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# Time-varying optimal distributed fusion white noise deconvolution estimator

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**Abstract** White noise deconvolution has a wide range of applications including oil seismic exploration, communication, signal processing, and state estimation. Using the Kalman filtering method, the time-varying optimal distributed fusion white noise deconvolution estimator is presented for the multisensor linear discrete time-varying systems. It is derived from the centralized fusion white noise deconvolution estimator so that it is identical to the centralized fuser, i.e., it has the global optimality. It is superior to the existing distributed fusion white noise estimators in the optimality and the complexity of computation. A Monte Carlo simulation for the Bernoulli-Gaussian input white noise shows the effectiveness of the proposed results.

**Keywords** multisensor information fusion, distributed fusion, white noise deconvolution, global optimality, Kalman filtering

## 1 Introduction

White noise deconvolution has important applications in oil seismic exploration [1–3] and occurs in many fields including communication, signal processing, and state estimation [4]. The optimal input white noise estimators based on Kalman filtering were presented in Refs. [1–3]. The general and unified white noise estimation theory based on Kalman filtering was presented in Ref. [5], which includes both the input white noise estimators and the measurement white noise estimators with the results in Refs. [1–3] as a special case. A unified white noise estimation theory based on the modern time series analysis method was presented in Ref. [4].

In order to improve the estimation accuracy based on a single sensor, the multisensor information fusion has received great attention in recent years, which has been widely applied to many fields including guidance, defence, robotics, integrated navigation, target tracking, GPS positioning, and signal processing. For Kalman filtering-based fusion, the basic fusion methods can be classified as the state fusion methods and measurement fusion methods [6,7]. The state fusion methods include the centralized and distributed fusion method. The centralized state fusion method simply combines all local measurement equations to obtain an augmented measurement equation, which accompanies the state equation to yield the global optimal Kalman filter [8]. However, its disadvantage is that the larger computational burden is required due to the higher dimension of the augmented measurement vector. The distributed state fusion methods weight or combine local Kalman filters to obtain the fused Kalman filter. They have the advantages that a lower computation and communication cost is required, and the fault detection and isolation are facilitated. Recently, the distributed information fusion white noise deconvolution filter weighted by scalars is given in Ref. [9], and three distributed fusion white noise deconvolution estimators weighted by scalars, diagonal matrices, and matrices are given in Refs. [10,11], but they are all globally suboptimal.

In this paper, using the Kalman filtering method, the time-varying optimal distributed fusion white noise deconvolution estimator is presented for the multisensor systems. It is completely equivalent to the centralized fusion white noise deconvolution estimator, so that it has the global optimality. Moreover, its complexity in computation is less than that in Refs. [9–11].

## 2 Problem formulation

Consider the multisensor linear discrete time-varying stochastic system

$$x(t+1) = \Phi(t)x(t) + \Gamma(t)w(t), \quad (1)$$

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$$y_i(t) = H_i(t)x(t) + v_i(t), \quad i = 1, 2, \dots, L, \quad (2)$$

where  $t$  is the discrete time;  $x(t) \in R^n$ ,  $y_i(t) \in R^{m_i}$ ,  $w(t) \in R^r$ , and  $v_i(t) \in R^{m_i}$  are the state, measurement, process, and measurement noises of the  $i$ th sensor subsystem, respectively; and  $\Phi(t)$ ,  $\Gamma(t)$ , and  $H_i(t)$  are time-varying matrices with compatible dimensions.

**Assumption 1**  $w(t) \in R^r$  and  $v_i(t) \in R^{m_i}$  are independent white noises with zero mean and

$$\begin{aligned} & \mathbb{E} \left\{ \begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix} \begin{bmatrix} w^T(k) & v_j^T(k) \end{bmatrix} \right\} \\ &= \begin{bmatrix} Q(t) & 0 \\ 0 & R_i(t)\delta_{ij} \end{bmatrix} \delta_{tk}, \end{aligned} \quad (3)$$

where  $\mathbb{E}$  denotes the mathematical expectation, superscript  $T$  denotes the transpose, and  $\delta_{\alpha\beta}$  is the Kronecker delta function, i.e.,  $\delta_{\alpha\alpha} = 1$ ,  $\delta_{\alpha\beta} = 0$  ( $\alpha \neq \beta$ ).

**Assumption 2**  $x(0)$  is uncorrelated with  $w(t)$  and  $v_i(t)$ , and  $\mathbb{E}x(0) = \mu$ ,  $\text{cov}x(0) = P_0$ , where  $\text{cov}$  denotes covariance.

**Remark 1** In fact, when the autoregressive moving average (ARMA) models with nonzero constant terms in both the autoregressive and moving average parts are transferred into the equivalent state space model, we will find that the cross-covariance matrices between the process noises and measurement noises are identical for all sensors at each time  $t$ . Therefore, Assumption 1 given in this paper is not restrictive for practical application.

The problem is to find the globally optimal distributed fusion white noise deconvolution fuser  $\hat{w}(t|t+N)$  ( $N > 0$  or  $N \leq 0$ ) based on the local Kalman filters  $\hat{x}_i(t|t)$  and predictors  $\hat{x}_i(t|t-1)$ .

### 3 Time-varying optimal distributed fusion white noise deconvolution estimator

#### 3.1 Centralized fusion white noise deconvolution estimator

Introducing an augmented measurement vector, we combine all the measurement equations into a centralized measurement fusion equation as

$$y(t) = H(t)x(t) + v(t), \quad (4)$$

with the definitions

$$y(t) = [y_1^T(t), y_2^T(t), \dots, y_L^T(t)]^T, \quad (5)$$

$$H(t) = [H_1^T(t), H_2^T(t), \dots, H_L^T(t)]^T, \quad (6)$$

$$v(t) = [v_1^T(t), v_2^T(t), \dots, v_L^T(t)]^T, \quad (7)$$

and the fused measurement white noise  $v(t)$  has the variance matrix  $R(t)$  as

$$R(t) = \text{diag}(R_1(t), R_2(t), \dots, R_L(t)). \quad (8)$$

For the centralized fusion system (1) and (4), applying the standard Kalman filter [12] with the initial time  $t_0 = 0$ , we can obtain the centralized Kalman predictor  $\hat{x}(t+1|t)$  and Kalman filter  $\hat{x}(t|t)$  as follows:

$$\hat{x}(t+1|t+1) = \hat{x}(t+1|t) + K_f(t+1)\varepsilon(t+1), \quad (9)$$

$$\varepsilon(t+1) = y(t+1) - H(t+1)\hat{x}(t+1|t). \quad (10)$$

The filtering gain  $K_f(t+1)$  is given as

$$K_f(t+1) = P(t+1|t)H^T(t+1)Q_\varepsilon^{-1}(t+1), \quad (11)$$

$$Q_\varepsilon(t+1) = H(t+1)P(t+1|t)H^T(t+1) + R(t+1), \quad (12)$$

$$\hat{x}(t+1|t) = \Psi_p(t)\hat{x}(t|t-1) + K_p(t)y(t), \quad (13)$$

with the definitions

$$K_p(t) = \Phi(t)K_f(t), \quad (14)$$

$$\Psi_p(t) = \Phi(t) - K_p(t)H(t), \quad (15)$$

where the prediction error variance matrix  $P(t+1|t)$  satisfies the time-varying Riccati equation:

$$P(t+1|t) = \Phi(t)P(t|t)\Phi^T(t) + \Gamma(t)Q(t)\Gamma^T(t). \quad (16)$$

The filtering error variance matrix  $P(t+1|t+1)$  is given by

$$\begin{aligned} & P(t+1|t+1) \\ &= [I_n - K_f(t+1)H(t+1)]P(t+1|t). \end{aligned} \quad (17)$$

The centralized fusion Kalman predictor and filter are globally optimal in the sense that their accuracy is higher than that of each local Kalman predictor and filter and is higher than that of the weighted state fusion Kalman predictor and filter [9–11].

For the centralized fusion system (1) and (4), applying the standard Kalman filter [11] with the initial time  $t_0 = 0$ , we can obtain the centralized fusion white noise deconvolution estimator  $\hat{w}(t|t+N)$  as follows:

$$\hat{w}(t|t+N) = 0, \quad N \leq 0, \quad (18)$$

$$\hat{w}(t|t+N) = \sum_{j=1}^N M(t|t+j)\varepsilon(t+j), \quad N > 0. \quad (19)$$

The estimation gain is given as

$$M(t|t+1) = Q(t)\Gamma^T(t)H^T(t+1)Q_\varepsilon^{-1}(t+1), \quad (20)$$

$$\begin{aligned} M(t|t+N) \\ = Q(t)\Gamma^T(t) \left\{ \prod_{j=1}^{N-1} \Psi_p^T(t+j) \right\} H^T(t+N)Q_\varepsilon^{-1}(t+N), \\ N > 1. \quad (21) \end{aligned}$$

The estimation error variance matrix is given as

$$P_w(t|t+N) = Q(t), \quad N \leq 0, \quad (22)$$

$$\begin{aligned} P_w(t|t+N) \\ = Q(t) - \sum_{j=1}^N M(t|t+j)Q_\varepsilon(t+j)M^T(t|t+j), \\ N > 0. \quad (23) \end{aligned}$$

### 3.2 Time-varying optimal distributed fusion white noise deconvolution estimator

**Lemma 1** [13] (Matrix Inversion Lemma) Assume that  $A$ ,  $B$ , and  $C$  are  $n \times n$ ,  $n \times m$ , and  $n \times m$  matrices, respectively, and the inverse matrices  $A^{-1}$  and  $(I_m + C^T A^{-1} B)^{-1}$  exist, then we have

$$\begin{aligned} (A + BC^T)^{-1} \\ = A^{-1} - A^{-1}B(I + C^T A^{-1}B)^{-1}C^T A^{-1}, \quad (24) \end{aligned}$$

where  $I_m$  is the  $m \times m$  identity matrix.

**Theorem 1** For the multisensor linear discrete system (1) and (2), the optimal distributed fusion Kalman predictor  $\hat{x}(t+1|t)$  and filter  $\hat{x}(t+1|t+1)$  are, respectively, given as

$$\begin{aligned} \hat{x}(t+1|t+1) \\ = \hat{x}(t+1|t) \\ + \sum_{i=1}^L [K_{fi}(t+1)(y_i(t+1) - H_i(t+1)\hat{x}(t+1|t))], \quad (25) \end{aligned}$$

$$\hat{x}(t+1|t) = \Phi(t)\hat{x}(t|t), \quad (26)$$

$$\begin{aligned} P(t+1|t+1) \\ = \left[ I_n - \sum_{i=1}^L K_{fi}(t+1)H_i(t+1) \right] P(t+1|t), \quad (27) \end{aligned}$$

$$K_{fi}(t+1) = P(t+1|t+1)H_i^T(t+1)R_i^{-1}(t+1), \quad (28)$$

with the prediction error variance matrix  $P(t+1|t)$  given by Eq. (16).

*Proof.* From Lemma 1, we have

$$\begin{aligned} (A + BC^T)^{-1} \\ = A^{-1} - A^{-1}B(I + C^T A^{-1}B)^{-1}C^T A^{-1}. \quad (29) \end{aligned}$$

Applying Eqs. (11) and (17), and taking  $A = P^{-1}(t|t-1)$ ,  $B = H^T(t)$ ,  $C^T = R^{-1}(t)H(t)$  at time  $t$ , we have

$$\begin{aligned} (P^{-1}(t|t-1) + H^T(t)R^{-1}(t)H(t))^{-1} \\ = P(t|t-1) - P(t|t-1)H^T(t)(I + R^{-1}(t)H(t)P(t|t-1)H^T(t))^{-1}R^{-1}(t)H(t)P(t|t-1) \\ = P(t|t-1) - P(t|t-1)H^T(t)[R(t)(I + R^{-1}(t)H(t)P(t|t-1)H^T(t))]^{-1}H(t)P(t|t-1) \\ = P(t|t-1) - P(t|t-1)H^T(t)[H(t)P(t|t-1)H^T(t) + R(t)]^{-1}H(t)P(t|t-1) \\ = [I - K_f(t)H(t)]P(t|t-1) \\ = P(t|t), \quad (30) \end{aligned}$$

which yields

$$P^{-1}(t|t) = P^{-1}(t|t-1) + H^T(t)R^{-1}(t)H(t). \quad (31)$$

Using Eqs. (11), (12), (16), and (31) yields

$$\begin{aligned}
K_f(t) &= P(t|t-1)H^T(t)[R(t) + H(t)P(t|t-1)H^T(t)]^{-1} \\
&= P(t|t-1)H^T(t)[R^{-1}(t) - R^{-1}(t)H(t)P(t|t)H^T(t)R^{-1}(t)] \\
&= P(t|t-1)[(I - H^T(t)R^{-1}(t)H(t)P(t|t)H^T(t)R^{-1}(t))] \\
&= P(t|t-1)[I - (P^{-1}(t|t) - P^{-1}(t|t-1))P(t|t)]H^T(t)R^{-1}(t) \\
&= P(t|t)H^T(t)R^{-1}(t).
\end{aligned} \tag{32}$$

From Eqs. (6), (8), and (32), we have

$$\begin{aligned}
K_f(t) &= P(t|t)H^T(t)R^{-1}(t) \\
&= P(t|t)[H_1^T(t)R_1^{-1}(t) \quad H_2^T(t)R_2^{-1}(t) \quad \cdots \quad H_L^T(t)R_L^{-1}(t)] \\
&= [K_{f1}(t) \quad K_{f2}(t) \quad \cdots \quad K_{fL}(t)].
\end{aligned} \tag{33}$$

From Eqs. (30) and (33), we can obtain Eq. (28). Applying Eqs. (5), (6), (9), and (10) yields Eq. (25). From Eqs. (9), (10), (14), and (15), we have Eq. (26). From Eqs. (6), (17), and (33), we can obtain Eq. (27). The proof is completed.

**Remark 2** For the multisensor system (1) and (2), applying the standard Kalman filter [12] with the initial time  $t_0 = 0$ , we can obtain the local Kalman predictor  $\hat{x}_i(t+1|t)$  and Kalman filter  $\hat{x}_i(t|t)$  with the forms similar to these in Eqs. (9)–(17). Furthermore, we can obtain the globally optimal distributed fusion Kalman filter and

predictor by Theorem 1. In Ref. [11], the Kalman fuser is obtained by weighting the local Kalman estimators. In order to compute the optimal weights, it is required to compute the local estimation error variance matrices and the local estimation error covariance matrices. Compared with Ref. [11], the complexity in computation is reduced.

**Theorem 2** For the multisensor linear discrete system (1) and (2), the globally optimal distributed fusion white noise deconvolution estimator  $\hat{w}(t|t+N)$  is given as

$$\hat{w}(t|t+N) = 0, \quad N \leq 0, \tag{34}$$

$$\hat{w}(t|t+N) = \sum_{j=1}^N \sum_{i=1}^L [M_i(t|t+j)(y_i(t+j) - H_i(t+j)\hat{x}(t+j|t+j-1))], \quad N > 0, \tag{35}$$

$$M_i(t|t+1) = Q(t)\Gamma^T(t)P^{-1}(t+1|t)P(t+1|t+1)H_i^T(t+1)R_i^{-1}(t+1), \tag{36}$$

$$\begin{aligned}
M_i(t|t+N) &= Q(t)\Gamma^T(t) \left\{ \prod_{j=1}^{N-1} \left[ \left( I - \sum_{k=1}^L K_{jk}(t+j)H_i(t+j) \right)^T \Phi^T(t+j) \right] \right\} \\
&\quad \times P^{-1}(t+N|t+N-1)P(t+N|t+N)H_i^T(t+N)R_i^{-1}(t+N).
\end{aligned} \tag{37}$$

The estimation error variance matrix is given as

$$P_w(t|t+N) = Q(t), \quad N \leq 0, \tag{38}$$

$$\begin{aligned}
P_w(t|t+N) &= Q(t) - \sum_{j=1}^N \left[ \left( \sum_{i=1}^L M_i(t|t+j)H_i(t+j) \right) P(t+j|t+j-1) \left( \sum_{i=1}^L H_i^T(t+j)M_i^T(t|t+j) \right) \right. \\
&\quad \left. + \sum_{i=1}^L M_i(t|t+j)R_i(t+j)M_i^T(t|t+j) \right].
\end{aligned} \tag{39}$$

*Proof.* From Eq. (32), we have

$$H^T(t)Q_\varepsilon^{-1}(t) = H^T(t)[R(t) + H(t)P(t|t-1)H^T(t)]^{-1} = P^{-1}(t|t-1)P(t|t)H^T(t)R^{-1}(t). \quad (40)$$

Applying Eqs. (20) and (40), it follows that

$$\begin{aligned} M(t|t+1) &= Q(t)\Gamma^T(t)P^{-1}(t+1|t)P(t+1|t+1)H^T(t+1)R^{-1}(t+1) \\ &= Q(t)\Gamma^T(t)P^{-1}(t+1|t)P(t+1|t+1) \\ &\quad \times [H_1^T(t+1)R_1^{-1}(t+1) \quad H_2^T(t+1)R_2^{-1}(t+1) \quad \cdots \quad H_L^T(t+1)R_L^{-1}(t+1)] \\ &= [M_1(t|t+1) \quad M_2(t|t+1) \quad \cdots \quad M_L(t|t+1)], \end{aligned} \quad (41)$$

with the definition of  $M_i(t|t+1)$  given by Eq. (36).

From Eqs. (6), (14), (15), (21), (33), and (40), we have

$$\begin{aligned} M(t|t+N) &= Q(t)\Gamma^T(t) \left\{ \prod_{j=1}^{N-1} \Psi_p^T(t+j) \right\} P^{-1}(t+N|t+N-1)P(t+N|t+N)H^T(t+N)R^{-1}(t+N) \\ &= Q(t)\Gamma^T(t) \left\{ \prod_{j=1}^{N-1} \left[ \left( I - \sum_{k=1}^L K_{jk}(t+j)H_k(t+j) \right)^T \Phi^T(t+j) \right] \right\} P^{-1}(t+N|t+N-1)P(t+N|t+N) \\ &\quad \times [H_1^T(t+N)R_1^{-1}(t+N) \quad H_2^T(t+N)R_2^{-1}(t+N) \quad \cdots \quad H_L^T(t+N)R_L^{-1}(t+N)] \\ &= [M_1(t|t+N) \quad M_2(t|t+N) \quad \cdots \quad M_L(t|t+N)], \end{aligned} \quad (42)$$

with the definition of  $M_i(t|t+N)$  given by Eq. (37).

From Eqs. (19), (41), and (42), we can obtain Eq. (35). Applying Eqs. (6), (8), (12), (41), and (42) yields Eq. (39). The proof is completed.

**Remark 3** For the multisensor system (1) and (2), applying the standard Kalman filter [11] with the initial time  $t_0 = 0$ , we can obtain the local white noise estimators  $\hat{w}_i(t|t+N)$  with the form similar to those in Eqs. (18)–(23). Furthermore, we can obtain the globally optimal distributed fusion white noise deconvolution estimator by Theorem 2, with the algorithm flow diagram and the fusion structure given by Fig. 1. In Ref. [11], the white noise deconvolution fuser is obtained by weighting the local white noise deconvolution estimators. In order to compute the optimal weights, it is required to compute the local estimation error variance matrices and the local estimation error covariance matrices. Compared with Ref. [11], the complexity in computation is reduced, which has important value in engineering applications.

**Remark 4** The distributed fusion white noise deconvolution estimator given by Theorem 2 is derived from the centralized fusion white noise deconvolution estimator so that it is identical to the centralized fuser, i.e., it has the global optimality. It is better than the existing globally suboptimal white noise deconvolution estimators [9–11].

## 4 Simulation example

In oil seismic exploration [1–3], the seismic waves are

reflected in different geological layers. The oil exploration is performed via the reflection coefficient sequence. The reflection coefficient sequence contains the important information for finding and discovering the oil field and determining its geometry shape, which can be described by Bernoulli-Gaussian white noise [1–3]. Therefore, estimating the Bernoulli-Gaussian input white noise becomes a key technical problem for oil seismic exploration. However, the proposed white noise fuser is not limited to handle the Gaussian signals.

Consider the multisensor linear discrete system with white measurement noises and three sensors:

$$x(t+1) = \Phi(t)x(t) + \Gamma(t)w(t), \quad (43)$$

$$y_i(t) = H_i(t)x(t) + v_i(t), \quad i = 1, 2, 3, \quad (44)$$

where  $x(t) \in R^2$  is the state,  $y(t) \in R$  is the measurement, and  $v_i(t) \in R$  is a white measurement noise with zero mean and variance  $R_i$ .  $w(t) = b(t)g(t)$  is a Bernoulli-Gaussian white noise, where  $b(t)$  is the Bernoulli white noise with the value as 1 or 0 and the probability  $P(b(t) = 1) = \lambda$ ,  $P(b(t) = 0) = 1 - \lambda$ , and  $g(t)$  is a Gaussian white noise with zero mean and variance  $Q_g$  and independent of  $b(t)$ . Obviously, there is the relation  $Q = \lambda Q_g$ . The objectives are to find local and the time-varying optimal distributed fusion white noise deconvolution smoothers  $\hat{w}_i(t|t+3)$ ,  $i = 1, 2, 3$ , and  $\hat{w}_d(t|t+3)$  for the input white noise  $w(t)$  based on the measurements  $(y(t+3), y(t+2), \dots, y(1))$ .

In simulation, we take

$$\Phi(t) = \begin{bmatrix} 0.1 & 0.25 \\ 0.1 + \cos\frac{2\pi t}{200} & 0 \end{bmatrix},$$

$$\Gamma(t) = \begin{bmatrix} 0.1\sin\frac{2\pi t}{200} \\ 0.2 \end{bmatrix},$$

$$H_1(t) = \begin{bmatrix} 1 + 0.2\sin\frac{2\pi t}{200} & 0 \end{bmatrix},$$

$$H_2(t) = \begin{bmatrix} 0 & 1 + 0.3\sin\frac{2\pi t}{200} \end{bmatrix},$$

$$H_3(t) = \begin{bmatrix} 1 & 1 + 0.1\sin\frac{2\pi t}{200} \end{bmatrix},$$

$$\lambda = 0.25,$$

$$Q_g = 1,$$

$$R_1 = 0.001,$$

$$R_2 = 0.003,$$

$$R_3 = 0.005. \quad (45)$$

The simulation results are shown in Figs. 2–4 and Tables 1 and 2. Figure 2 gives the input noise  $w(t)$  and the local

and distributed fusion white noise smoothers, respectively, where the vertical coordinates at the endpoints of solid lines denote the true values and the vertical coordinates of the dots denote the estimates. Figure 3 shows the comparison of curves of the accumulated error squares for the local and distributed fusion white noise smoothers. Figure 4 shows the comparison between the local and the fused mean square error (MSE) values of 200 Monte Carlo runs. The above figures show that the accuracy of the distributed fusion white noise smoother is higher than that of the local white noise smoothers. Table 1 gives the comparison of values for the centralized fusion white noise smoother  $\hat{w}_c(t|t+3)$  and the distributed fusion white noise smoother  $\hat{w}_d(t|t+3)$ . The comparison of variances for the local white noise smoothers  $P_w^{(i)}(t|t+3)$ ,  $i = 1, 2, 3$ , the centralized fusion white noise smoother  $P_{wc}(t|t+3)$ , and the distributed fusion white noise smoother  $P_{wd}(t|t+3)$  is given in Table 2. The above tables show that the accuracy of the distributed fusion white noise smoother is higher than that of the local white noise smoothers and the distributed fusion white noise deconvolution estimator is numerically identical to the centralized fusion white noise deconvolution estimator, i.e., it has the global optimality. All the results have important value in seismic exploration [1–3].

Here, we define MSE value at time  $t$  as follows:

$$\text{MSE}_i(t) = \frac{1}{n} \sum_{j=1}^n \left( w^{(j)}(t) - \hat{w}_i^{(j)}(t|t+3) \right)^2,$$

$$i = 1, 2, 3, d, \quad t = 1, 2, \dots, 200, \quad (46)$$

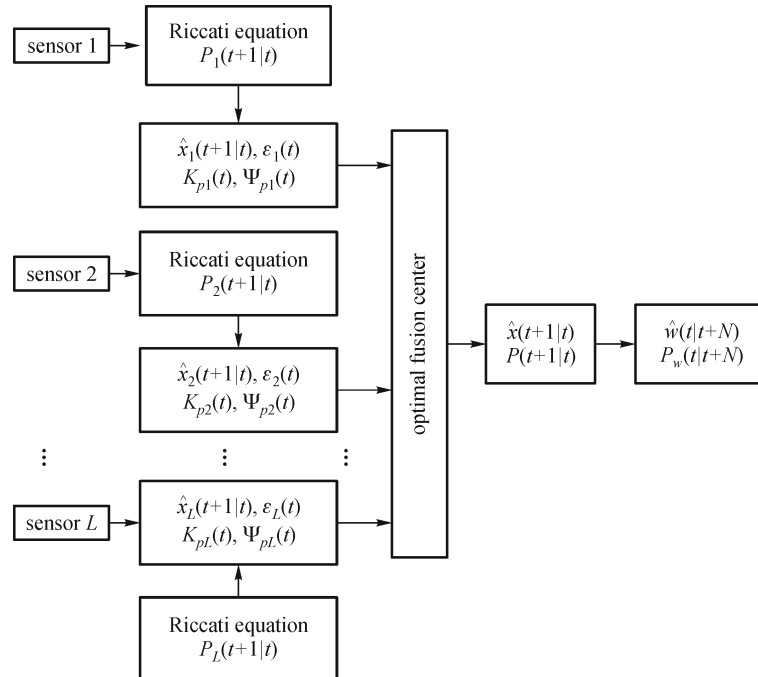
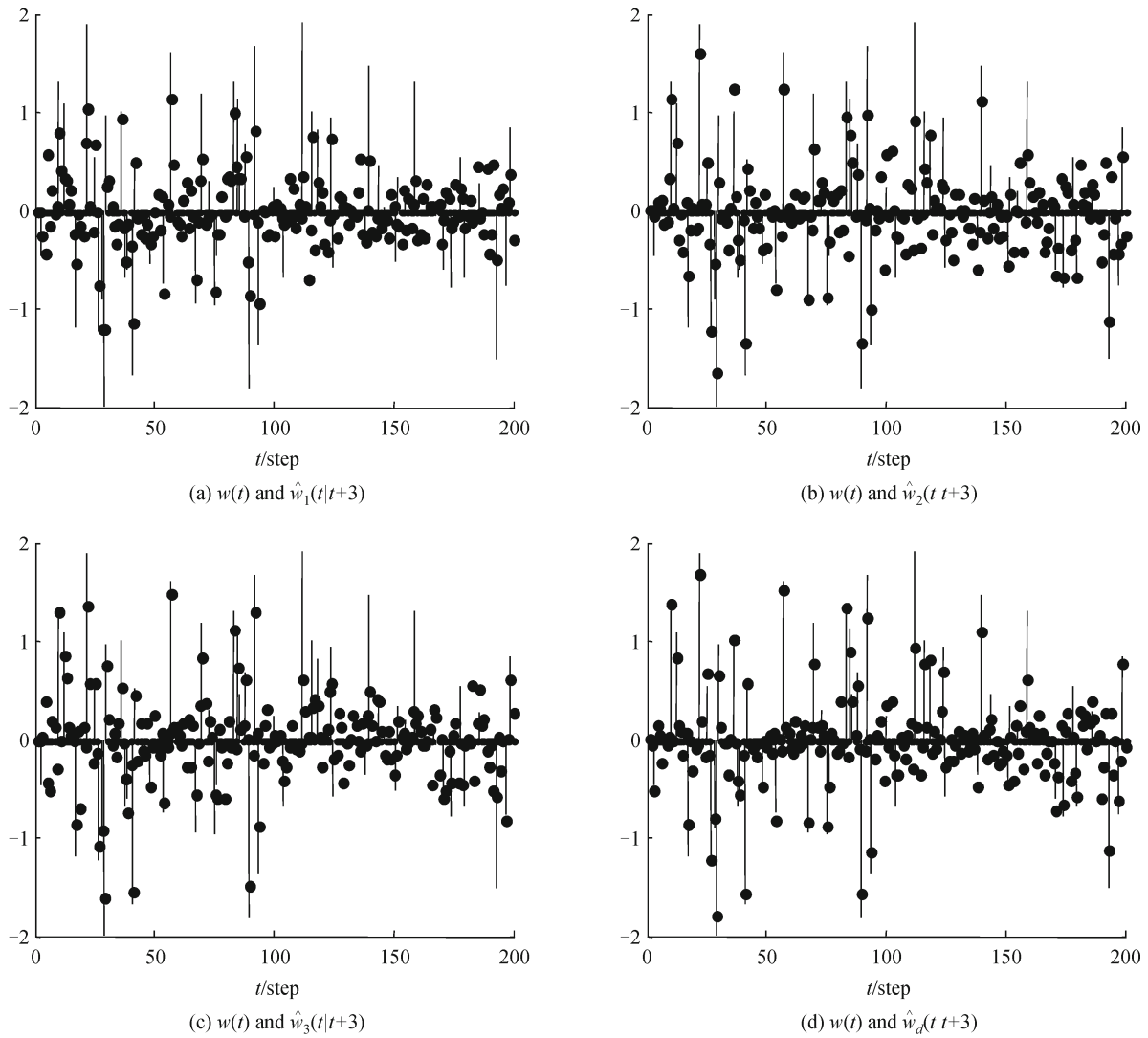
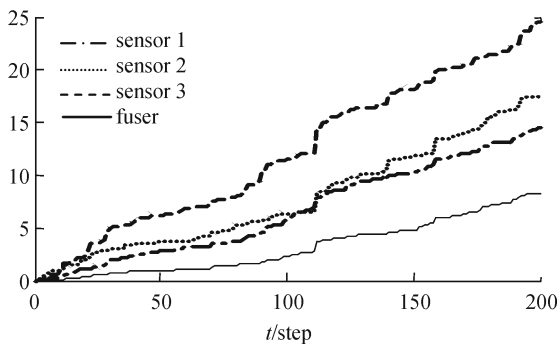


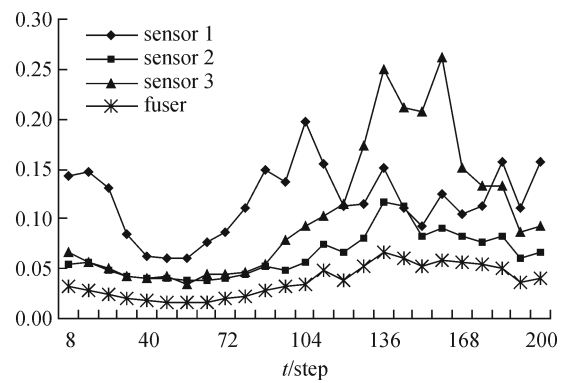
Fig. 1 Algorithm flow diagram and fusion structure of the optimal distributed fusion white noise deconvolution estimators



**Fig. 2**  $w(t)$  and local and optimal distributed fusion white noise deconvolution smoothers



**Fig. 3** Comparison of curves of accumulated error squares for local and distributed fusion white noise smoothers



**Fig. 4** Comparison of mean square error (MSE) curves for local and fused white noise smoothers

**Table 1** Comparison of values for centralized and distributed fusion white noise smoothers

| $t$                | 50        | 100      | 150       | 200       |
|--------------------|-----------|----------|-----------|-----------|
| $\hat{w}_d(t t+3)$ | -0.073084 | 0.345999 | -0.154367 | -0.067506 |
| $\hat{w}_c(t t+3)$ | -0.073085 | 0.349009 | -0.154408 | -0.058246 |

**Table 2** Comparison of variances for local, centralized, and distributed fusion white noise smoothers

| $t$                | 50       | 100      | 150      | 200      |
|--------------------|----------|----------|----------|----------|
| $P_w^{(1)}(t t+3)$ | 0.062155 | 0.156296 | 0.098047 | 0.153817 |
| $P_w^{(2)}(t t+3)$ | 0.037776 | 0.059894 | 0.094903 | 0.058704 |
| $P_w^{(3)}(t t+3)$ | 0.037889 | 0.085025 | 0.206825 | 0.084670 |
| $P_{wd}(t t+3)$    | 0.015644 | 0.037203 | 0.054858 | 0.036892 |
| $P_{wc}(t t+3)$    | 0.015644 | 0.037048 | 0.054862 | 0.036646 |

where  $w^{(j)}(t)$  is the  $j$ th sample of the stochastic process  $w(t)$  and  $\hat{w}_i^{(j)}(t|t+3)$  is the  $j$ th sample of  $\hat{w}_i(t|t+3)$ , respectively,  $j = 1, 2, \dots, n$ , and  $n = 200$  is the sampled number (i.e., 200 Monte Carlo runs).

## 5 Conclusion

Based on the Riccati equation, the optimal distributed fusion white noise deconvolution estimator is presented using the Kalman filtering method. It is completely equivalent to the centralized fusion white noise deconvolution estimator, so that it has the global optimality. The complexity in computation is reduced. The proposed results can be applied to oil seismic exploration, communication, signal processing, and state estimation.

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