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RESEARCH ARTICLE

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Efficiency measurement for mixed two-stage nonhomogeneous network processes with shared extra intermediate resources

Abstract Unreasonable allocation of shared resources reduces the system efficiency and is a considerable operational risk. Sub-processes with insufficient portion of shared resources could not help accomplish complicated tasks, and overstaffing and idle resources will occur in the sub-processes assigned with redundant shared resources. This unfair portion distribution may cause internal contradictions among sub-processes and even lead to the collapsing of the entire system. This study proposes a data-driven, mixed two-stage network data envelopment analysis model. This method aims to reasonably define the allocation portion of shared extra intermediate resources among several nonhomogeneous subsystems and measure the overall system performance. A data set of 58 international hotels is used to test the features of the proposed model.

Keywords shared resource allocation, mixed two-stage system, data envelopment analysis, efficiency

1 Introduction

Data envelopment analysis (DEA) is a data-driven approach that measures the relative efficiency or performance of a set of decision-making units (DMUs) with multiple inputs and outputs. This method was first proposed by Charnes et al. (1978). Numerous extension studies based on this approach have been conducted (Amin and Toloo, 2004; Chen et al., 2014; An et al., 2019; Song et al., 2020).

Traditional DEA studies regard each DMU as a black box, but the internal structure is ignored. Many studies investigated a simple two-stage network structure that uses intermediate measures as connection products between two stages to avoid the bias caused by ignoring the structure. Chen et al. (2006) evaluated the effect of IT on the two-stage bank's operation process. The first stage uses fixed assets, number of employees, and IT investment to generate deposit dollars. As an intermediate measure, deposit dollars are utilized to output securities and loans in the second stage. Several studies have also applied DEA to this simple two-stage process (Seiford and Zhu, 1999; Kao and Hwang, 2008; An et al., 2018).

The studies mentioned above are based on a simple two-stage structure and involve inputs, outputs, and intermediate measures but without any additional intermediate measure. However, many real-life cases must be represented by highly complicated two-stage structures. Yin et al. (2019) discussed the fundamental structure of hotel operations and management model by considering additional inputs and comparing it with the simple two-stage structure. Other types of two-stage process have been proposed to avoid misleading due to the ignored operations of internal processes (Liang et al., 2006; Cook et al., 2010; Halkos et al., 2014; Kao, 2014; Li et al., 2014).

Mixed two-stage network structure is a special structure that mixes series and parallel to a two-stage network process. This structure is common in real life, such as in airports, hospital operations, military systems, and hotel chains. In this study, we propose a mixed two-stage nonhomogeneous network process with shared extra intermediate resources. In the two serial stages, the second stage has several parallel nonhomogeneous sub decision-

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making units (SDMUs). These SDMUs consume the respective inputs from the first stage and have shared extra intermediate resources. This mixed structure can be applied to the real life. For example, in a complete supply chain, the first stage uses initial investment to produce various semi-finished products and distributes them to the second stage. The nonhomogeneous SDMUs of the second stage use respective intermediate measures to produce the final productions. These nonhomogeneous SDMUs also require shared extra intermediate inputs, such as advertising and management costs. These shared extra intermediate inputs cannot be conveniently divided but directly affect the performance evaluation.

The rest of this paper is organized as follows. Section 2 contains the literature review in the field of nonhomogeneous DEA and network systems. Section 3 proposes a mixed two-stage DEA model. Section 4 presents the empirical data of 58 Taiwan international hotels that are applied in the proposed model. Section 5 comprises the drawn conclusions.

2 Literature review

2.1 Nonhomogeneous DEA

Traditional DEA measures the performance of a set of homogeneous DMUs. However, this method cannot reasonably handle actual nonhomogeneous issues that remain to be measured. Cook et al. (2012) presented a case in which some DMUs did not produce certain outputs and therefore could be categorized into different nonhomogeneous groups. A DEA-type model was developed to measure this nonhomogeneous situation. Cook et al. (2013) also proposed a DEA-based methodology to solve this nonhomogeneous problem. Each DMU was divided into a set of business sub-units, and the weighted average of sub-group efficiency values was assumed as the overall efficiency for the DMU. Different from the above works that investigated the nonhomogeneous issue on the output side, Li et al. (2016) developed DEA models to handle nonhomogeneous DMUs with input configurations. Another nonhomogeneous problem is partial input-to-output effects. Imanirad et al. (2013) extended the traditional DEA methodology to estimate the technical efficiency score of nonhomogeneous DMUs with partial input-to-output effect. Imanirad et al. (2015) also extended their earlier work for efficiency measurement, in which the nonhomogeneous DMUs had different input/output profiles. Li et al. (2017) presented a model to measure the efficiency for nonhomogeneous DMU with given output bundles produced by multiple procedures or processes. Zhu et al. (2018) established a cross-like efficiency model to evaluate efficiency for homogenous DMUs. The proposed model can also be used to range the DMUs with missing outputs or inputs. For parallel systems with nonhomogeneous subunits, Du et al. (2015) proposed DEA models to measure the overall performance of entire system and the efficiency decomposition for each nonhomogeneous subunit.

2.2 Network systems

From the perspective of system structure, existing research on shared resources/inputs can mainly be divided into three categories, namely, two-stage series, parallel, and mixed structures. Numerous studies on shared resources are based on two-stage series process. In two-stage bank operation process, Chen et al. (2006) developed a nonlinear DEA programming model to measure the effect of IT considering three types of shared input in two stages. Chen et al. (2010) emphasized that in many real-life situations, some inputs cannot be split up, which would lead to inaccurate results in efficiency evaluation. They developed a set of DEA models to measure the efficiency for two-stage systems with shared input that cannot be split. Zha and Liang (2010) proposed a product-form cooperative efficiency model to measure the performance of a two-stage production process in series. Amirteimoori (2013) divided shared resources into two interdependent stages arranged in series. Wu et al. (2016b) indicated a situation in which the undesirable intermediate measure from the first stage could be used or processed by the second stage. New resources would be fed back to the first stage from the second stage. In this situation, shared resources were used by two stages, because the inputs cannot be conveniently split up.

Another part of research on shared resources is based on parallel structure systems. Cook and Tuenter (2000) and Cook and Hababou (2001) proposed DEA models to evaluate the sales and service efficiency for Canadian Bank branches with parallel structure. Jahanshahloo et al. (2004a; 2004b) measured the efficiency of a multi-component bank branch parallel system. Kao (2012) discussed the distribution of the same resources for a parallel system in his example, which was used by Beasley (1995). Rogge and de Jaeger (2012) presented a shared input DEA model to estimate the cost efficiency for municipality and each fraction. In their research, waste cost was shared among six parallel waste fractions. Bian et al. (2015) measured the efficiency for a general parallel system with shared inputs and outputs. In the general parallel system, each subsystem had a set of common, dedicated, and shared inputs/outputs. A general DEA model was proposed to handle the efficiency evaluation issue. Wu et al. (2016a) treated the transportation as a parallel system with shared resources. They proposed a DEA model to measure the performance of transportation systems in China.

The third type of research on shared resources is based on mixed structure. Yu and Lin (2008) measured the efficiency and effectiveness of railways' operations with a mixed structure by a multiactivity network DEA model. Yu and Fan (2009) presented a DEA model named MSNDEA to estimate the efficiency of multimode transit firm. The structure of transit firm was a mixed structure, in which the first stage was a parallel and was connected in series with the second stage. Huang et al. (2014) proposed a modified two-stage DEA approach based on the envelope form to evaluate the efficiency and effectiveness of Taiwan's international tourist hotels with a mixed structure framework. The first stage is a production process, and the second stage is a parallel system. As an intermediate input, marketing expense helps each parallel subsystem to produce their own output.

As reviewed in this section, the literature of serial system focuses on the internal structure. However, this type of system still ignores the internal details of each stage. As another type of network systems, parallel system usually limits that each parallel sub-process must be the same as others, i.e., the parallel system has insufficient ability to address the cases with parallel nonhomogeneous sub-processes. Mixed system is the integration version of series and parallel systems. Mixed system is suitable for complex cases and ensures that the parallel sub-processes are homogeneous. In the present study, a mixed system suitable for nonhomogeneous DMUs is considered. This structure compensates for the short board that the ordinary mixed structure cannot solve the nonhomogeneous problem. To the best of our knowledge, the sharing of extra intermediate resources among nonhomogeneous parallel sub-processes in the second stage of mixed two-stage systems has not been discussed yet. This study aims to propose a multiplier-based network DEA model to estimate the overall efficiency of mixed two-stage nonhomogeneous network processes with shared extra intermediate resources. We adopt the weighted additive efficiency decomposition method proposed by Chen et al. (2014) to calculate the efficiency for two stages. The optimal portion of shared resources for each sub-process is determined by the proposed model, which varies in accordance with intervals.

3 Mixed two-stage DEA models for shared extra intermediate resources

As shown in Fig. 1, a mixed two-stage nonhomogeneous network process has two stages that are connected in series. The second stage is composed of multiple parallel nonhomogeneous SDMUs. Among the nonhomogeneous SDMUs, their inputs and outputs are not exactly the same. Nevertheless, all SDMUs have shared extra intermediate resources.

DMUs with mixed two-stage network structure are supposed. The second stage of each DMU$_j$ (j = 1,..., n) has h parallel nonhomogeneous SDMUs, and each SDMU$_k$ (k = 1,..., h) produces its own products y_{r_k} ($r_k \in R_k$, k = 1,..., h) by consuming p shared extra intermediate resources e_t (t = 1,..., p). Q_k refers to the intermediate measures z_{q_k} ($q_k \in Q_k$, k = 1,..., h) that come from the first stage by using m initial inputs x_i (i = 1,..., m).

Intermediate resources e_t (t = 1,..., p) are shared by SDMUs; hence, we denote a_{tk} (t = 1,..., p; k = 1,..., h) as the portion of shared extra intermediate resources used by SDMU$_k$ (k = 1,..., h). Thus, we have $\sum_{k=1}^{h} a_{tk} = 1 \quad (t = 1, \ldots, p)$. Similar to Cook and Hababou (2001), the intervals of a_{tk} (t = 1,..., p; k = 1,..., h) are specified by users, namely, $L_{tk} \leqslant a_{tk} \leqslant U_{tk}$ (t = 1,..., p; k = 1,..., h).

The DMU being evaluated is denoted as DMU$_o$. We will discuss the additive efficiency of the parallel nonhomogeneous SDMUs in the second stage. The weighted average of the two-stage efficiency is used to calculate the total efficiency for DMU$_o$. The efficiency of the first stage can be measured under the assumption of constant returns to scale.

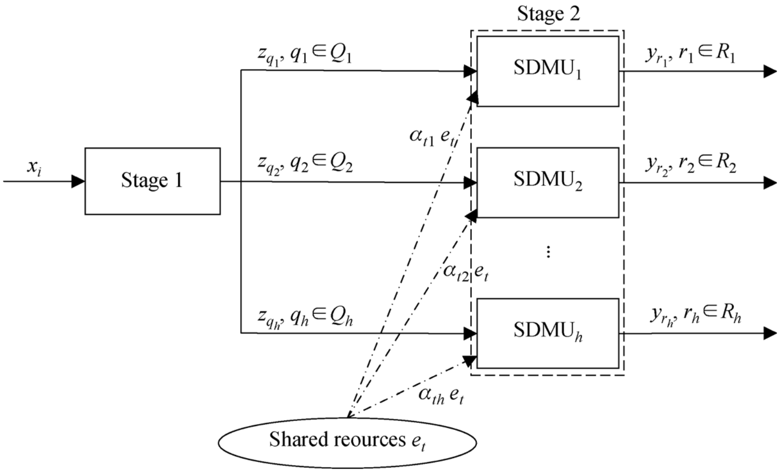

Fig. 1 Mixed two-stage nonhomogeneous network processes.

$$E_o^{(1)} = \max \frac{\sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o}}{\sum_{i=1}^{m} v_i x_{io}},$$

$$\text{s.t.} \quad \frac{\sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k j}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1 \quad (j = 1, ..., n),$$

$$w_{q_k}, v_i \geqslant 0 \quad (q_k \in Q_k, \ \ k = 1, ..., h; \ \ i = 1, ..., m), \qquad (1)$$

where w_{q_k} and v_i are unknown nonnegative weights associated with the intermediate measures z_{q_k} and initial inputs x_i, respectively.

In two-stage DEA study, Kao and Hwang (2008) and Liang et al. (2008) argued that the multipliers associated with the intermediate measures are the same in two stages. In parallel systems, Kao and Hwang (2010) and Kao (2012) emphasized that the same inputs (or outputs) should have the same multiplier, regardless of what process assumes (or produces). Hence, we can measure the efficiency of each parallel nonhomogeneous SDMU$_k$ (k = 1,..., h) in the second stage via model (2):

$$E_o^{(2k)} = \max \frac{\sum_{r_k \in R_k} u_{r_k} y_{r_k o}}{\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{t=1}^{p} g_t a_{tk} e_{to}},$$

$$\text{s.t.} \quad \frac{\sum_{r_k \in R_k} u_{r_k} y_{r_k j}}{\sum_{q_k \in Q_k} w_{q_k} z_{q_k j} + \sum_{t=1}^{p} g_t a_{tk} e_{tj}} \leqslant 1$$

$$(k = 1, ..., h; \ j = 1, ..., n),$$

$$\sum_{k=1}^{h} a_{tk} = 1 \quad (t = 1, ..., p),$$

$$L_{tk} \leqslant a_{tk} \leqslant U_{tk} \quad (t = 1, ..., p; \ k = 1, ..., h),$$

$$u_{r_k}, w_{q_k}, g_t, a_{tk} \geqslant 0 \quad (r_k \in R_k, \ \ q_k \in Q_k,$$

$$k = 1, ..., h; \ t = 1, ..., p), \qquad (2)$$

where u_{r_k} indicates the unknown weights associated with the final outputs of SDMU$_k$, w_{q_k} is the same as the multipliers of the intermediate measures in model (1), g_t denotes the unknown weights of the shared extra intermediate resources e_t, and a_{tk} represents the portions of shared extra intermediate resources e_t used by SDMU$_k$.

We take the approach of weighted average of efficiency for SDMU$_k$ to derive the aggregate measure of efficiency for the second stage, which was used to estimate the overall efficiency of nonhomogeneous parallel network processes by Du et al. (2015). Weight δ_k represents the relative importance of SDMU$_k$, where $\sum_{k=1}^{h} \delta_k = 1$. The aggregate efficiency of the second stage can then be estimated by model (3):

$$E_o^{(2)} = \max \sum_{k=1}^{h} \delta_k$$

$$\times \frac{\sum_{r_k \in R_k} u_{r_k} y_{r_k o}}{\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{t=1}^{p} g_t a_{tk} e_{to}},$$

$$\text{s.t.} \quad \frac{\sum_{r_k \in R_k} u_{r_k} y_{r_k j}}{\sum_{q_k \in Q_k} w_{q_k} z_{q_k j} + \sum_{t=1}^{p} g_t a_{tk} e_{tj}} \leqslant 1$$

$$(k = 1, ..., h; \ j = 1, ..., n),$$

$$\sum_{k=1}^{h} a_{tk} = 1 \quad (t = 1, ..., p),$$

$$L_{tk} \leqslant a_{tk} \leqslant U_{tk} \quad (t = 1, ..., p; \ k = 1, ..., h),$$

$$\sum_{k=1}^{h} \delta_k = 1,$$

$$u_{r_k}, w_{q_k}, a_{tk}, g_t, \delta_k \geqslant 0 \quad (r_k \in R_k, \ \ q_k \in Q_k,$$

$$k = 1, ..., h; \ t = 1, ..., p). \qquad (3)$$

Next, we will find a feasible scheme for weight δ_k (k = 1,..., h). As mentioned above, weight δ_k (k = 1,..., h) represents the relative importance of SDMU$_k$. This scheme is reasonable, considering it lets the proportion of the resources used by SDMU$_k$ to the total resources in the second stage represent the weight δ_k, which reflects the relative importance or relative operating scale for SDMU$_k$ (Chen et al., 2014; Wang and Chin, 2010). δ_k can be obtained by model (4):

$$\delta_k = \frac{\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{t=1}^{p} g_t a_{tk} e_{to}}{\sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{k=1}^{h}\sum_{t=1}^{p} g_t a_{tk} e_{to}} \quad (k = 1, ..., h), \qquad (4)$$

where the denominator represents the total resources for the second stage, including intermediate measures and shared extra intermediate resources, and the numerator represents the size of SDMU$_k$. The objective function in model (3) can be rewritten as follows:

$$E_o^{(2)} = \max \frac{\sum_{k=1}^{h}\sum_{r_k \in R_k} u_{r_k} y_{r_k o}}{\sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{k=1}^{h}\sum_{t=1}^{p} g_t a_{tk} e_{to}}. \tag{5}$$

The overall efficiency and stage efficiency for the mixed two-stage systems can be estimated by the additive efficiency decomposition approach proposed by Chen et al. (2014). $\phi^{(1)}$ and $\phi^{(2)}$ represent the relative weights for the first and second stages, respectively. Similar to weight δ_k, $\phi^{(1)}$ and $\phi^{(2)}$ can also be determined by the proportion of the resources used by the second stage to the total resources in the two-stage system. The weights $\phi^{(1)}$ and $\phi^{(2)}$ are accordingly displayed as follows:

$$\phi^{(1)} = \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{k=1}^{h}\sum_{t=1}^{p} g_t a_{tk} e_{to}}, \tag{6a}$$

$$\phi^{(2)} = \frac{\sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{k=1}^{h}\sum_{t=1}^{p} g_t a_{tk} e_{to}}{\sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{k=1}^{h}\sum_{t=1}^{p} g_t a_{tk} e_{to}}, \tag{6b}$$

where the denominators in models (6a) and (6b) represent all of the resources consumed by DMU$_o$, and the numerators in models (6a) and (6b) represent the resources used by the first and second stages, respectively.

The overall efficiency of DMU$_o$ is the weighted average of the two stages' efficiencies, which is solved by the following problem:

$$E_o = \max\left(\phi^{(1)} \times E_o^{(1)} + \phi^{(2)} \times E_o^{(2)}\right) = \max \frac{\sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{k=1}^{h}\sum_{r_k \in R_k} u_{r_k} y_{r_k o}}{\sum_{i=1}^{m} v_i x_{io} + \sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k o} + \sum_{k=1}^{h}\sum_{t=1}^{p} g_t a_{tk} e_{to}},$$

$$\begin{aligned}
s.t. \quad & \frac{\sum_{k=1}^{h}\sum_{q_k \in Q_k} w_{q_k} z_{q_k j}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1 \quad (j = 1, \ldots, n), \\
& \frac{\sum_{r_k \in R_k} u_{r_k} y_{r_k j}}{\sum_{q_k \in Q_k} w_{q_k} z_{q_k j} + \sum_{t=1}^{p} g_t a_{tk} e_{tj}} \leqslant 1 \\
& (k = 1, \ldots, h;\ j = 1, \ldots, n), \\
& \sum_{k=1}^{h} a_{tk} = 1 \quad (t = 1, \ldots, p), \\
& L_{tk} \leqslant a_{tk} \leqslant U_{tk} \quad (t = 1, \ldots, p;\ k = 1, \ldots, h), \\
& v_i, u_{r_k}, w_{q_k}, a_{tk}, g_t \geqslant 0 \quad (i = 1, \ldots, m;\ r_k \in R_k, \\
& q_k \in Q_k,\ k = 1, \ldots, h;\ t = 1, \ldots, p).
\end{aligned} \tag{7}$$

Under the Charnes–Cooper transformation (Charnes and Cooper, 1962), the fractional (7) can be converted into linear program model (8):

$$\begin{aligned}
E_o = \max\ & \sum_{k=1}^{h}\sum_{q_k \in Q_k} \omega_{q_k} z_{q_k o} \\
& + \sum_{k=1}^{h}\sum_{r_k \in R_k} \mu_{r_k} y_{r_k o}, \\
s.t. \quad & \sum_{k=1}^{h}\sum_{q_k \in Q_k} \omega_{q_k} z_{q_k j} - \sum_{i=1}^{m} \nu_i x_{ij} \leqslant 0 \\
& (j = 1, \ldots, n), \\
& \sum_{r_k \in R_k} \mu_{r_k} y_{r_k j} - \sum_{q_k \in Q_k} \omega_{q_k} z_{q_k j} \\
& - \sum_{t=1}^{p} \gamma_t a_{tk} e_{tj} \leqslant 0 \quad (k = 1, \ldots, h;\ j = 1, \ldots, n), \\
& \sum_{i=1}^{m} \nu_i x_{io} + \sum_{k=1}^{h}\sum_{q_k \in Q_k} \omega_{q_k} z_{q_k o} \\
& + \sum_{k=1}^{h}\sum_{t=1}^{p} \gamma_t a_{tk} e_{to} = 1, \\
& \sum_{k=1}^{h} a_{tk} = 1 \quad (t = 1, \ldots, p), \\
& L_{tk} \leqslant a_{tk} \leqslant U_{tk} \quad (t = 1, \ldots, p;\ k = 1, \ldots, h), \\
& \nu_i, \mu_{r_k}, \omega_{q_k}, a_{tk}, \gamma_t \geqslant 0 \quad (i = 1, \ldots, m;\ r_k \in R_k,
\end{aligned}$$

$$q_k \in Q_k, \ \ k = 1,...,h; \ \ t = 1,...,p). \qquad (8)$$

Obviously, model (8) is a nonlinear programming. We set $\gamma_t a_{tk} = \lambda_{tk}$ to linearize the nonlinear programming model (8). We have:

$$E_o = \max \sum\nolimits_{k=1}^{h} \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k o} + \sum\nolimits_{k=1}^{h} \sum\nolimits_{r_k \in R_k} \mu_{r_k} y_{r_k o},$$

$$s.t. \quad \sum\nolimits_{k=1}^{h} \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k j} - \sum\nolimits_{i=1}^{m} \nu_i x_{ij} \leqslant 0 \quad (j = 1,...,n),$$

$$\sum\nolimits_{r_k \in R_k} \mu_{r_k} y_{r_k j} - \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k j} - \sum\nolimits_{t=1}^{p} \lambda_{tk} e_{tj} \leqslant 0 \quad (k = 1,...,h; \ j = 1,...,n),$$

$$\sum\nolimits_{i=1}^{m} \nu_i x_{io} + \sum\nolimits_{k=1}^{h} \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k o} + \sum\nolimits_{k=1}^{h} \sum\nolimits_{t=1}^{p} \lambda_{tk} e_{to} = 1,$$

$$\sum\nolimits_{k=1}^{h} \lambda_{tk} - \gamma_t = 0 \quad (t = 1,...,p),$$

$$\gamma_t L_{tk} \leqslant \lambda_{tk} \leqslant \gamma_t U_{tk} \quad (t = 1,...,p; \ k = 1,...,h),$$

$$\nu_i, \mu_{r_k}, \omega_{q_k}, \gamma_t, \lambda_{tk} \geqslant 0 \quad (i = 1,...,m; \ r_k \in R_k,$$

$$q_k \in Q_k, \ \ k = 1,...,h; \ \ t = 1,...,p). \qquad (9)$$

$(\mu_{r_k}^*, \omega_{q_k}^*, \gamma_t^*, \lambda_{tk}^*)$ represents the optimal solution for model (9), then $a_{tk}^* = \lambda_{tk}^* / \gamma_t^*$. The optimal overall efficiency is

$$E_o^* = \sum\nolimits_{k=1}^{h} \sum\nolimits_{q_k \in Q_k} \omega_{q_k}^* z_{q_k o} + \sum\nolimits_{k=1}^{h} \sum\nolimits_{r_k \in R_k} \mu_{r_k}^* y_{r_k o}.$$

At last, the decomposition of stage efficiency can be obtained, when the second stage is given pre-emptive priority. Under the overall efficiency constant, the decomposition efficiency of the second stage is as follows:

$$E_o^{(2)*} = \sum\nolimits_{k=1}^{h} \sum\nolimits_{r_k \in R_k} \mu_{r_k} y_{r_k o},$$

$$s.t. \quad \sum\nolimits_{k=1}^{h} \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k j} - \sum\nolimits_{i=1}^{m} \nu_i x_{ij} \leqslant 0 \quad (j = 1,...,n),$$

$$\sum\nolimits_{r_k \in R_k} \mu_{r_k} y_{r_k j} - \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k j} - \sum\nolimits_{t=1}^{p} \lambda_{tk} e_{tj} \leqslant 0 \quad (k = 1,...,h; \ j = 1,...,n),$$

$$\sum\nolimits_{k=1}^{h} \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k o} + \sum\nolimits_{k=1}^{h} \sum\nolimits_{r_k \in R_k} \mu_{r_k} y_{r_k o} - E_o^* \sum\nolimits_{i=1}^{m} \nu_i x_{io} = E_o^*,$$

$$\sum\nolimits_{k=1}^{h} \sum\nolimits_{q_k \in Q_k} \omega_{q_k} z_{q_k o} + \sum\nolimits_{k=1}^{h} \sum\nolimits_{t=1}^{p} \lambda_{tk} e_{to} = 1,$$

$$\sum\nolimits_{k=1}^{h} \lambda_{tk} - \gamma_t = 0 \quad (t = 1,...,p),$$

$$\gamma_t L_{tk} \leqslant \lambda_{tk} \leqslant \gamma_t U_{tk} \quad (t = 1,...,p; \ k = 1,...,h),$$

$$\nu_i, \mu_{r_k}, \omega_{q_k}, \gamma_t, \lambda_{tk} \geqslant 0 \quad (i = 1,...,m; \ r_k \in R_k,$$

$$q_k \in Q_k, \ \ k = 1,...,h; \ \ t = 1,...,p). \qquad (10)$$

$E_o^{(1)**}$ represents the decomposition efficiency of the first stage with second-emptive priority. The first stage's efficiency is then calculated as:

$$E_o^{(1)**} = \frac{E_o^* - \phi^{(2)*} \times E_o^{(2)*}}{\phi^{(1)*}}, \qquad (11)$$

where $\phi^{(1)*}$ and $\phi^{(2)*}$ are the optimal weights obtained from models (6a) and (6b), respectively. When the first stage is given the pre-emptive priority, the model for decomposition efficiency of the second stage is similar to model (10); hence, we omit it.

All the steps of our method are summarized as follows to present our approach in this study clearly.

Algorithm of our approach:

Step 1: Measure the efficiency $E_o^{(1)}$ for the first stage using model (1).

Step 2: Measure the efficiency $E_o^{(2)}$ for the second stage using models (2–4).

Step 2.1: Measure efficiency $E_o^{(2k)}$ for each parallel nonhomogeneous SDMU$_k$ in the second stage using model (2).

Step 2.2: Calculate the relative weight δ_k for SDMU$_k$ using model (4).

Step 2.3: Measure the efficiency $E_o^{(2)}$ for the second stage using model (3).

Step 3: Calculate relative weights $\phi^{(1)}$ and $\phi^{(2)}$ for the first and second stages using models (6a) and (6b), respectively.

Step 4: Measure efficiency E_o for mixed structure system using model (7).

Step 5: Calculate the optimal solution a_{tk}^* for allocation portion of shared resources and the optimal stage efficiency $E_o^{(1)**}$ (or $E_o^{(1)*}$) and $E_o^{(2)*}$ (or $E_o^{(2)**}$).

In summary, our method is an integrated study of three types of problems, namely, serial system, parallel system, and nonhomogeneous DMU efficiency evaluation. Our

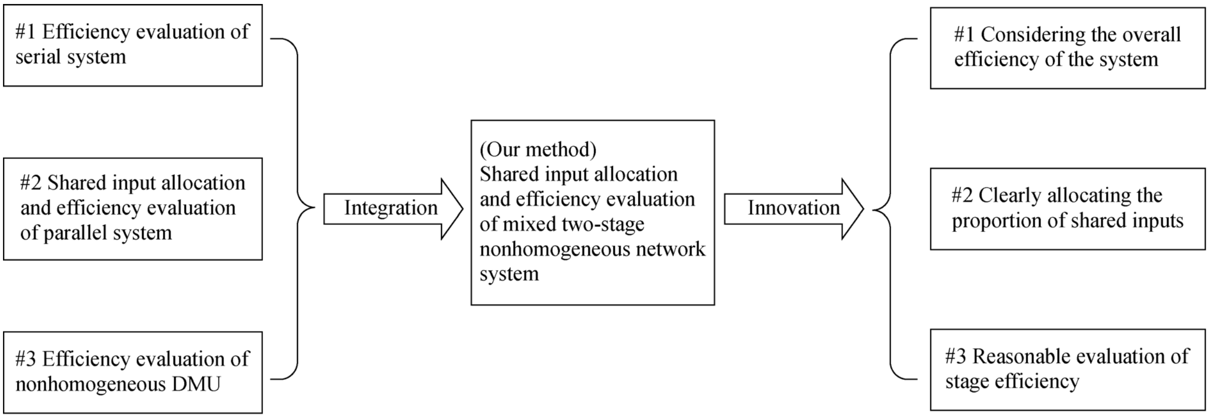

Fig. 2 Work of our method.

method is based on the principle of maximizing system efficiency, evaluating the overall efficiency of the system, rationally allocating extra intermediate resources, and obtaining appropriate stage efficiency (Fig. 2).

4 Illustrative application

4.1 Data description

In this section, we collate and use the empirical data provided by Huang et al. (2014) to test our models. The data consist of 58 Taiwan international tourist hotels. The authors considered mixed two-stage nonhomogeneous network processes, as depicted in Fig. 3. All factors are clarified as follows:

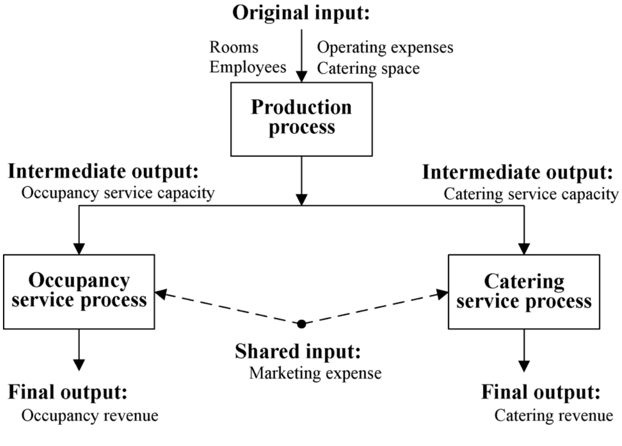

Fig. 3 Mixed two-stage structure of hotel's production.

- Operating expenses: The cost of hotel operation, including cost of food and beverage (F&B), maintenance expenses, and expenditures for water and electricity, which are all measured in millions New Taiwan Dollar (NT$).
- Rooms: The number of rooms supplied for renting.
- Catering space: The square of floor space for F&B, which is measured in square feet (ft^2).
- Employees: The number of employees, including workers in guest rooms and catering and management staff.
- Occupancy service capacity: The number of room nights available, which is the quotient of the number of rooms sold and the occupancy rate of the room.
- Catering service capacity: The product of catering space and the number of employees of F&B department.
- Marketing expense: The cost for marketing activity, such as advertisement and website operation, which is measured in millions NT$.
- Occupancy revenue: The operational revenue gained from room renting, which is measured in millions NT$.
- Catering revenue: The operational revenue gained from F&B sales, which is measured in millions NT$.

The first stage is the production process with four original inputs and two intermediate outputs. The two intermediate outputs are transported to two parallel and nonhomogeneous sub-processes, occupancy service process, and catering service process, which belong to the second stage. The two sub-processes use shared extra intermediate resources for marketing expense. The shard marketing expense affects the two sub-processes but cannot be conveniently split up. Finally, the two sub-processes produce their own output as the final outputs of the second stage. The data of the number of lodging guests, which is one factor of the final output of the occupancy division in the study of Huang et al. (2014), are missing; consequently, we have to omit the factor in our study. Table 1 shows the descriptive statistics of all factors.

4.2 Efficiency analysis

Table 2 reports the efficiencies estimated from the proposed model when the intervals of a_{tk} are set as [0.2, 0.8]. The number and the name of hotels are shown in the first and second columns, respectively. The overall

Table 1 Descriptive statistics for 58 Taiwan international tourist hotels

	Mean	SD	Maximum	Minimum
Original input				
Operating expenses ($\times 10^6$ NT$)	543.3	488	2227.1	45.4
Rooms	298.9	148.6	865	50
Catering space (ft^2)	4495.4	7094.8	52966.0	210
Employees	315.9	207.4	868	53
Intermediate output				
Occupancy service capacity	72.8	42	213.1	7.8
Catering service capacity	1038.6	2773.7	20286.0	4.1
Intermediate input				
Marketing expense ($\times 10^6$ NT$)	7	9.4	61.3	0.1
Final output of the occupancy division				
Occupancy revenue ($\times 10^6$ NT$)	245	224.8	1243.7	26.1
Final output of the catering division				
Catering revenue ($\times 10^6$ NT$)	270.3	269.8	1127.9	8.5

Table 2 Efficiency analysis

No.	Hotel	Ranking	E_o^*	$E_o^{(1)*}$	$E_o^{(2)**}$	$E_o^{(1)**}$	$E_o^{(2)*}$	$\phi^{(1)*}$	$\phi^{(2)*}$
1	Grand Hotel	18	0.6877	0.8853	0.4915	0.8743	0.5025	0.4941	0.5059
2	Ambassador	26	0.6449	0.9353	0.3573	0.8300	0.4617	0.4970	0.5030
3	Imperial Hotel	29	0.6228	0.7926	0.5177	0.7551	0.5409	0.5546	0.4454
4	Gloria Prince	24	0.6477	0.8625	0.4106	0.8536	0.4205	0.5228	0.4772
5	Emperor Hotel	10	0.7835	0.9105	0.7817	0.8740	0.7822	0.4638	0.5362
6	Hotel Riverview	28	0.6242	1.0000	0.2783	0.9998	0.2785	0.4792	0.5208
7	Caesar Park	13	0.7346	1.0000	0.4926	1.0000	0.4926	0.4745	0.5255
8	Golden China	7	0.8047	0.8900	0.7386	0.8831	0.7440	0.4273	0.5727
9	San Want Hotel	17	0.7063	0.8922	0.5139	0.8891	0.5171	0.5060	0.4940
10	Brother Hotel	30	0.6186	0.8084	0.4047	0.7834	0.4329	0.5289	0.4711
11	Santos Hotel	5	0.8612	1.0000	0.7639	0.9999	0.7639	0.4035	0.5965
12	Lands Hotel	25	0.6456	0.7703	0.4851	0.7668	0.4897	0.5624	0.4376
13	United Hotel	1	0.9443	0.9039	0.9776	0.9035	0.9779	0.4439	0.5561
14	Sheraton	14	0.7294	1.0000	0.4768	0.7836	0.6788	0.4829	0.5171
15	Hotel Royal	15	0.7231	0.9057	0.5569	0.9026	0.5597	0.4726	0.5274
16	Howard Hotel	19	0.6842	0.9912	0.3463	0.7841	0.5743	0.5120	0.4880
17	Grand Hyatt	57	0.4444	0.7586	0.0297	0.3791	0.5306	0.5689	0.4311
18	Formosa	11	0.7793	0.9535	0.5796	0.7519	0.8107	0.5322	0.4678
19	Sherwood Hotel	16	0.7105	0.8630	0.5153	0.7606	0.6463	0.5614	0.4386
20	Far Eastern Plaza	8	0.7999	0.9299	0.6409	0.7807	0.8234	0.5494	0.4506
21	Westin	4	0.8671	0.7433	0.9243	0.6742	0.9562	0.5620	0.4380
22	Miramar Garden	20	0.6798	1.0000	0.3831	1.0000	0.3831	0.4788	0.5212
23	Hotel Kingdom	27	0.6359	1.0000	0.3241	0.9997	0.3243	0.4614	0.5386
24	Holiday Garden	47	0.5615	0.8104	0.2540	0.7808	0.2906	0.5527	0.4473
25	Ambassador	45	0.5726	0.8456	0.2629	0.8355	0.2743	0.5302	0.4698
26	Grand Hi-Lai	21	0.6700	1.0000	0.3465	0.9772	0.3688	0.4943	0.5057
27	Howard Hotel	39	0.5904	0.9362	0.2100	0.8683	0.2847	0.5238	0.4762

(*Continued*)

No.	Hotel	Ranking	E_o^*	$E_o^{(1)*}$	$E_o^{(2)**}$	$E_o^{(1)**}$	$E_o^{(2)*}$	$\phi^{(1)*}$	$\phi^{(2)*}$
28	Splendor	44	0.5779	0.7915	0.3253	0.7421	0.3837	0.5418	0.4582
29	Han-Hsien Intl	34	0.6097	0.9331	0.2732	0.9190	0.2879	0.5098	0.4902
30	Lees Hotel	48	0.5536	0.7317	0.3355	0.7276	0.3405	0.5489	0.4511
31	Hotel National	53	0.5053	0.6666	0.2779	0.6642	0.2813	0.5851	0.4149
32	Plaza Intl	40	0.5893	0.9250	0.2283	0.8708	0.2866	0.5181	0.4819
33	Evergreen Laurel	43	0.5796	0.7702	0.3430	0.7086	0.4194	0.5528	0.4472
34	Howard Hotel	37	0.5967	0.7992	0.3806	0.7739	0.4076	0.5355	0.4645
35	Splendor	56	0.4595	0.5385	0.3124	0.5284	0.3312	0.6501	0.3499
36	Marshal Hotel	52	0.5160	0.7051	0.2534	0.6888	0.2760	0.5813	0.4187
37	Chinatrust Hotel	49	0.5465	0.7711	0.2763	0.7630	0.2861	0.5440	0.4560
38	Parkview Hotel	31	0.6150	0.8916	0.3463	0.8783	0.3592	0.5299	0.4701
39	Farglory Hotel	33	0.6130	0.6976	0.5054	0.5976	0.6325	0.5557	0.4443
40	Lands Resort	46	0.5693	0.4815	0.7175	0.4634	0.7480	0.6279	0.3721
41	The Lalu Hotel	3	0.8981	0.8246	0.9640	0.7882	0.9967	0.5434	0.4566
42	Fleur De Chine	35	0.6070	0.5522	0.7055	0.5451	0.7184	0.6423	0.3577
43	Hibiscus Resort	6	0.8556	0.4611	0.8616	0.3535	0.8632	0.0149	0.9851
44	Grand	38	0.5956	0.3976	0.6085	0.1353	0.6255	0.0610	0.9390
45	Caesar Park	12	0.7628	0.8808	0.6516	0.8760	0.6562	0.4792	0.5208
46	Howard Hotel	22	0.6569	0.8728	0.4174	0.8382	0.4558	0.5257	0.4743
47	Chihpen	23	0.6493	0.7662	0.4967	0.7576	0.5079	0.5663	0.4337
48	Silks Place	55	0.4721	0.4575	0.5003	0.4565	0.5022	0.6564	0.3436
49	CHIAO-HIS	9	0.7989	0.7657	0.8415	0.7539	0.8567	0.5614	0.4386
50	Taoyuan Hotel	36	0.6064	1.0000	0.2334	0.9999	0.2335	0.4866	0.5134
51	Ta Shee Resort	58	0.3235	0.3027	0.3458	0.2906	0.3587	0.7583	0.2417
52	Hotel Royal	42	0.5837	0.6773	0.4731	0.6749	0.4760	0.5631	0.4369
53	Ambassador	41	0.5868	0.7033	0.4205	0.6411	0.5092	0.5878	0.4122
54	Nice Prince	54	0.5011	0.6329	0.2915	0.6204	0.3114	0.6139	0.3861
55	Hotel Tainan	2	0.9431	0.6546	0.9487	0.4656	0.9523	0.0190	0.9810
56	Evergreen Plaza	32	0.6148	0.7454	0.4484	0.7098	0.4937	0.5601	0.4399
57	Tayih Landis	51	0.5258	0.7029	0.2746	0.6699	0.3214	0.5858	0.4142
58	Naruwan	50	0.5291	0.6289	0.4005	0.6242	0.4065	0.6145	0.3855

efficiency and the ranking-based overall efficiency for DMUs are listed in the fourth and third columns, respectively. The fifth column reports the efficiency of the first stage when the first stage is given pre-emptive priority and the overall efficiency keeps constant. The data in the sixth column are the efficiency of the second stage with secondary priority. The seventh and eighth columns are the reverse of the fifth and sixth columns, which show the efficiency of the first stage with secondary priority and the efficiency of the second stage with pre-emptive priority, respectively. The optimal weights of two stages are listed in the last two columns.

The overall efficiency differs, which illustrates that the proposed models have strong recognition capability. The maximum efficiency is 0.9443 (DMU_{13}), and the minimum is 0.3235 (DMU_{51}). All DMUs can be sorted in accordance with the obtained overall efficiency value. This finding indicates that the proposed method can be used to evaluate the performance of DMUs accurately, that is, it can clearly distinguish the small overall efficiency differences among DMUs.

On the premise that the overall efficiency remains unchanged, the efficiency of the two stages can also be obtained. Figures 4 and 5 are the scatter diagrams of the decomposition efficiency of two stages, where x-coordinate and y-coordinate are the overall efficiency and stage

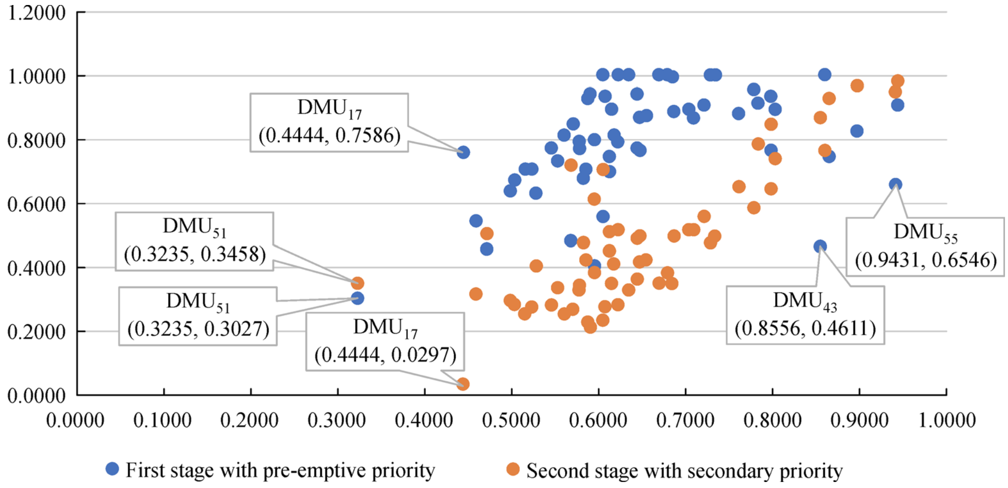

Fig. 4 Decomposition efficiency when the first stage is given pre-emptive priority.

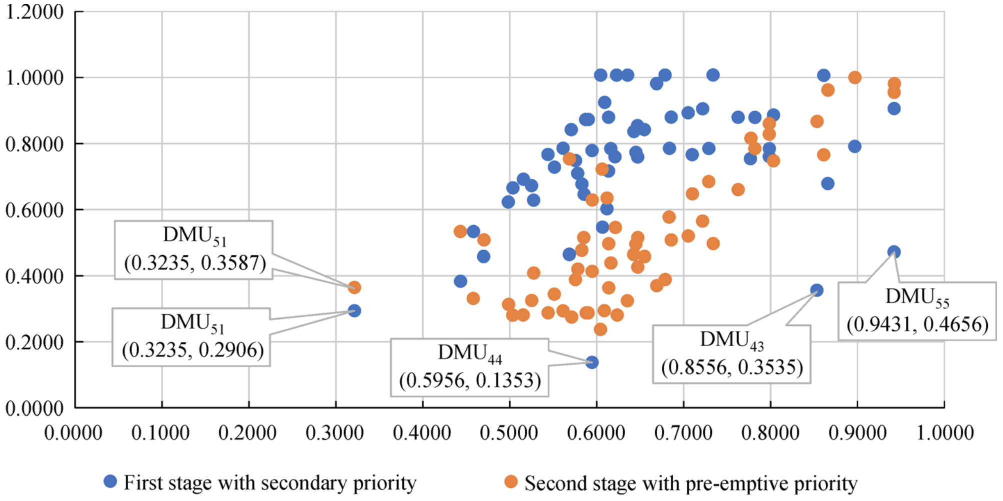

Fig. 5 Decomposition efficiency when the second stage is given pre-emptive priority.

efficiency, respectively. Figure 4 illustrates the efficiency of the distribution for two stages when the first stage is given pre-emptive priority. On the contrary, Fig. 5 is based on the situation that the second stage has pre-emptive priority. Comparison of the two figures shows that the dots in Fig. 5 are more concentrated, and fewer extreme dots exist. For example, six dots are clearly far from the population dots in Fig. 4, whereas five lonely dots are scattered in Fig. 5. These extreme dots are all marked out in Figs. 4 and 5. From the perspective of the number of extreme dots, the efficiency decomposition method that gives the second stage pre-emptive priority is more suitable for these hotels with mixed two-stage network structure. Nonetheless, the number of DMUs with extreme dots in Fig. 5 is the same as the number in Fig. 4. In other words, the effect of the two efficiency decomposition principles on DMU is similar. Most of the extreme dots in Fig. 5 are about the first stage, such as DMUs 51, 44, 43, and 55. The ordinates of these dots are significantly smaller in Fig. 5 than in Fig. 4. This condition may seem unreasonable, given the blue dots are generally above the orange dots from both figures. However, this result is caused by the principle of efficiency decomposition. As shown in Fig. 5, the second stage has higher priority than the first stage. The efficiency of the second stage will thus be maximized as much as possible when the efficiency is decomposed, and the efficiency of the first stage will naturally decline. In fact, both efficiency decomposition principles are valid. As mentioned above, from the overall trend, the results of the two methods are similar without very serious and large changes, that is, the blue dots on both pictures are above the orange dots on the whole. Therefore, when using this method, the decision maker should carefully choose which stage should be given greater priority.

Table 3 Interval corresponds to efficiency

No.	Hotel	$L_{tk} = 0.2$, $U_{tk} = 0.8$			$L_{tk} = 0.1$, $U_{tk} = 0.9$			$L_{tk} = 0$, $U_{tk} = 1$		
		E_o^*	$a^{(1)*}$	$a^{(2)*}$	E_o^*	$a^{(1)*}$	$a^{(2)*}$	E_o^*	$a^{(1)*}$	$a^{(2)*}$
1	Grand Hotel	0.6824	0.8	0.2	0.6877	0.9	0.1	0.6886	0.9283	0.0717
2	Ambassador	0.6447	0.8	0.2	0.6449	0.8126	0.1874	0.6449	0.8126	0.1874
3	Imperial Hotel	0.6097	0.8	0.2	0.6228	0.9	0.1	0.6383	0.9937	0.0063
4	Gloria Prince	0.6458	0.8	0.2	0.6477	0.9	0.1	0.6486	0.9606	0.0394
5	Emperor Hotel	0.7561	0.8	0.2	0.7835	0.9	0.1	0.8550	1	0
6	Hotel Riverview	0.6242	0.6568	0.3432	0.6242	0.6568	0.3432	0.6242	0.6568	0.3432
7	Caesar Park	0.7311	0.8	0.2	0.7346	0.9	0.1	0.7358	0.9442	0.0558
8	Golden China	0.7942	0.8	0.2	0.8047	0.9	0.1	0.8131	0.9945	0.0055
9	San Want Hotel	0.7034	0.8	0.2	0.7063	0.9	0.1	0.7076	0.9503	0.0497
10	Brother Hotel	0.6176	0.8	0.2	0.6186	0.8360	0.1640	0.6186	0.8360	0.1640
11	Santos Hotel	0.8459	0.8	0.2	0.8612	0.9	0.1	0.8647	0.9987	0.0013
12	Lands Hotel	0.6455	0.2	0.8	0.6456	0.1	0.9	0.6457	0.0220	0.9780
13	United Hotel	0.9305	0.8	0.2	0.9443	0.9	0.1	0.9942	0.9999	0.0001
14	Sheraton	0.7294	0.7445	0.2555	0.7294	0.7445	0.2555	0.7294	0.7445	0.2555
15	Hotel Royal	0.7176	0.8	0.2	0.7231	0.9	0.1	0.7262	0.9660	0.0340
16	Howard Hotel	0.6837	0.8	0.2	0.6842	0.8837	0.1163	0.6842	0.8837	0.1163
17	Grand Hyatt	0.4425	0.2	0.8	0.4444	0.1	0.9	0.4456	0.0234	0.9766
18	Formosa	0.7769	0.8	0.2	0.7793	0.8672	0.1328	0.7793	0.8672	0.1328
19	Sherwood Hotel	0.7105	0.5	0.5	0.7105	0.5	0.5	0.7105	0.5	0.5
20	Far Eastern Plaza	0.7997	0.2	0.8	0.7999	0.1	0.9	0.8001	0.0221	0.9779
21	Westin	0.8411	0.8	0.2	0.8671	0.9	0.1	0.8973	0.9972	0.0028
22	Miramar Garden	0.6786	0.8	0.2	0.6798	0.9	0.1	0.6806	0.9992	0.0008
23	Hotel Kingdom	0.6359	0.2447	0.7553	0.6359	0.2447	0.7553	0.6359	0.2447	0.7553
24	Holiday Garden	0.5615	0.8	0.2	0.5615	0.9	0.1	0.5626	0.9993	0.0007
25	Ambassador	0.5725	0.2	0.8	0.5726	0.1	0.9	0.5727	0.0065	0.9935
26	Grand Hi-Lai	0.6697	0.2	0.8	0.6700	0.1	0.9	0.6701	0.0229	0.9771
27	Howard Hotel	0.5904	0.5	0.5	0.5904	0.5	0.5	0.5904	0.5	0.5
28	Splendor	0.5779	0.7979	0.2021	0.5779	0.7979	0.2021	0.5779	0.7979	0.2021
29	Han-Hsien Intl	0.6096	0.8	0.2	0.6097	0.8247	0.1753	0.6097	0.8247	0.1753
30	Lees Hotel	0.5527	0.8	0.2	0.5536	0.8502	0.1498	0.5536	0.8502	0.1498
31	Hotel National	0.5053	0.7890	0.2110	0.5053	0.7890	0.2110	0.5053	0.7890	0.2110
32	Plaza Intl	0.5892	0.8	0.2	0.5893	0.8135	0.1865	0.5893	0.8135	0.1865
33	Evergreen Laurel	0.5791	0.8	0.2	0.5796	0.8397	0.1603	0.5796	0.8397	0.1603
34	Howard Hotel	0.5955	0.8	0.2	0.5967	0.8899	0.1101	0.5967	0.8899	0.1101
35	Splendor	0.4594	0.2	0.8	0.4595	0.1	0.9	0.4595	0.0220	0.9780
36	Marshal Hotel	0.5159	0.8	0.2	0.5160	0.8155	0.1845	0.5160	0.8155	0.1845
37	Chinatrust Hotel	0.5449	0.8	0.2	0.5465	0.8764	0.1236	0.5465	0.8764	0.1236
38	Parkview Hotel	0.6132	0.8	0.2	0.6150	0.9	0.1	0.6170	0.9905	0.0095
39	Farglory Hotel	0.6104	0.8	0.2	0.6130	0.9	0.1	0.6146	0.9818	0.0182
40	Lands Resort	0.5693	0.3922	0.6078	0.5693	0.3922	0.6078	0.5693	0.3922	0.6078
41	The Lalu Hotel	0.8964	0.8	0.2	0.8981	0.9	0.1	0.9072	0.9798	0.0202
42	Fleur De Chine	0.6070	0.2	0.8	0.6070	0.1	0.9	0.6070	0.0220	0.9780
43	Hibiscus Resort	0.8041	0.8	0.2	0.8556	0.9	0.1	0.9071	0.9999	0.0001

(*Continued*)

No.	Hotel	$L_{tk} = 0.2, U_{tk} = 0.8$			$L_{tk} = 0.1, U_{tk} = 0.9$			$L_{tk} = 0, U_{tk} = 1$		
		E_o^*	$a^{(1)*}$	$a^{(2)*}$	E_o^*	$a^{(1)*}$	$a^{(2)*}$	E_o^*	$a^{(1)*}$	$a^{(2)*}$
44	Grand	0.5612	0.2	0.8	0.5956	0.1	0.9	0.6293	0.0020	0.9980
45	Caesar Park	0.7569	0.8	0.2	0.7628	0.9	0.1	0.7668	0.9990	0.0010
46	Howard Hotel	0.6567	0.8	0.2	0.6569	0.8108	0.1892	0.6569	0.8108	0.1892
47	Chihpen	0.6493	0.5	0.5	0.6493	0.5	0.5	0.6493	0.5	0.5
48	Silks Place	0.4711	0.8	0.2	0.4721	0.8566	0.1434	0.4721	0.8566	0.1434
49	CHIAO-HIS	0.7984	0.2	0.8	0.7989	0.1	0.9	0.7992	0.0220	0.9780
50	Taoyuan Hotel	0.6064	0.4300	0.5700	0.6064	0.4300	0.5700	0.6064	0.4300	0.5700
51	Ta Shee Resort	0.3192	0.8	0.2	0.3235	0.9	0.1	0.3309	0.9950	0.0050
52	Hotel Royal	0.5814	0.8	0.2	0.5837	0.9	0.1	0.5852	0.9950	0.0050
53	Ambassador	0.5866	0.2	0.8	0.5868	0.1	0.9	0.5868	0.0494	0.9506
54	Nice Prince	0.5011	0.5	0.5	0.5011	0.5	0.5	0.5011	0.5	0.5
55	Hotel Tainan	0.8981	0.2	0.8	0.9431	0.1	0.9	0.9870	0.0024	0.9976
56	Evergreen Plaza	0.6144	0.8	0.2	0.6148	0.8262	0.1738	0.6148	0.8262	0.1738
57	Tayih Landis	0.5256	0.2	0.8	0.5258	0.1	0.9	0.5260	0.0060	0.9940
58	Naruwan	0.5290	0.8	0.2	0.5291	0.9	0.1	0.5317	0.9992	0.0008

4.3 Shared resource allocation

Next, we discuss the effect of the interval of a_{tk} on overall efficiency. In Table 3, when the interval of a_{tk} is $L_{tk} = 0.2$, $U_{tk} = 0.8$, the overall efficiency is listed in the third column, and the fourth and fifth columns report the portions of shared extra intermediate resources obtained by occupancy and catering service process, respectively. Similarly, the center-right and last sets of data are based on the a_{tk} interval of [0.1, 0.9] and [0, 1], respectively.

The absolute optimal efficiency depends on whether the optimal portion is in the interval of a_{tk}. For example, in three different interval settings, the efficiency of DMU_6 is always 0.6242, because the optimal portion of it is $(a^{(1)*}, a^{(2)*}) = (0.6568, 0.3432) \in [0.2, 0.8] \subset [0.1, 0.9] \subset [0, 1]$, which is the optimal portion locating among the three intervals at the same time. For DMU_1, however, its optimal overall efficiency varies with the intervals. When $L_{tk} = 0$, $U_{tk} = 1$, the maximum optimal of DMU_1 is 0.8550, and the optimal portion is $(a^{(1)*}, a^{(2)*}) = (0.9283, 0.0717) \in [0, 1]$. Nevertheless, $(0.9283, 0.0717) \notin [0.2, 0.8]$. As a result, when $[L_{tk}, U_{tk}] = [0.2, 0.8]$, the efficiency of DMU_1 is only 0.6824, and $(a^{(1)*}, a^{(2)*}) = (0.8, 0.2)$, which indicates that the efficiency is a locally optimal solution.

For DMU_5, the efficiency increases with the expansion of the interval. When $[L_{tk}, U_{tk}] = [0.2, 0.8]$, the efficiency $E_5^* = 0.7561$, and $(a^{(1)*}, a^{(2)*}) = (0.8, 0.2)$. When $[L_{tk}, U_{tk}] = [0, 1]$, the efficiency of DMU_5 becomes 0.8550, and $(a^{(1)*}, a^{(2)*}) = (1, 0)$. This result suggests that assigning all marketing expense to occupancy service process is the most beneficial way to maximize the efficiency for Emperor Hotel. The results of DMUs 13, 22, 24, 43, and 58 imply similar meaning. In turn, the results of DMUs 44, 49, 55, and 57 indicate that marketing expense mainly used for catering service process contributes considerably to efficiency. For DMUs 19, 27, 47, and 54, the marketing expense shares for both sub-processes is 50%, which means that the two sub-processes have the same effect on overall efficiency. Setting interval as [0, 1] gives the most accurate and reasonable result for all DMUs. Nonetheless, in some real-life cases, decision makers have to set an interval with nonzero limit to ensure the basic benefits of all sub-processes. Therefore, decision makers need to accord to the different results to specify the interval.

4.4 Summary of test results

The key factor affecting the accuracy and robustness of results is the interval setting of a_{tk}. From the perspective of accuracy, the definition of accuracy should be combined with the actual situation. If all shared resources can be given to a certain department, then the result is the most accurate when the interval of a_{tk} is [0, 1]. At this time, if the result obtained in the other interval of a_{tk} is different from the result obtained in [0, 1], it may be regarded as inadequately accurate. However, in practice, shared resources are often divided into various departments. The results are hence obtained within a reasonable interval of a_{tk}. Although they are not the most ideal results, they are the most objective and accurate results.

For robustness, we can conclude that the results obtained by our model are relatively robust. To illustrate the robustness clearly, we first make the following three definitions:

- Efficiency change rate β_1 = (efficiency value of interval [0.1, 0.9] / efficiency value of interval [0.2, 0.8] – 1) × 100%;
- Efficiency change rate β_2 = (efficiency value of interval [0, 1] / efficiency value of interval [0.1, 0.9] – 1) × 100%;
- Average efficiency change rate $\beta_0 = (\beta_1 + \beta_2)/2$

Among the 58 DMUs, the average efficiency change rate β_0 of 48 DMUs is less than 1% (see Table 4). The average efficiency change values β_0 of 12 DMUs among the 48 DMUs are zero. Most DMUs thus have good robustness under different values of a_{tk}. Among the other 10 DMUs, only 3 DMUs have an average efficiency change rate β_0 higher than 0.05, and the maximum value of β_0 is 0.06375. DMU_6 is taken as an example, and the efficiency change rates β_1 and β_2 are both 0, indicating that the set three intervals have no effect on the actual efficiency value of DMU_6. For DMU_{10}, the efficiency change rate β_1 = 0.00162, and the change rate β_2 = 0, indicating that the efficiency of DMU_{10} is affected in the range of [0.2, 0.8]. However, the efficiency performance area is robust when the interval is expanded to [0.1, 0.9] or more. For DMU_5,

Table 4 Efficiency change rate

No.	Hotel	Efficiency change rate β_1	Efficiency change rate β_2	Average efficiency change rate β_0
1	Grand Hotel	0.777%	0.131%	0.454%
2	Ambassador	0.031%	0	0.016%
3	Imperial Hotel	2.149%	2.489%	2.319%
4	Gloria Prince	0.294%	0.139%	0.217%
5	Emperor Hotel	3.624%	9.126%	6.375%
6	Hotel Riverview	0	0	0
7	Caesar Park	0.479%	0.163%	0.321%
8	Golden China	1.322%	1.044%	1.183%
9	San Want Hotel	0.412%	0.184%	0.298%
10	Brother Hotel	0.162%	0	0.081%
11	Santos Hotel	1.809%	0.406%	1.108%
12	Lands Hotel	0.015%	0.015%	0.015%
13	United Hotel	1.483%	5.284%	3.384%
14	Sheraton	0	0	0
15	Hotel Royal	0.766%	0.429%	0.598%
16	Howard Hotel	0.073%	0	0.037%
17	Grand Hyatt	0.429%	0.270%	0.350%
18	Formosa	0.309%	0	0.154%
19	Sherwood Hotel	0	0	0
20	Far Eastern Plaza	0.025%	0.025%	0.025%
21	Westin	3.091%	3.483%	3.287%
22	Miramar Garden	0.177%	0.118%	0.147%
23	Hotel Kingdom	0	0	0
24	Holiday Garden	0	0.196%	0.098%
25	Ambassador	0.017%	0.017%	0.017%
26	Grand Hi-Lai	0.045%	0.015%	0.030%
27	Howard Hotel	0	0	0
28	Splendor	0	0	0
29	Han-Hsien Intl	0.016%	0	0.008%
30	Lees Hotel	0.163%	0	0.081%
31	Hotel National	0	0	0
32	Plaza Intl	0.017%	0	0.009%
33	Evergreen Laurel	0.086%	0	0.043%

(*Continued*)

No.	Hotel	Efficiency change rate β_1	Efficiency change rate β_2	Average efficiency change rate β_0
34	Howard Hotel	0.202%	0	0.101%
35	Splendor	0.022%	0	0.011%
36	Marshal Hotel	0.019%	0	0.010%
37	Chinatrust Hotel	0.294%	0	0.147%
38	Parkview Hotel	0.294%	0.325%	0.309%
39	Farglory Hotel	0.426%	0.261%	0.343%
40	Lands Resort	0	0	0
41	The Lalu Hotel	0.190%	1.013%	0.601%
42	Fleur De Chine	0	0	0
43	Hibiscus Resort	6.405%	6.019%	6.212%
44	Grand	6.130%	5.658%	5.894%
45	Caesar Park	0.779%	0.524%	0.652%
46	Howard Hotel	0.030%	0	0.015%
47	Chihpen	0	0	0
48	Silks Place	0.212%	0	0.106%
49	CHIAO-HIS	0.063%	0.038%	0.050%
50	Taoyuan Hotel	0	0	0
51	Ta Shee Resort	1.347%	2.287%	1.817%
52	Hotel Royal	0.396%	0.257%	0.326%
53	Ambassador	0.034%	0	0.017%
54	Nice Prince	0	0	0
55	Hotel Tainan	5.011%	4.655%	4.833%
56	Evergreen Plaza	0.065%	0	0.033%
57	Tayih Landis	0.038%	0.038%	0.038%
58	Naruwan	0.019%	0.491%	0.255%

the efficiency change rate $\beta_2 = 0.09126$ is the maximum of all the change rates, indicating that the efficiency volatility is larger when the interval is expanded from [0.1, 0.9] to [0, 1] but still does not exceed 10%. In general, the obtained results have the accuracy to adapt to different actual situations and high robustness.

5 Conclusions

In this study, we propose a multiplier-based network DEA model to estimate the overall efficiency of mixed two-stage nonhomogeneous network processes with shared extra intermediate resources. Mixed two-stage nonhomogeneous network processes commonly occur in real life. We use the data set of 58 Taiwan international hotels to test the features of the proposed model. The results show that the proposed model can accurately determine the efficiency of each DMU and rank all DMUs based on their efficiencies. Different efficiency decomposition principles cause different stage efficiencies for the same DMU. The interval of the shared resource portion may affect the optimal efficiency values for certain DMUs. In future research, we will consider general system structures, such as multistage mixed structures. Panel data are also worthy of future study and analysis. Shared extra intermediate resources can be applied to other research fields, such as transportation (Zhen et al., 2019), environment (Yang et al., 2019), and supply chain (Brunaud et al., 2018).

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