

Khalil Gorgani FIROUZJAH, Abdolreza SHEIKHOLESAMI, Taghi BARFOROUSHI

# Multi-objective allocation of measuring system based on binary particle swarm optimization

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**Abstract** Due to the size and complexity of power network and the cost of monitoring and telecommunication equipment, it is unfeasible to monitor the whole system variables. All system analyzers use voltages and currents of the network. Thus, monitoring scheme plays a main role in system analysis, control, and protection. To monitor the whole system using distributed measurements, strategic placement of them is needed. This paper improves a topological circuit observation method to minimize essential monitors. Besides the observability under normal condition of power networks, the observability of abnormal network is considered. Consequently, a high level of system reliability is carried out. In terms of reliability constraint, identification of bad measurement data in a given measurement system by making them sure to be detectable is well done. Furthermore, it is maintained by a certain level of reliability against the single-line outages. Thus, observability is satisfied if all possible single line outages are plausible. Consideration of these limitations clears the role of utilizing an optimization algorithm. Hence, particle swarm optimization (PSO) is used to minimize monitoring cost and removing unobservable states under abnormal condition, simultaneously. The algorithm is tested in IEEE 14 and 30-bus test systems and Iranian (Mazandaran) Regional Electric Company.

**Keywords** optimal allocation, phasor measurement units, observability, binary particle swarm optimization

## 1 Introduction

The main challenge to evaluate power systems is the

measurement of the node voltages and line/load currents. Conventional measurements (power injections and flows) as well as phasor measurement units (PMUs) are used to perform system analyzer algorithms and detect/analyze power quality events. Although many softwares and devices are available to power system monitoring, use of PMUs can make it more accurate. PMUs can be freely installed in locations where the power utilities wish to measure the electrical parameters of their system.

Power system disturbances have forced the power networks to be armed with wide-area monitoring, protection, and control. Some papers in the literature suggest the installation of a monitor in each bus to measure voltages and currents of the entire system [1]. It should be mentioned that the cost of the PMU as a device, installation and related equipment constitute the main impediment for the utilities. Hence, it can be optimized by utilizing enough measurements placed strategically throughout the system. Therefore, determination of appropriate monitor locations with minimum numbers of them is a critical problem, as it is clearly related to the efficiency of the scheme. In fact, there is a tradeoff between the monitoring capability and cost efficiency, the proper number of monitors and their locations must be optimally determined [2–5]. The complete state of the power system (voltages and currents of the entire system) can be estimated from a relatively small number of synchronized, partial, and asymmetric measurements of phasor voltage and current at the selected bus bar. Power system state estimation (SE) refers to the collection of a redundant set of measurements from around the system and computing a state vector of the voltage at each observed bus. Estimation of the network state has been heavily used in industry since the 1960s [6].

A mathematical relation between system state variables and actual measurements plays the key role on state estimation [7]. Generally, state estimation problems are solved by using least squares (LS) methods [6,8–10]. In fact, the solution based on LS must be obtained through an iterative algorithm. In general, state estimation process consists of data acquisition, network topology,

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Khalil Gorgani FIROUZJAH (✉), Abdolreza SHEIKHOLESAMI, Taghi BARFOROUSHI  
Faculty of Electrical and Computer Engineering, Babol (Noshirvani) University of Technology, Babol, Iran  
E-mail: kgorgani@stu.nit.ac.ir, khalilgorgani@gmail.com

observability analysis, estimation of the state vector, and detection of bad measurement data. State estimation is a tool that is widely used in the network control centers to improve the quality of direct metered data, to provide a way for immediate monitoring of network conditions, which are not available directly to the power system dispatcher through telemetry, and to provide the best estimation of the network model that can be used as a starting point for power system application. In general, the models and estimation method shall be chosen adequately to get plenty of accuracy.

Many technologies have come along since the inception of state estimation that have improved its performance and drove it to become an integral part of control center operations. Today, PMU technology will serve as the next step in improving the quality of the observability of the system, providing operators with better information to maintain a high level of system reliability.

According to the mentioned before, the basis of the spread monitoring system is based on evaluating a few numbers of the voltages and currents in some busses and estimating the parameters of the other busses. Determining the number and the locations of monitors is known as an observability analysis. Observability analysis can be performed by well documented procedures, which are based on topological or numerical methods [11]. The advantage of this method in solving the observability problem is the low required input data. It needs only the existence matrix as an input, and it can get from this matrix all the required data in order to solve the problem. According to Eldery et al. [11], the observability constraints guarantee that all the state variables are measured or calculated by at least one monitor, and the constraints are obtained from Kirchhoff's current law (KCL) and Ohm's law (OL), resulting in two groups of constraints.

Related to the observability of the system, another benefit of using PMUs is their ability to directly measure the state of the network. A PMU placed at a given bus can provide the voltage phasor at the bus and the phasor currents of several or all lines incidents to that bus [12–14]. Therefore, given the widespread availability of PMUs in a power system, the state of the system can be directly obtained at a higher rate without estimation. It is noted such an approach also carries the risk of bad data.

To realize the system observability with considering the minimum cost, this paper focuses on guaranteed network observability, removing of critical measurements, robustness of the measurement system against contingencies and bad data detection. The cost of the distributed measurement method can be reduced by applying optimization techniques in order to keep the cost within an acceptable range.

The goal of the proposed method is to minimize the cost of the monitoring system. In this way, the role of utilizing the optimization algorithms is undeniable. Hence, the

topological observability of the system with any possible measurement set is evaluated to minimize the cost of monitoring achieve focused objective using genetic algorithm (GA) optimization [15]. Particle swarm optimization (PSO) is another evolutionary optimization method, which was inspired by the social behavior of bird flocking and fish schooling [16]. PSO has its roots in artificial life, as well as in engineering. To take advantage of this algorithm, a binary maintained PSO is utilized in this paper. The proposed technique consists in a robust cooperation of PSO and PMU placement term. Thus, the paper consists of two main sections:

- 1) Observability assessment during normal and abnormal conditions (Section 3);
- 2) Optimal searches of monitoring placements regarding to the observability constraints (Section 4).

The proposed optimal measurement placement algorithm is applied to the IEEE 14-bus and 30-bus test systems and Iranian (Mazandaran) Regional Electric Company 24-bus (MREC 24-bus) in Section 5. Finally, concluding remarks are made in Section 6.

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## 2 Objective functions in optimum monitors allocation

The optimum monitor allocation model determines the points where measurement devices should be installed, in order to maximize the monitored area over the power system that is being studied. A simple alternative to place measurement devices would be through considering minimum monitors; these should be installed in busses that would allow this equipment to observe the large number of busses and lines of the power system. This concept is established based on determination of the least number of meters required to monitor the whole power network with the lowest feasible redundancy. According to this concept, if each bus is monitored by at least one meter, no one monitor will be required. In other words, observability analysis is the evaluation of the ability of a given monitoring system to make the whole network observable through minimum measurement devices. In addition to this limitation, another objective of installing measurement devices is to improve the accuracy and reliability of the system state estimation.

Reliability limitation is to identify bad data in a given measurement system by making sure that all single bad data are detectable. Furthermore, the monitoring system must be maintained by a certain level of reliability against the branch outages. Single line outage leads to the outage of two or some bus connections. This changes system observability and state estimation. Thus, in order to reduce observability risk accompanied by the low cost, these constraints must be applied to the monitoring system as objective function.

### 3 Observability of system

Observability of any system deals with two system analysis viewpoints. These viewpoints can be categorized in two manners.

- 1) The system analysis under normal conditions;
- 2) The system analysis under abnormal conditions.

During normal conditions, the system is assumed without any fault, branch outage, and measurement current error; and vice versa, abnormal condition deals with these problems.

#### 3.1 Observability under normal condition

Observability of the system depends on the state equations dominated on it. These equations are based on the voltage and current considerations of the system. Kirchhoff's voltage and current laws are applicable on any electrical circuit analysis. Due to these concepts, if the voltage at one bus of the line and the current through it are observable, then the voltage at the other bus is observable. Furthermore, if the voltages across the line are observable, then the current through this line is observable [11]. By using the Kirchhoff's laws, graph theory, and line-bus incidence matrix, the observability analyzer algorithm can be performed. The algorithm should be able to evaluate any monitoring system with arbitrary placed monitors. Consideration of a given measurement equipment at a bus is performed base with a bit string with length of system busses. This string is called as measurement bit string. The role of observability analyzer algorithm is just the calculation of unmonitored bus data utilizing known and monitored busses data. Any suggested monitoring system is coded by the bit string where 1 means presence, and 0 means absence of the measurement device at a bus related to each bit.

It should be mentioned that placement of a given measurement equipment at a bus, the voltage of that bus, and currents of all connected branches will be monitored. According to the mentioned above, the observability analyzer algorithm is established as Fig. 1. This algorithm is constructed as following stages.

**Stage 1** The monitored busses (each column in measurement bit string set to 1) should be sorted in descending order based on maximum connectivity. According to this, among the suggested busses in measurement bit string, the bus with the maximum connected line will be selected as the first priority for monitoring allocation.

**Stage 2** According to Kirchhoff's voltage and current laws, the voltage of busses connected to the first monitored bus can be calculated using its voltage and related line currents.

**Stage 3** During Stages 1 and 2, some bus voltages and line currents are known. Thus, the network can be checked for possible busses with known voltage and unknown

current through the line connected them. As mentioned laws, the current through the line connected among two observed busses can be observable.

**Stage 4** By considering known currents during previous stages, the network can be checked for possible busses consisting of single unknown current. In other words, the busses checking is performed based on the capability of applying Kirchhoff's current law.

**Stage 5** According to the new line currents calculated in Stage 4, the unknown voltages of busses connected to these lines could be calculated.

**Stage 6** Stages 3 through 5 must be repeated until any new current and voltage are calculated.

**Stage 7** During Stage 6, if no more change in observability is found, the next suggested bus (according to the next monitoring bus in sorted measurement bit string) is considered. Afterwards, Stages 3 through 6 are accomplished respectively. It should be mentioned that all suggested busses for monitoring must be applied to observability assessment in a similar manner.

Considering plausible data redundancy is an indispensable point during these stages. The numbers of redundant data are the numbers of voltages and currents calculated on former stages and monitored through later stages. As mentioned above, observability assessment is constructed to check the capability of a given monitoring system (measurement bit string) to make the whole system observable. Here, the numbers of adjusted measurement devices and data redundancy are assigned as the fitness of a given measurement bit string on system observability.

#### 3.2 Observability under abnormal condition

System observability considers not only the capability of a given monitoring to monitor the whole system utilizing minimum measurement devices, but also their reliability. The reliability issue that is previously mentioned can be explained in cases which some plausible inconsistencies in measured data are possible. Detection and identification capability of these inconsistencies and removing of them could improve the reliability of a given monitoring scheme. Therefore, measurements and its precisions must be checked.

Observability can be affected by plausible system configuration changes. These changes that are mainly due to loss of transmission equipment could be called as contingency event. In these cases, a contingency analysis must be applied to reliability assessment for system observability checking. Contingency analysis involves the analysis of possible contingencies that are likely to occur to detect the potentially harmful ones, i.e., contingencies that, if they occurred, would lead the system into a state of emergency. The single line outage is considered as must occurrence event.

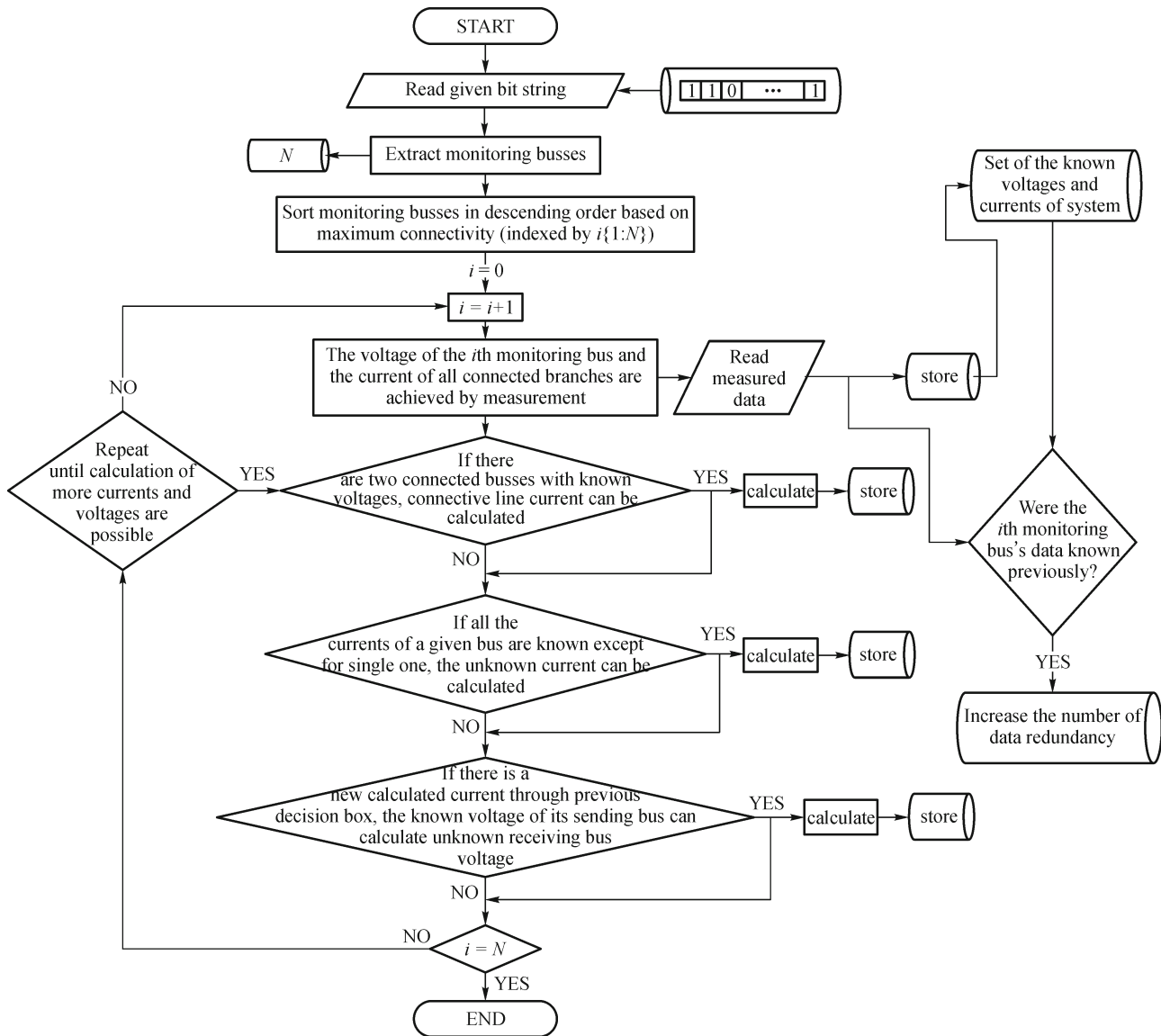


Fig. 1 Observability analyzer algorithm

### 3.2.1 Possibility of single line outage

Problems associated with contingencies have recently received greater attention. Single-line outages are more probable than double or multiple outages. Nevertheless, if the first outage is one of the critical lines, then subsequent outages are more likely. The observability analyzer algorithm is established based on line-bus incidence matrix. Thus, any fault that leads to line outage can change the incidence matrix relative to faulted section. Line-bus incidence matrix organizes the KVL and KCL to calculate unmonitored bus and its data. Hence, some changes in this matrix may leads to unobservable busses or lines.

As mentioned above, the system that is observed with a given monitoring scheme under normal condition, it must

still be observable even after every line outage. In fact, the system observability must be independent of the topological changing of the system under faults.

To satisfy this concept, the observability analyzer algorithm is compatible with the single line outages. In this way, every line assumed to be faulted in every analysis. It should be mentioned that faulted line are removed by breakers. Hence, several new networks derived from removing of single branch in the normal network are performed. The number of these new networks is the number of system branches.

According to the suggested algorithm in Fig. 2, for a given measurement bit string, several new networks are checked for observability. The number of unobservable (new) networks explains the number of incapability of the studied monitoring scheme under fault and single branch

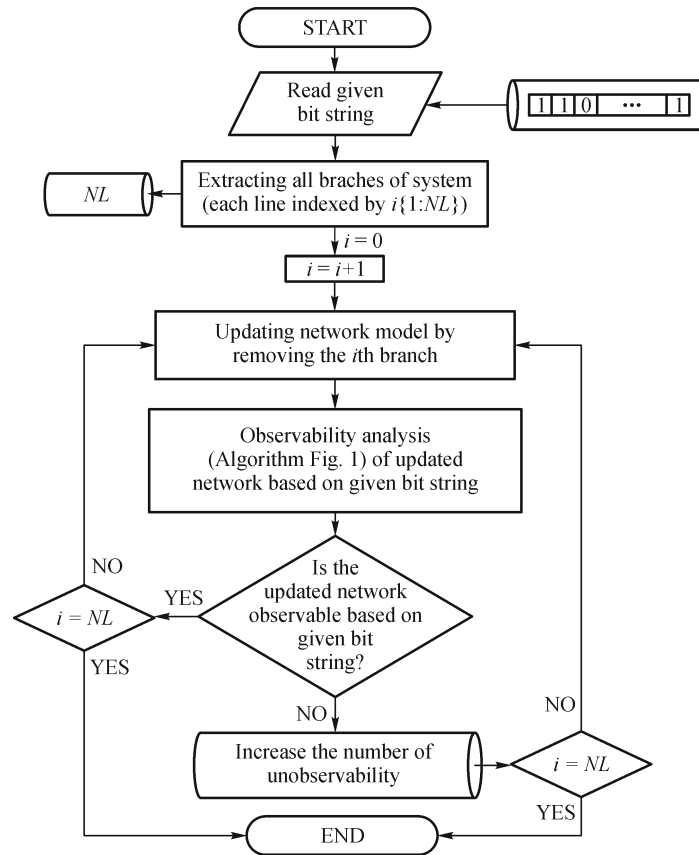


Fig. 2 Observability analyzer algorithm under single line outage

outage. The fitness function of this constraint is the reverse of the number of unobservability.

### 3.2.2 Possibility of single current error

Bad data detection is a main ability of any monitoring system. This concept plays a key role on conventional and phasor measurements because the existence of an error on measured data will have a significant impact on the calculated data for the whole system. The ability of error detection requires enough redundant measurement. The higher levels of redundancy will improve this function to act on the system with higher measurement error since the observability analyzer algorithm is improved by bad data detector function.

As the illustrated algorithm in Fig. 3, an arbitrary error is added to the measured current on a single branch. Thus, the given measurement bit string is loaded as initial string. Each possible current measurement assigned by measurement string supposed to be disturbed. Therefore, several independent observability analyses (with the number of possible current measurements) are performed, in which just single current error is adjusted. In other words, related to every suggested current measurement, an independent analysis is carried out. Each analysis involves single

current error.

Detection of single branch current error could be performed in a simple way. If the Kirchoff's current law at every monitoring bus is not satisfied, the current measurement of that bus is not true. However, the uncorrected current and related branch is undetectable. Thus, in order to identify the bad measured current, the current of every monitored branch must be achieved through another way. The current is free of error, if the calculation of it equals with the measured amount. The whole system observability is satisfied if all possible errors in measured current are detectable. When the system is observable under any possible current error, the number of adjusted measured current and measurement redundancy is assigned as the fitness of a given measurement bit string.

### 3.3 Observability assessment

Observability assessment of a given monitoring scheme can be achieved by consideration of the above concepts in a multi-objective manner. Observability under normal condition of any system through a given monitoring scheme plays a key role in accepting or rejecting that scheme. A given monitoring scheme can be studied from a reliability point of view, if it is able to monitor the whole

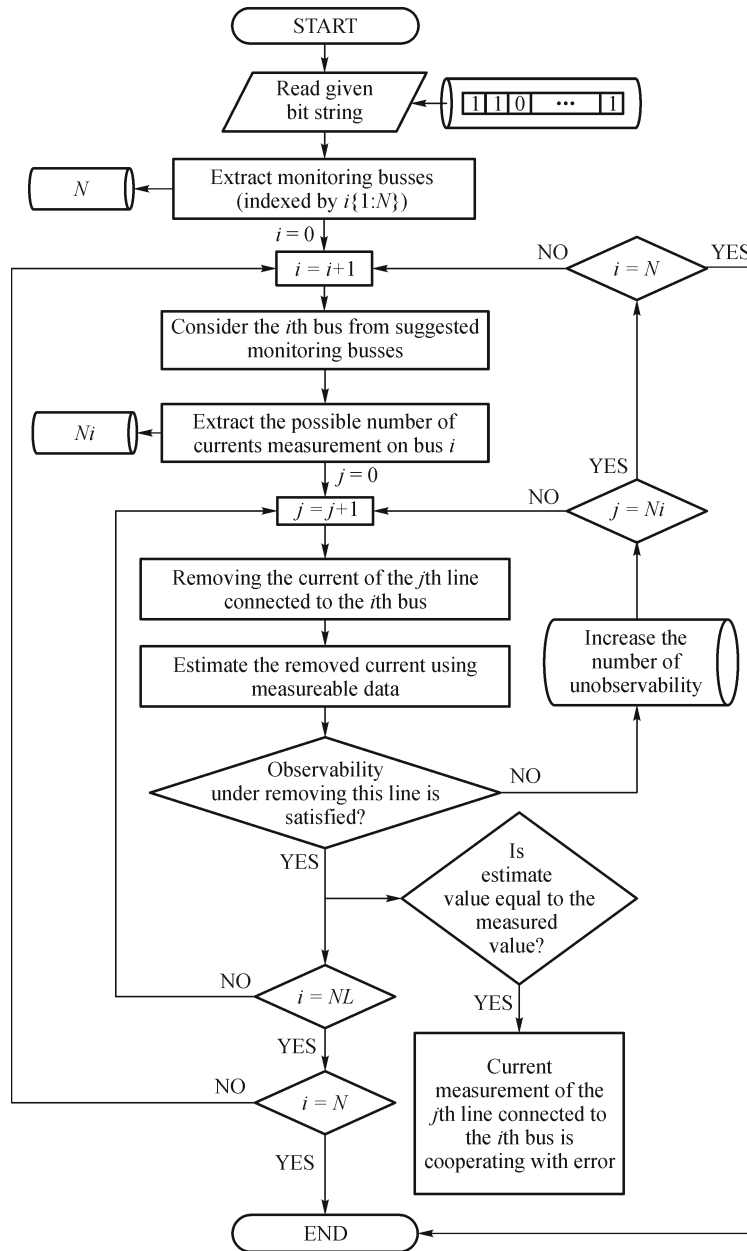


Fig. 3 Observability analyzer algorithm under single current error

system in normal condition. Therefore, observability under normal condition is a hard constraint. A given monitoring scheme can still be kept as the best monitoring allocation, when the observability under abnormal condition is satisfied with the lower data measurement and redundancy. The inference algorithm is shown in Fig. 4.

#### 4 Optimization technique

##### • Particle swarm optimization (PSO)

The PSO algorithm is a multi-agent parallel search technique which maintains a swarm of particles and each

particle represents a potential solution in the swarm. All particles fly through a multi-dimensional search space where each particle is adjusting its position according to its own experience and that of the neighbors. The main principle behind the enhanced PSO is that, when an individual particle is not able to find a feasible solution, it should use the knowledge of the feasible solution, if any, found by some other particles. When none of the particles have found a feasible solution, a random search enhances the possibility of quickly finding a feasible solution.

Suppose  $x_i^k$  denotes the position vector of the  $i$ th particle in the multi-dimensional search space at iteration  $k$ , then the position of each particle is updated in the search space

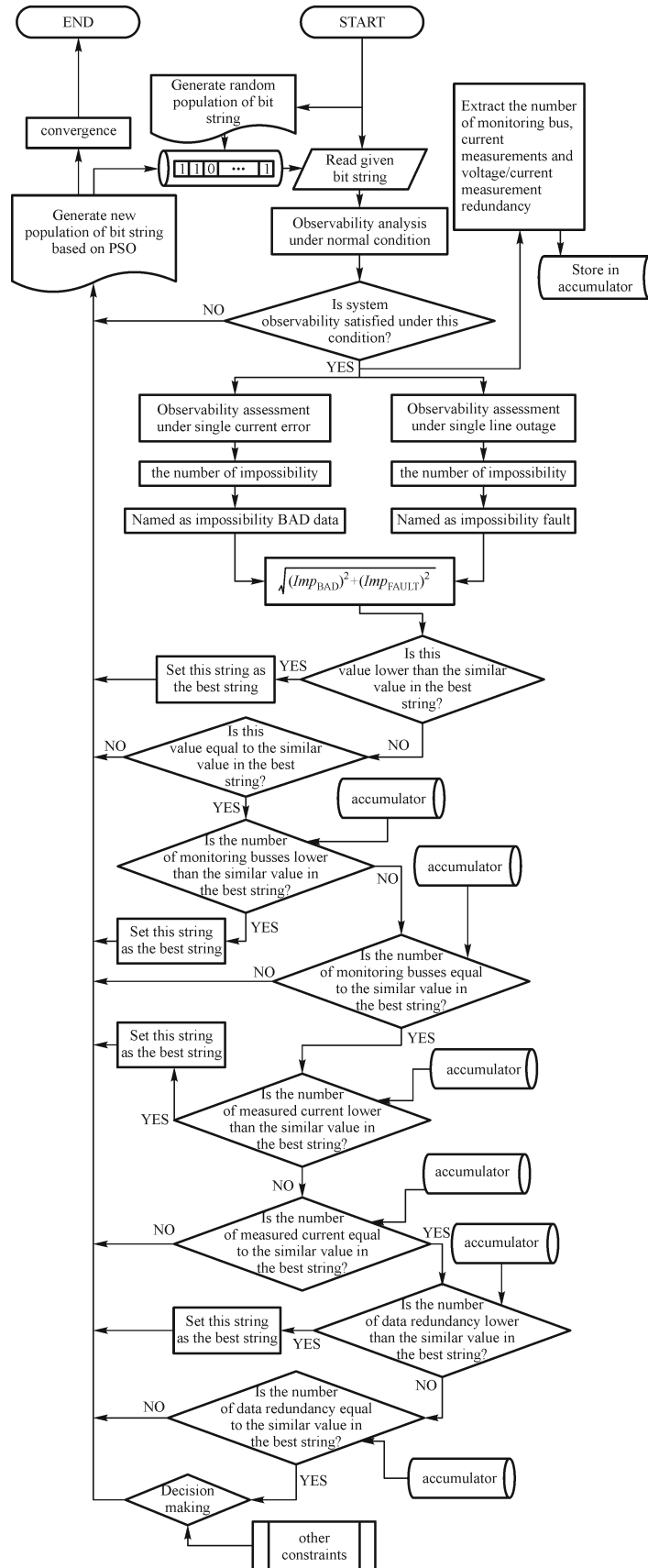


Fig. 4 Observability assessment under normal and abnormal conditions

by

$$x_i^{k+1} = x_i^k + v_i^{k+1} \text{ with } x_i^0 \sim U(x_{\min}, x_{\max}),$$

where  $v_i^k$  is the velocity vector of particle that drives the optimization process and reflects both the own experience knowledge and the social experience knowledge from all the particles;  $U(x_{\min}, x_{\max})$  is the uniform distribution where  $x_{\min}$  and  $x_{\max}$  are its minimum and maximum values, respectively.

Therefore, in a PSO method, all particles are initiated randomly and evaluated to compute fitness together with finding the personal best (best value of each particle) and global best (best value of particle in the entire swarm). After that, a loop starts to find an optimum solution. In the loop, the velocity of particles is updated by the personal and global bests, and then each particle's position is updated by the current velocity. The loop ends with a stopping criterion predetermined in advance [17]. Basically, two PSO algorithms, namely, the global best ( $g_{\text{best}}$ ) and local best ( $l_{\text{best}}$ ) PSO, have been developed which differ in the size of their neighborhoods.

On being the global best PSO (or  $g_{\text{best}}$  PSO), it is a method where the position of each particle is influenced by the best-fit particle in the entire swarm. The other is the local best PSO (or  $l_{\text{best}}$  PSO) method that only allows each particle to be influenced by the best-fit particle chosen from its neighborhood, and it reflects a ring social topology. There are two differences between the  $g_{\text{best}}$  PSO and the  $l_{\text{best}}$  PSO: One is that because of the larger particle interconnectivity of the  $g_{\text{best}}$  PSO, sometimes it converges faster than the  $l_{\text{best}}$  PSO. Another is due to the large diversity of the  $l_{\text{best}}$  PSO, it is less susceptible to being trapped in local minima [18].

The personal best position  $P_{\text{best},i}$  corresponds to the position in search space where particle had the smallest value as determined by the objective function  $f$ , considering a minimization problem. In addition, the position yielding the lowest value among all the personal best  $P_{\text{best},i}$  is called the global best position which is denoted by  $G_{\text{best}}$ .

For  $g_{\text{best}}$  PSO method, the velocity of particle  $i$  at iteration  $k+1$  is calculated by

$$v_{ij}^{k+1} = wv_{ij}^k + c_1r_{1j}^k[P_{\text{best},i}^k - x_{ij}^k] + c_2r_{2j}^k[G_{\text{best}}^k - x_{ij}^k],$$

where  $v_{ij}^k$  is the velocity vector of particle  $i$  in dimension  $j$  at iteration  $k$ ;  $x_{ij}^k$  is the position vector of particle  $i$  in dimension  $j$  at iteration  $k$ ;  $P_{\text{best},i}^k$  is the personal best position of particle  $i$  in dimension  $j$  found from initialization through iteration  $k$ ;  $G_{\text{best}}^k$  is the global best position of particle  $i$  in dimension  $j$  found from initialization through iteration  $k$ ;  $c_1$  and  $c_2$  are positive acceleration constants which are used to level the contribution of the cognitive and social components, respectively;  $r_{1j}^k$  and  $r_{2j}^k$  are random numbers from uniform distribution  $U(0,1)$  at iteration  $k$ ;  $w$  is the inertia weight that considers the influence of the

previous velocities in the process.

The constant  $c_1$  expresses how much confidence a particle has in itself, while  $c_2$  expresses how much confidence a particle has in its neighbors [16]. There are some properties of  $c_1$  and  $c_2$ . When  $c_1 = c_2$ , all particles are attracted toward the average of  $P_{\text{best},i}^k$  and  $G_{\text{best}}^k$ . From the different empirical researches, it has been proposed that the two acceleration constants should be  $c_1 = c_2 = 2$ .

In the beginning, PSO algorithm was developed for continuous-valued search spaces and most of its modified versions worked in the continuous space, which could not be used to optimize for a discrete-valued search spaces. In 1997, Kennedy and Eberhart first extended the basic PSO algorithm to the discrete space to solve this problem [16]. They developed the PSO to operate in the binary search spaces, because real-valued domains can be transformed into the binary-valued domains. The proposed algorithm is called binary PSO (BPSO) algorithm where the particles represent position in binary space and particle's position vectors can take on the binary value 0 or 1, i.e.,  $x_{ij} \in \{0,1\}$ .

In BPSO, a particle's velocity  $v_{ij}^k$  is connected to the possibility that the particle's position  $x_{ij}^k$  takes a value of 0 or 1. The update equation for the velocity does not change from that used in the original PSO.

Now, the  $j$ th bit of the  $i$ th particle,  $x_{ij}^k$ , is updated by

$$x_{ij}^k = \begin{cases} 1, & \text{if } u_{ij}^k < s_{ij}^k; \\ 0, & \text{if } u_{ij}^k \geq s_{ij}^k, \end{cases}$$

where  $u_{ij}^k$  is a random number selected from a uniform distribution in  $(0,1)$ , and  $s_{ij}^k$  is the sigmoid function, denoted by

$$s_{ij}^k = \frac{1}{1 + e^{-u_{ij}^{k+1}}}.$$

To control the balance between global and local exploration, to obtain quick convergence, and to reach an optimum, the inertia weight whose value decreases linearly with the iteration number is set according to the following equation:

$$w^{k+1} = w_{\max} - \frac{(w_{\max} - w_{\min})}{k_{\max}}k,$$

where  $w_{\max}$  and  $w_{\min}$  are the initial and final values of the inertia weight, respectively;  $k_{\max}$  is the maximum iteration number;  $k$  is the current iteration number. Commonly, the inertia weight decreases linearly from 0.9 to 0.4 over the entire run.

According to the mentioned above, the position vector of particle  $i$  in dimension  $j$  at the next iteration can be achieved by

$$x_{ij}^{k+1} = x_{ij}^k + v_{ij}^{k+1}.$$

## 5 Optimal measuring system placement

Optimization algorithms for monitoring system location must be performed, first, by a random bit string. This bit string assigns the distributed voltage and current sensors at the network busses randomly. The next step is established based on observability assessment of random generated measurement bit string in system monitoring. The illustrated algorithms in Figs. 1–4 are used to calculate the ability of suggested location of measurement devices. The ability of monitoring system is evaluated by the number of current and voltage measurement and data redundancy. It should be mentioned that when a monitoring scheme is not able to monitor the system entirely, its data redundancy and number of measurement are not considered into optimization. In fact, observability under normal condition plays a key role in the selection of a given monitoring scheme as a favorite scheme.

### 5.1 Network test cases

Bus/branch network models are most commonly used in the state estimation and power flow studies. The algorithms in this dissertation have been tested with IEEE 14 and 30-bus standard test systems that can be found in <http://www.ee.washington.edu/research/pstca/> (Data for the IEEE 14, 30, 57, 118, 300-bus test system. University of Washington). In addition to these networks, the proposed algorithm in measurement system allocation is applied to the Mazandaran Regional Electric Company of Mazandaran province in Iran. This network consists of 24 busses with two voltage levels: 230 and 400 kV. This network is illustrated in Fig. 5. Furthermore, related line-bus

incidence matrix is carried out in Table 1. Diagonal component of this matrix (component  $A_{ii}$ ) is performed by the number of loads and generators of each bus  $i$ . According to the IEEE 14-bus, IEEE 30-bus, and Mazandaran Regional Electric Company 24-bus (MREC 24-bus), some parameters can be extracted as noted in Table 2. These parameters consist of the number of busses, number of branches, number of busses with zero injection current, and the number of injection currents in excess of 2 at each bus.

For example, in MREC 24-bus network, the busses 3, 4, and 6 are busses with zero injection current. There are no load and generator on these busses. Single current of these busses is relative to other currents of them. In other words, there is a linear relation between the current variables in these busses. The number of injection currents in excess of 2 at bus 1 is three. In addition, this number of bus 2 is one. The calculation of this number at bus  $i$  can be done equal to diagonal component of line\_bus incidence matrix ( $A_{ii}$ ) – 1.

Consequently, the number of independent state variable can be achieved:

- number of bus voltage
- + number of injection currents in excess of 2 at each bus
- number of busses with zero injection current.

In addition to these variables, degree of redundancy is usually expressed in terms of the ratio number of meters to the number of states. The redundancy ratio is a very important quantity; more redundant measurements give more chances for bad data to be detected.

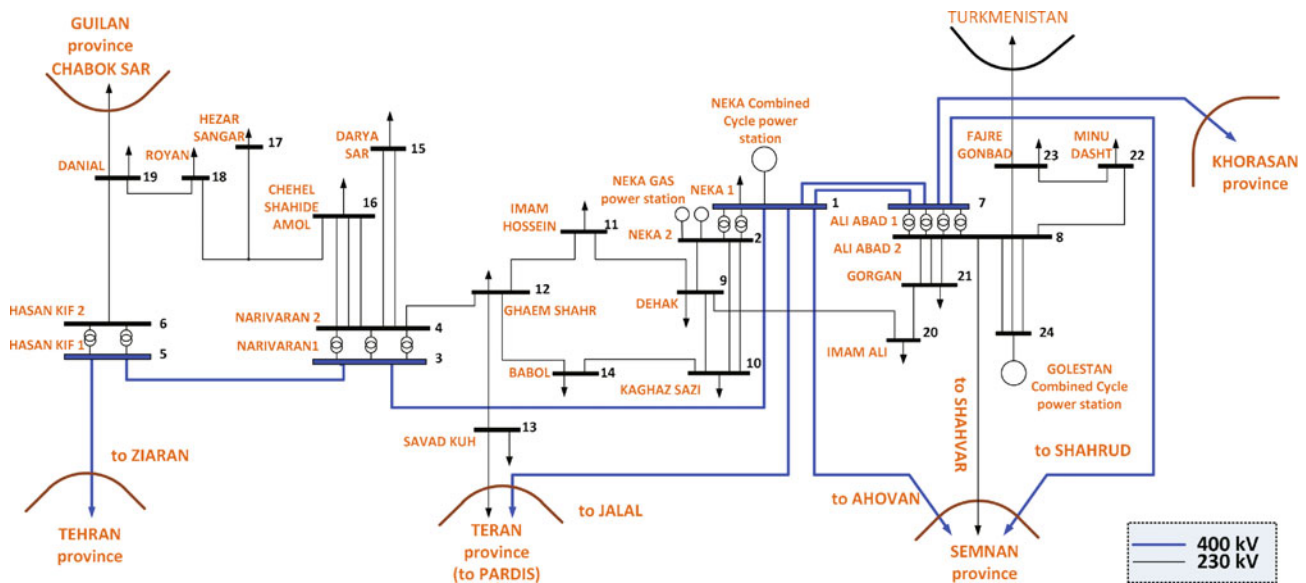


Fig. 5 Mazandaran Regional Electric Company 24-bus (MREC 24-bus)

**Table 1** Line-bus incidence matrix of MREC 24-bus transmission network

bus number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	4	2	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	2	2	0	0	0	0	0	0	2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	3	0	0	0	0	0	0	0	0	1	0	0	2	3	0	0	0	0	0	0	0	0
5	0	0	1	0	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
7	2	0	0	0	0	0	2	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	4	1	0	0	0	0	0	0	0	0	0	0	0	0	3	1	1	3
9	0	2	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
10	0	2	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	1	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0
15	0	0	0	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
16	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
18	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	2	0	0	0	0	0	0
19	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
20	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
21	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
22	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
23	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0
24	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

**Table 2** Number of state variables in the test systems

No. of state variables	No. of busses with zero injection current	No. of injection currents in excess of 2 at busses	No. of busses	test system
$14 + 4 - 1 = 17$	1	4	14	IEEE 14-bus
$30 + 6 - 8 = 28$	8	6	30	IEEE 30-bus
$24 + 9 - 3 = 30$	3	9	24	MREC 24-bus

## 5.2 Optimization result

### 5.2.1 Optimization under normal condition

Optimization under normal condition of the power system is performed based on the ability of monitoring scheme in observability. The number of current and voltage measurements and their redundancy are considered as optimization constraint. Monitoring cost consists of the number of monitoring busses, communication equipments, and the number of employed CT and CVT in measurement. Thus, monitoring scheme is optimized to reduce the number of measurement devices.

The main conclusions of this concept, performed on the IEEE 14-bus test systems, IEEE 30-bus test systems, and MREC 24-bus transmission network, will be summarized next.

Optimization process using PSO is done in 1000 iterations. However, in order to illustrate the result clearly, it has bent focused at most 50 or 150 illustrate iterations.

#### • IEEE 14-bus test system

In this case, optimal allocation is assigned to busses 2/3/6/9. It should be mentioned that existence of load and synchronous condenser in bus 3 enforces a separated measurement device to monitor their currents. However, some researchers consider these currents as a unit

measurable current and state variable. Therefore, optimum monitoring location is reduced into three busses: 2/6/9. The results of these optimizations are illustrated in Fig. 6. The number of current measurements in suggesting monitoring schemes and the number of redundant voltage and current measurements are shown in these figures. The redundancy ratio is 1.58.

- **IEEE 30-bus test system**

Assigned monitoring busses on this system are consisting of busses 2/5/10/12/19/24/30. The redundancy ratio is 1.60. The optimization process is shown in Fig. 6.

- **MREC 24-bus system**

Unlike IEEE 14 and 30-bus test systems, MREC 24-bus transmission network involves higher independent state variables. In spite of lower busses of MREC's network, it involves higher interconnections. Also, the number of busses with injection currents in excess of 2 is higher in MREC's network. The optimum number of selected busses as monitoring bus is equal to 11 (busses: 1/2/4/7/12/13/18/19/20/23/24). The redundancy ratio is 2.40. The redundancy ratio of this network in comparison with IEEE test systems has larger amount. In fact, interconnections of MREC's network are higher than IEEE test systems'. The optimization process is shown in Fig. 6.

The total result of the observability assessment under normal condition is summarized in Table 3.

### 5.2.2 Optimization under single line outage

As mentioned above, single line outage can change the observable network into unobservable system. It means that outage of some branches leads to lack of some monitored currents in the estimation process. Thus, other variables related to lose current will be unmonitored. Optimization under this condition is performed based on the ability of monitoring scheme on observability (in lack of each measured current). In fact, outage of some lines may lead to unobservable condition of the whole system. This means that a given monitoring scheme has some critical current measurements, which impress the estimation process in the rest of the system. Therefore, in addition to the previous constraint of Section 5.1, the number of impossible observation under single line outage is considered to be minimized.

- **IEEE 14-bus test system**

In this case, optimal allocation is assigned to busses 2/3/6/8/9/11/13. The results of these optimizations are illustrated in Fig. 7. The redundancy ratio is 2.29. In comparison with normal condition, this amount is increased (changes from 1.58 to 2.29).

In spite of increasing of the number of monitoring busses and measured variables, observability is still kept under any possible single line outage.

- **IEEE 30-bus test system**

Applying the suggested algorithms, result of optimal

placements for measurement systems are at busses 2/3/5/10/11/13/15/16/19/24/26/28/29. Optimization process using PSO algorithm is shown in Fig. 7. The redundancy ratio is 2.35. In comparison with normal condition, this amount is increased (changes from 1.60 to 2.35).

- **MREC 24-bus system**

Similar to the pervious test systems, a same procedure is applied to an optimum allocation measurement system in MREC 24-bus transmission network under single line outage. The outage is possible for each branch. As obtained results, suggested monitoring busses are 1/2/5/7/8/11/13/14/15/16/18/19/20/23. The redundancy ratio is 2.90. Similar to the previous section, the redundancy ratio of this network (in this case) in comparison with IEEE test systems has larger amount. The optimization process is shown in Fig. 7.

The total result of observability assessment under this condition is summarized in Table 3.

### 5.2.3 Optimization under single current error

The performance of the distributed placed monitoring scheme depends on the accuracy of the received measurement data. The measured data are subject to noise or errors in the metering system or communication process. With proper redundancy in the measurement configuration the state estimator can perform the bad data processing to detect, identify, and eliminate the bad measurements in the measurement set. The optimization process is carried out by evaluating several analyses (with a number of suggested current measurements by a given monitoring scheme). Thus, each analysis is performed by assigning a random error to just single measured current. Hence, related to the number of suggested current measurements, these analyses are checked for the number of probable unobservable conditions. The number of unobservable conditions relates to the number of cases that a given single line current error could not be detected and corrected. Similar to the previous section, this number called as impossible observation under single current error. Consequently, in addition to the previous constraint of Section 5.1, the number of impossible observation under single current error is considered to be minimized.

- **IEEE 14-bus test system**

In this case, optimal allocation is assigned to seven busses consisting of busses 2/3/6/8/9/10/13. The results of these optimizations are illustrated in Fig. 8. The redundancy ratio is 2.29. In comparison with normal condition, this amount is increased (changes from 1.58 to 2.29).

In spite of increasing of the number of monitoring busses and measured variables, observability is still kept under any possible single current error.

- **IEEE 30-bus test system**

Applying the suggested algorithms, result of optimal placements for measurement systems are at busses: 2/5/6/

8/10/14/15/16/19/24/27/29. Optimization process using the PSO algorithm in 1000 iterations is done. The results are zoomed in 50 iterations in Fig. 8. The redundancy ratio is 2.50. In comparison with normal condition, this amount is increased (changes from 1.60 to 2.50).

• **MREC 24-bus system**

Similar to the pervious test systems, a same procedure is applied to an optimum allocation measurement system in MREC 24-bus transmission network under single current error. The error is possible for each branch that is suggested by monitoring places. As obtained results, suggested monitoring busses are 1/2/4/7/8/11/13/14/17/18/19/20/23. The redundancy ratio is 2.86. Similar to the previous section, the redundancy ratio of this network (in this case) in comparison with IEEE test systems has larger amount. The optimization process is shown in Fig. 8.

The total result of observability assessment under this condition is summarized in Table 3.

### 5.2.4 Optimization under reliability constraints

In real case, in order to achieve an accurate and reliable monitoring scheme, establishment of the mentioned optimization constraint is required. Hence, optimization must be performed by considering observability under normal and abnormal conditions, simultaneously.

The optimization process is carried out to minimize the number of monitoring busses, measured current, voltage/current measurement redundancy, the number of impossible observation under single line outage and impossible observation under single current error. Convergence of the result is shown in Fig. 9, for IEEE 14-bus test system, IEEE 30-bus test system, and MREC 24-bus transmission network, respectively.

• **IEEE 14-bus test system**

In this case, optimal allocation is assigned to seven busses consisting of busses 2/3/6/8/9/11/13. The results of

these optimizations are illustrated in Fig. 9. Convergence of the result is checked during 1000 iterations and shown in 50 iterations. The redundancy ratio is 2.29. In comparison with normal condition, this amount is increased (changes from 1.58 to 2.29).

• **IEEE 30-bus test system**

In this case, optimum allocation is assigned to 16 busses: 2/3/5/6/8/10/11/13/14/15/16/19/24/26/29/30. Optimization process using the PSO algorithm in 1000 iterations is done. The results are zoomed in 50 iterations in Fig. 9. The redundancy ratio is 2.92. In comparison with normal condition, this amount is increased (changes from 1.60 to 2.92).

• **MREC 24-bus system**

In this case, optimal allocation is assigned to 14 busses: 1/2/5/7/8/11/13/14/15/16/18/19/20/23. The error and outage are possible for each branch that is suggested by monitoring places. The redundancy ratio is 2.90. The optimization process is shown in Fig. 9.

The total result of the observability assessment under normal and abnormal conditions is summarized in Table 3.

## 6 Conclusion

Conventional measurements considered measurement devices for the whole system. The cost and complexity of these methods lead to improve distributed monitoring. Observability assessment is the main function in the strategic placement of limited monitors. This paper establishes based on a simple and well known topological method. This method utilizes the allocated busses data to calculate the rest busses states by Kirchhoff's current and voltage laws. Observability assessment during normal condition of the system is introduced in the first part. The second part is established based on a plausible probability of system configuration changes and measured data errors.

**Table 3** Summarized optimization results in measurement system allocation

redundancy ratio	No. of current measurement	No. of voltage measurement	No. of monitoring busses	No. of state variables	No. of network busses	assessment type	test system
1.58	23	4	4	17	14	normal	IEEE 14-bus
2.29	32	7	7	17	14	line outage	
2.29	32	7	7	17	14	current error	
2.29	32	7	7	17	14	reliability	
1.60	38	7	7	28	30	normal	IEEE 30-bus
2.35	53	13	13	28	30	line outage	
2.50	58	12	12	28	30	current error	
2.92	66	16	16	28	30	reliability	
2.40	61	11	11	30	24	normal	MREC 24-bus
2.90	73	14	14	30	24	line outage	
2.86	73	13	13	30	24	current error	
2.90	73	14	14	30	24	reliability	

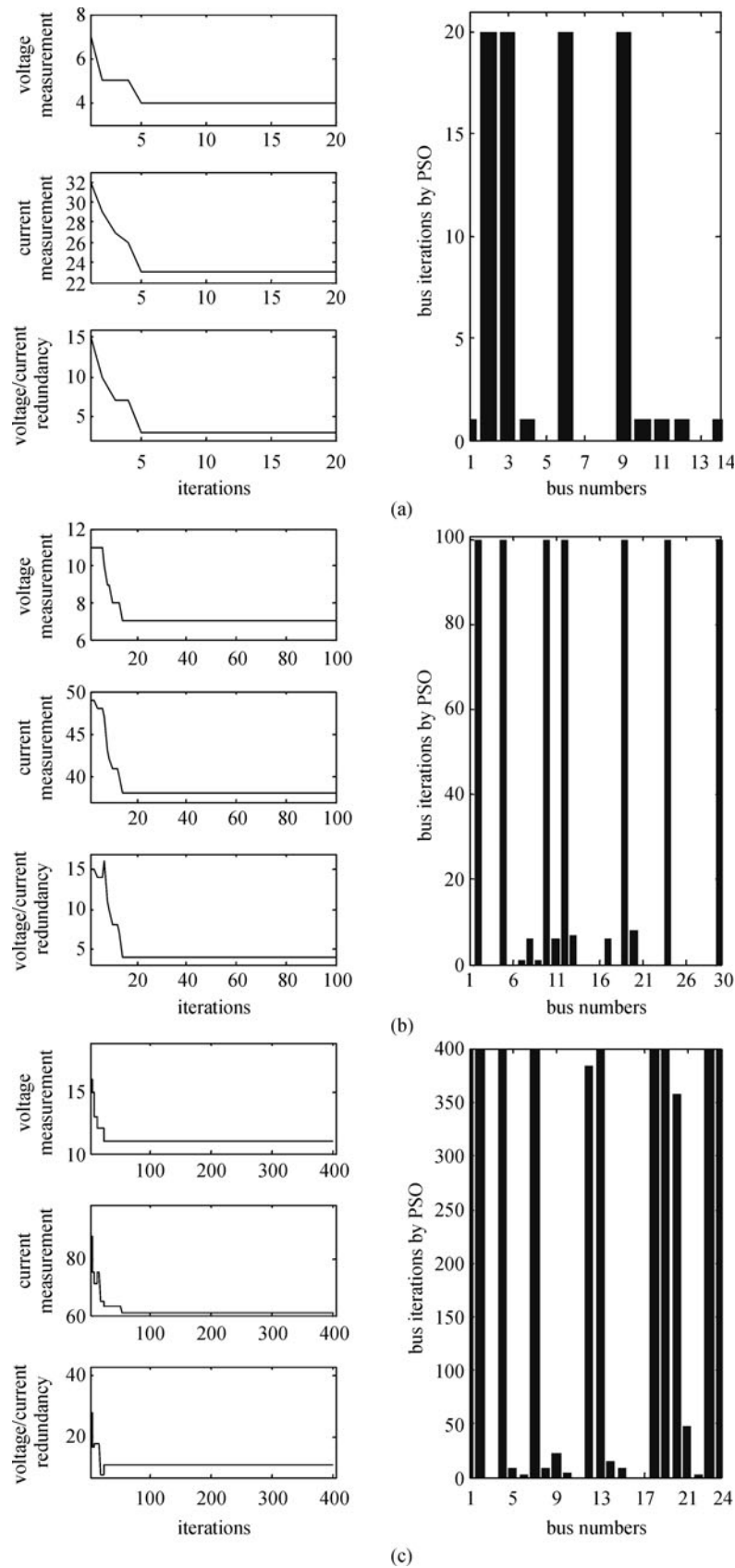


Fig. 6 Optimization under normal condition. (a) IEEE 14-bus test system; (b) IEEE 30-bus test system; (c) MREC 24-bus system

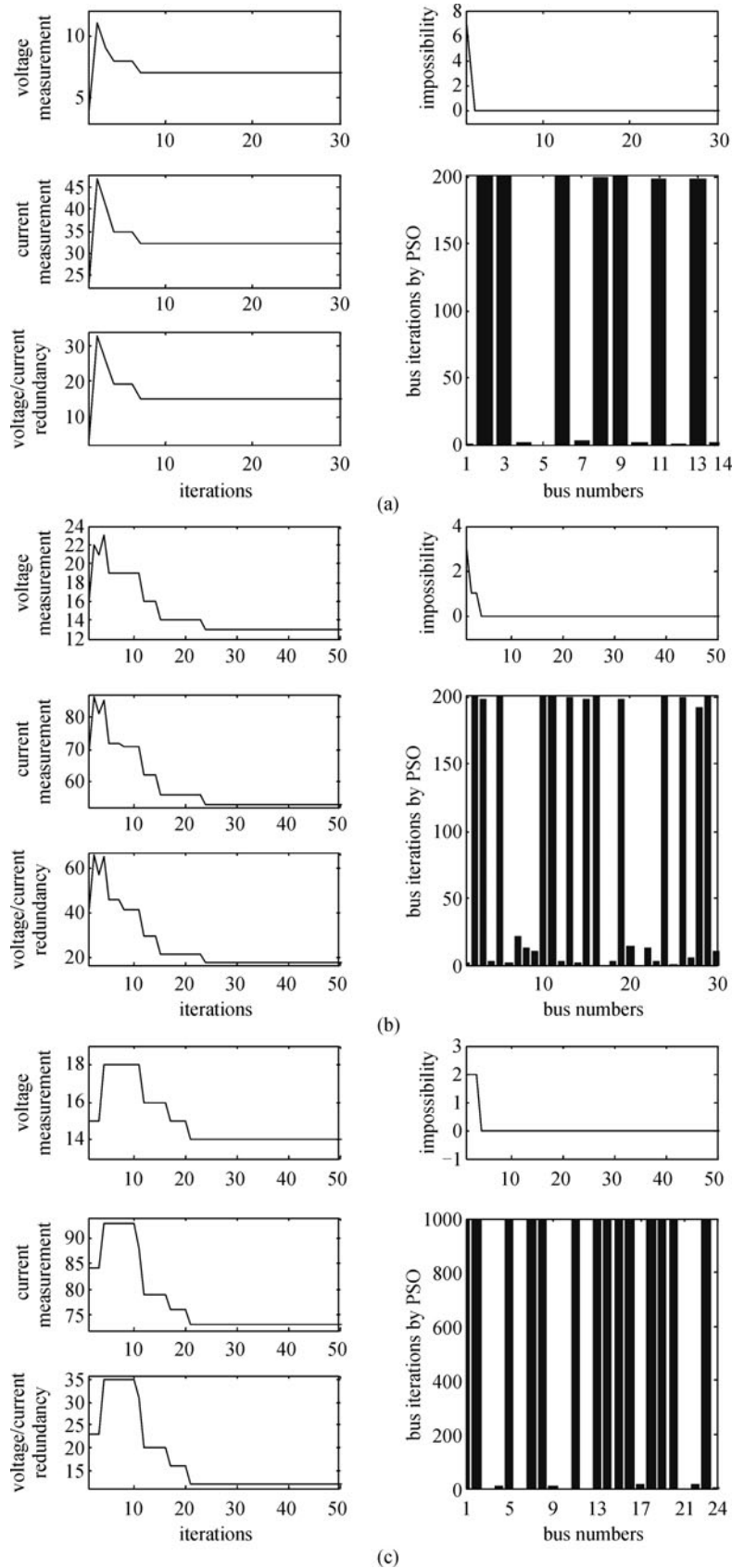


Fig. 7 Optimization under single line outage. (a) IEEE 14-bus test system; (b) IEEE 30-bus test system; (c) MREC 24-bus system

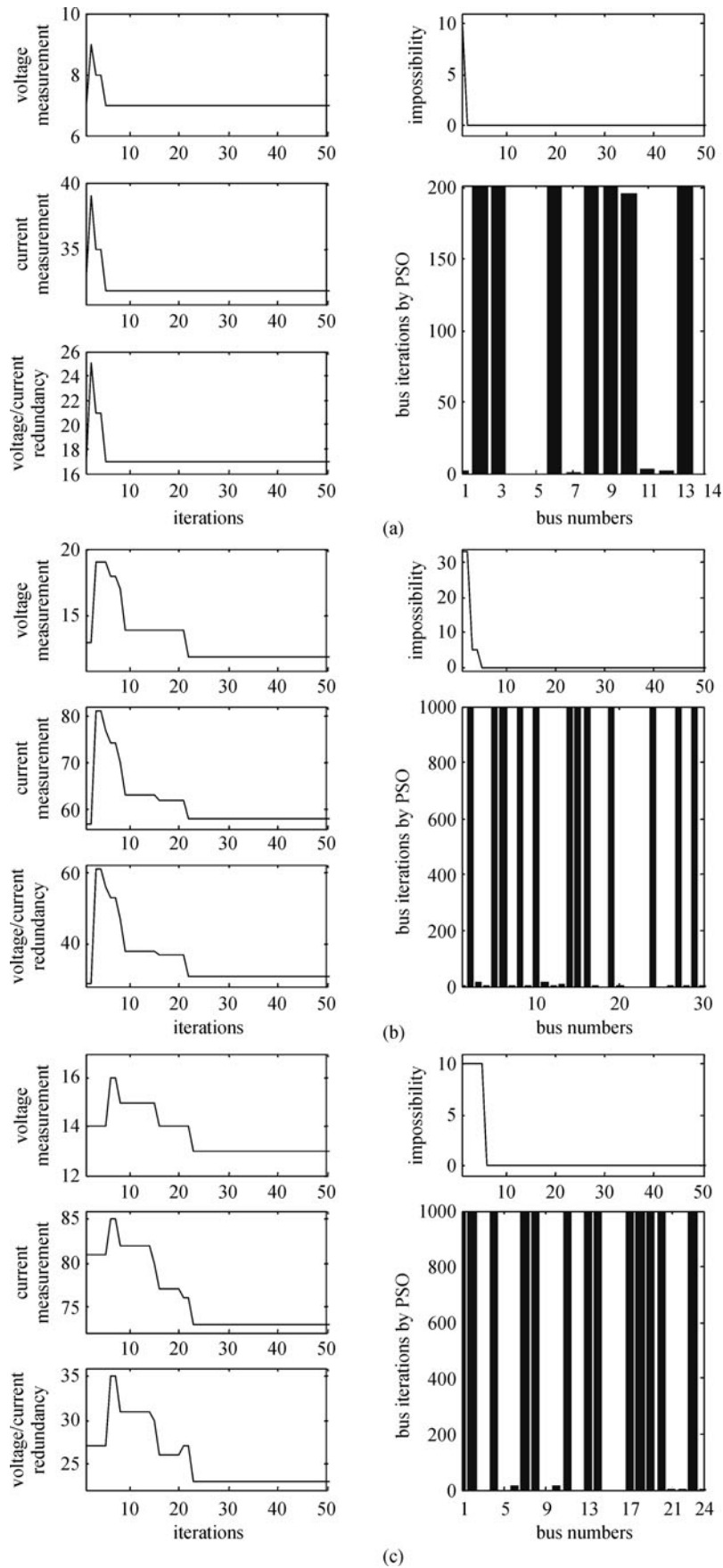
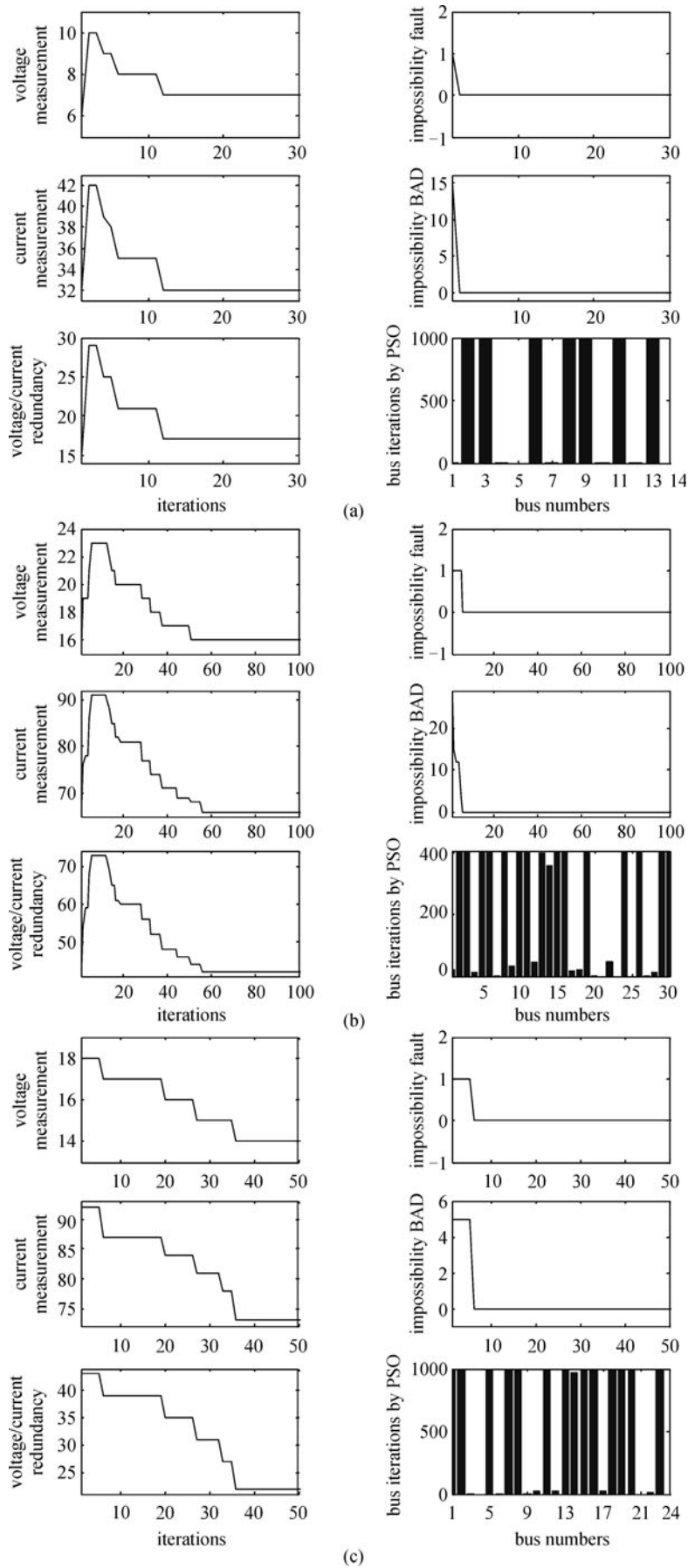


Fig. 8 Optimization under single current error. (a) IEEE 14-bus test system; (b) IEEE 30-bus test system; (c) MREC 24-bus system



**Fig. 9** Optimization under reliability constraints. (a) IEEE 14-bus test system; (b) IEEE 30-bus test system; (c) MREC 24-bus system

Hence, observability is checked in abnormal condition. The results show that if the observability is aimed at abnormal system, some redundant measurement is needed. Thus, the redundancy ratio is increased in keeping observability under any condition. Abnormal condition is modeled by the single line outage due to a random fault and random measurement error. Optimal searching of monitoring placements is carried out by binary particle swarm optimization. The proposed algorithm is tested by three test cases consists of IEEE 14 and 30-bus test systems and Iranian (Mazandaran) Regional Electric Company 24-bus (MREC 24-bus). The allocated measurements in IEEE 14-bus test system during normal and abnormal conditions are in 4 and 7 busses, respectively. Clearly the data redundancy is increased. However, the reliability is complied. A similar process is applied to IEEE 30-bus test system. According to the results, optimal monitoring bus is increased from 7 to 16, during normal to abnormal conditions. In addition, the proposed algorithm is applied to MREC consisting of 400 and 230 kV busses. The result shows that the higher interconnections in the system lead to the higher state variable and data redundancy.

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