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RESEARCH ARTICLE

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Real option-based optimization for financial incentive allocation in infrastructure projects under public–private partnerships

Abstract Financial incentives that stimulate energy investments under public–private partnerships are considered scarce public resources, which require deliberate allocation to the most economically justified projects to maximize the social benefits. This study aims to solve the financial incentive allocation problem through a real option-based nonlinear integer programming approach. Real option theory is leveraged to determine the optimal timing and the corresponding option value of providing financial incentives. The ambiguity in the evolution of social benefits, the decision-maker's attitude toward ambiguity, and the uncertainty in social benefits and incentive costs are all considered. Incentives are offered to the project portfolio that generates the maximum total option value. The project portfolio selection is formulated as a stochastic knapsack problem with random option values in the objective function and random incentive costs in the probabilistic budget constraint. The linear probabilistic budget constraint is subsequently transformed into a nonlinear deterministic one. Finally, the integer non-linear programming problem is solved, and the optimality gap is computed to assess the quality of the optimal solution. A case study is presented to illustrate how the limited financial incentives can be optimally allocated under uncertainty and ambiguity, which demonstrates the efficacy of the proposed method.

Keywords financial incentives, public–private partnerships, energy infrastructure projects, real option, optimization, uncertainty

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1 Introduction

Owing to soaring infrastructure demands and constrained public budgets, public–private partnerships (PPP) emerge as an appealing approach to leverage private sector's financial resources and expertise to deliver infrastructure projects (Li et al., 2017; 2019; Cai et al., 2019). A PPP is a long-term contractual agreement between a public agency and a private sector, which allows greater participation of the private sector in the provision of public infrastructure and service compared with traditional contracts. The International Energy Agency (2014) estimated that a cumulative investment of $ 48 trillion is required in energy supply and efficiency projects between 2014 and 2035. Approximately 60% of the investments require financing by private sectors (McInerney and Johannsdottir, 2016).

Governments typically provide various financial supports, such as equity, subsidy, and government guarantee to induce private investments under PPPs (Li and Cai, 2017). Two primary reasons justify the provision of these financial incentives. First, an economically justified project does not necessarily mean it can be commercially financed. Governments must provide subsidies for private sectors as compensations. Second, incentives can induce timely private investments by demonstrating the governments' commitments in risk-sharing (EPEC, 2011). Governments generally provide interest-free loans and inject equities to facilitate private investments. Financial incentives are common and substantial in the energy sector. The International Monetary Fund estimated the global energy subsidy to be $ 5.3 trillion in 2015 or 6.5% of global gross domestic product (Coady et al., 2015). The US Energy Information Administration (2015) reported that the federal energy subsidies in renewable energy increased from $ 8.6 billion in 2011 to $ 13.2 billion in 2013. Owing to the large number of projects and the depletion of stimulus funds, financial incentives are considered scarce resources requiring deliberate allocation to maximize its value.

Under budget constraints, supporting all proposed projects may be difficult for the government. Hence, the problem confronting the government involves how to select a portfolio of PPP projects, for which financial support is provided under budget constraints to maximize social benefits with given uncertainties. Social benefits are maximized when the financial incentives are allocated to the most economically justified projects. Accordingly, this study aims to create an optimization model that supports the incentive allocation decision. The proposed method can aid policy-makers in deciding when and which projects should be offered financial incentives to promote private investments. The real option technology is leveraged to determine the optimal timing and the corresponding option value of providing financial incentives. The ambiguity in the social benefit evolution and the attitude of the decision-maker toward such ambiguity are considered. The uncertainty is incorporated into social benefits and incentive costs through probability density functions. Furthermore, the maximum total option value is generated on the basis of the incentives offered to the project portfolio. The project portfolio selection is treated as a stochastic knapsack problem.

The remainder of this paper is organized as follows. The next section reviews relevant studies and identifies knowledge gaps. Subsequently, a real option-based optimization model for incentive allocation is formulated, and an algorithm is suggested to solve the model. A case study is presented to demonstrate the efficacy of the proposed method. Finally, the research contributions are highlighted, and the limitations and future research directions are pointed out.

2 Literature review

2.1 Real option-based project evaluation

The use of net present value (NPV) and discounted cash flow (DCF) analysis remain conventional doctrines for project evaluation (Wibowo and Kochendoerfer, 2011; Arnold and Yildiz, 2015). Schachter and Mancarella (2016) identified three limitations of DCF method. First, DCF analysis assumes that investments are reversible. However, investments in infrastructure projects are partially or completely irreversible. Second, DCF analysis assumes that the discount rate and future cash flows are known in advance. Valuing projects based on deterministic future scenarios will mislead decision-makers. Third, DCF approach completely omits the managerial flexibility to postpone and adjust investment decisions, which result in undervaluation of projects with high uncertainty and flexibility.

Owing to the above limitations, real option analysis (ROA) emerges as an alternative method for project evaluation. Fernandes et al. (2011), Ceseña et al. (2013), and Schachter and Mancarella (2016) reviewed the application of ROA in energy project investments. Lee (2011) investigated the feasibility of using ROA to value renewable energy investment. In addition, Santos et al. (2014) compared ROA with traditional methods in assessing renewable energy projects. Moreover, Zhang et al. (2014a) proposed an ROA-based model for renewable energy policy evaluation. All these studies ascertained that ROA is more effective than NPV analysis when dealing with irreversibility, uncertainty, and managerial flexibility in a project.

Zhang et al. (2014b) developed a trinomial tree model based on ROA to evaluate a carbon capture and storage (CCS) investment, which considered the uncertainties in carbon price, government incentives, annual operating time, power plant lifespan, and technological improvements. Wang and Du (2016) extended the study of Zhang et al. (2014b) by proposing a quadrinomial model based on ROA to evaluate the investments in CCS retrofitting for coal-fired power plants in China. By comparing the results with those obtained from NPV method, the study proved that ROA is more appropriate than the traditional NPV method in addressing uncertainty, and the government subsidy will significantly reduce the critical carbon price in the CCS investment. Reuter et al. (2012) and Kroniger and Madlener (2014) used ROA, which considers the environmental and economic uncertainty to investigate the economic viability of power-to-power operation, such as hydrogen or pumped hydropower storage for excess wind power production. Both studies concluded that power-to-power operation is unprofitable without substantial public support. Moreover, Kim et al. (2014) and Wesseh and Lin (2015) evaluated the economic value and optimal timing of investing in R&D for renewable energy technologies using ROA, considering the market and environmental uncertainty. Both studies indicate the economic attractiveness of the investments.

Almassi et al. (2013) developed a computationally efficient valuation tool using finite-difference method based on ROA for the valuation of government guarantees in infrastructure projects; subsequently, they modeled the project risk using a new continuous stochastic differential equation. However, the new method cannot perform efficiently under high dimension risks than with Monte-Carlo (MC) simulation. Lin and Wesseh (2013) quantified the benefits of feed-in-tariff for solar power generation by using ROA to evaluate the value of solar energy technologies in the presence of uncertainty in fossil fuel price and learning effect in solar energy technologies. Ritzenhofen and Spinler (2016) used ROA to examine the optimal design of feed-in-tariff for stimulating renewable energy investments under regulatory uncertainty. Wesseh and Lin (2016) considered the valuation of wind energy technologies and used ROA to further probe if the benefits from the feed-in-tariffs outweigh costs. Jeon et al. (2015) integrated system dynamics and real option to estimate the optimal financial subsidy and public R&D investment. Furthermore, Park et al. (2014) proposed the use of ROA to evaluate clean development mechanism projects and investigate how uncertain energy policies affect the financial viability of the projects.

2.2 Optimal project portfolio selection

Meier et al. (2001) formulated a capital budgeting problem to determine the portfolio of options that exhibits maximum value and satisfies budget constraints. Wibowo and Kochendoerfer (2011) used NPV as a measure of economic benefits and employed a chance-constraint goal programming method in selecting projects to receive government guarantees. In addition, Wu et al. (2011) investigated the problem of how to select build-operate-transfer (BOT) projects to improve social benefits while ensuring the marketability of the selected projects. This problem was solved using a mixed integer program with equilibrium constraints. Li et al. (2010) modeled the project selection puzzle as a multi-dimensional knapsack problem and developed an efficient heuristic solution algorithm using Lagrange relaxation technique. Moreover, Eckhause et al. (2012) created an integer programming approach to optimally select a portfolio of R&D projects.

Tavana et al. (2015) examined the project selection and resource allocation problem. They employed data envelopment analysis for initial project screening, the technique utilized for the order of preference by similarity to the ideal solution for project ranking, and linear integer programming for project selection in a fuzzy environment according to organizational objectives. Huang and Zhao (2016) developed an optimization model to consider both the selection of new projects and the adjustment of existing ones, and genetic algorithm is used to solve the model. Sefair et al. (2017) measured the risk using the semi-variance of the portfolio's NPV and the reward using the portfolio's expected NPV. Afterwards, mixed integer quadratic programming technique is leveraged to obtain the optimal risk-reward project portfolio.

2.3 Knowledge gaps

The literature review shows two main knowledge gaps. First, majority of the mentioned studies adopted ROA without considering the ambiguity of the uncertainty in the valuation of energy infrastructure projects. These studies assumed that underlying uncertainty is characterized by a single probability measure. In practice, decision-makers may be unsure about the selected probability measure. They evaluate investments using another possible probability measure, known as ambiguity, also referred to as Knightian uncertainty in economics and decision science fields. The notion of ambiguity refers to the existence of a multitude of probability distributions that capture the return volatility of an investment project to describe future returns. Ambiguity can be characterized by a set of probability measures (Nishimura and Ozaki, 2007; Schröder, 2011). Uncertainty may enhance the value of investment, whereas ambiguity will negatively affect the project valuation (Nishimura and Ozaki, 2007). Therefore, incorporating ambiguity in the standard ROA enables decision-makers to capture their uncertainty regarding the future evolution of the project, thereby performing an accurate project valuation. In addition, the estimates of the initial social benefits and required public investments are subject to high uncertainties, which have been neglected in the previous studies.

The second knowledge gap is the lack of an optimization model and a solution algorithm that incorporate uncertain real option values for project portfolio selection. Despite the widespread use of NPV, studies, such as Meier et al. (2001) and Eckhause et al. (2012), attempted using option values as project selection criteria. Nevertheless, their optimization models did not explicitly obtain the inherent uncertainties in option values. Moreover, two existing technical barriers prevent previous studies from achieving a reliable and optimality-aware solution. The first barrier is the treatment of budget constraints in the optimization model. Using expected values of investment costs may result in a high risk of budget excess, which renders the obtained solution unreliable. Second, previous studies scarcely evaluated the goodness of a candidate solution, hence providing limited insights and confidence for decision-makers to implement the selected project portfolio. Accordingly, this study aims to fill the two knowledge gaps.

3 Methodology

The government targets at providing financial support to the most economically justified projects to maximize the total social benefits. Figure 1 presents an overview of the methodology and illustrates how the proposed method can address the two knowledge gaps. The method involves two steps, i.e., single project evaluation and project portfolio selection.

For single project evaluation, real option theory is leveraged to determine the optimal timing for providing financial incentives to induce private investments and the corresponding option value. The evolution of social benefit throughout the project period is captured using geometric Brownian motion (GBM). In reality, owing to the lack of perfect information and subjectivity in forecasting the social benefit, decision-makers may not have full

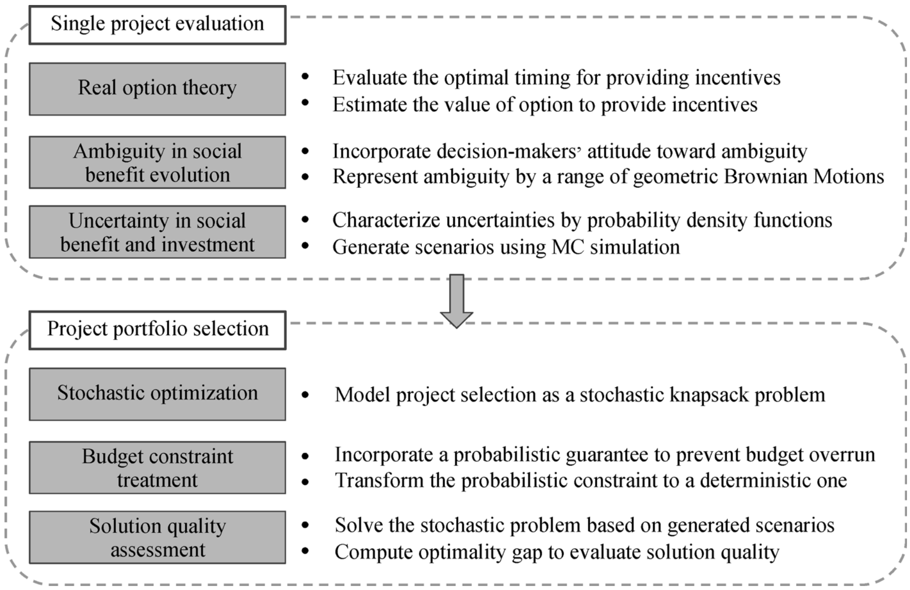


Fig. 1 Methodology overview.

confidence about the extent of single GBM's description of the future social benefit. Hence, they may adopt a range of GBMs to represent such ambiguity of the probability measure. The decision-maker's attitude toward ambiguity is incorporated in computing the optimal timing and the option value of providing financial incentives. In addition, the uncertainty in estimating the initial social benefit and the public costs of financial incentives are modeled by probability density functions (PDFs). Alternately, MC simulation can generate several possible scenarios for the optimization in the second step. The project portfolio selection can be modeled as a stochastic knapsack problem with random option values in the objective function and random public costs in the budget constraint. A probabilistic guarantee of not exceeding the budget limit is included in the constraint. The probabilistic budget constraint is then transformed into a deterministic one. Finally, the integer non-linear programming problem is solved, and the optimality gap is computed to assess the quality of the optimal solution. Table 1 presents a list of parameters.

Table 1 Definition of key parameters

Symbol	Definition
S	Social benefits
Q	Probability measure
μ	Drift rate
σ	Volatility
dt	Length of time interval
B_t	Standard Brownian motion
θ	Density generators
Θ	Set of density generators
α	Decision-makers' attitude toward ambiguity
W_t	Present value of social benefits
r	Discount rate
V_t	Value of providing financial incentives
I	Public costs of financial incentives
W^*	Critical value between postponing and exercising the provision of financial incentives
p_j	Given real numbers
ε	Degree of uncertainty
η	Symmetrically distributed random variable with zero mean and support [–1, 1]
ϕ	Positive parameter corresponding to the probabilistic guarantee
B	Deterministic budget limit
U_m	Expectation of empirical distributions
z^*	Optimal solution value
L_m	Objective value
G	Optimality gap
$\overline{G}(n)$	Mean of the optimality gap
$S_G^2(n)$	Variance of the optimality gap

3.1 Project evaluation based on ROA with ambiguity and uncertainty

The evolution of social benefits S generated by a project is assumed to follow a GBM under a probability measure Q and defined in Eq. (1), where μ is the drift rate which indicates the expected growth rate in social benefits, σ is the volatility, dt is the length of time interval, and B_t is the standard Brownian motion. One can estimate the μ and σ from relevant literature, historical data of similar projects, or from an expert's judgment based on the project characteristics and socio-economic environment. Casparri and Fronti (2012) and Thompson and Green (1998) also modeled the evolution of social benefits as GBMs. Owing to ambiguity, decision-makers may also consider other possible GBMs under a set of probability measures $P = \{Q^\theta | \theta \in \Theta\}$ that deviate from the original probability measure Q. Θ is the set of density generators θ, which represents the ambiguity (Gao and Driouchi, 2013). Then, Eq. (1) can be extended to Eq. (2), where a range of GBMs are used to represent the ambiguous social benefit evolution. The density generate θ is restricted to the non-stochastic interval $[-\kappa, \kappa]$ with $\kappa \geqslant 0$ (Chen and Epstein, 2002). B_t^θ is the standard Brownian motion with respect to Q^θ, derived on the basis of Girsanov's theorem, i.e., $\mathrm{d}B_t^\theta = \mathrm{d}B_t + \theta_t \mathrm{d}t$ (See Nishimura and Ozaki (2007) for a detailed derivation). Equation (2) incorporates the degree of ambiguity into the formulation of drift rate, illustrating decision-maker's uncertainty about the probability distribution of the expected growth of the social benefits generated by the project.

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t, \tag{1}$$

$$\mathrm{d}S_t = (\mu - \sigma\theta) S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t^\theta. \tag{2}$$

Decision-makers can either be pessimistic or optimistic about the evolution of social benefits generated by a project. $\alpha \in \{0, 1\}$ represents the decision-makers' attitude toward ambiguity. $\alpha = 0$ indicates that the decision-maker is pessimistic, whereas $\alpha = 1$ implies that the decision-maker is optimistic. Next, the option value of providing financial incentives is derived under pessimism. To simplify the derivation, the energy infrastructure lifespan is assumed infinite, and the opportunity for providing financial incentives never expires. The present value of social benefits, denoted by W_t, is calculated in Eq. (3), where r is the discount rate. Under pessimism, decision-makers choose an optimal timing for providing incentives over the worse scenario. Moreover, Eq. (4) calculates V_t, the value of providing financial incentives, where I is the public costs of financial incentives. Nishimura and Ozaki (2007) derived an important second-order differential equation in Eq. (5) to solve the optimal timing and option value.

$$W_t = \int_t^\infty S_t \mathrm{e}^{(-r+\mu-\kappa\sigma)(x-t)} \mathrm{d}x = \frac{S_t}{r - (\mu - \kappa\sigma)}, \tag{3}$$

$$V_t = \max_{t' \geqslant t} \min_{Q \in \mathrm{P}} E^Q \left[\int_{t'}^\infty \mathrm{e}^{-r(x-t)} S_x \mathrm{d}x - e^{-r(t'-t)} I \right], \tag{4}$$

$$\frac{1}{2}\sigma^2 W_t^2 V''(W_t) + (\mu - \kappa\sigma) W_t V'(W_t) - rV(W_t) = 0. \tag{5}$$

Equations (6) to (8) are boundary conditions for solving Eq. (5) (Dixit and Pindyck, 1994). Equation (6) indicates that if the present value of social benefits is 0, then, the corresponding value of the option to provide financial incentives must be 0. The government should not provide any financial support for a project that cannot generate any social benefit. Equation (7) concerns the critical value W^*, which marks the boundary between postponing and exercising the provision of financial incentives. At this point, postposing the provision of incentives does not add any further value, thus, the value of the option $V(W^*)$ equals the NPV $W^* - I$. Equation (8) is the so-called smooth-pasting condition that ensures the regularity of the solution.

$$V(0) = 0, \tag{6}$$

$$V(W^*) = W^* - I, \tag{7}$$

$$V'(W^*) - 1 = 0. \tag{8}$$

In the case of optimism, decision-makers choose to provide incentives over the best scenario; therefore, Eq. (4) is replaced by $V_t = \max_{t' \geqslant t} \max_{Q \in \mathrm{P}} E^Q \left[\int_{t'}^\infty \mathrm{e}^{-r(x-t)} S_x \mathrm{d}x - \mathrm{e}^{-r(t'-t)} I \right]$. Solving Eq. (5) with the three boundary conditions and incorporating the attitude indicator α, the optimal investment timing is determined in Eq. (9), where $\beta = \frac{1}{2} - \frac{\gamma}{\sigma^2} + \sqrt{\left(\frac{\gamma}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$ and $\gamma = \alpha(\mu + \sigma\kappa) + (1-\alpha)(\mu - \sigma\kappa)$. Notably, γ is used to generalize the expression of two scenarios. When the decision-maker is pessimistic, $\alpha = 0$ and $\gamma = \mu - \sigma\kappa$; when the decision-maker is optimistic, $\alpha = 1$ and $\gamma = \mu + \sigma\kappa$ (See Gao and Driouchi (2013) for a detailed derivation). In addition, the corresponding option value is calculated in Eq. (10).

$$S^* = \frac{\beta(\rho - \gamma)}{\beta - 1} I, \tag{9}$$

$$V(W_t) = \begin{cases} \dfrac{(\beta-1)^{\beta-1}}{I^{(\beta-1)}\beta^\beta} W_t^\beta & \text{if} \quad S_t < S^*, \\ W_t - I & \text{if} \quad S_t \geqslant S^*. \end{cases} \tag{10}$$

Estimates of the initial present value of social benefits W_0 and public costs of financial incentives I are subject to high uncertainties that can be characterized by PDFs. If historical data are unavailable, a knowledge-based empirical approximation procedure can be adopted to derive the PDFs. Stakeholders and experts are solicited to provide worst, base, and best scenarios for the estimates. These values can be interpreted as quantiles of a PDF. For example, the best scenario value for the social benefit can be specified at the 95% quantile. Then, a specific PDF function can be characterized on the basis of the empirical values and their quantiles. Arnold and Yildiz (2015) described the procedure to determine the PDFs of normal and log-normal distributions based on the values and their quantiles. After obtaining the PDFs, MC simulation technique can be leveraged to obtain the empirical distribution of the uncertain option values and public costs.

3.2 Optimization model and solution

The project portfolio selection problem is formulated as a stochastic knapsack problem in Eq. (11). The N represents candidate projects. x_j is a binary decision variable. $x_j = 1$ indicates that project j is selected, whereas $x_j = 0$ indicates that project j is excluded. $\tilde{V}(W_{j,0})$ is the random value of option to provide financial incentives for project j, which is calculated using Eq. (10). The objective is to maximize the expected total option value of project portfolio. $\tilde{I}_j$ is the uncertain public cost of providing financial incentive for project j. B is the deterministic budget limit.

$$\begin{aligned} &\max\ E\left[\sum_{j=1}^{N}\tilde{V}\left(\tilde{W}_{j,0}\right)x_j\right] \\ &s.t.\quad \sum_{j=1}^{N}\tilde{I}_j x_j \leqslant B, \\ &x_j \in \{0,1\} \quad \forall j. \end{aligned} \tag{11}$$

Directly using the expected value of $\tilde{I}_j$ in the constraint for optimization is inappropriate. To illustrate, suppose that $\tilde{I}_j$ follows a normal distribution; therefore, the obtained optimal solution will have a 50% chance of exceeding the budget if the expected value of $\tilde{I}_j$ is used. Such high risk of budget overrun is unacceptable in practice. Therefore, the first step in solving the optimization model is to transform the budget constraint by including a contingency term that corresponds to a probabilistic guarantee of not exceeding the budget. To facilitate this transformation, the random cost is further assumed to be characterized by Eq. (12), where I_j is the mean value, ε_j is the degree of uncertainty, and $\tilde{\eta}_j$ is a symmetrically distributed random variable with zero mean and support $[-1, 1]$. Inspired by Ng et al. (2011), we transform the original budget constraint in Eq. (13) that consists of the expected cost $\sum_{j=1}^{N} I_j x_j$ and the contingency term $\phi\sqrt{\sum_{j=1}^{N}(I_j\varepsilon_j x_j)^2}$, where ϕ is a positive parameter corresponding to the probabilistic guarantee.

$$\tilde{I}_j = I_j\left(1 + \varepsilon_j\tilde{\eta}_j\right), \tag{12}$$

$$\sum_{j=1}^{N} I_j x_j + \phi\sqrt{\sum_{j=1}^{N}(I_j\varepsilon_j x_j)^2} \leqslant B. \tag{13}$$

The next step is to derive the relationship between the parameter ϕ and the probabilistic guarantee. $\Pr\left(\sum_{j=1}^{N}\tilde{I}_j x_j > B\right)$ is denoted as the probability of exceeding the budget; then, Eq. (14) holds. The first equality is followed by the definition of $\tilde{I}_j$. Given that x_j is a feasible solution, it must satisfy Eq. (13). Replacing B with $\sum_{j=1}^{N} I_j x_j + \phi\sqrt{\sum_{j=1}^{N}(I_j\varepsilon_j x_j)^2}$ leads to the second line in Eq. (14). The last line follows owing to the well-known fact proved in Ben-Tal and Nemirovski (2000). Let p_j be the given real numbers and $\tilde{\eta}_j$ the independent random variables symmetrically distributed in $[-1, 1]$. Subsequently, for every $\phi > 0$, one has $\Pr\left\{\sum_j \tilde{\eta}_j p_j > \phi\sqrt{\sum_j p_j^2}\right\} \leqslant \exp(-\phi^2/2)$.

$$\begin{aligned} &\Pr\left(\sum_{j=1}^{N}\tilde{I}_j x_j > B\right) \\ &= \Pr\left(\sum_{j=1}^{N} I_j\left(1+\varepsilon_j\tilde{\eta}_j\right)x_j > B\right) \\ &\leqslant \Pr\left(\sum_{j=1}^{N} I_j\left(1+\varepsilon_j\tilde{\eta}_j\right)x_j > \sum_{j=1}^{N} I_j x_j + \phi\sqrt{\sum_{j=1}^{N}(I_j\varepsilon_j x_j)^2}\right) \\ &= \Pr\left(\sum_{j=1}^{N} I_j\varepsilon_j\tilde{\eta}_j x_j > \phi\sqrt{\sum_{j=1}^{N}(I_j\varepsilon_j x_j)^2}\right) \\ &\leqslant \exp(-\phi^2/2). \end{aligned} \tag{14}$$

Equation (14) enables decision-makers to set a specific value of ϕ in constraint Eq. (13); the latter guarantees that the probability of budget excess is at most $\exp(-\phi^2/2)$. Specifically, if decision-makers want to guarantee that the probability of budget excess is at most α (5%), then, $\phi = \sqrt{-2\log\alpha}$ ($\phi = 2.45$).

With the treatment of the budget constraint, the original integer linear programming problem becomes an integer non-linear programming problem with random variables in the objective function. To solve this stochastic optimization model, an approximation procedure is adopted to replace the actual distributions of option values with empirical distributions generated by MC simulation. MC simulation generates independent and identically distributed scenarios for the option value $V^i(W_{j,0})$, where, $i = 1,\ldots, m$, and m is the total number of scenarios. These m scenarios constitute an empirical distribution. With a modest value of m, the original problem can be reasonably approximated by a computationally tractable optimization model (Morton and Wood, 1998).

The approximating problem based on empirical distributions is formulated in Eq. (15). Mak et al. (1999) proved that the expectation of U_m is an upper bound of the optimal solution value z^* as indicated in Eq. (16). Estimating the upper bound of the optimal solution value is necessary for ascertaining the quality of a candidate solution $\hat{x}$. The objective value L_m given the candidate solution $\hat{x}$ is provided in Eq. (17).

$$\begin{aligned} U_m &= \max_x \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{N} V^i(W_{j,0})x_j \\ s.t. \quad & \sum_{j=1}^{N} I_j x_j + \phi\sqrt{\sum_{j=1}^{N}(I_j\varepsilon_j x_j)^2} \leqslant B, \\ & x_j \in \{0,1\} \quad \forall j, \end{aligned} \tag{15}$$

$$\begin{aligned} z^* &= \max_x \sum_{j=1}^{N} V(W_{j,0})x_j \\ &= \max_x E\left[\frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{N} V^i(W_{j,0})x_j\right] \\ &\leqslant E\left[\max_x \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{N} V^i(W_{j,0})x_j\right] = EU_m, \end{aligned} \tag{16}$$

$$L_m = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{N} V^i(W_{j,0})\hat{x}_j. \tag{17}$$

Equation (18) defines the optimality gap that can be used to ascertain the quality of a candidate solution. Based on Mak et al. (1999) and Morton and Wood (1998), a one-sided confidence interval of the optimality gap can be constructed using the method of batch means. The same streams of random numbers are used to estimate the upper and lower bounds to compute G_m.

$$\begin{aligned} G_m &= U_m - L_m \\ &= \max_x \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{N} V^i(W_{j,0})x_j - \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{N} V^i(W_{j,0})\hat{x}_j. \end{aligned} \tag{18}$$

The steps for solving the stochastic optimization problem in Eq. (15) and computing the optimality gap are summarized as follows.

Step 1: Approximate the actual distributions by generating m scenarios for the option values of each candidate project. To obtain a good candidate solution, m should be relatively large.

Step 2: Based on the generated scenarios, solve the optimization problem in Eq. (15) using an integer nonlinear programming technique and obtain a candidate solution $\hat{x}$.

Step 3: Generate n streams of random option values to compute the optimality gap G_m^i, $i = 1,..., n$ based on Eq. (18).

Step 4: Construct an approximate $1-\alpha$ level confidence interval $[0, \overline{G}(n) + \varepsilon_G]$ on the optimality gap based on Eqs. (19) to (21). $\overline{G}(n)$ and $S_G^2(n)$ are the mean and variance of the optimality gap, while z_α can be obtained in a statistical table.

$$\overline{G}(n) = \frac{1}{n}\sum_{i=1}^{n} G_m^i, \tag{19}$$

$$S_G^2(n) = \frac{1}{n-1}\sum_{i=1}^{n}\left[G_m^i - \overline{G}(n)\right]^2, \tag{20}$$

$$\varepsilon_G = z_\alpha S_G(n)/\sqrt{n}. \tag{21}$$

4 Case study

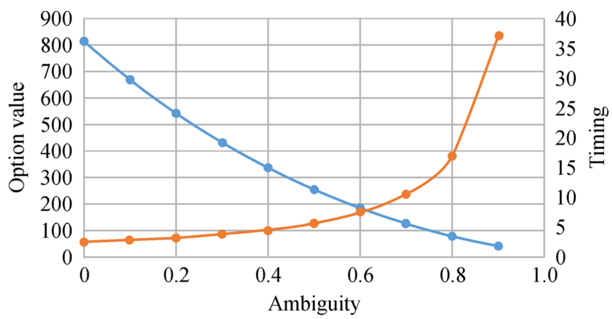
To further illustrate the proposed method, this section presents a case study in which a portfolio of renewable energy projects must be selected to provide financial incentives. Table 2 presents the initial social benefit estimates, benefit evolution and ambiguity, decision-maker's attitude toward ambiguity, and incentive cost estimates. In this case study, the initial social benefit and the incentive cost are both further assumed to follow normal distributions. The random variable η_j thus follows a truncated standard normal distribution with support $[-1, 1]$. The discount rate is assumed to be 12.5%.

Using Project 1 as example, Fig. 2 presents the option values and investment timing with different ambiguities and decision-makers' attitudes with the mean initial social benefit and incentive cost. Apparently, the ambiguity and decision-maker's attitude significantly affect the option value, highlighting their necessity in the project evaluation. When the decision-makers are pessimistic, they tend to dislike ambiguity. Thus, the option value decreases, and the timing for incentive provision increases with increasing ambiguity. By contrast, when the decision-makers are optimistic, the option value increases and the timing for incentive provision decreases with increasing ambiguity.

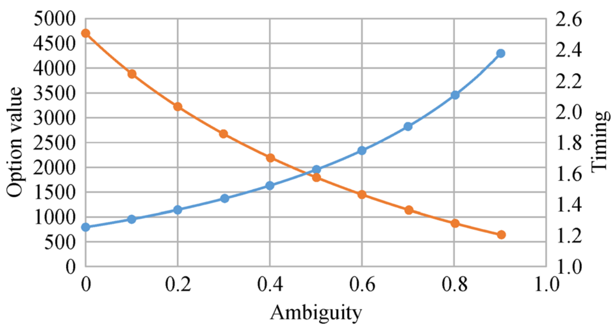
Using Project 15 as an example when uncertainties in estimating initial social benefits and incentive costs are considered, Figs. 3 and 4 present three examples of option value distributions with different ambiguities. The option values are asymmetrically distributed. When the decision-maker is pessimistic, with increasing ambiguity, the distribution shifts to the left and possesses a fatter tail in the right. Conversely, when the decision-maker is optimistic, with increasing ambiguity, the distribution is

Table 2 Social benefits and public investment data of candidate projects

Project ID	Social benefit		GMB		Ambiguity	Investment cost	
	Mean	Standard deviation	Drift rate	Volatility		Mean	Degree of uncertainty
1	175	5	0.05	0.05	0.2	1550	0.05
2	180	4	0.05	0.05	0.2	1600	0.15
3	25	3	0.06	0.08	0.5	500	0.4
4	200	20	0.03	0.06	0.25	1650	0.35
5	155	35	0.06	0.1	0.25	1900	0.15
6	50	4	0.08	0.15	0.1	950	0.15
7	300	7.5	0.03	0.10	0.5	2150	0.1
8	225	15	0.07	0.09	0.35	1550	0.5
9	330	60	0.04	0.05	0.45	2850	0.2
10	125	5	0.1	0.05	0.4	2000	0.3
11	220	5	0.05	0.07	0.1	1800	0.1
12	280	10	0.06	0.08	0.2	3300	0.25
13	120	15	0.07	0.05	0.75	1200	0.2
14	250	50	0.08	0.1	0.25	3350	0.05
15	360	15	0.1	0.1	0.1	4500	0.05



(a)



(b)

Fig. 2 Timing and option values with varying ambiguity.

shifted to the right and is more spread out. In addition, the average option value increases with the decrease in ambiguity for ambiguity-averse decision-maker, whereas it decreases when the decision-maker is ambiguity-seeking. The ambiguity and decision-maker's attitude is further proved to have a substantial influence on the project selection. The distributions of option values are not directly characterized, thereby requiring further justification in the adoption of the approximation procedure and the computation of the optimality gap to solve the optimization model.

Table 3 lists the optimal solutions with various probabilistic guarantees of not exceeding the budget of 18000, when the decision-maker is pessimistic about all the projects. Figure 5 plots the optimal value and the optimality gap at the 95% confidence level. The value of m is set to be 10000, whereas the value of n is set to be 20. The sample size in the calculation of the optimality gap is set to be 20.

Table 4 lists the optimal solutions with various probabilistic guarantees of not exceeding the budget of 18000, when the decision-maker is optimistic about all the projects. Figure 6 plots the optimal value and the optimality gap at the 95% confidence level.

5 Discussion of results

Comparing the optimal solutions obtained under pessimism and optimism, one can observe that the ambiguity, the decision-maker's attitude toward ambiguity, the probabilistic guarantee, and the project characteristics significantly affect the project selection. For example, the project with the largest ambiguity is Project 13. When the

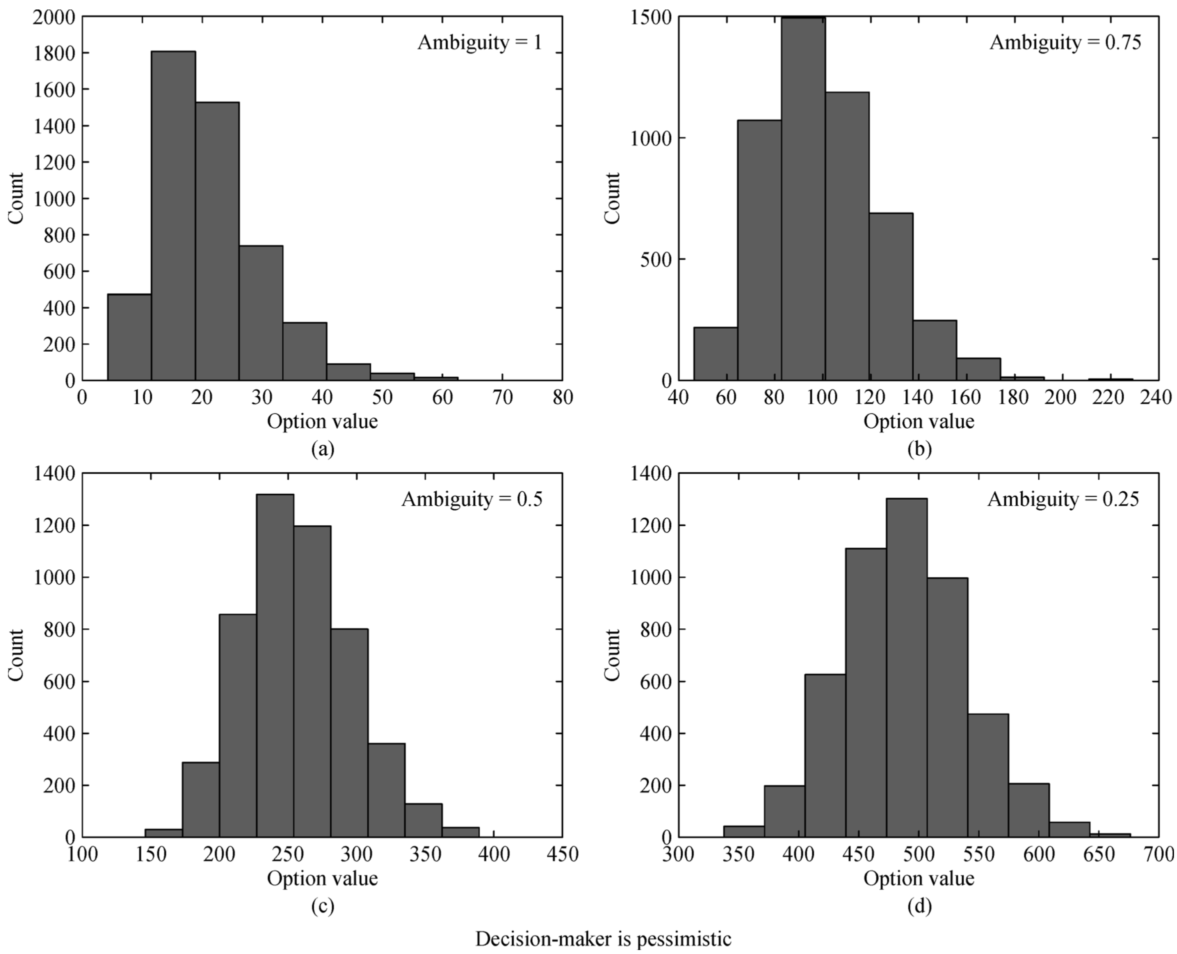


Fig. 3 Option value distributions with varying ambiguity under pessimism.

decision-maker is optimistic, Project 13 is constantly selected for various probabilistic guarantees as the decision-maker believes that it generates promising benefits. However, when the decision-maker is pessimistic, Project 13 is not selected, except when the probability of exceeding budget is guaranteed to be less than 5%. Such selection is attributed to the intrinsic characteristics of the candidate projects. The incentive costs for Projects 3, 6, and 13 are the lowest. Compared with Projects 3 and 6, although Project 13 has a higher incentive cost, it yields much more benefit than Projects 3 and 6. Hence, although the decision-maker is pessimistic and the ambiguity of Project 13 is high, Project 13 is selected when the decision-maker requires a high probability of satisfying the budget constraint.

Projects 11 and 15 have the lowest ambiguity. Project 15 is constantly selected under pessimism and optimism with various probabilistic guarantees because of its low ambiguity, low degree of uncertainty in incentive cost, and large social benefits. Project 11 is constantly selected under pessimism and is constantly discarded under optimism. A possible explanation is that when the decision-maker is pessimistic, small ambiguity makes the project preferable to the decision-maker. Conversely, when the decision-maker is optimistic, small ambiguity indicates a small option value. This finding implies that the decision-maker does not believe that this project will generate promising social benefits; hence, the project will not be selected. This phenomenon implies that the high ambiguity project is more likely to be selected when the decision-maker is optimistic than when the decision-maker is pessimistic. Moreover, high ambiguity project may be selected when the pessimistic decision-maker requires a strong probabilistic budget guarantee. With respect to small ambiguity, it is more likely to be selected under pessimism and discarded under optimism. Therefore, the degree of ambiguity and the decision-maker's attitude toward ambiguity should be considered for project selection.

6 Conclusions

Governments provide various financial incentives, such as subsidy and government guarantee to induce private investments under PPPs. However, owing to the large

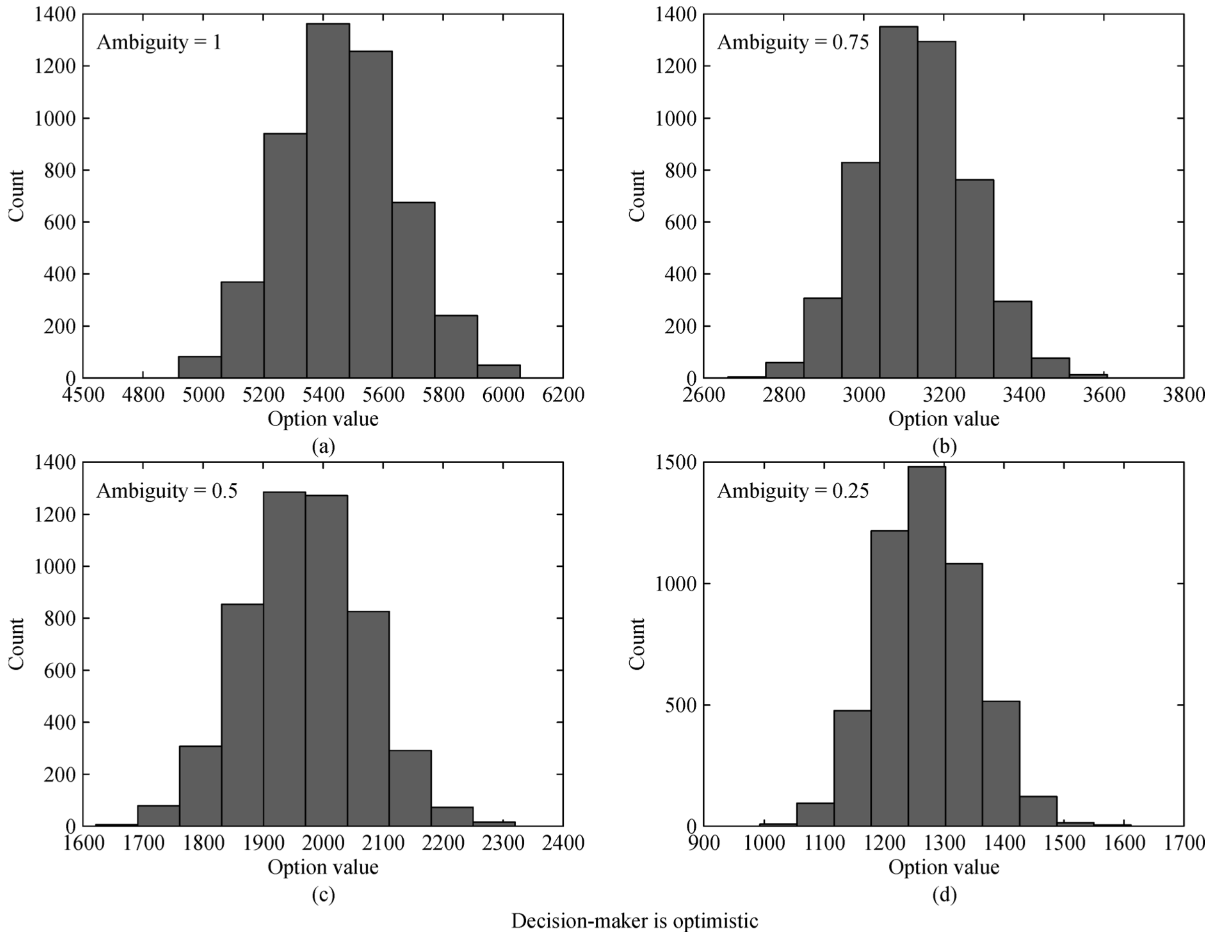


Fig. 4 Option value distributions with varying ambiguity under optimism.

Table 3 Optimal solutions with various probabilistic budget guarantees under pessimism

Project ID	α									
	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
1	1	1	0	0	0	0	1	1	1	1
2	1	0	1	1	1	1	1	1	1	1
3	0	0	0	0	0	1	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	1	1	1	1	1	1	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	1	1	1	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0
13	1	0	0	0	0	0	0	0	0	0
14	0	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1

Note: “1” means the project is selected; “0” means the project is excluded.

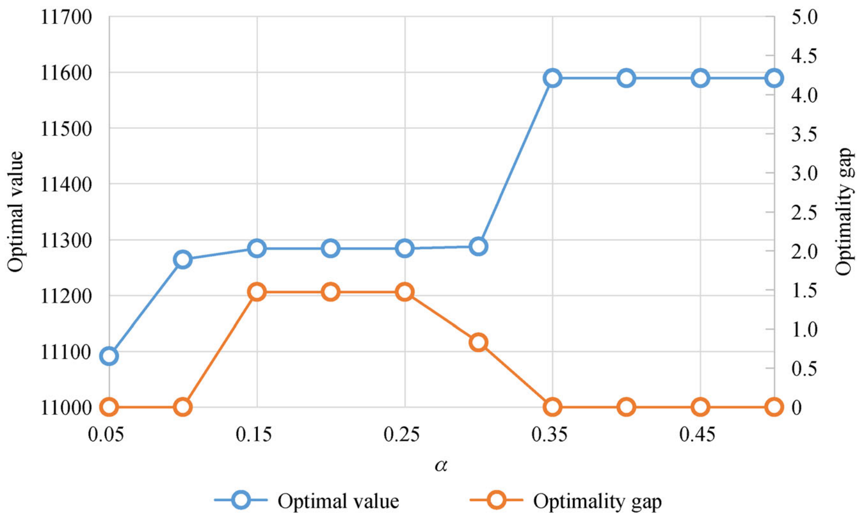


Fig. 5 Optimal value and the optimality gap when decision-maker is pessimistic.

Table 4 Optimal solutions with various probabilistic budget guarantees under optimism

Project ID	α									
	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	1	1	0	0	0	1	1	1	1	0
4	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1
6	0	0	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	0	0
10	1	1	1	1	1	1	1	1	1	1
11	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0
13	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1

Note: “1” means the project is selected; “0” means the project is excluded.

number of projects and the depletion of stimulus funds, financial incentives are scarce resources requiring deliberate allocation to maximize the total social benefits. This study proposes a real option-based nonlinear integer programming approach to allocate the financial incentives to the most economically justified projects. The method involves two steps, i.e., single project evaluation and project portfolio selection. Considering the ambiguity in social benefit evolution, the decision-maker’s attitude toward ambiguity, and the uncertainty involved in the social benefit and incentive cost, real option theory is leveraged to determine the optimal timing and option value of providing financial incentives. MC simulation can generate several scenarios for the project portfolio selection. The project portfolio selection is formulated as a stochastic knapsack problem with random option values in the objective function and random public costs in the budget constraint. A probabilistic guarantee of not exceeding the budget limit is included in the constraint. The probabilistic budget constraint is subsequently transformed into a deterministic one. Finally, the integer non-linear programming problem is solved, and the optimality gap is computed to assess the quality of the optimal solution. Moreover, the case study demonstrated

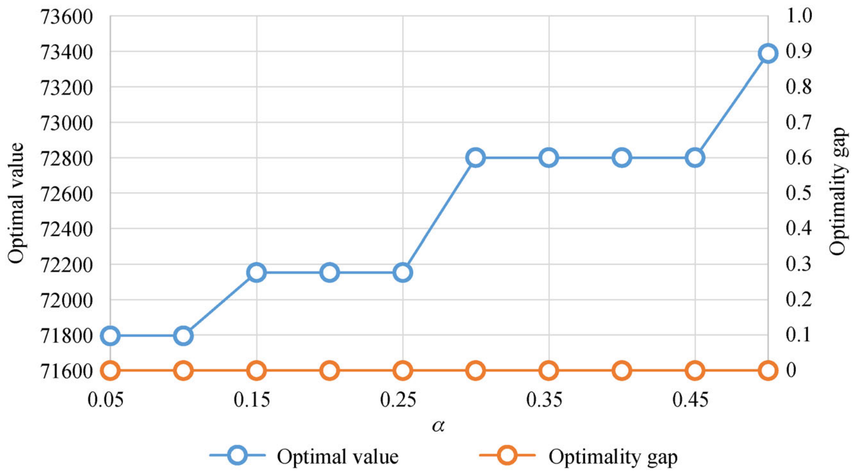


Fig. 6 Optimal value and the optimality gap when decision-maker is optimistic.

the efficacy of the proposed method. Results suggest that the degree of ambiguity and the decision-maker's attitude toward ambiguity significantly affect the selection of projects to receive financial incentives. In addition, the probabilistic guarantee of not exceeding the budget constraint also largely determines the optimal project portfolio.

Certain limitations remain and deserve further research efforts. First, to simplify the derivation, the lifespan of the infrastructure is assumed to be infinite and the opportunity for providing incentives never expires. Future research may restrict this assumption and generalize the model to incorporate finite lifespan scenario. Second, the risk attitudes of decision-makers considered in this study are two extreme cases, either pessimistic or optimistic, i.e., $\alpha \in \{0, 1\}$. Future efforts may consider different levels of optimism, i.e., $\alpha \in [0, 1]$, for an improved depiction of reality.

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