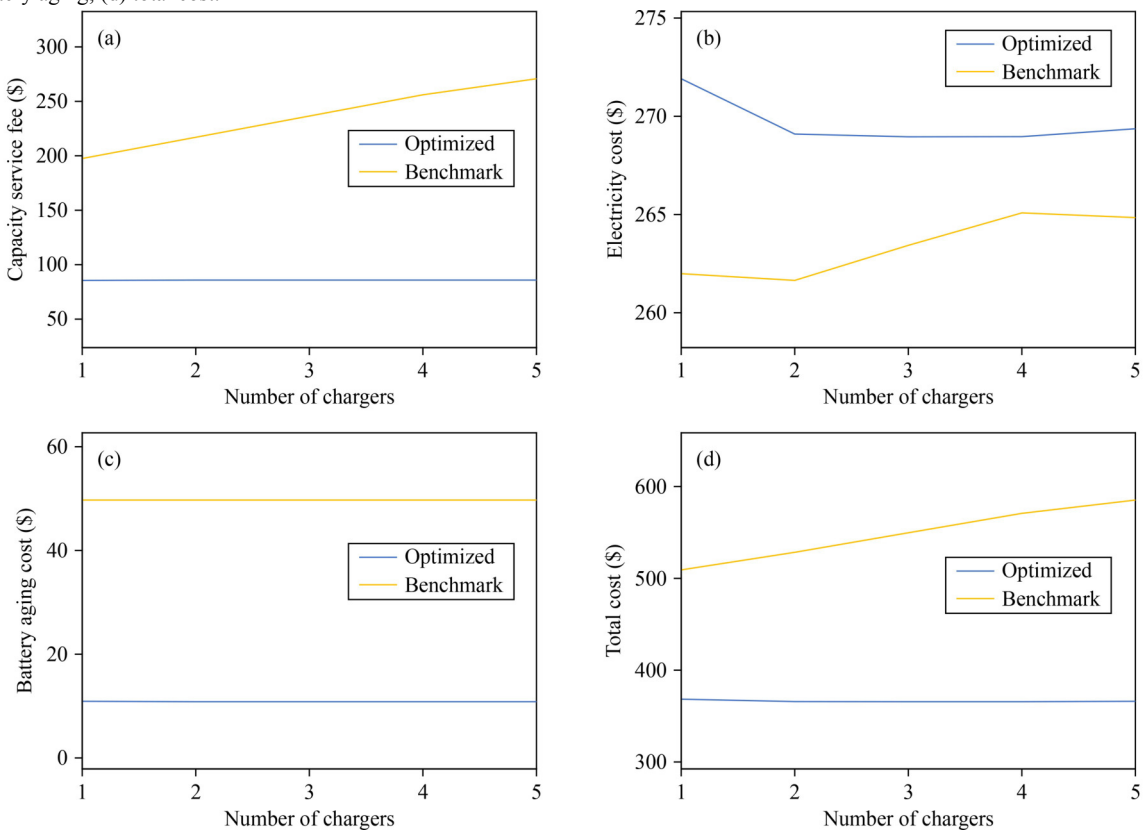
Subsequently, we delve into the sensitivity of the number of chargers in the transit center to various cost factors, as displayed in Fig. 14. It is evident that the electricity and battery aging costs are less sensitive to the number of chargers, with the impact on the total cost primarily arising from the effect on the capacity service charges. As the number of chargers increases, the capacity service charges in the benchmark case show nearly linear growth. Additional chargers facilitate the simultaneous charging of more buses, reduce the total charging time and increase the instantaneous charging power, thereby increase the capacity service fee. Conversely, a reduction in the number of chargers leads to staggered charging of buses, flattening the load profile and decreasing the capacity service charges.

In the optimized case, precise adjustment of the charging power at each time slot ensures that the capacity service charges do not increase with an increase in the number of chargers, resulting in minimal impact on the overall cost. This finding underscores the significance of advanced energy management strategies for abundant charging facilities.

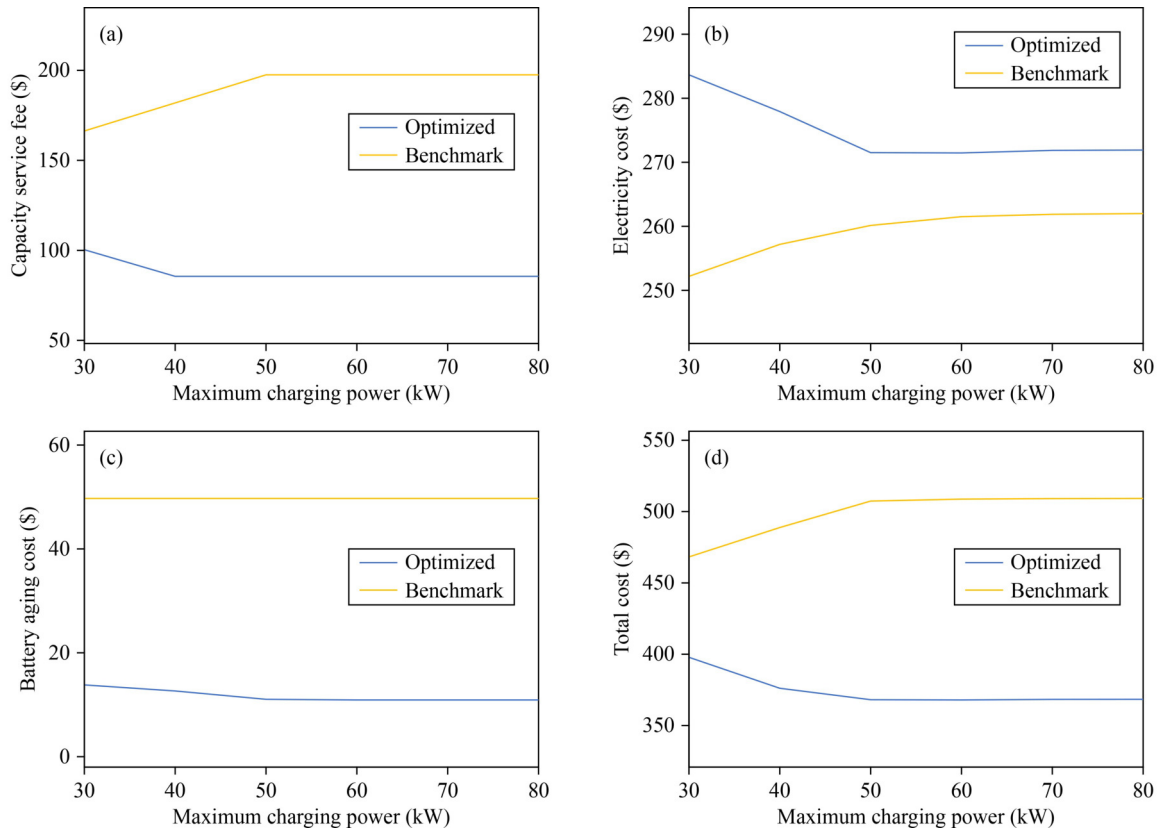
Similarly, the impact of charger capacity on costs can be observed in Fig. 15. A higher charging power also presents the challenge of a concentrated charging load on



**Fig. 13** Sensitivity analysis of each cost to the number of energy storage system batteries: (a) capacity service fee; (b) electricity cost; (c) battery aging; (d) total cost.



**Fig. 14** Sensitivity analysis of each cost to the number of chargers: (a) capacity service fee; (b) electricity cost; (c) battery aging; (d) Total cost.



**Fig. 15** Sensitivity analysis of each cost to the maximum charging power for each charger: (a) capacity service fee; (b) electricity cost; (c) battery aging; (d) total cost.

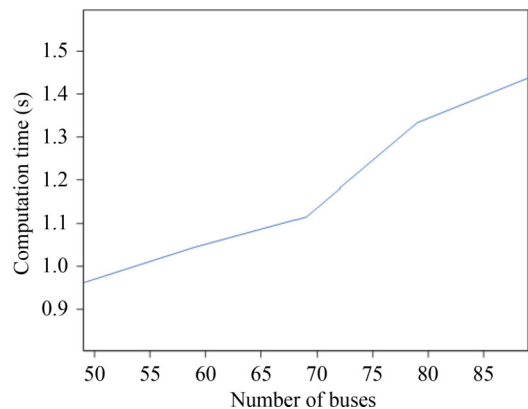
the benchmark. However, the relationship between increased charging power and the benchmark capacity service fee is not linear. Once the maximum charging power exceeds 50 kW, the capacity service fee for the benchmark no longer increases. Increasing the maximum charging power for the optimized case reduces all three costs until it reaches 50 kW. Beyond 50 kW, the charging power no longer affects costs.

By optimizing the charging power in each time slot, it is evident that increasing the charger's capacity can provide more peak-shaving capabilities, consequently reducing costs. In this case study, a charging power of 50 kW or more is sufficient to meet peak shaving requirements (after which the capacity service fee no longer decreases). In the future, fast chargers can bring significant flexibility to the power system with the proliferation of Level 3 charging and highly optimized orderly charging.

The five aforementioned sensitivity analyses demonstrate that having more abundant hardware facilities can help reduce operating costs. However, the improvement compared to that of the optimized model is relatively limited.

The five sensitivity analyses reveal that while having richer hardware facilities can contribute to cost reduction, it is only effective when combined with precise power control strategies.

Regarding the changes in computational performance



**Fig. 16** Sensitivity analysis of the computational time to the number of buses.

with varying problem sizes, we evaluated the model with bus counts ranging from 49 to 89 and recorded the corresponding solution times, as shown in Fig. 16. The results indicate that even at the largest scale, the solution times remain within an acceptable range. Additionally, it is important to consider the practical constraints of daily operational hours when determining the number of time steps. Extending the timeframe beyond typical daily operations is unnecessary for the scope of the model. Moreover, the current time step granularity is sufficiently

detailed for intraday scheduling purposes. Considering the inherent uncertainty in vehicle arrival times, further reducing the time step size may not significantly enhance the accuracy of the scheduling results. Therefore, the study does not explore smaller time steps beyond the current configuration.

## 5 Conclusions and future work

In this paper, we explore the joint optimization problem of electric buses and energy storage systems. Our study proposes a mixed-integer planning model that regulates energy storage system charging and discharging while rationalizing charging strategies to minimize the total system cost. This cost includes the electricity cost, battery aging cost, and capacity service charge. We fully consider uncertainties such as PVs and office loads, as well as time-of-use electric prices. To ensure a fast solution, we linearize the proposed model through a series of equivalent transformations and relaxations.

We conducted numerical simulations using a real-world transit system and actual PV and load data. Additionally, we developed a heuristic model as a benchmark to simulate the process of creating a scheduling plan without complex optimization calculations. Our results demonstrate that the optimized scheduling scheme can achieve significant cost savings, with the largest reduction observed in capacity service charges. A more detailed analysis reveals that our proposed method can increase the charging time of buses to the load trough period when electricity prices are lower. The optimized charging power trajectory is symmetric to the load and PV trajectory axes of the same period, thereby ensuring a smooth total load trajectory and reducing the peak load. During peak load periods, all bus charging is halted to lower power consumption, and the energy storage system discharge is utilized to further reduce the total transit center power.

Through Monte Carlo analysis, we determined that the total cost of the optimized case is \$368.4, representing a 27.7% decrease compared to the benchmark. Battery aging costs are also significantly reduced, showing a 78.0% decrease compared to the benchmark. Although the electricity cost increases by 3% (reaching \$271.9) compared to the benchmark, the capacity service charge experiences a substantial reduction of 56.7% (reaching \$85.56). Moreover, our proposed method demonstrates robustness to power uncertainty fluctuations. Although the total cost of both the proposed model and the benchmark increases as the PV and load uncertainties increase, the optimized scheduling scheme consistently outperforms the benchmark.

A sensitivity analysis revealed that larger fleet sizes and greater bus battery capacities can somewhat reduce the total cost of optimization. However, this effect eventually reaches its limits, and costs cannot be indefinitely

reduced. Additionally, the inclusion of more energy storage system batteries reduces capacity service charges, but it also increases battery aging costs, providing limited cost reduction benefits. Future research could expand upon the findings of this paper in two ways.

On the one hand, it is proposed to expand the optimization of operating costs to include the optimization of the entire life cycle costs (Zeng and Qu, 2023; Zeng et al., 2023). This involves optimizing the construction planning and scheduling schemes for both the fleet and station facilities. On the other hand, more realistic factors, such as the nonlinear nature of the battery aging cost curve, step electricity prices, and even real-time electricity markets, should be considered.

Our research could be further developed in several compelling directions, which have been highlighted by recent technological advancements that are pertinent to our primary focus. For example, investigating the use of language models to optimize urban delivery routes could provide valuable insights into more efficient scheduling and routing of electric buses within city environments (Liu et al., 2023a, 2023b). Empowering highway networks through optimal deployment and strategies for dynamic wireless charging lanes could significantly alleviate range anxiety and enhance the appeal of electric buses (Pei et al., 2023). Additionally, enhancing the reliability of electric vehicle charging infrastructure with cross-linguistic deep learning approaches could streamline the integration of diverse data sources, potentially improving the accuracy of energy demand prediction and the resilience of charging networks (Lin et al., 2023; Qu et al., 2023; Wang et al., 2023). By embracing these avenues, our future research endeavors could not only enhance the performance and applicability of the current model but also contribute to a more sustainable and efficient public transportation ecosystem.

**Competing Interests** The authors declare that they have no competing interests.

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## System Integration

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