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RESEARCH ARTICLE

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Robust excitation control of multi-machine multi-load power systems using Hamiltonian function method

Abstract Using an energy-based Hamiltonian function method, this paper investigates the robust excitation control of multi-machine multi-load power systems described by a set of uncertain differential algebraic equations. First, we complete the dissipative Hamiltonian realization of the power system and adjust its operating point by the means of pre-feedback control. Then, based on the obtained Hamiltonian realization, we discuss the robust excitation control of the power system and put forward an H_∞ excitation control strategy. Simulation results demonstrate the effectiveness of the control scheme.

Keywords nonlinear differential algebraic systems, multi-machine multi-load power systems, dissipative Hamiltonian realization, robust excitation control

1 Introduction

By nature, power systems are continually experiencing disturbances, such as short-circuits caused by lighting or other fault conditions, changes of network parameters, mechanical oscillations, et al. Recent trends toward the full utilization of existing generation and transmission systems have increased the effects of these disturbances on the stability of power systems. To improve the stability of power systems under disturbances, many nonlinear control techniques were used to design robust excitation controllers [1–4]. However, most of these methods were based on the feedback linearization techniques, i.e., they transformed the system under consideration to a feedback equivalent linear system by canceling the inherent nonlinearities of the system. Therefore, the internal structure properties of the power systems cannot be effectively used during the controller design.

Recently, the Hamiltonian function method has drawn a considerable attention and achieved great success in the performance enhancement control of power systems [5–11]. Different from the linearization-based approaches, Hamiltonian function method employs an energy-based design principle and constructs the feedback controllers by extensively using the internal structural properties of power systems. Furthermore, the controller designed by this method is relatively simple in form. Unfortunately, most of the power system models adopted in the energy-based excitation control were the classical power system model, which were obtained from the differential algebraic system model under constant impedance load assumption [12]. This will put stringent limitation on the control performance because almost all the real loads are nonlinear.

It is known that the key procedure of the Hamiltonian function method is to draw an equivalent representation of the system under consideration as a dissipative Hamiltonian system, which can be viewed as open systems interacting with external environments through port power conjugated variables and satisfy the energy balance equation [13,14]. For the differential algebraic system model of power systems, the algebraic equations, or the power flow equations, describe the internal energy balance in the systems. Therefore, the introduction of algebraic equations will not influence the energy balance between the system and external world. Motivated by this intuition, Liu et al. proposed a dissipative Hamiltonian realization for nonlinear differential algebraic system and gave some results on the stabilization and robust controller design [15] and proposed an excitation controller for multi-machine multi-load power systems [16].

In the paper, we investigate the H_∞ robust control of

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multi-machine multi-load power systems described by uncertain nonlinear differential algebraic equations. First, we transform the considered power system to its dissipative Hamiltonian system formulation and adjust its operating point by the means of pre-feedback control. Then, we construct a robust excitation controller for the power system and analyze the stability of the closed-loop system. The proposed control scheme can extensively utilize the internal structural properties, especially the internal energy balance of the differential algebraic power system. Simulation on a six-machine eight-load power system shows the effectiveness of the proposed controller. Different from that in Ref. [16], this paper concentrates on the H_∞ control of multi-machine multi-load power systems by the means of Hamiltonian function method and put forward a decentralized robust excitation controller.

The rest of the paper is organized as follows. In Sect. 2, we propose the dynamic model of multi-machine multi-load power systems with external disturbances and get its Hamiltonian realization. Section 3 considers the H_∞ control of the power systems and constructs an excitation control strategy. In Sect. 4, we simulate a power system to illustrate the effectiveness of the proposed control scheme. Section 5 summarizes the results and draws the conclusion.

2 Dissipative Hamiltonian realization of multi-machine multi-load power systems

2.1 Dynamics of multi-machine multi-load power systems

Consider an $n+m+1$ bus power system suffered to external disturbances. The network composes of n machines and m loads connected by lossless transmission lines. Let $n+1$ be the reference node. Suppose that all the load buses are PQ buses, that is, each load corresponds to a constant active P_d and reactive Q_d demand. The dynamic model of the power system includes the dynamics of the generators and the constraints of power flows at their terminal buses and the load buses [12]:

1) The dynamics of the ith generator ($i=1,2,\ldots,n$) is given as follows:

$$\begin{cases} \dot{\delta}_i = \omega_0(\omega_i - 1), \\ \dot{\omega}_i = -\dfrac{D_i}{M_i}(\omega_i - 1) + \dfrac{1}{M_i}(P_{mi} - P_{ei}) + \dfrac{1}{M_i}d_{i1}, \\ \dot{E}'_{qi} = -\dfrac{1}{T'_{d0i}}E_{qi} + \dfrac{E_{fdi}}{T'_{d0i}} + \dfrac{1}{T'_{d0i}}d_{i2}, \\ P_{ei} = E'_{qi}I_{qi} - \left(x_{qi} - x'_{di}\right)I_{qi}I_{di}, \\ E_{qi} = E'_{qi} - (x_{di} - x'_{di})I_{di}, \end{cases} \tag{1}$$

where δ_i and ω_i are the power angle and rotor speed of the ith generator with $\omega_0 = 2\pi f_0$; E_{qi} (E'_{qi}) is the q-axis internal (transient) voltage of the ith generator; x_{qi} (x_{di}) is the q-axis (d-axis) reactance of the ith generator, while x'_{di} is the corresponding d-axis transient reactance. The generator parameters M_i is the inertia coefficient, D_i is the damping constant, and T'_{d0i} is the d-axis transient open-circuit time constant. P_{mi} is the mechanical power, assumed to be constant, and P_{ei} is the active electrical power. I_{qi} and I_{di} are the terminal bus currents of the ith generator. The control input E_{fdi} represents the voltage of the field circuit of the ith generator. d_{i1} and d_{i2} represent the external mechanical power disturbance and the electro-magnetic disturbance in the excitation circuit, respectively.

In many circumstances, one can ignore the salient pole effects, i.e., $x_{qi} \approx x'_{di}$. Let $V_i e^{j\theta_i} = (V_{qi} + jV_{di})e^{j\theta_i}$ are the terminal bus voltage magnitude and phase angle of the ith generator, in which $V_{qi} = E'_{qi} + x'_{di}I_{di}$ and $V_{di} = -x_{qi}I_{di}$ are the q-axis and d-axis terminal bus voltage amplitudes of the ith generator, we can simplify the generator dynamics as follows:

$$\begin{cases} \dot{\delta}_i = \omega_0(\omega_i - 1), \\ \dot{\omega}_i = -\dfrac{D_i}{M_i}(\omega_i - 1) + \dfrac{1}{M_i}(P_{mi} - P_{ei}) + \dfrac{1}{M_i}d_{i1}, \\ \dot{E}'_{qi} = -\dfrac{1}{T'_{d0i}}E_{qi} + \dfrac{E_{fdi}}{T'_{d0i}} + \dfrac{1}{T'_{d0i}}d_{i2}, \end{cases} \tag{2}$$

with

$$P_{ei} = \frac{1}{x'_{di}}E'_{qi}V_i\sin(\delta_i - \theta_i),$$

$$E_{qi} = \frac{x_{di}}{x'_{di}}E'_{qi} - \frac{x_{di} - x'_{di}}{x'_{di}}V_i\cos(\delta_i - \theta_i).$$

2) The terminal bus $i=1,2,\ldots,n$ of the generators satisfies the power flow equations:

$$\begin{aligned} 0 &= P_{Ti}(\delta, E'_q, \theta, V, \varphi) \\ &= \frac{1}{x'_{di}}E'_{qi}V_i\sin(\theta_i - \delta_i) + \sum_{j=1, j\neq i}^{n+1} B_{ij}V_iV_j\sin(\theta_i - \theta_j) \\ &\quad + \sum_{k=n+2}^{n+m+1} B_{ik}V_iV_k\sin(\theta_i - \varphi_k), \end{aligned} \tag{3}$$

$$\begin{aligned} 0 &= Q_{Ti}(\delta, E'_q, \theta, V, \varphi) \\ &= \frac{1}{x'_{di}}V_i^2 - \frac{1}{x'_{di}}E'_{qi}V_i\cos(\theta_i - \delta_i) - B_{ii}V_i^2 \\ &\quad - \sum_{j=1, j\neq i}^{n+1} B_{ij}V_iV_j\cos(\theta_i - \theta_j) \\ &\quad - \sum_{k=n+2}^{n+m+1} B_{ik}V_iV_k\cos(\theta_i - \varphi_k). \end{aligned} \tag{4}$$

3) All the load buses $k=n+2,n+3,\ldots,n+m+1$ satisfy the power flow equations as follows:

$$\begin{aligned} 0 &= -P_{dk} + P_{Lk}(\delta,E_q',\theta,V,\varphi) \\ &= -P_{dk} + \sum_{i=1}^{n+1} B_{ki} V_k V_i \sin(\varphi_k - \theta_i) \\ &\quad + \sum_{l=n+2,l\neq k}^{n+m+1} B_{kl} V_k V_l \sin(\varphi_k - \varphi_l), \end{aligned} \tag{5}$$

$$\begin{aligned} 0 &= -Q_{dk} + Q_{Lk}(\delta,E_q',\theta,V,\varphi) \\ &= -Q_{dk} - B_{kk} V_k^2 - \sum_{i=1}^{n+1} B_{ki} V_k V_i \cos(\varphi_k - \theta_i) \\ &\quad - \sum_{l=n+2,l\neq k}^{n+m+1} B_{kl} V_k V_l \cos(\varphi_k - \varphi_l), \end{aligned} \tag{6}$$

where V_k and φ_k are the voltage magnitude and voltage phase angle of the kth load bus, respectively. B_{ij} is the susceptance of the linear transmission line connecting bus i and j.

Equations (2)–(6) constitute the dynamic model of the multi-machine multi-load power system. Choose the state variables $x = (x_1^{\mathrm{T}},x_2^{\mathrm{T}},\ldots,x_n^{\mathrm{T}})^{\mathrm{T}}$ with $x_i = (\delta_i,\omega_i,E_{qi}')^{\mathrm{T}}$, and algebraic variables $z = (z_{g1}^{\mathrm{T}},\ldots,z_{gn}^{\mathrm{T}},z_{l,n+2}^{\mathrm{T}},\ldots,z_{l,n+m+1}^{\mathrm{T}})^{\mathrm{T}}$ with $z_{gi} = (\theta_i,\upsilon_i)^{\mathrm{T}}$ and $z_{lk} = (\varphi_k,\upsilon_k)^{\mathrm{T}}$, where $\upsilon_i = \ln V_i$, $\upsilon_k = \ln V_k$. Denote the control parameters by $u = (u_1,u_2,\ldots,u_n)^{\mathrm{T}}$ with $u_i = E_{fdi}$ and the external disturbance input variables by $d = (d_1,d_2,\ldots,d_n)^{\mathrm{T}}$ with $d_i = (d_{i1},d_{i2})^{\mathrm{T}}$. Then, the model of multi-machine multi-load power system can be compactly expressed as the following nonlinear differential algebraic system with disturbances:

$$\begin{cases} \dot{x} = f(x,z) + g(x,z)u + \tilde{g}(x,z)d, \\ 0 = \sigma(x,z), \end{cases} \tag{7}$$

where the smooth vector functions in the dynamical equations $f = [f_1^{\mathrm{T}},f_2^{\mathrm{T}},\ldots,f_n^{\mathrm{T}}]^{\mathrm{T}}$, $g = \mathrm{diag}(g_1,g_2,\ldots,g_n)$, and $\tilde{g} = \mathrm{diag}(\tilde{g}_1,\tilde{g}_2,\ldots,\tilde{g}_n)$ are

$$f_i = \begin{pmatrix} \omega_0(\omega_i - 1) \\ -\dfrac{D_i}{M_i}(\omega_i - 1) + \dfrac{1}{M_i}(P_{mi} - P_{ei}) \\ -\dfrac{1}{T_{d0i}'} E_{qi} \end{pmatrix},$$

$$g_i = \begin{pmatrix} 0 \\ 0 \\ \dfrac{1}{T_{d0i}'} \end{pmatrix},$$

$$\tilde{g}_i = \begin{pmatrix} 0 & 0 \\ \dfrac{1}{M_i} & 0 \\ 0 & \dfrac{1}{T_{d0i}'} \end{pmatrix}.$$

The functions in the algebraic equations are $\sigma = [\sigma_1^{\mathrm{T}},\ldots,\sigma_n^{\mathrm{T}},\sigma_{n+2}^{\mathrm{T}},\ldots,\sigma_{n+m+1}^{\mathrm{T}}]^{\mathrm{T}}$ with $\sigma_i = [P_{Ti},Q_{Ti}]^{\mathrm{T}}$ and $\sigma_k = [-P_{dk} + P_{Lk}, -Q_{dk} + Q_{Lk}]^{\mathrm{T}}$ $(i = 1,2,\ldots,n,\ k = n+2,n+3,\ldots,n+m+1)$.

To avoid the generation of impulses in the solution of the power system, we assume that system (7) is of index one in the neighborhood Ω of the desired equilibrium point (x_e, z_e), i.e., rank $\dfrac{\partial\sigma(x,z)}{\partial z}|_{(x_e,z_e)} = 2(n+m)$, $\forall(x,z) \in \Omega$. The assumption can be easily met because it is only invalid in the so-called impasse surface where the equilibrium point of the system is normally unlikely to reside [17].

2.2 Hamiltonian realization of multi-machine multi-load power systems

This subsection considers the dissipative Hamiltonian realization of the multi-machine multi-load power system. First, we have the following definition for the dissipative Hamiltonian realization of nonlinear differential algebraic systems [15]:

Definition 1 Consider the following nonlinear differential algebraic system:

$$\begin{cases} \dot{x} = f(x,z) + g(x,z)u, \\ 0 = \sigma(x,z), \end{cases} \tag{8}$$

where $x \in \mathbb{R}^n$, $z \in \mathbb{R}^s$, and $u \in \mathbb{R}^m$ represent the system state variables, the algebraic variables, and the control parameters, respectively. $f(\cdot)$, $g(\cdot)$, and $\sigma(\cdot)$ are sufficiently smooth vector functions with proper dimensions. Suppose there exists a continuous differentiable function $H(x,z)$ such that the nonlinear differential algebraic system (8) can be represented as

$$\begin{cases} \dot{x} = \Big(J(x,z) - R(x,z)\Big)\nabla_x H(x,z) + g(x,z)u, \\ 0 = \nabla_z H(x,z), \end{cases} \tag{9}$$

where $\nabla_x H(x,z)$ and $\nabla_z H(x,z)$ are gradient vectors of $H(x,z)$ with respect to x and z, respectively. If, pointwisely, $J(x,z)$ is skew-symmetric and $R(x,z)$ is positive semi-definite, Eq. (9) is called a dissipative Hamiltonian realization of nonlinear differential algebraic system (8), and $H(x,z)$ is the corresponding Hamiltonian function.

Noting that the Hamiltonian function generally indicates the total energy stored in the system, we can choose the widely used energy function in power system analysis as a Hamiltonian function candidate and select matrix $R(x,z)$ and $J(x,z)$ such that the matrix equation $(J(x,z)-R(x,z))\nabla_x H(x,z) = f(x,z)$ and $\nabla_z H(x,z) = \sigma(x,z)$ satisfy. For the multi-machine multi-load power system, choose the Hamiltonian function as follows:

$$\overline{H}(\delta,E_q',\theta,V,\varphi) = \sum_{i=1}^{n}\frac{1}{2}M_i\omega_0(\omega_i-1)^2 + P(\delta,E_q',\theta,V,\varphi), \tag{10}$$

where $\sum_{i=1}^{n}\frac{1}{2}M_i\omega_0(\omega_i-1)^2$ is the energy, and $P(\cdot)$ is the potential energy with the form of

$$\begin{aligned} &P(\delta,E_q',\theta,V,\varphi)\\ &=-\sum_{i=1}^{n}P_{mi}\delta_i-\sum_{k=n+2}^{n+m+1}(P_{dj}\varphi_j+Q_{dj}\upsilon_j)\\ &-\sum_{i=1}^{n}\frac{E_{qi}'e^{\upsilon_i}\cos(\delta_i-\theta_i)}{x_{di}'}-\sum_{i=1}^{n}\frac{E_{qi}'^2x_{di}}{2x_{di}'(x_{di}-x_{di}')}\\ &+\sum_{i=1}^{n}\frac{e^{2\upsilon_i}}{2}\left(\frac{1}{x_{di}'}-B_{ii}\right)-\sum_{i<j}^{n+1}B_{ij}e^{\upsilon_i+\upsilon_j}\cos(\theta_i-\theta_j)\\ &-\sum_{i=1}^{n+1}\sum_{k=n+2}^{n+m+1}B_{ik}e^{\upsilon_i+\upsilon_k}\cos(\theta_i-\varphi_k)-\sum_{k=n+2}^{n+m+1}\frac{1}{2}B_{kk}e^{2\upsilon_k}\\ &-\sum_{k<l}^{n+m+1}B_{kl}e^{\upsilon_k+\upsilon_l}\cos(\varphi_k-\varphi_l). \end{aligned} \tag{11}$$

Direct calculation shows that

$$\nabla_{xi}\overline{H} = \begin{bmatrix} -P_{mi}+p_{ei} \\ M_i\omega_0(\omega_i-1) \\ \dfrac{1}{x_{di}-x_{di}'}E_{qi} \end{bmatrix},$$

$$\nabla_{z_i}\overline{H} = \begin{bmatrix} P_{Ti} \\ Q_{Ti} \end{bmatrix},$$

$$\nabla_{z_k}\overline{H} = \begin{bmatrix} -P_{dk}+P_{Lk} \\ -Q_{dk}+Q_{Lk} \end{bmatrix}.$$

Therefore, the equation $\nabla_z\overline{H}(x,z) = \sigma(x,z)$ satisfies. Furthermore, choose matrix $J=\text{diag}(J_1,J_2,\ldots,J_n)$, $R=\text{diag}(R_1,R_2,\ldots,R_n)$ with

$$J_i = \begin{bmatrix} 0 & \dfrac{1}{M_i} & 0 \\ -\dfrac{1}{M_i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \dfrac{D_i}{M_i^2} & 0 \\ 0 & 0 & \dfrac{x_{di}-x_{di}'}{T_{d0i}'} \end{bmatrix}. \tag{12}$$

We can verify that the power system (3)–(6) can be expressed as

$$\begin{cases} \dot{x} = (J-R)\nabla_x\overline{H} + gu + \tilde{g}d, \\ 0 = \nabla_z\overline{H}. \end{cases} \tag{13}$$

Obviously, J is skew symmetric as well as J_i's and $R\geqslant 0$ as well as R_i's. Therefore, Eq. (13) is a dissipative Hamiltonian realization of the multi-machine multi-load power system.

Generally, the Hamiltonian function does not possess a strict minimum at the desired equilibrium point. Thus, before the end of this section, let us discuss the operating point adjustment of the system. Generally, we need to insert a predetermined excitation voltage E_{fd0i} to operate the power system at the desired equilibrium point (x_e,z_e), which was determined by

$$\begin{cases} 0 = \omega_0(\omega_i-1), \\ 0 = -\dfrac{D_i}{M_i}(\omega_i-1) + \dfrac{1}{M_i}(P_{mi}-P_{ei}), \\ 0 = -\dfrac{1}{T_{d0i}'}E_{qi} + \dfrac{1}{T_{d0i}'}E_{fd0i}. \end{cases} \tag{14}$$

From above equation, we have

$$E_{fd0i} = -\frac{x_{di}}{x_{di}'}E_{qie}'-\frac{x_{di}-x_{di}'}{x_{di}'}V_{ie}\cos(\delta_{ie}-\theta_{ie}), \tag{15}$$

where (V_{ie},θ_{ie}) is implicitly determined by the algebraic Eqs. (3)–(6) and generator dynamic variables $(\delta_{ie},1,E_{qie}')$. Therefore, in order to operate the power system at the desired equilibrium point, the following pre-feedback control

$$u_i = \upsilon_i + E_{fd0i}, \quad i = 1,2,\ldots,n, \tag{16}$$

is needed, where υ_i is the new reference input.

For the power system with the pre-feedback controller (16), we define a new Hamiltonian function as

$$\begin{aligned} &H(\delta,E_q',\theta,V,\varphi)\\ &= \overline{H}(\delta,E_q',\theta,V,\varphi)-\sum_{i=1}^{n}\frac{1}{x_{di}-x_{di}'}E_{qi}'E_{fd0i}. \end{aligned} \tag{17}$$

The gradient of $H(\cdot)$ becomes

$$\nabla_{x_i}H = \begin{bmatrix} -P_{mi}+P_{ei} \\ M_i\omega_0(\omega_i-1) \\ \dfrac{1}{x_{di}-x_{di}'}(E_{qi}-E_{fd0i}) \end{bmatrix},$$

$$\nabla_{z_i}H = \begin{bmatrix} P_{Ti} \\ Q_{Ti} \end{bmatrix},$$

$$\nabla_{z_k} H = \begin{bmatrix} -P_{dk} + P_{Lk} \\ -Q_{dk} + Q_{Lk} \end{bmatrix}.$$

Therefore, the nonlinear differential algebraic power system under the pre-feedback control can be expressed as

$$\begin{cases} \dot{x} = (J-R)\nabla_x H + gv + \tilde{g}d, \\ 0 = \nabla_z H, \end{cases} \tag{18}$$

where the matrix R and J are same as Eq. (12). Obviously, Eq. (18) is also a dissipative Hamiltonian realization.

Remark 1 From the dissipative Hamiltonian realization of the power system, we can see that the equilibrium point of the system is adjustable with the selection of the constant reference input E_{fd0i}, that is, for a predetermined equilibrium point, we can insert a constant reference input to adjust the system equilibrium point. Conversely, if E_{fd0i} $(i=1,2,\ldots,n)$ is selected in advance, they determine an equilibrium point.

3 H_∞ Excitation control of multi-machine multi-load power systems

Now, consider the H_∞ control of multi-machine multi-load power system suffered to external disturbances, that is, for the given $\gamma > 0$ and the penalty variable η, we will seek a state feedback control law $u = \alpha(x,z)$, $\alpha(0,0)=0$ such that the L_2 gain from the disturbance d to the estimation variable η is no more than γ, and the closed-loop system in absence of disturbances is asymptotically stable.

Before giving the main result of the paper, we have the following results for the L_2 gain analysis of uncertain nonlinear differential algebraic systems, which are a direct extension of the result of L_2 gain analysis for nonlinear systems and is useful in the robust excitation control design of multi-machine multi-load power systems:

Lemma 1 Consider the following nonlinear differential algebraic system with external disturbances:

$$\begin{cases} \dot{x} = f(x,z) + g(x,z)d, \\ 0 = \sigma(x,z), \\ \eta = h(x,z), \end{cases} \tag{19}$$

where d and η are the disturbance and estimation signal, respectively. Let $V(x,z)$ be a nonnegative definite differentiable function and satisfies $V(x_e,z_e)=0$, $\nabla_z V(x,z) = \sigma(x,z)$. For the given $\gamma > 0$, if the following Hamiltonian-Jacobi inequality holds:

$$\nabla_x^{\mathrm{T}} V(x,z) f(x,z) + \frac{1}{2\gamma^2}\nabla_x^{\mathrm{T}} V(x,z) g(x,z) g^{\mathrm{T}}(x,z) + \frac{1}{2} h^{\mathrm{T}}(x,z) h(x,z) \leqslant 0, \tag{20}$$

the L_2 gain of system (19) is no more than γ.

Generally, it is difficult to find a robust controller for uncertain nonlinear differential algebraic system. However, the problem can be solved conveniently if the system can be transformed to its Hamiltonian realization formulation. For the considered multi-machine multi-load power system, based on its dissipative Hamiltonian realization, we have the following result for the robust excitation control.

Theorem 1 For the given $\gamma > 0$ and the penalty signal

$$\eta_i = -\frac{r_i}{x_{di} - x'_{di}} \times \left[\frac{x_{di}}{x'_{di} T'_{d0i}} E'_{qi} + \frac{x_{di} - x'_{di}}{x'_{di} T'_{d0i}} V_i \cos(\delta_i - \theta_i) + \frac{E_{fd0i}}{T'_{d0i}}\right], \tag{21}$$

where $r = \mathrm{diag}(r_1, r_2, \ldots, r_n)$. If the following inequality holds:

$$\gamma \geqslant \gamma^* = \max_{1 \leqslant i \leqslant n} \frac{1}{\sqrt{2D_i}}, \tag{22}$$

an H_∞ excitation controller of the multi-machine multi-load power system can be constructed as follows:

$$\begin{aligned} u_i &= E_{fd0i} - \frac{1}{2}\left(r_i^2 + \frac{1}{\gamma^2}\right) g_i^{\mathrm{T}} \nabla_{x_i} H \\ &= E_{fd0i} + \frac{1}{2T'_{d0i}}\left(r_i^2 + \frac{1}{\gamma_i^2}\right) \\ &\quad \times \left[-\frac{x_{di}}{x'_{di}} E'_{qi} + \frac{x_{di} - x'_{di}}{x'_{di}} V_i \cos(\delta_i - \theta_i) + E_{fd0i}\right]. \end{aligned} \tag{23}$$

Proof Note that the penalty signal (21) presents the error between the excitation signal and its referee signal and thus has a significant physical meaning. Furthermore, it is easy to verify that the penalty signal can be rewritten in the form of

$$\eta_i = r_i g_i^{\mathrm{T}} \nabla_{x_i} H. \tag{24}$$

Therefore, the closed-loop system under the controller (23) can be expressed as

$$\begin{cases} \dot{x} = \left[J - R - \frac{1}{2} g\left(r^{\mathrm{T}} r + \frac{1}{\gamma^2} I_m\right) g^{\mathrm{T}}\right] \nabla_x H + \tilde{g}d, \\ 0 = \nabla_z H, \\ \eta = r g^{\mathrm{T}} \nabla_x H, \end{cases} \tag{25}$$

where $r = \mathrm{diag}(r_1, r_2, \ldots, r_n)$.

Obviously, the closed-loop system is also a dissipative Hamiltonian differential algebraic system in presence of disturbances, and Eq. (23) will be a desired H_∞ robust excitation controller if the following properties hold for the closed-loop system:

1) The Hamiltonian function $H(\cdot)$ achieves a local strict minimum at the desired equilibrium point.

2) The closed-loop system in absence of disturbances is asymptotically stable.

3) The L_2 gain of the closed-loop system is no more than γ.

For condition 1), it has been verified in Ref. [16]. We include it here for the completeness. Noting that the potential energy stored in a lossless transmission line is equal to half of the reactive power loss in the line [12], we can rewrite the Hamiltonian function (17) as follows:

$$
\begin{aligned}
H(x,z) =& \sum_{i=1}^{n}\frac{1}{2}M_i\omega_0(\omega_i-1)^2-\sum_{i=1}^{n}P_{mi}\delta_i-\sum_{j=n+2}^{n+m+1}P_{dj}\varphi_j \\
&-\sum_{j=n+2}^{n+m+1}Q_{dj}\upsilon_j+\sum_{i=1}^{n}\frac{e^{2\upsilon_i}}{2x'_{di}} \\
&-\sum_{i=1}^{n}\frac{V_iE'_{qi}\cos(\delta_i-\theta_i)}{x'_{di}} \\
&+\sum_{i<j}^{n}B_{ij}\left[\frac{V_i^2+V_j^2}{2}-V_iV_j\cos(\theta_i-\theta_j)\right] \\
&+\sum_{i=1}^{n}B_{i,n+1}\left[\frac{V_i^2}{2}-V_i\cos\theta_i\right] \\
&+\sum_{i=1}^{n}\sum_{k=n+2}^{n+m+1}B_{ik}\left[\frac{V_i^2+V_k^2}{2}-V_iV_k\cos(\theta_i-\varphi_k)\right] \\
&+\sum_{k=n+2}^{n+m+1}B_{k,n+1}\left[\frac{V_k^2}{2}-V_k\cos\varphi_k\right] \\
&+\sum_{k<l}^{n+m+1}B_{kl}\left[\frac{V_k^2+V_l^2}{2}-V_kV_l\cos(\varphi_k-\varphi_l)\right] \\
&+\sum_{i=1}^{n}\left[\frac{x_{di}E'^2_{qi}}{2x'_{di}(x_{di}-x'_{di})}-\frac{E'_{qi}E_{fd0i}}{x_{di}-x'_{di}}\right].
\end{aligned} \tag{26}
$$

It is easy to verify that

$$H_\alpha(x,z)+c\leqslant H(x,z)\leqslant H_\beta(x,z)+c, \tag{27}$$

where

$$
\begin{aligned}
H_\alpha(x,z) =& \sum_{i=1}^{n}\frac{1}{2}M_i\omega_0(\omega_i-1)^2-\sum_{i=1}^{n}P_{mi}\delta_i-\sum_{j=n+2}^{n+m+1}P_{dj}\varphi_j \\
&-\sum_{j=n+2}^{n+m+1}Q_{dj}\upsilon_j+\sum_{i<j}^{n}\frac{1}{2}B_{ij}(V_i-V_j)^2 \\
&+\sum_{i=1}^{n}\frac{1}{2}B_{i,n+1}(V_i-1)^2 \\
&+\sum_{i=1}^{n}\sum_{k=n+2}^{n+m+1}\frac{1}{2}B_{ik}(V_i-V_k)^2 \\
&+\sum_{k=n+2}^{n+m+1}\frac{1}{2}B_{k,n+1}(V_k-1)^2 \\
&+\sum_{k<l}^{n+m+1}\frac{1}{2}B_{kl}(V_k-V_l)^2 \\
&+\sum_{i=1}^{n}\frac{1}{x'_{di}}(V_i-E'_{qi})^2 \\
&+\sum_{i=1}^{n}\frac{1}{2(x_{di}-x'_{di})}(E'_{qi}-E_{fd0i})^2,
\end{aligned} \tag{28}
$$

$$
\begin{aligned}
H_\beta(x,z) =& \sum_{i=1}^{n}\frac{1}{2}M_i\omega_0(\omega_i-1)^2-\sum_{i=1}^{n}P_{mi}\delta_i-\sum_{j=n+2}^{n+m+1}P_{dj}\varphi_j \\
&-\sum_{j=n+2}^{n+m+1}Q_{dj}\upsilon_j+\sum_{i<j}^{n}\frac{1}{2}B_{ij}(V_i+V_j)^2 \\
&+\sum_{i=1}^{n}\frac{1}{2}B_{i,n+1}(V_i+1)^2 \\
&+\sum_{i=1}^{n}\sum_{k=n+2}^{n+m+1}\frac{1}{2}B_{ik}(V_i+V_k)^2 \\
&+\sum_{k=n+2}^{n+m+1}\frac{1}{2}B_{k,n+1}(V_k+1)^2 \\
&+\sum_{k<l}^{n+m+1}\frac{1}{2}B_{kl}(V_k+V_l)^2 \\
&+\sum_{i=1}^{n}\frac{1}{x'_{di}}(V_i+E'_{qi})^2 \\
&+\sum_{i=1}^{n}\frac{1}{2(x_{di}-x'_{di})}(E'_{qi}-E_{fd0i})^2,
\end{aligned} \tag{29}
$$

$$c=-\sum_{i=1}^{n}\frac{1}{2}B_{i,n+1}-\sum_{k=n+2}^{n+m+1}\frac{1}{2}B_{k,n+1}-\sum_{i=1}^{n}\frac{E^2_{fd0i}}{2(x_{di}-x'_{di})}. \tag{30}$$

Because $\delta_i\in[-\pi,\pi]$ and $\varphi_k\in[-\pi,\pi]$, $H_\alpha(x,z)$ is bounded from below. From Eq. (27), the Hamiltonian function

$H(x,z)$ is also bounded from below, and for $l > 0$, the set $\{(x,z)|H(x,z) \leqslant l\}$ is compact. Noticing that every equilibrium point of the power system is also an extremum point of $H(x,z)$, and for a real power system, there exists only one desired operating point in the interested region, we can know that $H(x,z)$ has a strict local minimum at the desired operating point (x_e,z_e).

For the property 2), choose the Lyapunov function $V(x,z) = H(x,z) - H(x_e,z_e)$, then the derivative of $H(x,z)$ along the trajectories of closed-loop system (25) in absence of disturbances is

$$\dot{V} = -\sum_{i=1}^{n}\left\{D_i[M_i\omega_0(\omega_i-1)]^2 + \frac{T_{d0i}'^2}{(x_{di}-x_{di}')^2}\left[\frac{x_{di}-x_{di}'}{T_{d0i}'} + \frac{1}{T_{d0i}'^2}\left(r_ir_i^{\mathrm{T}} + \frac{1}{\gamma_i^2}\right)\right] \times(-E_{qi}+E_{fd0i})^2\right\} \leqslant 0. \tag{31}$$

The trajectory of the closed-loop system (25) converges to the largest invariant set contained in

$$S = \left\{(x,z) : \dot{V}(x,z) = 0,\ \sigma(x,z) = 0\right\} = \{(x,z) : \omega_i = 1,\ P_{mi} = P_{ei},\ -E_{qi} + E_{fd0i} = 0,\ \sigma(x,z) = 0,\ i = 1,2,...,n\}. \tag{32}$$

It is obvious that S contains no trajectories other than the desired equilibrium point. According to the LaSalle's invariance principle of nonlinear differential algebraic systems, the closed-loop system is asymptotically stable.

As to condition 3), according to Lemma 1, we need to verify that inequality (20) holds. Note that for the closed-loop system (25), the inequality (20) is actually

$$-\nabla_x^{\mathrm{T}}H\left(R + \frac{1}{2}gr^{\mathrm{T}}rg^{\mathrm{T}} + \frac{1}{2\gamma^2}gg^{\mathrm{T}}\right)\nabla_xH + \frac{1}{2\gamma^2}\nabla_x^{\mathrm{T}}H\tilde{g}\tilde{g}^{\mathrm{T}}\nabla_xH + \frac{1}{2}\nabla_x^{\mathrm{T}}Hgr^{\mathrm{T}}rg^{\mathrm{T}}\nabla_xH = -\nabla_x^{\mathrm{T}}H\left[R + \frac{1}{2\gamma^2}\left(gg^{\mathrm{T}} - \tilde{g}\tilde{g}^{\mathrm{T}}\right)\right]\nabla_xH.$$

It is easy to verify that

$$R + \frac{1}{2\gamma^2}\left(gg^{\mathrm{T}} - \tilde{g}\tilde{g}^{\mathrm{T}}\right) = \mathrm{diag}(R_1,R_2,...,R_n) + \frac{1}{2\gamma^2}\mathrm{diag}\left(g_1g_1^{\mathrm{T}} - \tilde{g}_1\tilde{g}_1^{\mathrm{T}}, g_2g_2^{\mathrm{T}} - \tilde{g}_2\tilde{g}_2^{\mathrm{T}},...,g_ng_n^{\mathrm{T}} - \tilde{g}_n\tilde{g}_n^{\mathrm{T}}\right).$$

The main diagonal blocks in the above matrix is

$$R_i + \frac{1}{2\gamma^2}\left(g_ig_i^{\mathrm{T}} - \tilde{g}_i\tilde{g}_i^{\mathrm{T}}\right) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \dfrac{D_i}{M_i^2} - \dfrac{1}{2\gamma^2M_i^2} & 0 \\ 0 & 0 & \dfrac{x_{di}-x_{di}'}{T_{d0i}'} \end{pmatrix}.$$

Because $\gamma > \gamma^* = \max_{1\leqslant i\leqslant n}\dfrac{1}{\sqrt{2D_i}}$, we can know that $R_i + \dfrac{1}{2\gamma^2}\left(g_ig_i^{\mathrm{T}} - \tilde{g}_i\tilde{g}_i^{\mathrm{T}}\right) \geqslant 0$, and $R + \dfrac{1}{2\gamma^2}\left(gg^{\mathrm{T}} - \tilde{g}\tilde{g}^{\mathrm{T}}\right) \geqslant 0$, so the L_2 gain of the closed-loop system is no more than γ.

Remark 2 The realization of the control scheme (23) mainly depends on the realization of E_{fd0i}, E_{qi}', and $V_i\cos(\delta_i-\theta_i)$. E_{fd0i} is predetermined by the desired operating point and set in advance. Noting that for each generator, we have $P_{ei} = \dfrac{1}{x_{di}'}E_{qi}'\sin(\delta_i-\theta_i)$ and $Q_{ei} = \dfrac{V_i}{x_{di}'} - \dfrac{1}{x_{di}'}E_{qi}'V_i\cos(\delta_i-\theta_i)$, E_{qi}' and $V_i\cos(\delta_i-\theta_i)$ can be realized by the measurable variables P_{ei}, Q_{ei}, and V_i. Furthermore, the measurement errors can be regarded as equivalent external disturbances, so the proposed control scheme can work well if some of the measurements are not so accurate.

4 Simulation

A six-machine eight-load power system [18], as shown in Fig. 1, is chosen as an example to demonstrate the effectiveness of the proposed control strategy, where No. 6 machine is a synchronous condenser, and No. 1 generator represents an equivalent large power system, which is used as the reference here. The simulation is completed by the PSASP package, which is a professional testing system for power systems designed by the China Electrical Power Research Institute, Beijing, China.

We made comparisons among the following control configurations to show the difference of the effectiveness of control strategies:

Case 1 Generators No. 2–5 are equipped with the conventional nonlinear robust excitation controller (CNREC) [18].

Case 2 Generators No. 2–5 are equipped with H_∞ nonlinear excitation controller (HNEC) proposed in the paper with $r_i = 0.5$, $i = 1,2,3,4$ and $\gamma = 2.2$.

In the simulation, a three-phase temporary short-circuit fault is assumed to occur at the middle site of the transmission line between buses 11 and 12 during the time period 0.1–0.3 s, and a 5% load perturbation occurs at the same time. The simulation results are shown in Figs. 2–5,

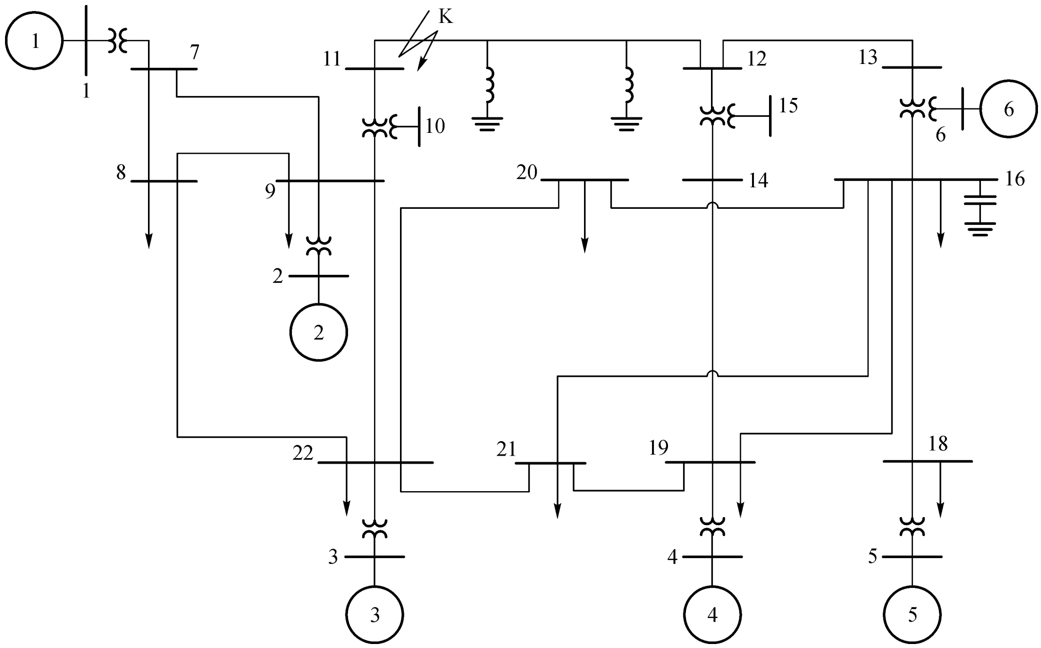

Fig. 1 Network structure

where Figs. 2 and 3 are the responses of the rotor angles under the above two control configurations ($\delta_{i1}=\delta_i-\delta_1$, $i=2,3,4,5$), Figs. 4 and 5 are the voltage responses of bus 11 and bus 18, respectively.

From above figures, it can be seen that the proposed H_∞ excitation control scheme is more effective to improve the transient stability and dynamic performance of the power systems.

5 Conclusion

The robust excitation control of multi-machine multi-load power systems is investigated based on a generalized dissipative Hamiltonian realization framework for nonlinear differential algebraic systems. First, we transform the considered power system to a Hamiltonian realization formulation. Then, we put forward a nonlinear H_∞ excitation control strategy by utilizing the internal structure properties and especially the internal energy balance of the power systems. Simulations on a six-machine eight-load power system show the effectiveness of the proposed control scheme.

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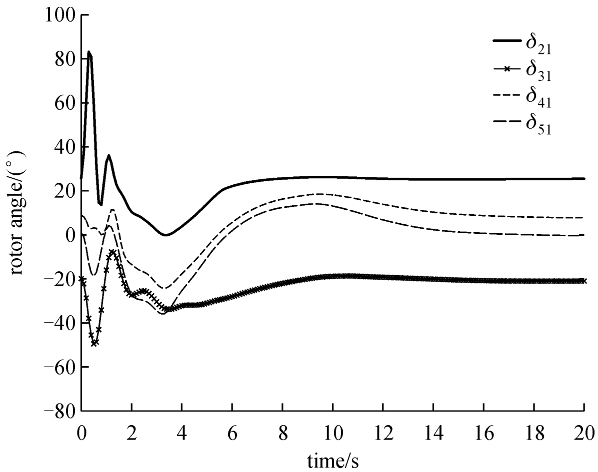

Fig. 2 Responses of generator angles under CNREC

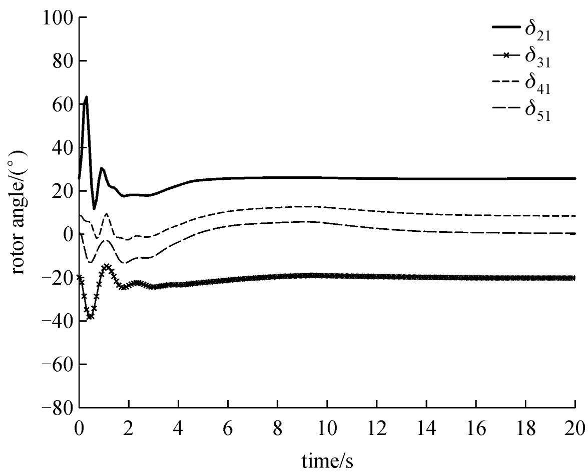

Fig. 3 Responses of generator angles under HNEC

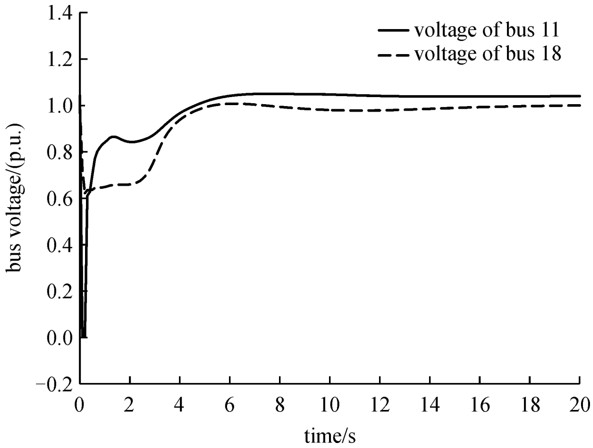

Fig. 4 Responses of bus voltages under CNREC

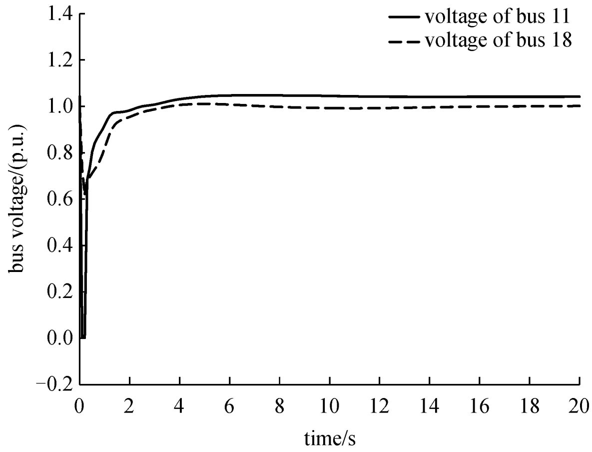

Fig. 5 Responses of bus voltages under HNEC

References

1. Wang Y, Gao G, Hill D J. Robust decentralized nonlinear controller design for multimachine power systems. Automatica, 1997, 33(9): 1725–1733
2. Akhrif O, Okou F A, Dessaint L A, Chmpagne R. Application of a multivariable feedback linearization scheme for rotor angle stability and voltage regulation of power systems. IEEE Transactions on Power Systems, 1999, 14(2): 620–628
3. Sun C, Zhao Z, Sun Y, Lu Q. Design of nonlinear robust excitation control for multimachine power systems. IEE Proceedings — Generation, Transmission and Distribution, 1996, 143(3): 253–257
4. Li G J, Lie T T, Soh C B, Yang G H. Decentralized nonlinear H_∞ control for stability enhancement in power systems. IEE Proceedings — Generation, Transmission and Distribution, 1999, 146(1): 19–24
5. Cheng D, Xi Z, Hong Y, Qin H. Energy-based stabilization in power systems. In: Proceedings of the 14th IFAC World Congress. 1999, 297–303
6. Wang Y, Cheng D, Hong Y. Stabilization of synchronous generators with the Hamiltonian function approach. International Journal of Systems Science, 2001, 32(8): 971–978
7. Galaz M, Ortega R, Bazanella A S, Stankovic A M. An energy-shaping approach to the design of excitation control of synchronous generators. Automatica, 2003, 39(1): 111–119
8. Xi Z, Cheng D. Passivity-based stabilization and H_∞ control of the Hamiltonian control systems with dissipation and its applications to power systems. International Journal of Control, 2000, 73(18): 1686–1691
9. Ortega R, Galaz M, Astolfi A, Sun Y, Shen T. Transient stabilization of multimachine power systems with nontrivial transfer conductances. IEEE Transactions on Automatic Control, 2005, 50(1): 60–75
10. Xi Z, Cheng D, Lu Q, Mei S. Nonlinear decentralized controller design for multimachine power systems using Hamiltonian function method. Automatica, 2002, 38(3): 527–534
11. Wang Y, Cheng D, Liu Y, Li C. Adaptive H_∞ excitation control of multimachine power systems via the Hamiltonian function method. International Journal of Control, 2004, 77(4): 336–350
12. Tsolas N, Arapostathis A, Varaiya P. A structure preserving energy function for power system transient stability analysis. IEEE Transactions on Circuits and Systems, 1985, 32(10): 1041–1049
13. Ortega R, van der Schaft A J, Maschke B, Escobar G. Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems. Automatica, 2002, 38(4): 585–596
14. Ortega R, van der Schaft A J, Mareels I, Maschke B. Putting energy back in control. IEEE Control Systems, 2001, 21(2): 18–33
15. Liu Y H, Li C W, Wu R B. Feedback control of nonlinear differential algebraic systems using Hamiltonian function method. Science in China, Series F: Information Sciences, 2006, 49(4): 436–445
16. Liu Y H, Li C W, Wang Y Z. Decentralized excitation control of multi-machine multi-load power systems using Hamiltonian function method. Acta Automatica Sinica, 2009, 35(7): 919–925
17. Hiskens I A, Hill D J. Energy functions, transient stability and voltage behavior in power systems with nonlinear loads. IEEE Transactions on Power Systems, 1989, 4(4): 1525–1533
18. Lu Q, Mei S, Sun Y. Nonlinear Control of Power Systems. Beijing: Tsinghua University Press, 2008 (in Chinese)