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RESEARCH ARTICLE

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On modeling and control of a flexible air-breathing hypersonic vehicle based on LPV method

Abstract This article develops a polytopic linear parameter varying (LPV) model and presents a non-fragile H_2 gain-scheduled control for a flexible air-breathing hypersonic vehicle (FAHV). First, the polytopic LPV model of the FAHV can be obtained by using Jacobian linearization and tensor-product (TP) model transformation approach, simulation verification illustrates that the polytopic LPV model captures the local nonlinearities of the original nonlinear system. Second, based on the developed polytopic LPV model, a non-fragile gain-scheduled control method is proposed in order to reduce the fragility encountered in controller implementation, a convex optimisation problem with linear matrix inequalities (LMIs) constraints is formulated for designing a velocity and altitude tracking controller, which guarantees H_2 control performance index. Finally, numerical simulations have demonstrated the effectiveness of the proposed approach.

Keywords linear parameter varying (LPV), non-fragile, gain-scheduled control, flexible air-breathing hypersonic vehicle (FAHV)

Nomenclature

$C_D(\alpha, \delta_e)$ = drag coefficient
$C_D^{\alpha_i}$ = the ith order coefficient of α contribution to $C_D(\alpha, \delta_e)$
$C_D^{\delta_e^i}$ = the ith order coefficient of δ_e contribution to $C_D(\alpha, \delta_e)$
C_D^0 = constant term in $C_D(\alpha, \delta_e)$
$C_L(\alpha, \delta_e)$ = lift coefficient
$C_L^{\alpha_i}$ = the ith order coefficient of α contribution to $C_L(\alpha, \delta_e)$
$C_L^{\delta_e}$ = coefficient of δ_e contribution to $C_L(\alpha, \delta_e)$
C_L^0 = constant term in $C_L(\alpha, \delta_e)$
$C_{M,Q}(\alpha, Q)$ = contribution to moment due to pitch rate
$C_{M,\alpha}(\alpha)$ = contribution to moment due to angle of attack
$C_{M,\delta_e}(\delta_e)$ = control surface contribution to moment
$C_{M,\alpha}^{\alpha_i}$ = the ith order coefficient of α contribution to $C_{M,\alpha}(\alpha)$
$C_{M,\alpha}^0$ = constant term in $C_{M,\alpha}(\alpha)$
$C_T^{\alpha_i}(\Phi)$ = the ith order coefficient of α in T
$\overline{c}$ = mean aerodynamic chord
c_c = canard coefficient in $C_{M,\delta_e}(\delta_e, \delta_c)$
c_e = elevator coefficient in $C_{M,\delta_e}(\delta_e, \delta_c)$
D = drag
g = acceleration due to gravity
h = altitude
I_{yy} = moment of intertia
L = lift
M_{yy} = pitching moment
m = vehicle mass
N_i = the ith generalised force
$N_i^{\alpha_j}$ = the jth order contribution of α to N_i
N_i^0 = constant term in N_i
$N_2^{\delta_e}$ = contribution of δ_e to N_2
θ = pitch angle
Q = pitch rate
$\overline{q}$ = dynamic pressure
S = reference area
T = thrust
V = velocity
x = state of the control-oriented model
α = angle of attack
$\beta_i(h, \overline{q})$ = the ith thrust fit parameter
γ = flight path angle
δ_c = canard angular deflection
δ_e = elevator angular deflection
η_i = the ith generalised elastic coordinate
ρ = density of air

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Φ = stoichiometrically normalised fuel-to-air ratio
ζ_i = damping ratio for elastic mode η_i
ω_i = natural frequency for elastic mode η_i
$1/h_s$ = air density decay rate

1 Introduction

Air-breathing hypersonic vehicles (AHVs) provide a promising and cost-effective technology to meet the objectives of commercial as well as military applications for space access and rapid global reach capabilities [1,2]. Research into AHVs started in the 1960s and continued through the 1990s with the National Aerospace Plane. More recently, NASA has successfully designed and flown the X-43A, which has affirmed the feasibility of this technology. Compared with the traditional aircraft, there are much stronger coupling effects among the aerodynamics, propulsion system and the elastic vibrations for the scramjet powered AHVs [3,4]. The problem of modeling for AHVs has drawn much attention in the past few years. A linear model for AHVs using small deviation linearized method was developed [5], the presented linear model only contained five rigid modes. Actually, full simulation models for AHVs include intricate couplings between the engine and flight dynamics, along with complex interplay between flexible and rigid modes of AHVs. A flexible air-breathing hypersonic vehicle (FAHV) model, which includes the flexible dynamics, was developed in Ref. [6]. The dynamic of this model become exceedingly complex when flexibility effects are considered, so the model can be used only for simulations or validation purposes. In Ref. [7], the authors presented a control-oriented model of AHVs considering five rigid modes and four flexible modes. The new control-oriented model is derived by replacing complex force and moment functions with curve-fitted approximation and neglecting certain weak coupling and slower portions of the system dynamics. On the basis of the control-oriented model, a quasi-linear parameter varying (quasi-LPV) model of AHVs is developed according to a first-principle model in Ref. [8]. However, these models are suitable for simulation and not useful for model-based control because of the complicated dynamic [9,10].

On the other hand, flight control design for AHVs is highly challenging because of strong coupling effects, the variable operating conditions and parametric uncertainties of flight condition. Recently, some representative control approaches have been proposed for the longitudinal dynamic model of AHVs, such as robust linear control technique [11,12], nonlinear robust adaptive control [13–15], backstepping method [16,17], nonlinear dynamic inversion method [18,19], etc. However, most of these results are based on the accurate feedback control. As a matter of fact, in the course of the actualisation of the controller, because of the existence of the parameter drift, accuracy problem and other factors, it is possible for the parameters of the controller to accrue some parameter variations, which may induce the controller fragility [20,21].

Gain-scheduled control is an engineering design approach applied widely to deal with nonlinear time-varying systems, its principle is to design a local linear controller for parameter-dependent or nonlinear time-varying systems and then obtain the global controller by using interpolation approach [22–26].

Motivated by the aforementioned reasons, first, this paper is concerned with the problem of LPV modeling for an FAHV. The developed polytopic LPV model is obtained by using Jacobian linearization and tensor-product (TP) model transformation approach and simulation verification illustrates that it captures the local nonlinearities of the origin hypersonic vehicle system; then, a non-fragile H_2 gain-scheduled control method is presented in order to reduce the fragility encountered in controller implementation. Finally, nonlinear simulation results show the excellent altitude and velocity tracking performance of the presented method.

The rest of the paper is organized as follows, the nonlinear longitudinal model and polytopic LPV model of an FAHV are presented in Section 2. Section 3 gives the main results on non-fragile H_2 gain-scheduled control for a polytopic LPV system. Simulation results are presented in Section 4. The concluding remarks are given in Section 5.

We emphasize the main contributions of this paper with the following points.

1) A polytopic LPV model is developed by using Jacobian linearization and TP model transformation approach. The resulting polytopic LPV model is suitable for linear matrix inequality (LMI)-based robust control design.

2) In order to solve the controller fragility, a non-fragile gain-scheduled control method is proposed and the designed controller guarantees H_2 control performance index. The simulation results show the effectiveness.

2 Nonlinear longitudinal model of a flexible air-breathing hypersonic vehicle

The FAHV model considered in this paper was developed by Bolender and Doman [6,27], the longitudinal sketch for the vehicle is given in Fig. 1 [28]

As discussed in Ref. [7], the weaking coupling between the rigid mode and flexible mode can be eliminated, then we can obtain the following nonlinear longitudinal model of a generic hypersonic vehicle:

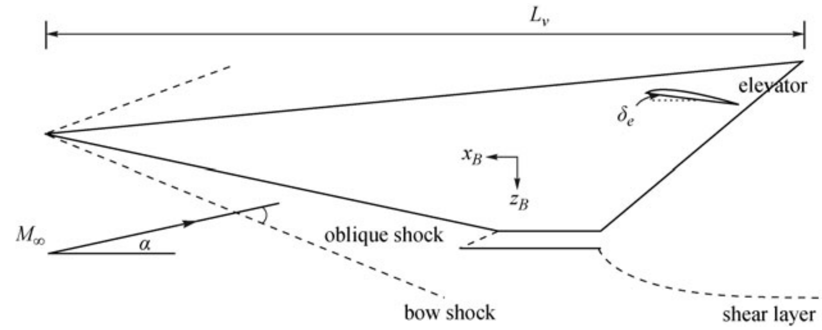

Fig. 1 Geometry of flexible air-breathing hypersonic vehicle model

$$\begin{cases} \dot{h} = V\sin(\theta-\alpha), \\ \dot{V} = \dfrac{T\cos\alpha - D}{m} - g\sin(\theta-\alpha), \\ \dot{\alpha} = \dfrac{1}{mV}(-T\sin\alpha - L) + Q + \dfrac{g}{V}\cos(\theta-\alpha), \\ \dot{\theta} = Q, \\ \dot{Q} = \dfrac{M_{yy}}{I_{yy}}, \\ \ddot{\eta}_1 = -2\zeta_1\omega_1\dot{\eta}_1 - \omega_1^2\eta_1 + N_1, \\ \ddot{\eta}_2 = -2\zeta_2\omega_2\dot{\eta}_2 - \omega_2^2\eta_2 + N_2, \end{cases} \tag{1}$$

where T, D, L, and M_{yy} denote the thrust, drag, lift, and pitching moment and are defined as follows:

$$\begin{aligned} & T \approx C_T^{\alpha^3}\alpha^3 + C_T^{\alpha^2}\alpha^2 + C_T^{\alpha}\alpha + C_T^0, \\ & D \approx \frac{1}{2}\rho V^2 S C_D(\alpha,\delta_e), \\ & L \approx \frac{1}{2}\rho V^2 S C_L(\alpha,\delta_e), \\ & M_{yy} \approx z_T T + \frac{1}{2}\rho V^2 S\bar{c}[C_{M,\alpha}(\alpha) + C_{M,\delta_e}(\delta_e)], \\ & N_1 \approx N_1^{\alpha^2}\alpha^2 + N_1^{\alpha}\alpha + N_1^0, \\ & N_2 \approx N_2^{\alpha^2}\alpha^2 + N_2^{\alpha}\alpha + N_2^{\delta_e}\delta_e + N_2^0, \end{aligned}$$

where S denotes the reference area of the vehicle, C_T, C_D, C_L, ρ are the corresponding thrust, drag, lift coefficients and the density of air, which are calculated as follows:

$$\begin{aligned} & C_T^{\alpha^3} = \beta_1(h,\bar{q})\Phi + \beta_2(h,\bar{q}), \\ & C_T^{\alpha^2} = \beta_3(h,\bar{q})\Phi + \beta_4(h,\bar{q}), \\ & C_T^{\alpha} = \beta_5(h,\bar{q})\Phi + \beta_6(h,\bar{q}), \\ & C_T^{0} = \beta_7(h,\bar{q})\Phi + \beta_8(h,\bar{q}), \\ & \bar{q} = \frac{1}{2}\rho V^2, \\ & C_D = C_D^{\alpha^2}\alpha^2 + C_D^{\alpha}\alpha + C_D^{\delta_e^2}\delta_e^2 + C_D^{\delta_e}\delta_e + C_D^0, \\ & C_L = C_L^{\alpha}\alpha + C_L^{\delta_e}\delta_e + C_L^0, \\ & \rho = \rho_0\exp\left(\frac{-(h-h_0)}{h_s}\right), \\ & C_{M,\alpha} = C_{M,\alpha}^{\alpha^2}\alpha^2 + C_{M,\alpha}^{\alpha}\alpha + C_{M,\alpha}^0, \\ & C_{M,\delta_e} = c_e\delta_e. \end{aligned}$$

This model contains nine state variables h, V, α, θ, Q, η_1, $\dot{\eta}_1$, η_2, and $\dot{\eta}_2$. The control input Φ and δ_e do not occur explicitly in the equations of general longitudinal dynamics for the FAHV model in Eq. (1); however, they appear through the forces and moments, which are denoted by T, L, D, M, N_1 and N_2. The miscellaneous coefficient values can be found in Appendix A.

2.1 LPV modeling for a flexible air-breathing hypersonic vehicle

The three approaches used to obtain the LPV model are Jacobian linearisation, state transformation, and function substitution. Among them, the Jacobian linearisation method is the most widespread methodogy. It can be used to obtain an LPV model based on a family of plants linearized with respect to a set of equilibrium points and the resulting model is a local approximation to the dynamics of nonlinear system around this set of equilibrium points. In this section, the LPV model of hypersonic vehicle will be obtained using Jacobian linearisation method.

The first step that we need to do is calculate the set point in order to obtain the LPV model of the hypersonic vehicle, so, assume the following equalities hold and choose scheduling variable vector $\rho(t) = [V \;\; h]^{\mathrm{T}}$:

$$\begin{cases} f_1 = \dot{h} = V\sin(\theta-\alpha), \\ f_2 = \dot{V} = \dfrac{T\cos\alpha - D}{m} - g\sin(\theta-\alpha), \\ f_3 = \dot{\alpha} = \dfrac{1}{mV}(-T\sin\alpha - L) + Q + \dfrac{g}{V}\cos(\theta-\alpha), \\ f_4 = \dot{\theta} = Q, \\ f_5 = \dot{Q} = \dfrac{M_{yy}}{I_{yy}}, \\ f_6 = \dot{\eta}_1, \\ f_7 = \ddot{\eta}_1 = -2\zeta_1\omega_1\dot{\eta}_1 - \omega_1^2\eta_1^2 + N_1^2, \\ f_8 = \dot{\eta}_2, \\ f_9 = \ddot{\eta}_2 = -2\zeta_2\omega_2\dot{\eta}_2 - \omega_2^2\eta_2^2 + N_2^2. \end{cases} \tag{2}$$

The LPV model of the hypersonic vehicle can be written as the following form by Jacobian linearisation method:

$$\begin{cases} \delta\dot{x}(t) = A(V,h)\delta x + B(V,h)\delta u, \\ \delta y(t) = C(V,h)\delta x, \end{cases} \tag{3}$$

where

$$\begin{aligned} &x = [h \quad V \quad \alpha \quad \theta \quad Q \quad \eta_1 \quad \dot{\eta}_1 \quad \eta_2 \quad \dot{\eta}_2]^{\mathrm{T}}, \\ &u = [\Phi \quad \delta_e]^{\mathrm{T}}, \\ &f(x_{eq}, u_{eq}) = 0, \\ &y_{eq} = g(x_{eq}, u_{eq}), \\ &\delta_x = x - x_{eq}(V,h), \\ &\delta_u = u - u_{eq}(V,h), \\ &\delta_y = y - y_{eq}(V,h). \end{aligned}$$

$(\cdot)_{eq}$ denotes the value of equilibrium point, the expression of elements of matrices $A(V,h)$, $B(V,h)$ and $C(V,h)$ are given in Appendix B.

For the tracking control of the hypersonic vehicle, the reference output signal can be defined as $r(t) = [V_r \;\; h_r]^{\mathrm{T}}$ and the actual output is $y(t) = [V_t \;\; h_t]^{\mathrm{T}}$; the tracking control problem can be solved by designing the state feedback controller such that the output of the closed system could track the given reference output signal.

$$\lim_{t\to\infty} [y(t) - r(t)] = 0. \tag{4}$$

It is desirable to include integral control action into the feedback control to eliminate the steady state error, we defined the following error integral action:

$$x_e = \int_0^t [y(\tau) - r(\tau)]\mathrm{d}\tau, \tag{5}$$

we could obtained the following augmented system:

$$\begin{cases} \dot{\hat{x}}(t) = \hat{A}(V,h)\hat{x}(t) + \hat{B}(V,h)u(t) + B_1 r(t), \\ y(t) = \hat{C}(V,h)\hat{x}(t), \end{cases} \tag{6}$$

where

$$\begin{aligned} &\hat{x}(t) = [x(t) \quad x_e]^{\mathrm{T}}, \\ &\hat{A}(V,h) = \begin{bmatrix} A(V,h) & 0 \\ C(V,h) & 0 \end{bmatrix}, \\ &\hat{B}(V,h) = \begin{bmatrix} B(V,h) \\ 0 \end{bmatrix}, \\ &\hat{C}(V,h) = \begin{bmatrix} C(V,h) & 0 \end{bmatrix}, \\ &B_1 = \begin{bmatrix} 0 \\ -I \end{bmatrix}. \end{aligned}$$

Based on TP model transformation approach [29], the LPV augmented system (6) can be written as

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ y(t) \end{bmatrix} = S \mathop{\otimes}_{n=1}^{N} A_n(p_n(t)) \begin{bmatrix} \hat{x}(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} r(t), \tag{7}$$

where row vector $A_n(p_n) \in R^{I_n}$ $(n = 1,2,\ldots,N)$ contains one bounded variable and continuous weighting functions $\alpha_{n,i_n}(p_n)$ $(i_n = 1,2,\ldots,I_n)$, $S \in R^{I_1\times I_2\times\cdots\times I_N\times O\times I}$ is constructed from LTI vertex systems $S_{i_1 i_2\cdots i_N} \in R^{O\times I}$ and $p_n(t) \in P = [V_{\min}, V_{\max}] \times [h_{\min}, h_{\max}]$.

According to Eq. (7), the LPV augmented system (6) can be transformed into the following polytopic system:

$$\begin{aligned} &S(p(t)) \\ &= \sum_{r=1}^{R} \alpha_r(p(t))S_r + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} r(t) \\ &= \begin{cases} \dot{\hat{x}}(t) = \displaystyle\sum_{i=1}^{N} \alpha_i(V,h)(\hat{A}_i\hat{x}(t) + \hat{B}_i u(t)) + B_1 r(t), \\ y(t) = \displaystyle\sum_{i=1}^{N} \alpha_i(V,h)C_i\hat{x}(t), \end{cases} \\ &= \begin{cases} \dot{\hat{x}}(t) = \bar{A}(V,h)\hat{x}(t) + \bar{B}(V,h)u(t) + B_1 r(t), \\ y(t) = \bar{C}(V,h)\hat{x}(t). \end{cases} \end{aligned} \tag{8}$$

Our control objective is to design the following non-fragile H_2 gain-scheduled state feedback controller, which make the LPV polytopic system (8) asymptotic:

$$u(t) = K(V,h)x(t) = \sum_{i}^{N} \alpha_i(t)(K_i + \Delta K)x(t), \tag{9}$$

where the gain variation of the controller $\Delta K = HPE$, $P^{\mathrm{T}}P \leqslant I$, $\sum_{i=1}^{N} \alpha_i(t) = 1$ $(N = 2^r)$, H and E are determined constant matrices.

Controller (9) is applied in system (8) and the closed-loop system can be written as

$$\begin{cases} \dot{\hat{x}}(t) = \bar{A}_c(V,h)\hat{x}(t) + B_1 r(t), \\ y(t) = \bar{C}(V,h)\hat{x}(t), \end{cases} \tag{10}$$

where $\bar{A}_c(V,h) = \bar{A}(V,h) + \bar{B}(V,h)K(V,h)$.

2.2 Verification of flexible air-breathing hypersonic vehicle polytopic LPV model

In this section, we will check if the obtained polytopic LPV model of hypersonic vehicle captures the local nonlinearities of the origin system. We take the flight condition $V = 2347$ m/s, $h = 25899$ m as an example. The two command inputs Φ and δ_e are defined as follows:

$$\Phi = \begin{cases} 0.25, & 0 \leqslant t < 2, \\ 0.35, & t \geqslant 2, \end{cases}$$

$$\delta_e = \begin{cases} 11.20, & 0 \leqslant t < 2, \\ 11.35, & t \geqslant 2. \end{cases}$$

Figures 2 and 3 show the time responses of the two command inputs for the nonlinear model and the LPV model linearized at the equilibrium point ($V = 2347$ m/s, $h = 25899$ m). From these simulation results, it is

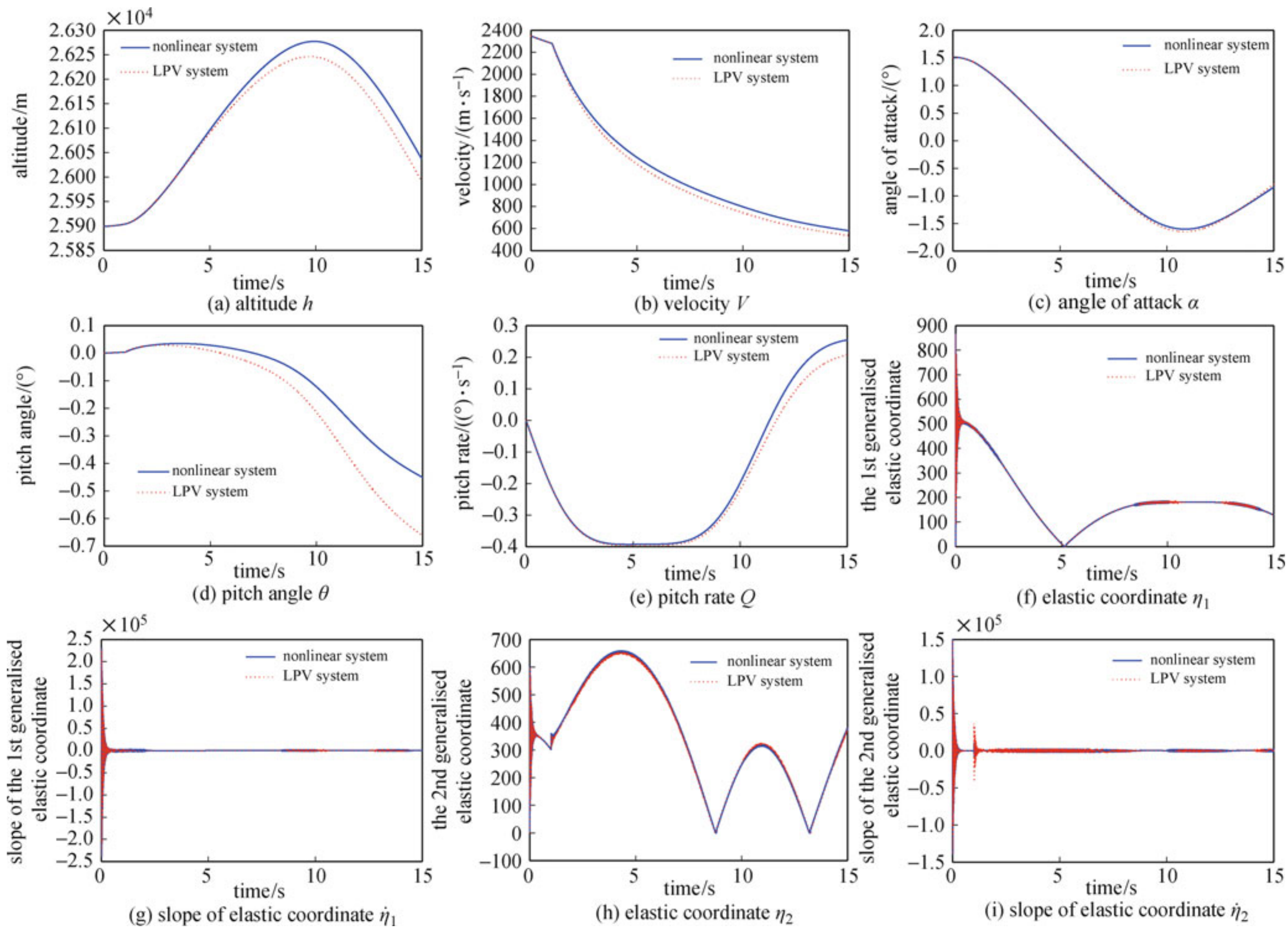

Fig. 2 Time responses for command input Φ

observed that the LPV model follows the nonlinear model quite closely, and we conclude that this model is suitable for model-based control.

3 Problem statement and main results

3.1 Preliminaries

As the foundation of the research, some important mathematical preliminaries will be presented in this section. Consider the following linear parameter-varying system:

$$\begin{cases} \dot{x}(t) = A(\theta)x(t) + B(\theta)w(t), \\ z(t) = C(\theta)x(t), \end{cases} \tag{11}$$

where $x(t) \in \Re^n$ is the state, $w(t) \in \Re^{n_w}$ is the disturbance input, $z(t) \in \Re^{n_z}$ is the controlled output. The system matrices $(A(\theta), B(\theta), C(\theta))$ are parameter-dependent matrices of compatible dimensions of time-varying parameter $\theta(t) = [\theta_1(t) \ \ \theta_2(t) \ \ \cdots \ \ \theta_r(t)]^{\mathrm{T}} \in \Re^r$. Moreover, we have the following assumption:

1) The state-space matrices $(A(\theta), B(\theta), C(\theta))$ are continuous and bounded functions and depend affinely on $\theta(t)$.

2) The real parameters $\theta(t)$, that can be known by on-line measurement values, exist in LPV plant and vary in a polytope Θ as

$$\theta(t) \in \Theta : = Co\{\omega_1, \omega_2, \ldots, \omega_N\} = \left\{ \sum_{i=1}^{N} \alpha_i(t) = 1, N = 2^r \right\}, \tag{12}$$

and the rate of variation $\dot{\theta}(t)$ are well defined at all times and vary in a polytope Θ_v as

$$\dot{\theta}(t) \in \Theta_v : = Co\{v_1, v_2, \ldots, v_N\} = \left\{ \sum_{k=1}^{N} \beta_k(t) = 1, N = 2^r \right\}. \tag{13}$$

With the above assumptions, the LPV system is called polytopic, when it ranges in a matrix polytope, the LPV system $P(\theta(t))$ can be expressed as

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & 0 \end{bmatrix} = \sum_{i=1}^{N} \alpha_i(t) \begin{bmatrix} A(\omega_i) & B(\omega_i) \\ C(\omega_i) & 0 \end{bmatrix}, \quad i = 1, 2, \ldots, N, \alpha_i \geqslant 0, \sum_{i=1}^{N} \alpha_i(t) = 1. \tag{14}$$

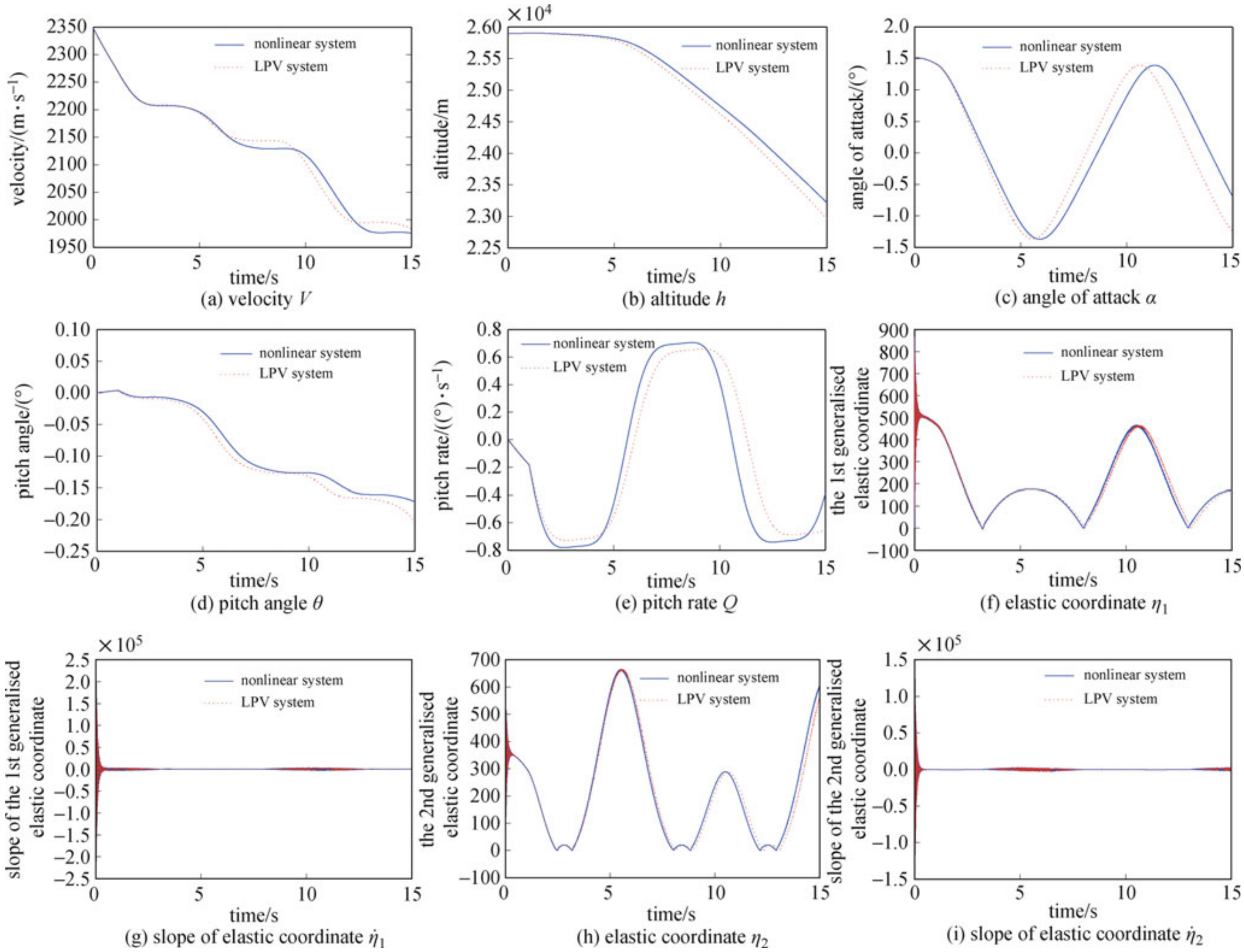

Fig. 3 Time responses for command input δ_e

Definition 1 [30] Let the system G be exponentially stable. The H_2 norm of G is define by

$$\|G\|_2^2 := \lim_{h\to\infty} E\left\{\frac{1}{h}\int_0^h z^{\mathrm{T}}(t)z(t)\mathrm{d}t\right\}, \tag{15}$$

when $x(0)=0$ and w is a zero-mean white process with an identity power spectrum density matrix, where in the above $E\{\cdot\}$ denotes mathematical expectation.

Lemma 1 [31] *Given the system G, the following conditions hold:*

1) *If the system G is exponentially stable, then*

$$\|G\|_2^2 = \lim_{h\to\infty}\frac{1}{h}\int_0^h \mathrm{Tr}[C(t)X(t)C^{\mathrm{T}}(t)]\mathrm{d}t, \tag{16}$$

where $X(t)$ satisfies

$$\begin{cases}\dot X(t) = A^{\mathrm{T}}(t)X(t) + X(t)A^{\mathrm{T}}(t) + B(t)B^{\mathrm{T}}(t),\\ X(0)=0.\end{cases} \tag{17}$$

2) *If there exists a positive definite matrix such that for all $t\in[0,\infty)$,*

$$A(t)P + PA^{\mathrm{T}}(t) + B(t)B^{\mathrm{T}}(t) < 0, \tag{18}$$

then the system G is exponentially stable and

$$\|G\|_2^2 < \lim_{h\to\infty}\frac{1}{h}\int_0^h \mathrm{Tr}[C(t)PC^{\mathrm{T}}(t)]\mathrm{d}t. \tag{19}$$

Lemma 2 [32] *If $N=N^{\mathrm{T}}$, P,Q are constant matrices of appropriate dimensions and $\Sigma\Sigma^{\mathrm{T}}\leqslant I$, then,* a1) *is equivalent to* b1).

a1) $N + P\Sigma Q + Q^{\mathrm{T}}\Sigma^{\mathrm{T}}P^{\mathrm{T}} < 0$.

b1) *There exists a constant ε such that* $N + \varepsilon PP^{\mathrm{T}} + \varepsilon^{-1}Q^{\mathrm{T}}Q < 0$.

3.2 Non-fragile H_2 gain-scheduled control design

For non-fragile gain-scheduled controller (9), consider the following LPV closed-loop system:

$$\begin{cases}\dot x(t) = A_c(\theta)x(t) + B_w(\theta)w(t),\\ z(t) = C(\theta)x(t),\end{cases} \tag{20}$$

where $A_c(\theta) = A(\theta) + B_u(\theta)K(\theta)$.

Theorem 1 *For a given state feedback non-fragile state feedback controller* (9), *the closed-loop system* (20) *is exponentially stable and* $\|G\|_2 < \gamma$, *if there exist matrices* $P_1 > 0$, Y_i, W_i *and* δ_j $(i, j = 1, 2, \ldots, 2^r)$ *such that the following LMIs hold:*

$$\begin{bmatrix} \Pi & B_{wi} & \delta_1 B_{ui} H & P_1 E^{\mathrm{T}} \\ B_{wi}^{\mathrm{T}} & -I & 0 & 0 \\ \delta_1 H^{\mathrm{T}} B_{ui}^{\mathrm{T}} & 0 & -\delta_j I & 0 \\ E P_1 & 0 & 0 & -\delta_j I \end{bmatrix} < 0, \tag{21}$$

$$\begin{bmatrix} P_1 & P_1 C_i \\ C_i P_1 & W_i \end{bmatrix} > 0, \tag{22}$$

$$\mathrm{Tr}(W_i) < 0. \tag{23}$$

Then, the nominal controller gain of non-fragile gain-scheduled controller $K_i = P_1 {Y_i}^{-1}$, *where* $\Pi = A_i P_1 + P_1 A_i + B_{ui} Y_i + {Y_i}^{\mathrm{T}} B_{ui}^{\mathrm{T}}$.

Proof By Lemma 1, we know the closed-loop system (20) $\|G\|_2 < \gamma$, there exist a matrix $P_1 > 0$ and $W(\theta)$ satisfying the following inequalities:

$$\begin{bmatrix} A_c(\theta) P + P_1 A_c^{\mathrm{T}}(\theta) & B_w(\theta) \\ B_w^{\mathrm{T}}(\theta) & -I \end{bmatrix} < 0, \tag{24}$$

$$\begin{bmatrix} P_1 & P_1 C(\theta) \\ C(\theta) P_1 & W(\theta) \end{bmatrix} > 0, \tag{25}$$

$$\mathrm{Tr}(W(\theta)) < \gamma^2. \tag{26}$$

We know Eq. (24) is equivalent to the following inequality:

$$\begin{bmatrix} \Pi_1 & B_w(\theta) \\ B_w^{\mathrm{T}}(\theta) & -I \end{bmatrix} < 0$$

$$\Leftrightarrow \sum_{i=1}^{N} a_i(t) \sum_{j=1}^{N} a_j(t) \begin{bmatrix} \Pi_2 & B_{wi} \\ B_{wi}^{\mathrm{T}} & -I \end{bmatrix} < 0, \tag{27}$$

where

$$\Pi_1 = [A(\theta) + B_u(\theta) K(\theta)] P_1 + P_1 [A(\theta) + B_u(\theta) K(\theta)]^{\mathrm{T}},$$
$$\Pi_2 = A_i P_1 + B_{ui}(K_i + \Delta K) P_1 + P_1 A_i^{\mathrm{T}} + P_1 [B_{ui}(K_i + \Delta K)]^{\mathrm{T}}.$$

Also, Eq. (27) can be written as

$$\begin{bmatrix} A_i P_1 + B_{ui} K_i P_1 + P_1 A_i^{\mathrm{T}} + P_1 K_i^{\mathrm{T}} B_{ui}^{\mathrm{T}} & B_{wi} \\ B_{wi}^{\mathrm{T}} & -I \end{bmatrix} + \begin{bmatrix} B_{ui} \Delta K P_1 + P_1 \Delta K^{\mathrm{T}} B_{ui}^{\mathrm{T}} & 0 \\ 0 & 0 \end{bmatrix} < 0. \tag{28}$$

By Lemma 2 and Eq. (28), we know that there exist δ_j $(j = 1, 2, \ldots, 2^r)$ such that the following inequality holds:

$$\begin{bmatrix} A_i P_1 + B_{ui} K_i P_1 + P_1 A_i^{\mathrm{T}} + P_1 K_i^{\mathrm{T}} B_{ui}^{\mathrm{T}} & B_{wi} \\ B_{wi}^{\mathrm{T}} & -I \end{bmatrix} + \begin{bmatrix} \delta_j B_{ui} H H^{\mathrm{T}} B_{ui}^{\mathrm{T}} + {\delta_j}^{-1} P_1 E^{\mathrm{T}} E P_1 & 0 \\ 0 & 0 \end{bmatrix} < 0. \tag{29}$$

Also, Eq. (29) can be written as

$$\begin{bmatrix} A_i P_1 + B_{ui} K_i P_1 + P_1 A_i^{\mathrm{T}} + P_1 K_i^{\mathrm{T}} B_{ui}^{\mathrm{T}} & B_{wi} \\ B_{wi}^{\mathrm{T}} & -I \end{bmatrix} + \begin{bmatrix} \delta_j B_{ui} H & P_1 E^{\mathrm{T}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} {\delta_j}^{-1} I & 0 \\ 0 & {\delta_j}^{-1} I \end{bmatrix} \cdot \begin{bmatrix} \delta_j H^{\mathrm{T}} B_{ui}^{\mathrm{T}} & 0 \\ E P_1 & 0 \end{bmatrix} < 0. \tag{30}$$

By the Schur complement lemma, we have

$$\begin{bmatrix} \Pi_3 & B_{wi} & \delta_j B_{ui} H & P_1 E^{\mathrm{T}} \\ B_{wi}^{\mathrm{T}} & -I & 0 & 0 \\ \delta_j H^{\mathrm{T}} B_{ui}^{\mathrm{T}} & 0 & \delta_j I & 0 \\ E P_1 & 0 & 0 & \delta_j I \end{bmatrix} < 0, \tag{31}$$

where

$$\Pi_3 = A_i P_1 + B_{ui} K_i P_1 + P_1 A_i^{\mathrm{T}} + P_1 K_i^{\mathrm{T}} B_{ui}^{\mathrm{T}}.$$

Set $Y_i = K_i P_1$, we can obtain LMI (21) and the nominal controller gain of non-fragile gain-scheduled controller $K_i = P_1 {Y_i}^{-1}$.

On the other hand, Eqs. (25) and (26) can be written as

$$\sum_{j=1}^{N} a_j(t) \begin{bmatrix} P_1 & P_1 C_i \\ C_i P_1 & W_i \end{bmatrix} > 0, \tag{32}$$

$$\sum_{j=1}^{N} a_j(t) \mathrm{Tr}(W_i) < 0. \tag{33}$$

From Eqs. (32) and (33), we can obtain Eqs. (22) and (23), this proof is completed.

Now, based on Theorem 1, we can design non-fragile gain-scheduled H_2 controller for FAHV.

4 Numerical simulation

To illustrate the performances attainable with the proposed method, the numerical simulations are given in this section. In this paper, The flight envelop covers altitude of $h \in [15000, 35000]$ and velocity $V \in [3000, 3400]$, the derivative of varying parameters $\dot{h} \in [-100, 100]$ and $\dot{V} \in [-10, 10]$, $\gamma = 0.8$, $H = I_2$, $E = I_{11}$, the trimmed cruise condition of the nominal flight of the vehicle are set as follows: $M = 9$, $h = 3.4 \times 10^4$ m, $V = 3200$ m/s, $\alpha = 0.02°$, $\theta = 0°$, $Q = 0°/\mathrm{s}$, $\eta_1 = 2.1$, $\dot{\eta}_1 = 0$, $\eta_2 = 1.5$, $\dot{\eta}_2 = 0$, $\Phi = 0.65$, $\delta_e = 9.81$.

The two representative cases are to show the vehicle's tracking performances for time-varying velocity and altitude command, respectively. The velocity and altitude reference trajectory are generated via the following filter:

$$H(s) = \frac{w^2}{s^2 + 2\varsigma w s + w^2}, \tag{34}$$

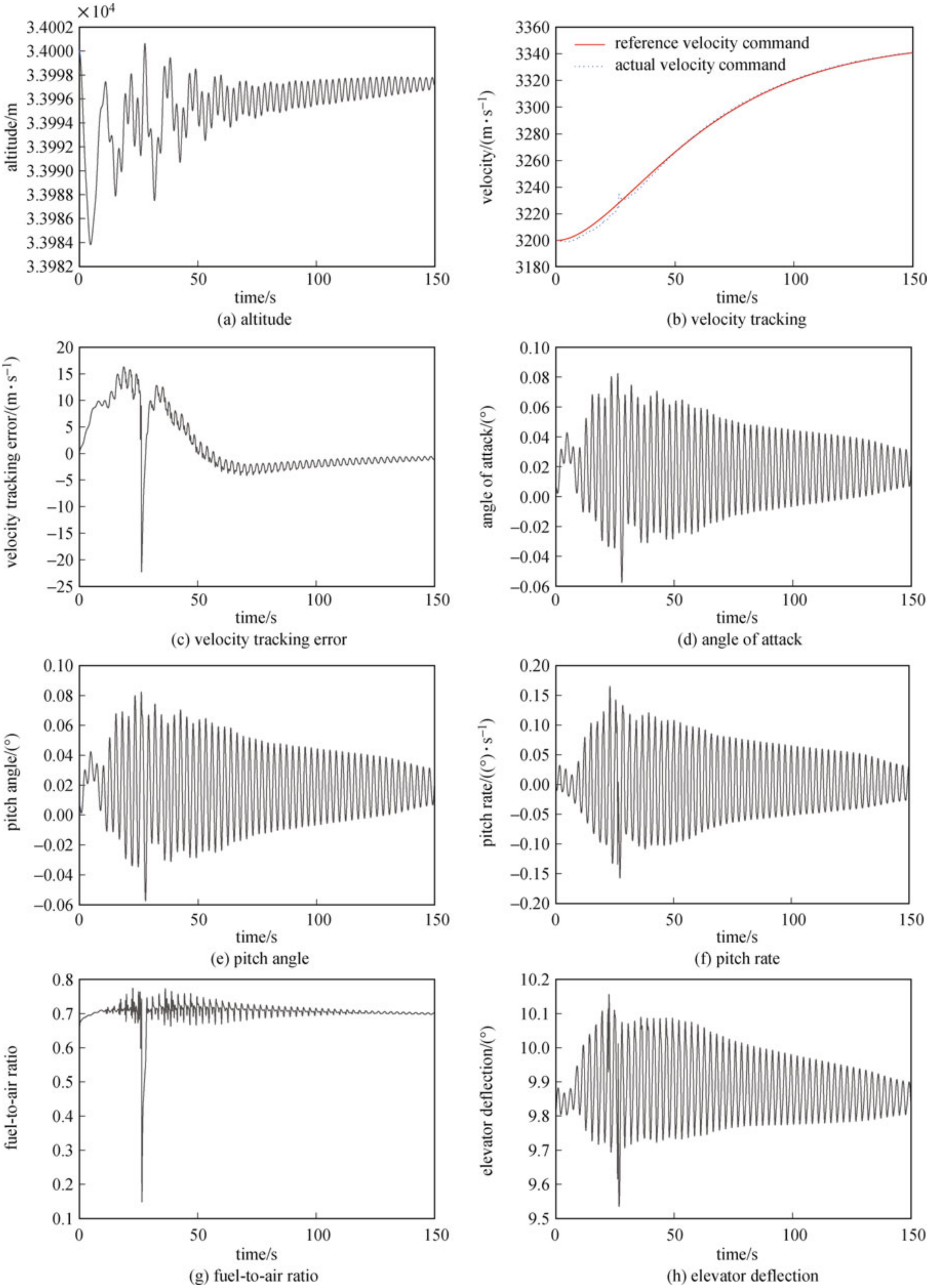

Fig. 4 Velocity tracking responses

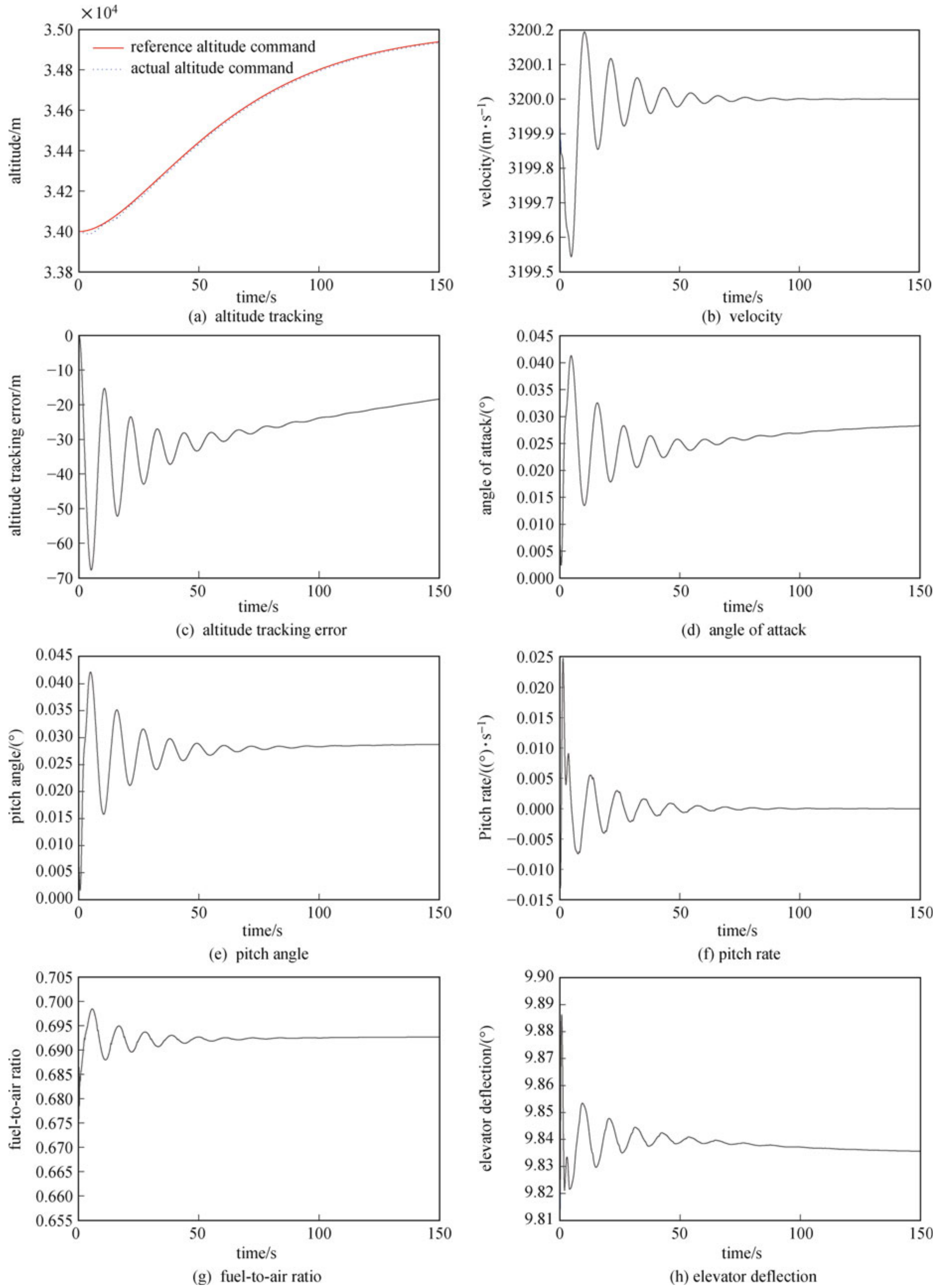

Fig. 5 Altitude tracking responses

where $\varsigma = 0.1$ and $w = 0.15$ denote damping ratio and natural frequency, respectively. By solving LMIs (21)–(23), the nominal controller gain K_i $(i = 1, 2, 3, 4)$ are given as follows:

$$K_1 = \begin{bmatrix} 0.812 & -7.1203 & 0.523 & 11.54 & 2.343 & \leftrightarrow \\ 12.63 & -0.001 & 3.6561 & 9.822 & -0.021 & \end{bmatrix}$$
$$\begin{bmatrix} -0.012 & 0.812 & 5.133 & 0.004 & -6.23 & 1.256 \\ 0.653 & -9.13 & -7.11 & 0.7453 & 0.892 & -0.002 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.728 & -8.22 & 0.925 & 9.215 & 3.227 & \leftrightarrow \\ 11.67 & -0.11 & 2.922 & 11.83 & -0.092 & \end{bmatrix}$$
$$\begin{bmatrix} -0.108 & 0.652 & 6.32 & 0.021 & -5.98 & 1.823 \\ 0.781 & -8.09 & -7.14 & 0.929 & 0.562 & -0.03 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 1.192 & -4.603 & 0.827 & 9.63 & 1.988 & \leftrightarrow \\ 12.02 & -0.013 & 2.978 & 9.101 & -0.152 & \end{bmatrix}$$
$$\begin{bmatrix} -0.025 & 0.936 & 5.821 & 0.003 & -5.92 & 0.981 \\ 0.498 & -11.25 & -6.99 & 0.925 & 0.801 & -0.021 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 2.385 & -5.726 & 1.881 & 12.37 & 2.035 & \leftrightarrow \\ 18.26 & -0.198 & 3.921 & 11.83 & -0.241 & \end{bmatrix}$$
$$\begin{bmatrix} -0.184 & 1.541 & 7.23 & 0.015 & -7.99 & 1.306 \\ 0.923 & -19.3 & -8.26 & 1.128 & 1.135 & -0.194 \end{bmatrix},$$

and α_i $(i = 1, 2, 3, 4)$ can be obtained by Eq. (9):

$$\alpha_1(t) = \frac{(h_{\max} - h)(V_{\max} - V)}{(h_{\max} - h_{\min})(V_{\max} - V_{\min})},$$
$$\alpha_2(t) = \frac{(h_{\max} - h)(V - V_{\min})}{(h_{\max} - h_{\min})(V_{\max} - V_{\min})},$$
$$\alpha_3(t) = \frac{(h - h_{\min})(V - V_{\min})}{(h_{\max} - h_{\min})(V_{\max} - V_{\min})},$$
$$\alpha_4(t) = \frac{(h - h_{\min})(V_{\max} - V)}{(h_{\max} - h_{\min})(V_{\max} - V_{\min})}.$$

Figures 4 and 5 show that the tracking performance in closed-loop for velocity and altitude, respectively. It can be seen that both altitude and velocity tracking have a satisfactory performance, the tracking errors for velocity and altitude converge to a reasonable value. Moreover, angle of attack, pitch angle, pitch rate, fuel-to-air ratio and elevator deflection are also showed satisfactory performances.

5 Conclusion

A polytopic linear parameter varying model is developed for a flexible air-breathing hypersonic vehicle and the verification of this polytopic LPV model illustrated that the developed model captures the local nonlinearities of origin nonlinear system, also, the developed polytopic LPV model is suitable for LMI-based robust control design. In addition, we present a non-fragile H_2 gain-scheduled control method based on this polytopic LPV model and take the gain variation of the controller into account in order to reduce the fragility encountered in controller implementation, the existing conditions of the desired controller are deduced by linear matrix inequalities. Nonlinear simulation results have demonstrated the effectiveness of the presented method.

Appendix A

Table A1 Miscellaneous coefficient values

coefficient	value	unit
S	1.7000×10^1	$\mathrm{ft}^2 \cdot \mathrm{ft}^{-1}$
ρ_0	6.7429×10^{-5}	$\mathrm{slug} \cdot \mathrm{ft}^{-3}$
h_0	8.5000×10^4	ft
h_s	2.1358×10^4	ft
C_L^{α}	4.6773×10	rad^{-1}
$C_L^{\delta_e}$	7.6224×10^{-1}	rad^{-1}
C_L^0	-1.8714×10^{-2}	—
$C_D^{\alpha^2}$	5.8224×10	rad^{-2}
C_D^{α}	-4.5315×10^{-2}	rad^{-1}
$C_D^{\delta_e^2}$	8.1993×10^{-1}	rad^{-2}
$C_D^{\delta_e}$	2.7699×10^{-4}	rad^{-1}
C_D^0	1.0131×10^{-2}	—
z^T	8.3600×10^0	ft
$\bar{c}$	1.7000×10^1	ft
$C_M^{\alpha^2}$	6.2926×10^0	rad^{-2}
C_M^{α}	2.1335×10^0	rad^{-1}
C_M^0	1.8979×10^{-1}	—
C_e	-1.2897×10^0	rad^{-1}
β_1	-3.7693×10^5	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{rad}^{-3}$
β_2	-3.7225×10^4	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{rad}^{-3}$
β_3	2.6814×10^4	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{rad}^{-2}$
β_4	-1.7277×10^4	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{rad}^{-2}$
β_5	3.5542×10^4	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{rad}^{-1}$
β_6	-2.4216×10^3	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{rad}^{-1}$
β_7	6.3785×10^3	$\mathrm{lb} \cdot \mathrm{ft}^{-1}$
β_8	-1.0090×10^2	$\mathrm{lb} \cdot \mathrm{ft}^{-1}$
$N_1^{\alpha^2}$	1.4013×10^3	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{slug}^{-0.5} \cdot \mathrm{rad}^{-2}$
N_1^{α}	4.5737×10^3	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{slug}^{-0.5} \cdot \mathrm{rad}^{-1}$
N_1^0	1.1752×10^2	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{slug}^{-0.5}$
$N_2^{\alpha^2}$	-5.0227×10^3	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{slug}^{-0.5} \cdot \mathrm{rad}^{-2}$
N_2^{α}	2.8633×10^3	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{slug}^{-0.5} \cdot \mathrm{rad}^{-1}$
$N_2^{\delta_e}$	1.2465×10^3	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{slug}^{-0.5} \cdot \mathrm{rad}^{-1}$
N_2^0	-4.4201×10^1	$\mathrm{lb} \cdot \mathrm{ft}^{-1} \cdot \mathrm{slug}^{-0.5}$

Appendix B

$$A(V,h) = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & a_{73} & 0 & 0 & a_{76} & a_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & a_{93} & 0 & 0 & 0 & 0 & a_{98} & a_{99} \end{bmatrix},$$

$$B(V,h) = \begin{bmatrix} 0 & 0 \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ 0 & 0 \\ b_{51} & b_{52} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & b_{92} \end{bmatrix},$$

$$C(V,h) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where

$$a_{12} = \sin[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{13} = -V\cos\left[\theta_e(V,h) - \alpha_e(V,h)\right],$$
$$a_{14} = V\cos[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{21} = \frac{1}{m}\left[\frac{\partial T}{\partial h}\cos\alpha_e(V,h) - \frac{\partial D}{\partial h}\right] + \frac{\partial g}{\partial h}\sin[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{22} = \frac{1}{m}\left[\frac{\partial T}{\partial V}\cos\alpha_e(V,h) - \frac{\partial D}{\partial V}\right],$$
$$a_{23} = \frac{1}{m}\left[\frac{\partial T}{\partial \alpha}\cos\alpha_e(V,h) - T\sin\alpha_e(V,h) - \frac{\partial D}{\partial \alpha}\right] + g\cos[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{24} = -g\cos[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{31} = -\frac{1}{mV}\left[\frac{\partial L}{\partial h} + \frac{\partial T}{\partial h}\sin\alpha_e(V,h)\right] + \frac{\partial g}{\partial h}\frac{\cos\theta_e(V,h)}{V},$$
$$a_{32} = \frac{1}{mV^2}[L + T\sin\alpha_e(V,h)] - \frac{1}{mV}\left[\frac{\partial L}{\partial V} + \frac{\partial T}{\partial V}\sin\alpha_e(V,h)\right] - \frac{g}{V^2}\cos[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{33} = -\frac{1}{mV}\left[\frac{\partial L}{\partial \alpha} + \frac{\partial T}{\partial \alpha}\sin\alpha_e(V,h) + T\cos\alpha_e(V,h)\right] + \frac{g}{V}\sin[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{34} = -\frac{g}{V}\sin[\theta_e(V,h) - \alpha_e(V,h)],$$
$$a_{51} = \frac{1}{I_{yy}}\frac{\partial M_{yy}}{\partial h},$$
$$a_{52} = \frac{1}{I_{yy}}\frac{\partial M_{yy}}{\partial V},$$
$$a_{53} = \frac{1}{I_{yy}}\frac{\partial M_{yy}}{\partial \alpha_e(V,h)},$$
$$a_{73} = \frac{\partial N_1}{\partial \alpha_e(V,h)},$$
$$a_{76} = -{\omega_1}^2,$$
$$a_{77} = -2\zeta_1\omega_1,$$
$$a_{93} = \frac{\partial N_2}{\partial \alpha_e(V,h)},$$
$$a_{98} = -{\omega_2}^2,$$
$$a_{99} = -2\zeta_2\omega_2,$$
$$b_{21} = \frac{1}{m}\frac{\partial T}{\partial \Phi_e(V,h)}\cos\alpha_e(V,h),$$
$$b_{22} = -\frac{1}{m}\frac{\partial D}{\partial \delta_{e,e}(V,h)},$$
$$b_{31} = -\frac{1}{m}\frac{\partial T}{\partial \Phi_e(V,h)}\sin\alpha_e(V,h),$$
$$b_{32} = -\frac{1}{mV}\frac{\partial L}{\partial \delta_{e,e}(V,h)},$$
$$b_{51} = \frac{1}{I_{yy}}\frac{\partial M_{yy}}{\partial \Phi_e(V,h)},$$
$$b_{52} = \frac{1}{I_{yy}}\frac{\partial M_{yy}}{\partial \delta_{e,e}(V,h)},$$
$$b_{92} = \frac{\partial N_2}{\partial \delta_{e,e}(V,h)}.$$

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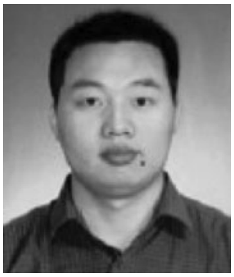

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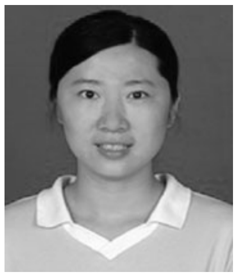

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