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RESEARCH ARTICLE

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Robust radar automatic target recognition algorithm based on HRRP signature

Abstract Automatic target recognition (ATR) is an important function for modern radar. High resolution range profile (HRRP) of target contains target structure signatures, such as target size, scatterer distribution, etc., which is a promising signature for ATR. Statistical modeling of target HRRPs is the key stage for HRRP statistical recognition, including model selection and parameter estimation. For statistical recognition algorithms, it is generally assumed that the test samples follow the same distribution model as that of the training data. Since the signal-to-noise ratio (SNR) of the received HRRP is a function of target distance, the assumption may be not met in practice. In this paper, we present a robust method for HRRP statistical recognition when SNR of test HRRP is lower than that of training samples. The noise is assumed independent Gaussian distributed, while HRRP is modeled by probabilistic principal component analysis (PPCA) model. Simulated experiments based on measured data show the effectiveness of the proposed method.

Keywords radar target recognition, high resolution range profile (HRRP), probabilistic principal component analysis (PPCA)

1 Introduction

High resolution range profile (HRRP) of a target is the amplitude of coherent summations of the complex time return from target scatterers in each range cell, as shown in Fig. 1, which is shown to be a promising signature for automatic target recognition (ATR) [1–7]. Statistical recognition methods have been extensively studied and successfully applied to HRRP-based ATR area [1–7]. For HRRP-based statistical recognition, one key problem is to choose an appropriate model that can describe HRRP's statistical property accurately. There are several issues needed to be considered. The first is that the data between range cells are statistically correlated or not. The second is that the data follow which kind of distribution model. The third is that how to build statistical model for the HRRP data according to different target orientations. In radar automatic target recognition (RATR) applications, the amplitude of received HRRP signal is a function of radar-target distance, namely, the signal-to-noise ratio (SNR) of HRRP signal will be much lower for the target at a longer distance. Therefore, it is very important for the RATR algorithms to have a robust performance for the low SNR data, in other words, to obtain satisfactory recognition performance for the target at longer distance.

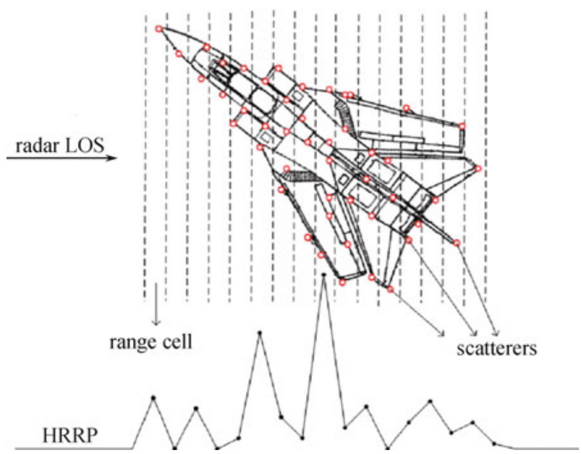

Fig. 1 Illustration of target HRRP [7]

The remainder of this paper is organized as follows. In Section 2, we give a brief review of some existing statistical models for HRRP data. In Section 3, we propose a robust recognition algorithm for low SNR data. Based on measured HRRP data, some results are presented in Section 4. Finally, some remarks and discussion are drawn in Section 5.

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2 Statistical modeling of radar HRRP data

The statistical modeling of radar HRRP data includes model selection and parameters estimation. In this section, we will give a brief review of some existing statistical modeling algorithms for HRRP data.

2.1 Statistical independent Gaussian model

Based on the assumptions that the HRRP data are Gaussian distributed and statistical independent between different range cells, we can estimate the mean vector and the covariance matrix (actually is a diagonal covariance under this assumption) as

$$\tilde{\boldsymbol{m}}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} \boldsymbol{x}_{j,i}, \tag{1}$$

$$\tilde{\Sigma}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} (\boldsymbol{x}_{j,i} - \boldsymbol{m}_i)^{\mathrm{T}} (\boldsymbol{x}_{j,i} - \boldsymbol{m}_i), \tag{2}$$

where $\boldsymbol{x}_{j,i}$ is the jth HRRP sample of the ith target, and N_i is the number of training data sample. Based on the estimated parameters, for a test data sample $\boldsymbol{x}_t$, one can determine its label by using the distance metric as shown below:

$$D_i = \det\left|\tilde{\Sigma}_i\right|^{-1/2} + (\boldsymbol{x}_t - \tilde{\boldsymbol{m}}_i)^{\mathrm{T}}\,\tilde{\Sigma}_i^{-1}\,(\boldsymbol{x}_t - \tilde{\boldsymbol{m}}_i), \quad i = 1, 2, ..., I. \tag{3}$$

2.2 Statistical independent non-Gaussian model

For wideband radar, it is well known that the target echoed signal is generally non-Gaussian distributed. Therefore, the Gaussian distributed assumption cannot get a satisfactory recognition performance. In Refs. [1] and [2], Gamma distribution is used to describe the HRRP data. Actually, the distribution model of HRRP data in different range cells may be different. We divide the range cells into three types according to different scatterer distribution structures [3]:

a) The first type of range cells: There are a large number of small scatterers and no predominant scatterers in this type of range cell. Under the assumption that the intensities of the small scatterers are almost the same and the number of the small scatterers is large enough, the central limit theorem holds. The inphase or quadrature component of the complex echo of this type of cell will follow Gaussian distribution with zero mean, and its amplitude will follow Rayleigh distribution as

$$P_{\mathrm{Rayleigh}}(x) = \begin{cases} \dfrac{x}{b^2}\exp\left(-\dfrac{x^2}{2b^2}\right), & x > 0, \\ 0, & x \leqslant 0, \end{cases} \tag{4}$$

where b is the scale parameter.

b) The second type of range cells: This type of range cell consists of a predominant scatterer and a large number of small scatterers. Under the hypothesis similar to the first type, the inphase or quadrature component of the complex echo of this type of cell will follow Gaussian distribution with the mean determined by the echo's inphase or quadrature component of the predominant scatterer, in accordance with the central limit theorem. Thus, its amplitude follows a Ricean distribution amplitude as

$$p_{\mathrm{Rice}}(x) = \begin{cases} \dfrac{x}{b^2}\exp\left(-\dfrac{x^2+|v|^2}{2b^2}\right) I_0\left(\dfrac{x\cdot|v|}{b^2}\right), & x > 0, \\ 0, & x \leqslant 0, \end{cases} \tag{5}$$

where b is the scale parameter, v is the noncentrality parameter, and $I_0(\cdot)$ is the modified Bessel function of zero order.

c) The third type of range cells: There are a large number of small scatterers and several predominant scatterers in this type of range cell. Its statistical property can be approximately represented by a multimodal distribution, which depends on the particular geometrical distribution of the predominant scatterers.

We can use Gamma distribution to describe the statistical property of the first type and the second type of range cells and Gaussian mixture model to describe the third type of range cells.

The next question is how to determine the range cells type given by the training data set. We use the rival penalized competitive learning (RPCL) algorithm [8], which is an efficient clustering algorithm, to determine the type of a range cell. Moreover, the parameters of Gamma distribution and Gaussian mixture distribution are estimated by the maximum likelihood (ML) method and the expectation-maximization (EM) algorithm, respectively [3].

2.3 Statistical correlated Gaussian model

Theoretically speaking, the target can be regarded as consists of multiple isolated scatterers; therefore, the HRRP data can be assumed as statistical independent between range cells. However, considering the multiple reflections phenomena, target structure similarity between range cells, etc., this independence assumption is not true. One can use joint Gaussian distribution directly to model the range cells correlation property of HRRP data. However, the model parameter dimension will increase significantly. This will introduce difficulty in parameter estimation and larger memory and computational complexity requirement. Therefore, lower dimension model is required.

a) Probabilistic principal component analysis (PPCA) model: For an HRRP sample $\boldsymbol{x}$ with dimension of d, it can be written as

$$\boldsymbol{x} = \boldsymbol{A}_k \boldsymbol{y}_k + \boldsymbol{\mu}_k + \boldsymbol{\varepsilon}_k, \quad (6)$$

where k is target label, $\boldsymbol{\mu}_k$ is a d-dimensional mean vector, $\boldsymbol{A}_k \in \Re^{d\times q}$ is the weight matrix with column orthogonal to each other, the latent variable $\boldsymbol{y}_k \in \Re^q$ follows the Gaussian distribution, and $\boldsymbol{\varepsilon}_k \in \Re^d \sim N(0, \sigma_k \boldsymbol{I}_d)$ is the Gaussian distributed noise variable. Therefore, the distribution of $\boldsymbol{x}$ can be written as

$$p(\boldsymbol{x}|k) = (2\pi)^{-d/2} \left|\sigma_k^2 \boldsymbol{I}_d + \boldsymbol{A}_k \boldsymbol{A}_k^{\mathrm{T}}\right|^{-1/2} \cdot \mathrm{e}^{\left\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_k)^{\mathrm{T}}(\sigma_k^2 \boldsymbol{I}_d + \boldsymbol{A}_k \boldsymbol{A}_k^{\mathrm{T}})^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_k)\right\}}. \quad (7)$$

b) Factor analysis (FA) model: FA model has the same formation with PPCA model, as shown in Eq. (6). PPCA model requires orthogonal bases in the signal subspace and independent elements in the noise variable with the same variances, while FA model requires independent bases in the signal subspace and independent elements in the noise variable with different variances. In the FA model, the noise variable $\boldsymbol{\varepsilon}_k \in \Re^d \sim N(0, \boldsymbol{\Psi}_k)$, where $\boldsymbol{\Psi}_k$ is a diagonal matrix with different elements. Therefore, the distribution of $\boldsymbol{x}$ can be written as

$$p(\boldsymbol{x}|k) = (2\pi)^{-d/2} \left|\boldsymbol{\Psi}_k + \boldsymbol{A}_k \boldsymbol{A}_k^{\mathrm{T}}\right|^{-1/2} \cdot \mathrm{e}^{\left\{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu}_k)^{\mathrm{T}}\left(\boldsymbol{\Psi}_k + \boldsymbol{A}_k \boldsymbol{A}_k^{\mathrm{T}}\right)^{-1}(\boldsymbol{x}-\boldsymbol{\mu}_k)\right\}}. \quad (8)$$

Since that the FA model has more degrees of freedom to describe the data than that of PPCA model, our experimental results show that the recognition performance of FA model is better than that of PPCA model [5]. Detail approaches about the model selection and parameters estimation methods can be found in Ref. [5].

2.4 Statistical correlated non-Gaussian model

As indicated before, the HRRP data are generally non-Gaussian distributed, and FA model actually is an approximately model to describe the statistical property of HRRP data. Considering that the statistical correlation between range cells, a statistical model which can describe the statistical correlated non-Gaussian HRRP data are desired.

Local factor analysis (LFA) model is introduced into radar HRRP recognition [6]. In LFA model, the data can be represented as the summation of multiple FA model:

$$p(\boldsymbol{x}|k) = \sum_{l=1}^{L} \alpha_l p(\boldsymbol{x}|k, l), \quad (9)$$

where $\forall \alpha_l \geqslant 0$, $\sum_{l=1}^{L} \alpha_l = 1$. In the FA model, the latent variable dimension needs to be determined by model selection algorithm, and the loading matrix $\boldsymbol{A}_k$, the statistical parameters of $\boldsymbol{\mu}_k, \boldsymbol{y}_k, \boldsymbol{\varepsilon}_k$ need to be estimated. For LFA model, the model selection and parameters estimation are much more complex than that of FA model. Due to space limit, the detail approach is not introduced here, one can find them in Ref. [6].

2.5 Statistical model by using temporal correlation

In applications, radar generally can obtain multiple HRRP during a period of time. One can use multiple HRRP observations together to improve the recognition performance. One direct approach is to regard the multiple HRRP data as statistical independent, and fuse the recognition results of each HRRP data together to get the final result. However, since that target-radar orientation varies continuously during the observation, the HRRP sequence may be statistically correlated. It is expected that the recognition performance can be further improved by using the correlation property between HRRP data. Motivated by this, we introduce the temporal factor analysis (TFA) model [9,10] for HRRP recognition application. In this model, the HRRP data can be written as

$$\begin{aligned}
&\boldsymbol{x}_{t,k} = \boldsymbol{A}_k \boldsymbol{y}_{t,k} + \boldsymbol{\mu}_k + \boldsymbol{\varepsilon}_{t,k}, \\
&\boldsymbol{y}_{t,k} = \tilde{\boldsymbol{B}}_k \boldsymbol{y}_{t-1,k} + \boldsymbol{\omega}_{t,k},\ t = 1, 2, .., T, \\
&\boldsymbol{\varepsilon}_{t,k} \sim N(\boldsymbol{\varepsilon}_{t,k}|0,\ \boldsymbol{\Psi}_k),\ \boldsymbol{\omega}_{t,k} \sim N(\boldsymbol{\omega}_{t,k}|0,\ \boldsymbol{\Omega}_k),
\end{aligned} \quad (10)$$

where time is indexed by discrete t, parameters $\boldsymbol{A}$, $\boldsymbol{\mu}$, $\boldsymbol{\Psi}$ have the same meaning as in Eq. (6), $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\omega}_t$ are assumed to be uncorrelated random noises, and $\boldsymbol{y}_t$ and $\boldsymbol{\omega}_t$ are uncorrelated with $\boldsymbol{y}_{t-1}$. Moreover, $\tilde{\boldsymbol{B}}$ is an $m \times m$ transition matrix, The temporal relation is embodied in the first-order vector autoregressive process in the hidden state sequence $\{\boldsymbol{y}_0, \boldsymbol{y}_1, ..., \boldsymbol{y}_T\}$. Based on further constraints that $\tilde{\boldsymbol{B}}, \boldsymbol{\Omega}, \boldsymbol{\Psi}$ are all diagonal matrices, in Ref. [7], a model selection and parameters estimation approach is proposed.

3 Robust recognition algorithm in low SNR situation

Almost all the existing statistical models for HRRP data assume that the training data and test data are obtained under the similar measurement circumstance. In other words, the test data and training data share the same distribution model and the same statistical parameters. In real applications, the training data are usually collected via some cooperative measurement experiments or directly via simulations, which generally has high SNR; while the test samples are usually obtained in the non-cooperative circumstance, where the high SNR cannot be guaranteed due to the severe measurement conditions, such as the large distance between the non-cooperative targets and radar. To handle this problem, one method is to improve the SNR at signal processing stage. However, for radar application, it is always the first desirability to find the target and identify it at longer distance. Therefore, the robust recognition algorithm at low SNR situation is desired.

For the simple statistical models used in Refs. [1–3], since there is no model variable to describe the noise term, it is difficult to represent different noise background. For the PPCA model, from Eq. (6), we can see that the model parameters $\boldsymbol{A}_k$, $\boldsymbol{\mu}_k$ are determined by the signal of interest component, and the statistical parameters of latent variable $\boldsymbol{y}_k$ are also determined by the signal component. Only the statistical parameters of $\boldsymbol{\varepsilon}_k$ are determined by the noise. In other words, for different SNR levels, if the statistical parameters of $\boldsymbol{\varepsilon}_k$ are updated, the model can be used for different SNR cases. The conclusion also applies to the FA and LFA models.

Motivated by the above analysis, we develop a noise adaptive HRRP statistical recognition algorithm. The statistical model is trained by the high SNR data. When a test data sample comes, we first keep the signal-related parameters unchanged, and use the test sample to estimate the noise-related statistical variable. Then, we use the updated statistical model with the estimated noise variable to perform the recognition.

Without loss of generality, the returned HRRP echo consists of signal and noise. The noise is assumed added white Gaussian noise with equal power for every range cell. Two situations, high SNR which is $\boldsymbol{x}^+ = \boldsymbol{s}^+ + \boldsymbol{w}^+$ in formula formation and low SNR which is $\boldsymbol{x}^- = \boldsymbol{s}^- + \boldsymbol{w}^-$, are considered. The signal and noise power are P_s^+, P_s^-, P_w^+, and P_w^-, respectively, where superscript "+" denotes high SNR and "–" denotes low SNR. Since HRRP's amplitude must be normalized to equal energy in effects to overcome amplitude sensitivity, all samples used to train templates are normalized samples, i.e., $\bar{\boldsymbol{x}}^+ = \boldsymbol{x}^+/\sqrt{(\boldsymbol{x}^+)^{\mathrm{T}}\boldsymbol{x}^+}$ and $\bar{\boldsymbol{x}}^- = \boldsymbol{x}^-/\sqrt{(\boldsymbol{x}^-)^{\mathrm{T}}\boldsymbol{x}^-}$. Because $\boldsymbol{s}^+ \propto \boldsymbol{s}^-$, we just simply let $\boldsymbol{s}^+ = \boldsymbol{s}^- = \boldsymbol{s}$; thus, $P_s^+ = P_s^- = P_s$ and $P_w^+ < P_w^-$.

Based on the PPCA model, for high SNR situation, the mean vector and covariance matrix of training data samples can be approximated as follows:

$$\begin{aligned}\bar{\boldsymbol{m}}^+ &= \frac{1}{N}\sum_{i=1}^{N}\bar{\boldsymbol{x}}_i^+ = \frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{x}_i^+}{\sqrt{(\boldsymbol{x}_i^+)^{\mathrm{T}}\boldsymbol{x}_i^+}}\\ &\approx \frac{\sum_{i=1}^{N}\boldsymbol{s}_i^+}{N\cdot E\left(\sqrt{(\boldsymbol{x}_i^+)^{\mathrm{T}}\boldsymbol{x}_i^+}\right)}\\ &= \frac{1}{N\cdot\sqrt{P_s+P_w^+}}\sum_{i=1}^{N}\boldsymbol{s}_i,\end{aligned} \tag{11}$$

$$\begin{aligned}\bar{\boldsymbol{s}}^+ &= \sum_{i=1}^{N}\frac{(\bar{\boldsymbol{x}}_i-\bar{\boldsymbol{m}}^+)(\bar{\boldsymbol{x}}_i-\bar{\boldsymbol{m}}^+)^{\mathrm{T}}}{N}\\ &\approx \frac{\sum_{i=1}^{N}\boldsymbol{x}_i^+(\boldsymbol{x}_i^+)^{\mathrm{T}}}{N\cdot E\left((\boldsymbol{x}_i^+)^{\mathrm{T}}\boldsymbol{x}_i^+\right)} - \frac{\left(\sum_{i=1}^{N}\boldsymbol{s}_i\right)\left(\sum_{i=1}^{N}\boldsymbol{s}_i\right)^{\mathrm{T}}}{N^2\left(P_s+P_w^+\right)}\\ &= \frac{\boldsymbol{C}_{ss}+P_w^+\boldsymbol{I}_D}{P_s+P_w^+},\end{aligned} \tag{12}$$

where

$$\boldsymbol{C}_{ss} = \frac{\sum_{i=1}^{N}\boldsymbol{s}_i\boldsymbol{s}_i^{\mathrm{T}}}{N} - \frac{\left(\sum_{i=1}^{N}\boldsymbol{s}_i\right)\left(\sum_{i=1}^{N}\boldsymbol{s}_i\right)^{\mathrm{T}}}{N^2}.$$

Let $\omega_1, \omega_2, \ldots, \omega_D$ ($\omega_1 \geqslant \omega_2 \geqslant \cdots \geqslant \omega_D$) denote $\boldsymbol{C}_{ss}$'s eigenvalues, $\boldsymbol{U}$ be a full matrix whose columns are the corresponding eigenvectors, so that $\boldsymbol{C}_{ss} = \boldsymbol{U}\boldsymbol{\Omega}\boldsymbol{U}^{\mathrm{T}}$, where $\boldsymbol{\Omega}$ is a diagonal matrix with $\omega_1, \omega_2, \ldots, \omega_D$ on the diagonal. Thus,

$$\bar{\boldsymbol{s}}^+ = \boldsymbol{U}\frac{\boldsymbol{\Omega}+P_w^+\boldsymbol{I}_D}{P_s+P_w^+}\boldsymbol{U}^{\mathrm{T}}.$$

According to PPCA parameter estimation algorithm, the mean estimator has already been addressed by Eq. (11), and the other estimators are given as follows:

$$\left(\sigma^+\right)^2 = \frac{\sum_{i=d+1}^{D}\omega_i}{(D-d)\left(P_s+P_w^+\right)} + \frac{P_w^+}{P_s+P_w^+}, \tag{13}$$

$$\begin{aligned}\boldsymbol{A}^+ &= \boldsymbol{U}_d\left[\frac{\boldsymbol{\Omega}_d}{P_s+P_w^+} + \frac{P_w^+\boldsymbol{I}_d}{P_s+P_w^+} - \left(\sigma^+\right)^2\boldsymbol{I}_d\right]^{1/2}\boldsymbol{T}\\ &= \boldsymbol{U}_d\left[\frac{1}{P_s+P_w^+}\left(\boldsymbol{\Omega}_d - \frac{\sum_{i=d+1}^{D}\omega_i\boldsymbol{I}_d}{D-d}\right)\right]^{1/2}\boldsymbol{T},\end{aligned} \tag{14}$$

where $\boldsymbol{\Omega}_d$ is a diagonal matrix with $\omega_1, \omega_2, \ldots, \omega_d$ on the diagonal. From Eqs. (13) and (14), we can deduce

$$\frac{\sum_{i=d+1}^{D}\omega_i}{D-d} = \left(P_s+P_w^+\right)\left(\sigma^+\right)^2 - P_w^+, \tag{15}$$

$$\boldsymbol{\Omega}_d = \left(P_s+P_w^+\right)\left(\boldsymbol{A}^+\right)^{\mathrm{T}}\boldsymbol{A} + \frac{\sum_{i=d+1}^{D}\omega_i\boldsymbol{I}_d}{D-d}. \tag{16}$$

Similarly, for low SNR case, the estimators can be obtained by the following equations:

$$\bar{\boldsymbol{m}}^- \approx \frac{\sum_{i=1}^{N}\boldsymbol{s}_i}{N\sqrt{P_s+P_w^-}} = \bar{\boldsymbol{m}}^+\frac{\sqrt{P_s+P_w^+}}{\sqrt{P_s+P_w^-}}, \tag{17}$$

$$\begin{aligned}\left(\sigma^-\right)^2 &= \frac{\sum_{i=d+1}^{D}\omega_i/(D-d)+P_w^-}{P_s+P_w^-}\\ &= \frac{\left(P_s+P_w^+\right)\left(\sigma^+\right)^2+P_w^- - P_w^+}{P_s+P_w^-},\end{aligned} \tag{18}$$

$$\begin{aligned}\boldsymbol{A}^- &= \boldsymbol{U}_d\left[\frac{\boldsymbol{\Omega}_d + P_w^-\boldsymbol{I}_d}{P_s + P_w^-} - \left(\sigma^-\right)^2\boldsymbol{I}_d\right]^{1/2}\boldsymbol{T}\\ &= \boldsymbol{A}^+\frac{\sqrt{P_s+P_w^+}}{\sqrt{P_s+P_w^-}}\boldsymbol{T}.\end{aligned} \tag{19}$$

Therefore, a discriminant function for low SNR is obtained as follows:

$$\begin{aligned}d =& -N\ln\left(P_s+P_w^-\right)+\ln\left|\boldsymbol{C}^-\right|\\ &+\left(\sqrt{P_s+P_w^-}\boldsymbol{x}-\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\right)^{\mathrm{T}}\boldsymbol{U}\left(\boldsymbol{C}^-\right)^{-1}\boldsymbol{U}^{\mathrm{T}}\\ &\times\left(\sqrt{P_s+P_w^-}\boldsymbol{x}-\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\right),\end{aligned} \tag{20}$$

where

$$\boldsymbol{C}^- = \begin{bmatrix}\boldsymbol{\Omega}_d + P_w^-\boldsymbol{I}_d & \\ & \left(\dfrac{\sum_{i=d+1}^{D}\omega_i}{D-d}+P_w^-\right)\boldsymbol{I}_{D-d}\end{bmatrix}.$$

Note that there is only a single unknown variable P_w^- in the above equation, our scheme is to find P_w^- by minimizing Eq. (20) given a test data sample. Obviously, some single-variable minimization method can be used directly to solve this problem. Considering that the above algorithm is time-consuming and may suffer from local minimum, an approximated solution is suggested in the following, which is much faster though not optimal. Let us rewrite Eq. (20) as a sum of individuals along HRRP range cells,

$$\begin{aligned}d =& \sum_{i=1}^{D}\Bigg[-\ln\left(P_s+P_w^-\right)+\ln\left(\omega_i'+P_w^-\right)\\ &+\frac{1}{\omega_i'+P_w^-}\left(\sqrt{P_s+P_w^-}\boldsymbol{x}-\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\right)^{\mathrm{T}}\\ &\times\boldsymbol{u}_i\boldsymbol{u}_i^{\mathrm{T}}\left(\sqrt{P_s+P_w^-}\boldsymbol{x}-\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\right)\Bigg]\\ \triangleq& \sum_{i=1}^{D}d_i,\end{aligned} \tag{21}$$

where $\omega_i' = \omega_i$, if $i \leqslant d$; otherwise, $\omega_i' = \sum_{i=d+1}^{D}\omega_i/(D-d)$. $\boldsymbol{u}_i$ is the ith eigenvector of sample covariance matrix at high SNR. As a matter of fact, it is difficult to minimize Eq. (21) through usual algebra method, because the derivative of Eq. (21) is a fraction having a polynomial of P_w^-'s $4.5(D-1)$ powers as denominator and a polynomial of P_w^-'s $(3)^D$ powers as numerator. Nevertheless, a phenomenon we notice is that the value P_w^{-*} at which d is minimum and $P_{w,i}^{-*}$ $(i=1,2,\ldots,D)$ at which d_i $(i=1,2,\ldots,D)$ is minimum are quite close, and the former is often in the middle of the latter. Therefore, we try to find $P_{w,i}^{-*}$ and then approximated P_w^{-*} by using $P_{w,i}^{-*}$.

$$\begin{aligned}\frac{\partial d_i}{\partial P_w^-} =& -\frac{1}{P_s+P_w^-}\\ &+\frac{1}{\omega_i'+P_w^-}\left[1+\left(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{u}_i\right)^2-\frac{\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\boldsymbol{u}_i\boldsymbol{x}^{\mathrm{T}}\boldsymbol{u}_i}{\sqrt{P_s+P_w^-}}\right]\\ &-\frac{1}{(\omega_i'+P_w^-)^2}\left(\sqrt{P_s+P_w^-}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{u}_i-\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\boldsymbol{u}_i\right)^2.\end{aligned} \tag{22}$$

Let $\sqrt{P_s+P_w^-} = a$, then $\omega_i'+P_w^- = \omega_i'-P_s+a^2$, wherein deduction leads to

$$\begin{aligned}&\frac{\partial d_i}{\partial P_w^-}\\ &= \frac{1}{a^2(\omega_i'-P_s+a^2)^2}\Big\{a^3\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\boldsymbol{u}_i\boldsymbol{x}^{\mathrm{T}}\boldsymbol{u}_i\\ &\quad - a(\omega_i'-P_s)\sqrt{P_s+P_w^+}\bar{\boldsymbol{m}}^+\boldsymbol{u}_i\boldsymbol{x}^{\mathrm{T}}\boldsymbol{u}_i\\ &\quad + a^2[(\omega_i'-P_s)((\boldsymbol{x}^{\mathrm{T}}\boldsymbol{u}_i)^2-1)-(P_s+P_w^+)(\bar{\boldsymbol{m}}^+\boldsymbol{u}_i)^2]\\ &\quad - (\omega_i'-P_s)^2\Big\}.\end{aligned} \tag{23}$$

Since $a^2(\omega_i'-P_s+a^2)^2 > 0$, the minimum is achieved when the numerator equals 0. This cubic equation can be solved by Cardano formula. By analysis of the three roots and the boundary points (it is lengthy but of common mathematical method, we do not list them here to save space), $P_{w,i}^{-*}$ is obtainable. In this paper, we simply approximately P_w^{-*} by

$$P_w^- = \prod_{i=1}^{D}(p_{w,i}^-)^{1/D}. \tag{24}$$

4 Experimental results

The measured data used to evaluate the performance of proposed algorithm in this paper is same as the data used in Ref. [7]. In Fig. 2, we give the average correct

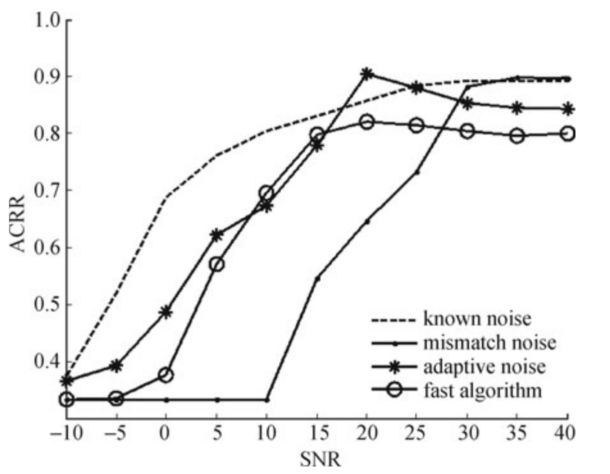

Fig. 2 Recognition performance for different algorithms under noisy case

recognition rate (ACRR) for different recognition algorithms for different SNR levels. "Known noise" means that we get different SNR level training data and test the classifier by using the test samples with the same SNR level. "Adaptive noise" denotes using training data with high SNR and estimating $P_{w,i}^{-*}$ by minimizing Eq. (20) for test data. "Fast algorithm" denotes using training data with high SNR and estimating $P_{w,i}^{-*}$ by using approximated algorithm based on Eq. (24). "Mismatch noise" denotes using training data with high SNR and applying the classifier directly to the low SNR data. The superiority of our two robust algorithms is clear: they are close to the "known noise" case, and much better than the "mismatch noise" case, especially in SNR range [5 dB, 25 dB] which may be the most useful SNR level in practice. However, the performance of fast algorithm seems not quite good at high SNR level, because the parameters estimated based on single test data sample may have larger variance values, comparing with that of the parameters estimated by the high SNR training data samples. In real applications, one can choose the recognition algorithm based on a rough SNR estimate results.

5 Conclusion

We give a brief overview for the statistical modeling algorithms for HRRP recognition. For applications, it is very important to identify a target at longer distance; therefore, noise robust recognition algorithm is practical useful. We developed a noise robust algorithm for PPCA model, in which, the noise-related parameters are estimated based on the test data sample, and the classifier is performed by using the updated model with the estimated parameter. The simulated results based on measured HRRP data show that the proposed methods have inspiring recognition performance for the low SNR data.

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