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RESEARCH ARTICLE

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Wireless multicarrier digital transmission via frames: Capacity analysis and optimization design

Abstract By treating information transmission as tiling over the time-frequency plane, we propose a digital signal transmission scheme employing *overcomplete* frames as modulation pulses. The new scheme can achieve a signaling rate larger than the Nyquist rate. We first analyze the capacity performance of the frame transmission scheme over additive white Gaussian noise (AWGN) channels. It proves that the proposed scheme can achieve the Shannon capacity asymptotically. Next, we design the Gabor frame system parameters in time-frequency dispersive channels. It is shown that the pulses shape and the time-frequency separation should be matched to the channel dispersion parameters to achieve the minimum energy perturbation. Numerical results are presented to verify the theoretical findings.

Keywords overcomplete, frame, Nyquist rate, signaling efficiency, capacity

1 Introduction

Two major technical challenges in the design of future broadband wireless networks are the need for improving the spectral efficiency and alleviating the impairment of the propagation channel. It has been recognized for long that the orthogonality of transmitted signals is one of the necessary conditions for reliable information transmission or complete reconstruction [1]. In orthogonal transmission system, the transmitted symbols are modulated using a set of orthogonal basis waveforms. Specifically, in the frequency division multiplex (FDM) and time division multiplex (TDM) modes, information symbols are transmitted in different channels that occupy separated frequency bands and different time slots, respectively, while, in the code division multiplex (CDM) mode, different information symbols are encoded with orthogonal codes for reliable separation at the receiving end. Similarly, orthogonal frequency division multiplexing (OFDM), which is regarded as a promising technique for future communications [2], also requires subcarriers to be orthogonal to each other, although it allows the aliasing of the spectra of adjacent subcarriers. Orthogonality requires the signal waveforms to be well separated in some domain — time, frequency, or code. Thus, for future broadband wireless networks with high data rate, the scarcity of the spectrum will become more and more apparent.

When transmitted over mobile radio channels, the signals usually suffer from two kinds of impairments: the intersymbol interference (ISI) caused by the multipath propagation and the interchannel interference (ICI) by the Doppler effect due to the time-varying property of each path. The ISI and ICI can be viewed as the distortion in the time and frequency domain, respectively. Due to the time and frequency dispersion, the energy of one transmitted symbol spills over into neighbouring symbols at the receiver, which causes loss of the orthogonality among these waveforms and degrades the system performance.

In Ref. [3], Kozek and Molisch proposed the non-orthogonal frequency division multiplexing (NOFDM) scheme, where time-frequency (T-F) well-localized non-orthogonal pulses are allowed for perfect transmultiplexing. It was shown [3] that nonorthogonal pulses are superior to existing orthogonal pulses (such as the Hermite functions [4]) in combating both ISI and ICI introduced by the joint T-F dispersive channels.

In conclusion, in all the existing multicarrier transmission schemes, including Strohmer's lattice-OFDM [5], the underlying requirement is that the pulses must be orthogonal or biorthogonal for perfect transmultiplexing. From the function space theory, this corresponds to critical ($TF=1$) or undercritical ($TF>1$) T-F grid in the phase

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space[1)], where T and F are the time and frequency separation, respectively. In this paper, we propose a multicarrier communication system based on overcomplete frame, which is a family of functions generated by a single window function with time-translations and frequency-modulations in the phase space. The proposed system can be viewed as an important generalization of OFDM and NOFDM in the case of $TF < 1$. This results in a higher spectral efficiency. In summary, the main contribution of this work is the use of Gabor frame with well localized pulses for digital signal transmission in order to obtain larger spectral efficiency and more robustness to the T-F doubly dispersive channels.

The rest of this paper is organized as follows. After presenting the principle of the novel wireless digital transmission via overcomplete frames in Sect. 2, we analyze its capacity performance in Sect. 3. Section 4 deals with the choice of the system parameters that are optimum in the sense of minimizing the impairments caused by the mobile radio channels. Some numerical results are provided in Sect. 5. Finally, this paper is concluded in Sect. 6.

2 Digital transmission via frames

It is commonly realized that reliable communication requires the information-bearing modulation waveforms be independent of (or even orthogonal to) one another. Namely, they constitute a Riesz (orthogonal) basis for their closed span. From the linear algebra, this is true for general symbols that are taken from a continuous real line or complex plane. However, for practical digital data taken from discrete and finite alphabets, perfect reconstruction may be achieved by correlated pulses. This is accomplished by employing *frames* as the modulation waveforms, where frame is a set of overcomplete and redundant basis functions. Conceptually, the frame is defined as follows.

Definition 1 [6] A sequence of functions $\{g_n(t)\}_{n\in\mathscr{J}}$ ($\mathscr{J}$ is a countable index set) belonging to a separable Hilbert space $\mathcal{H}$ is said to be a *frame* for $\mathcal{H}$ if there exist positive constants B_L and B_U (*frame bounds*) such that

$$B_\mathrm{L}\|g\|^2 \leqslant \sum_{n\in\mathscr{J}} |\langle g, g_n\rangle|^2 \leqslant B_\mathrm{U}\|g\|^2$$

for every $g \in \mathcal{H}$, where $\|\cdot\|$ and $\langle \cdot, \cdot \rangle$ denote the Euclidean norm and inner product, respectively. Particularly, if $B_\mathrm{L}= B_\mathrm{U}$, the frame is called *tight*.

When the frame functions $g_n(t)$ are generated from a prototype $g(t)$ by time-translations and frequency-modulations with a sufficiently high density in the T-F plane, the resulting frame $\{g_n(t)\}$ is called a *Weyl-Heisenberg* or *Gabor frame*. In recent years, frame theory has been used in digital communications for pulse shaping of multicarrier transmission [7–9], optimization of the signatures in code-division multiple-access (CDMA) systems [5,10], multiple description coding for erasure channels [11,12], design of the unitary space-time constellations [13], linear dispersion codes [14,15], etc. Next, we will show the principle of digital transmission via frames.

In linearly modulated communication systems, the general baseband transmission signal can be expressed as

$$s(t) = \sum_n c_n g_n(t), \qquad (1)$$

where c_n is the real or complex coded symbol sequence, which is usually taken from some discrete and finite alphabet Ω, and $g_n(t)$ is the modulation waveform associated with c_n. Thus, the transmitted signal $s(t)$ can be viewed as a linear combination of the function set $\{g_n(t)\}$ weighted by $\{c_n\}$. In the conventional single carrier transmission, $g_n(t)$ is usually a time-translated version of the prototype $g(t)$, that is, $g_n(t) = g(t-nT_\mathrm{s})$, where T_s is the symbol period. Without loss of generality, we assume that the elementary pulse $g(t)$ has unit energy, namely, $\int_{-\infty}^{\infty} g^*(t)g(t)\mathrm{d}t = 1$ with the "$*$" denoting complex conjugation.

The characteristic and performance of the communication system in Eq. (1) are primarily determined by the underlying function set $\{g_n(t)\}$. In the classic communication theory, it is believed that the linear independence among $\{g_n(t)\}$ is one of the necessary conditions for perfect transmultiplexing, that is, $\{g_n(t)\}$ should form an orthogonal or Riesz basis for their closed span. We are interested in the case when $g_n(t)$, $n \in \mathscr{J}$ are mutually correlated and overcomplete, namely, the number of the transmitted symbols c_n is larger than the dimensionality of the space spanned by $\{g_n(t)\}$.

When $\{g_n(t)\}$ is overcomplete, they may constitute a frame of $L^2(\mathbb{R})$, the square-integrable function space over the real number field $\mathbb{R}$. It was shown in Ref. [16] that, for digital symbols taken from discrete and finite alphabets, perfect transmultiplexing can be achieved by overcomplete frames.

Theorem 1 [16] Let $\{g_n(t)\}$ be an overcomplete function set that constitutes a frame for their closed span, and the transmitted symbols c_n come from some discrete and finite alphabet excluding zero, then $\{c_n\}$ can be perfectly recovered from the synthesized signal $s(t) = \sum_n c_n g_n(t)$.

Remark 1 When the symbols are transmitted at a Nyquist rate, $\{g_n(t)\}$ reduces to an orthogonal or Riesz basis for their closed span. In this case, the dual frame

1) The term *phase space*, which was first used in quantum mechanics, was borrowed in signal processing community to represent the time-frequency plane, considered as one geometric whole [6].

$\{\gamma_n(t)\}$ is just $\{g_n(t)\}$ itself or its biorthogonal basis, which is unique. The corresponding analysis operator L_γ maps $L^2(\mathbb{R})$ onto the whole space $l^2(\mathbb{Z})$. This means that no matter the symbol c_n is discrete or continuous, it can be completely recovered from $s(t)$ by using L_γ. Therefore, it is unnecessary to make subspace classification for the transmitted symbol vectors.

3 Capacity analysis in AWGN channels

In this section, we analyze the capacity performance of the frame transmission with equiprobable, discrete, and finite alphabets.

3.1 Nyquist rate and Shannon capacity

It is well known that the maximum reliable information transmission rate for an additive white Gaussian noise (AWGN) channel with bandwidth W is [17]

$$C = W\log\left(1+\frac{P}{WN_0}\right)\ \text{bit/s}, \tag{2}$$

where P and N_0 are the signal power and one-sided noise spectral density, respectively.[2)] Gaussian input distributions achieve this ultimate transmission rate. However, since the transmission signals are usually digital symbols taken from discrete and finite alphabets, a gap stands between the theoretical limit and the actual performance (or "constrained" information transmission capability). Unfortunately, Shannon did not provide any constructive transmission scheme on how to achieve such a limit, thereby giving birth to the discipline of channel coding.

In the derivation of the capacity formula (2), the sampling theorem was used to convert the continuous waveform channel to a set of parallel discrete channels. The sampling theorem was first suggested by Nyquist in Ref. [18]. In the classic paper [18], Nyquist investigated the problem of maximizing the symbol transmission rate under the constraint that the pulses caused no ISI at the sampling instant k/T, $k=0,\pm1,\pm2,\ldots$. In his study of the problem of distortionless transmission, Nyquist established that the maximum signaling rate that can be supported by a channel with bandwidth W Hz is $2W$ pulses per second. Such a maximal signaling rate was later named *Nyquist rate*.

In fact, the Nyquist criterion means that the maximal symbol density in the T-F plane (*signaling efficiency*) is 1 symbol/(s·Hz). Therefore, from the function space theory[3)], the Nyquist rate implies that the transmission waveforms must constitute an orthogonal basis of the T-F space band-limited to W Hz, such that the information symbols can be reliably retrieved from the synthesized signal.

In the above discussion, including Nyquist's original work, the underlying assumption is that the symbols are taken from the whole real line or complex plane. Shannon's capacity formula suggests that to achieve the maximum information transmission rate, the signal in each channel use should take on a Gaussian-like distribution. However, as shown in the above section, when the information symbols are taken from discrete alphabets, overcomplete frames can also achieve complete transmultiplexing. In fact, a frame transmission system achieves a signaling rate larger than the Nyquist rate. It actually serves as a bridge between the digital modulation and the Shannon capacity.

3.2 Capacity analysis

The received baseband signal can be written as

$$r(t) = A\sum_n c_n g_n(t) + w(t), \tag{3}$$

where A is the amplitude of each modulation pulse controlling the signal power, and $w(t)$ is the AWGN with zero mean and power spectral density N_0. For packet transmissions, we assume that the transmitted signal $s(t) = A\sum_{n=1}^{K} c_n g_n(t)$ is band limited to W Hz with essentially temporal support of T seconds, where K is the number of modulation waveforms within each packet. Thus, the dimensionality of the T-F space occupied by $s(t)$ is approximately $N=2TW$. Let $\{\varphi_i(t)\}_{i=1}^{N}$ represent an orthonormal basis for such a T-F space. By defining $r_i = \langle r(t),\varphi_i(t)\rangle$, $g_{i,n} = \langle g_n(t),\varphi_i(t)\rangle$, and $w_i = \langle w(t),\varphi_i(t)\rangle$, then Eq. (3) can be expressed in a vector form as

$$\boldsymbol{r} = A\boldsymbol{G}\boldsymbol{c} + \boldsymbol{w}, \tag{4}$$

where $\boldsymbol{r}$, $\boldsymbol{c}$, $\boldsymbol{w}$, and $\boldsymbol{G}$ are stacked by r_i, c_n, w_i, and $g_{i,n}$, respectively. Furthermore, $\boldsymbol{w}$ is a zero mean Gaussian random vector with covariance

$$\mathrm{E}\{\boldsymbol{w}\boldsymbol{w}^{\mathrm{T}}\} = \frac{N_0}{2}\boldsymbol{I}_N, \tag{5}$$

where the superscript $(\cdot)^{\mathrm{T}}$ represents the transpose and $\boldsymbol{I}_N$ is the $N\times N$ identity matrix. In order to demonstrate the capacity-approaching essence of the proposed frame-transmission scheme, here we further assume that the modulation matrix $\boldsymbol{G}$ is a random matrix independent of both $\boldsymbol{c}$ and $\boldsymbol{w}$, with i.i.d. entries. That is, the elements $g_{i,n}$ are independently selected from a fixed but arbitrary distribution with zero mean, variance $1/N$ and finite higher moments. Moreover, the realizations of the transmission matrix $\boldsymbol{G}$ and the energy-dispersion sequences $\{d_n\}$ are assumed to be available at the receiver.

2) Here and hereafter, the base of the logarithm function is set to 2.

3) The function space representation of communication systems was first used in Ref. [19], where the capacity formula (2) was derived geometrically.

Lemma 1 [20] If the entries of an $N \times K$ random matrix $\boldsymbol{H}$ are i.i.d. with zero mean and unit variance, then the empirical eigenvalue distribution[4)] of $\frac{1}{K}\boldsymbol{H}\boldsymbol{H}^{\mathrm{T}}$ converges almost surely as $N, K \to \infty$ with $N/K \to \gamma$ to the Marčenko-Pastur law whose density is given by

$$f_\gamma(\lambda) = \left(1 - \frac{1}{\gamma}\right)_+ \delta(\lambda) + \frac{\sqrt{(\lambda - \alpha) + (\beta - \lambda)_+}}{2\pi\gamma\lambda}, \ \alpha \leqslant \lambda \leqslant \beta, \tag{6}$$

where $(\cdot)_+ = \max\{0, \cdot\}$, $\delta(\cdot)$ is the Dirac delta function, and $\alpha = (1 - \sqrt{\gamma})^2$, $\beta = (1 + \sqrt{\gamma})^2$.

Some $f_\gamma(\lambda)$ with $\gamma < 1$ are illustrated in Fig. 1. Based on Lemma 1, we can establish the main result of this section.

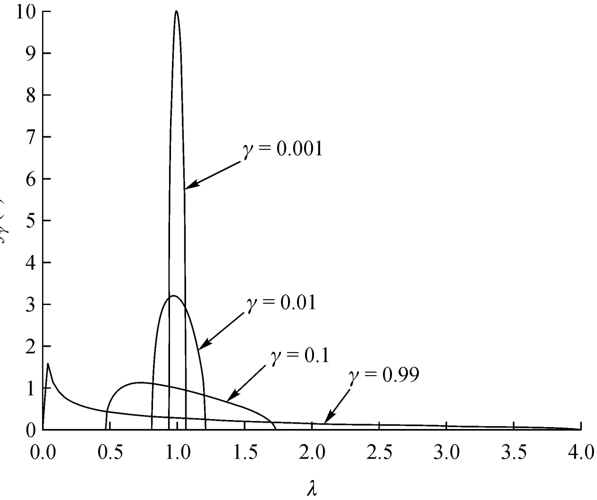

Fig. 1 Illustration of Marčenko-Pastur distribution density functions $f_\gamma(\lambda)$ for various γ

Theorem 2 The digital transmission scheme via overcomplete frames with a uniform, discrete, and finite alphabet asymptotically achieves the capacity of band-limited AWGN channels, when the data packet is of sufficient length such that the frame waveforms are sufficiently redundant or overcomplete.

Proof Let $\boldsymbol{s} = A\boldsymbol{G}\boldsymbol{c}$ be the transmitted signal vector. The Gaussian distribution of $s_i = A\sum_{n=1}^{K} g_{i,n}c_n$ as $K \to \infty$ is the immediate consequence of the central limit theorem.

On the other hand, by Lemma 1, the empirical eigenvalue distribution of $\frac{1}{K}\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}$ converges almost surely as $N, K \to \infty$ with $N/K = \gamma$ to the Marčenko-Pastur law $f_\gamma(\lambda)$ (6). Since $\left(1 - \frac{1}{\gamma}\right)_+ = 0$ when $\gamma < 1$, the first term on the right-hand side of Eq. (6) tends to zero as $\gamma \to 0$. Note also that, when $\gamma \to 0$, we have $\alpha \to 1$ and $\beta \to 1$. Since $\alpha \leqslant \lambda \leqslant \beta$, it is straightforward to deduce that $\lambda \to 1$ as $\gamma \to 0$. Combining the above results, it is concluded that $f_\gamma(\lambda)$ tends to a point mass at $\lambda = 1$, i.e., $f_\gamma(\lambda) \to \delta(\lambda - 1)$ as $\gamma \to 0$ (as evidenced in Fig. 1). That is to say, all the eigenvalues of $\frac{1}{K}\boldsymbol{G}\boldsymbol{G}^{\mathrm{T}}$ tend to be identical under the above conditions, which means that the rows of $\boldsymbol{G}$ tend to be mutually orthogonal as $N, K \to \infty$ with $N/K = \gamma \to 0$. Therefore, the columns of $\boldsymbol{G}$ tend to be a tight frame of the spanned space [6]. This implies the i.i.d. characteristic of s_i, $i = 1, 2, \ldots, N$.

Thus, for a random frame transmission, the transmitted signal vector $\boldsymbol{s}$ tends to be a Gaussian-like random vector with zero mean and covariance matrix $\frac{K}{N}A^2\boldsymbol{I}_N$ as $N, K \to \infty$. Then, the information capacity of Eq. (4) can be expressed as [21,22]

$$C = \frac{1}{2N}\log\det\left(\boldsymbol{I}_N + \frac{K}{N}\frac{A^2}{N_0/2}\boldsymbol{I}_N\right) = \frac{1}{2}\log\left(1 + \frac{K}{N}\frac{A^2}{N_0/2}\right) \text{ bit/dimension}, \tag{7}$$

where $\det(\cdot)$ denotes the matrix determinant. By noting that $KA^2/N = P/2W$ with P being the transmitted signal power, Eq. (7) can be further reduced to

$$C = \frac{1}{2}\log\left(1 + \frac{P}{WN_0}\right) \text{ bit/dimension}, \tag{8}$$

which is just the Shannon capacity of the band-limited AWGN channel with bandwidth W, signal power P, and noise power spectral density N_0.

Remark 2 From the coding point of view, we can treat each random transmission matrix $\boldsymbol{G}$ as one realization of random block coding matrix under the given constraint. Therefore, the above analysis is somewhat similar to the technique of random coding [17,22]. By the proof of Theorem 2, although the input symbols are taken from a discrete and finite alphabet, the resulting transmission signal in each dimension takes on a Gaussian-like distribution. Therefore, the capacity-achieving Gaussian random variables s_i, $i = 1, 2, \ldots, N$, are a set of *underlying* variables rather than explicit input symbols as in the conventional concept. It is also worth stressing that the Gaussian essence of $s(t)$ is *not* achieved by the conventional signal shaping technologies but by the modulation via overcomplete frame waveforms.

4) The empirical eigenvalue distribution of an $N \times N$ (Hermitian) matrix $\boldsymbol{S}$ denoted $F_{\boldsymbol{S}}^N$ is defined as $F_{\boldsymbol{S}}^N = \frac{1}{N}\sum_{n=1}^{N} 1\{\lambda_n(\boldsymbol{S}) \leqslant x\}$, where $\lambda_1(\boldsymbol{S}), \lambda_2(\boldsymbol{S}), \ldots, \lambda_N(\boldsymbol{S})$ are the eigenvalues of $\boldsymbol{S}$ and $1\{\cdot\}$ is the indicator function. So $F_{\boldsymbol{S}}^N(x)$ is the proportion of eigenvalues of $\boldsymbol{S}$ less than or equal to x.

4 System design in dispersive channels

In the previous section, we showed that frame transmission scheme can achieve the Shannon capacity of AWGN channels. In this section, we focus upon the design of Gabor system over T-F dispersive channels. Let us first review some notations and properties of the mobile channels that are necessary for system parameters design.

4.1 Characteristics of mobile radio channels

The baseband T-F (doubly) dispersive channel can be modeled as a random linear operator H with kernel $H(\tau,\upsilon)$ [23]

$$\mathrm{H}[s(t)] = \int_0^{\tau_{\max}} \int_{-f_{\mathrm{d}}}^{f_{\mathrm{d}}} H(\tau,\upsilon)s(t-\tau)\mathrm{e}^{\mathrm{j}2\pi\upsilon t}\mathrm{d}\tau\mathrm{d}\upsilon, \quad (9)$$

where $s(t)$ is the channel input, and the kernel $H(\tau,\upsilon)$, which is the Fourier transform of the time-varying impulse response of the channel $h(t,\tau)$ with respect to t, is called the delay-Doppler spread function; $\tau_{\max}$ and f_{d} are the maximum multipath delay spread and the maximum Doppler frequency, respectively. Therefore, the output signal $\mathrm{H}[s(t)]$ can be formulated as a weighted combination of the T-F shifted version of the input signal $s(t)$. In general, the time-varying multipath channel in wireless communication environment satisfies the assumption of wide sense stationary uncorrelated scattering (WSSUS), under which different delays and Doppler shifts are uncorrelated, namely,

$$\mathrm{E}\{H(\tau,\upsilon)H^*(\tau',\upsilon')\} = S_H(\tau,\upsilon)\delta(\tau-\tau',\upsilon-\upsilon'), \quad (10)$$

where $S_H(\tau,\upsilon)$ is the scattering function and characterizes the statistics of the WSSUS channel. Projections of the scattering function along τ and υ produce the delay power profile and the Doppler power spectrum, respectively. Without loss of generality, $H(\tau,\upsilon)$ is assumed to have zero mean and unit variance, i.e., $\int_0^{\tau_{\max}} \int_{-f_{\mathrm{d}}}^{f_{\mathrm{d}}} S_H(\tau,\upsilon)\mathrm{d}\tau\mathrm{d}\upsilon = 1$.

4.2 Choice of system parameters

In the previous subsection, we saw that when a signal is transmitted over mobile radio channels, the energy of one data bit will be spread out to neighbouring symbols due to the time and frequency dispersion, which produces ISI/ICI and degrades the system performance. Such energy perturbation is mainly determined by two factors: 1) energy concentration of the elementary modulation pulse — a better T-F concentrated pulse would lead to more robustness against the energy leakage, and 2) separation between the transmission pulses in the T-F plane — it is obvious that, the larger the separation, the less the perturbation among the transmitted symbols. On the other hand, the data bit rate in a Gabor transmission system depends critically on the grid parameters T and F. The product TF is a measure for the T-F density of the grid shown in Fig. 1 [24] and also determines the spectral efficiency that is approximately given by $\kappa/(TF)$, where κ is the number of bits per symbol [4]. In this paper, we only consider $\rho = 1/(TF)$ and coin the name *signaling efficiency* to represent the number of symbols per T-F unit in the phase space. Obviously, the larger the value of ρ is, the more efficiently the time and frequency resource is utilized.

In the literature, several researchers have touched the issue of matching rules of pulse shaping and T-F lattice, see, e.g., Refs. [25, ch. 10] and [26]. However, the two matching rules are derived separately in the existing literature. For example, in Refs. [27,28], only the pulse design was dealt with, while the T-F lattice matching rule was left untouched. In Ref. [25, ch. 10], the T-F lattice matching rule was suggested intuitively. As mentioned above, the ISI/ICI are determined not only by the pulse shape but also by the T-F grid parameters T and F. Moreover, most of the previous works limited the scattering function to be finitely supported and flat/constant in a rectangular or elliptic area. In the following, we will tackle the problem on how to select pulse shape and T-F grid parameters *jointly* for optimizing the system performance, given channel fading characteristics and signaling efficiency.

It is well known that Gaussian pulse

$$g^{\sigma}(t) = (2/\sigma)^{1/4}\mathrm{e}^{-(\pi/\sigma)t^2} \quad (11)$$

has the best energy concentration in the sense that it achieves the equality in the Heisenberg uncertainty principle $W_tW_f \geqslant 1/(4\pi)$, where W_t^2 and W_f^2 are the centralized temporal and spectral second-order moments, respectively [29]. In Eq. (11), the parameter σ controls the energy distribution of the Gaussian pulse in the joint time and frequency directions. To be more specific, we have $\sigma = W_t/W_f$. The Gabor system formed by the Gaussian pulse is referred to as canonical coherent states in quantum mechanics. Although the Gaussian pulse is optimal in the sense of minimum energy spread, by the Balian-Low theorem, it is unable to constitute an orthonormal basis or even Riesz basis for $L^2(\mathbb{R})$ via its T-F shifted versions [6,25]. Thus, it has to be excluded from the orthogonal transmultiplexing systems with signaling efficiency 1. Nevertheless, due to its excellent T-F concentration property, the Gaussian pulse has been chosen as the initial pulse shape to constitute a set of orthogonal functions by orthogonalization procedure [5,26]. However, as stated by Strohmer [30], the resulting orthogonalized functions will undesirably lose some T-F localization and thus lower the resistance against the ISI and ICI. Fortunately, it has been proven in Ref. [31] that the Gabor system constituted by Gaussian pulse with $TF < 1$ is surely a frame for $L^2(\mathbb{R})$. Hence, the Gaussian pulse is the reasonable choice in our system in T-F doubly dispersive channels.

Consider the energy perturbation of the appointed symbol $c_{m,n}$ from other symbols in time-varying multipath fading channel $H(\tau,\upsilon)$. For parameter optimization, we assume a data packet with an infinite number of symbols, and the cost function is given by

$$\varepsilon_I = \mathrm{E}\left\{\left|\sum_{m'=-\infty}^{+\infty}\sum_{n'=-\infty}^{+\infty} c_{m',n'}\left\langle \mathrm{H}[g_{m',n'}(t)], g_{m,n}(t)\right\rangle\right|^2\right\},$$

$$(m',n') \neq (m,n)$$

$$= \mathrm{E}\left\{\left|\sum_{m'=-\infty}^{+\infty}\sum_{n'=-\infty}^{+\infty} c_{m',n'}\left\langle \mathrm{H}[g_{m',n'}(t)], g_{m,n}(t)\right\rangle - c_{m,n}\left\langle \mathrm{H}[g_{m,n}(t)], g_{m,n}(t)\right\rangle\right|^2\right\}. \quad (12)$$

Obviously, the smaller ε_I is, the more robust to the mobile radio channels the system will be. This means that when designing a system, we should optimize (σ,T,F) to minimize the cost function ε_I, given the statistical characteristics of $H(\tau,\upsilon)$ and the prescribed signaling efficiency ρ. Under the assumptions of independent symbols and WSSUS channels, one can deduce that

$$\varepsilon_I = \sigma_c^2 \int_\tau\int_\upsilon S_H(\tau,\upsilon) \times \left[\sum_{m'=-\infty}^{+\infty}\sum_{n'=-\infty}^{+\infty} |A_g((m'-m)T+\tau,(n'-n)F+\upsilon)|^2 - |A_g(\tau,\upsilon)|^2\right] \mathrm{d}\tau\mathrm{d}\upsilon, \quad (13)$$

where

$$A_g(\tau,\upsilon) = \int_{-\infty}^{+\infty} g(t)g^*(t-\tau)\mathrm{e}^{-\mathrm{j}2\pi\upsilon t}\mathrm{d}t \quad (14)$$

is the *ambiguity function* of $g(t)$, which can be viewed as the mismatching between $g(t)$ and its T-F shifted version by τ and υ in the phase space. It can be easily checked that the ambiguity function of the Gaussian pulse is given by

$$A_{g^\sigma}(\tau,\upsilon) = \mathrm{e}^{-\frac{\pi}{2}\left(\frac{1}{\sigma}\tau^2+\sigma\upsilon^2\right)}\mathrm{e}^{-\mathrm{j}\pi\tau\upsilon} \quad (15)$$

with exponential decay in τ^2 and υ^2. Next, we will evaluate the system parameters of the proposed multicarrier transmission employing Gaussian pulse that is best localized in the T-F plane. According to the form of the channel scattering functions, we proceed with the discussion in two cases.

Case 1 Doubly dispersive channels with uniform delay power profile and uniform Doppler power spectrum.

In this case, the scattering function is expressed as

$$S_H(\tau,\upsilon) = \frac{1}{2\tau_{\max} f_\mathrm{d}}, \quad 0 \leqslant \tau \leqslant \tau_{\max}, \quad -f_\mathrm{d} < \upsilon < f_\mathrm{d}. \quad (16)$$

Then, the symbol energy perturbation function ε_I can be written as

$$\varepsilon_I = \frac{\sigma_c^2}{\tau_{\max} f_\mathrm{d}}\left\{\sum_{p\geqslant 1}\int_{(pT-\tau_{\max})/\sqrt{\sigma}}^{(pT+\tau_{\max})/\sqrt{\sigma}} \mathrm{e}^{-\pi\tau^2}\mathrm{d}\tau \times \sum_{q\geqslant 1}\int_{\sqrt{\sigma}\left(\frac{q}{\rho T}-f_\mathrm{d}\right)}^{\sqrt{\sigma}\left(\frac{q}{\rho T}+f_\mathrm{d}\right)} e^{-\pi\upsilon^2}\mathrm{d}\upsilon + \int_0^{\tau_{\max}/\sqrt{\sigma}} \mathrm{e}^{-\pi\tau^2}\mathrm{d}\tau \sum_{q\geqslant 1}\int_{\sqrt{\sigma}\left(\frac{q}{\rho T}-f_\mathrm{d}\right)}^{\sqrt{\sigma}\left(\frac{q}{\rho T}+f_\mathrm{d}\right)} \mathrm{e}^{-\pi\upsilon^2}\mathrm{d}\upsilon + \sum_{p\geqslant 1}\int_{(pT-\tau_{\max})/\sqrt{\sigma}}^{(pT+\tau_{\max})/\sqrt{\sigma}} \mathrm{e}^{-\pi\tau^2}\mathrm{d}\tau \int_0^{\sqrt{\sigma}f_\mathrm{d}} \mathrm{e}^{-\pi\upsilon^2}\mathrm{d}\upsilon\right\}, \quad (17)$$

which is minimized at $\sigma = T/F = \tau_{\max}/f_\mathrm{d}$. Since $\sigma = W_t/W_f$ for Gaussian pulse, one has

$$\frac{T}{F} = \frac{W_t}{W_f} = \frac{\tau_{\max}}{f_\mathrm{d}}, \quad (18)$$

which indicates that for uniform scattering function the time and frequency spacings between the symbols in the phase space and the durations of the individual pulse in the temporal and spectral direction should be matched to the maximum multipath delay and the maximum Doppler frequency, respectively.

Remark 3 A similar suggestion is given by Kozek in Ref. [25, ch. 10], where $W_t/W_f = \tau_{\max}/f_\mathrm{d}$ (atom adaptation) was derived asymptotically as $\tau_{\max} f_\mathrm{d} \rightarrow 0$ in the context of approximate diagonalization of trace-class underspread operators, while $T/F = \tau_{\max}/f_\mathrm{d}$ (grid adaptation) was drawn in an intuitive way. Interestingly, by considering the joint optimization of both the grid parameters and the atom shape from the minimum energy perturbation point of view, we obtain the two matching criteria *simultaneously* without deploying the constraint of $\tau_{\max} f_\mathrm{d} \rightarrow 0$.

Remark 4 A particular form of the uniform scattering is $S_H(\tau,\upsilon) = \delta(\tau)\delta(\upsilon)$, representing no time and frequency dispersion. In this case, the matching criteria (18) is reduced to $T/F = W_t/W_f$.

Case 2 Doubly dispersive channels with exponential delay power profile and U-shape Doppler power spectrum.

In this case, the scattering function is given by

$$S_H(\tau,\upsilon) = \frac{1}{\tau_{\mathrm{rms}}}\mathrm{e}^{-\tau/\tau_{\mathrm{rms}}}\frac{1/(\pi f_\mathrm{d})}{\sqrt{1-(\upsilon/f_\mathrm{d})^2}}, \ \tau > 0, \ |\upsilon| < f_\mathrm{d}, \quad (19)$$

where τ_{rms} is called the root-mean-square (rms) delay spread. Then, the symbol energy perturbation function ε_I can be written as follows: for $(m', n')\neq(m, n)$,

$$\varepsilon_I = \frac{\sigma_c^2}{\tau_{\rm rms}\pi f_d}\sum_{m'=-\infty}^{+\infty}\sum_{n'=-\infty}^{+\infty}\int_0^{+\infty}\int_{-f_d}^{f_d}\frac{e^{-\tau/\tau_{\rm rms}}}{\sqrt{1-(\upsilon/f_d)^2}} \times e^{-\pi\left[\frac{1}{\sigma}((m'-m)T+\tau)^2+\sigma((n'-n)F+\upsilon)^2\right]}d\tau d\upsilon$$
$$= \frac{\sigma_c^2}{\tau_{\rm rms}\pi f_d}\sum_{p=-\infty}^{+\infty}\sum_{q=-\infty}^{+\infty}\int_0^{+\infty}\int_{-f_d}^{f_d}\frac{e^{-\tau/\tau_{\rm rms}}}{\sqrt{1-(\upsilon/f_d)^2}} \times e^{-\pi\left[\frac{1}{\sigma}(pT+\tau)^2+\sigma(qF+\upsilon)^2\right]}d\tau d\upsilon, \quad (p,q)\neq(0,0)$$
$$= \frac{\sigma_c^2}{\tau_{\rm rms}\pi f_d}\left\{\sum_p\int_0^{+\infty}e^{-\frac{\tau}{\tau_{\rm rms}}}e^{-\frac{\pi}{\sigma}(pT+\tau)^2}d\tau \times\sum_q\int_{-f_d}^{f_d}\frac{e^{-\pi\sigma(qF+\upsilon)^2}}{\sqrt{1-(\upsilon/f_d)^2}}d\upsilon - \int_0^{+\infty}e^{-\frac{\tau}{\tau_{\rm rms}}}e^{-\frac{\pi}{\sigma}\tau^2}d\tau\int_{-f_d}^{f_d}\frac{e^{-\pi\sigma\upsilon^2}}{\sqrt{1-(\upsilon/f_d)^2}}d\upsilon\right\}. \quad (20)$$

It seems hard to arrive at an analytical expression of the optimal parameters triplet (σ,T,F) for this case. Alternatively, we will proceed by numerical method. Note that the integral kernels in Eq. (20) have exponential decay in $(pT+\tau)^2$ and $(qF+\upsilon)^2$, respectively. Moreover, for multicarrier transmission in underspread channels ($\tau_{\rm rms}f_d \ll 1$), the conditions $T\gg\tau_{\rm rms}$, $F\gg f_d$ are usually satisfied. Thus, when we evaluate the optimal system parameters, it is sufficient to consider only the energy perturbation from the neighbouring symbols with negligible performance loss.

We first evaluate ε_I for $\tau_{\rm rms}$ = 2 μs, f_d = 100 Hz, and ρ = 1.5, and find that the minimum of ε_I exists at least in the considered ranges of T and σ. A further investigation shows that, for a fixed T, the minimum of the energy perturbation function is almost achieved at $\sigma=\rho T^2$, which implies the matching criteria $W_t/W_f = T/F$. By calculating the ratios $\frac{\sigma_{\rm opt}}{\rho T_{\rm opt}^2}$ and $\frac{\sigma_{\rm opt}}{\tau_{\rm rms}/f_d}$ for different signaling efficiencies ρ's and various channel fading conditions ($\tau_{\rm rms}/f_d$ ranges from 0.5×10^{-8} to 4×10^{-8}), it is seen that $\frac{\sigma_{\rm opt}}{\rho T_{\rm opt}^2}$ is almost fixed to 1 and $\frac{\sigma_{\rm opt}}{\tau_{\rm rms}/f_d}$ is centered about 1.5 in all cases regardless of $\tau_{\rm rms}$, f_d, and ρ. Therefore, the optimal system parameters for the T-F dispersive channels with exponential delay power profile and U-shape Doppler power spectrum can be approximately expressed as

$$\sigma = \frac{W_t}{W_f} = \frac{T}{F} = 1.5\frac{\tau_{\rm rms}}{f_d}. \quad (21)$$

5 Numerical results

In this section, we present some numerical results to demonstrate the above theoretical findings.

5.1 Capacity-approaching property of frame transmission schemes

In this subsection, we study the capacity-approaching properties of the frame transmission schemes in terms of spectral efficiency. Here, the temporal interval T and bandwidth W are set to 0.05 ms and 1 MHz, respectively. Therefore, the dimensionality of the corresponding T-F space is $N=2TW=100$. A binary alphabet $\Omega=\{+1,-1\}$ is assumed. Moreover, the AWGN spectral density N_0 is set to −174 dBm/Hz.

Under the assumption of Gaussian distribution of the transmitted signals s_i, $i=1,2,...,N$, the bandwidth efficiency of Eq. (4) is calculated as [21]

$$C(\boldsymbol{G}) = \frac{1}{N}\log\det\left(\boldsymbol{I}_N + \frac{A^2}{N_0/2}\boldsymbol{G}\boldsymbol{G}^{\rm T}\right) = \frac{1}{N}\sum_{n=1}^{N}\log\left[1+\frac{A^2}{N_0/2}\lambda_n(\boldsymbol{G}\boldsymbol{G}^{\rm T})\right]\ \text{bit/(s·Hz)}, \quad (22)$$

where $C(\boldsymbol{G})$ denotes the empirical capacity in terms of bits per second per hertz, which depends on the realization of $\boldsymbol{G}$, and $\lambda_n(\boldsymbol{G}\boldsymbol{G}^{\rm T})$ is the nth eigenvalue of $\boldsymbol{G}\boldsymbol{G}^{\rm T}$. As before, here the entries of $\boldsymbol{G}$ are assumed to be independently taken from the Gaussian distribution with zero mean and variance $1/N$. The resulting bandwidth efficiency versus E_b/N_0 is plotted in Fig. 2, where E_b is the energy per information bit. In the simulation, we fix K to 2×10^4. Moreover, since we have $KA^2=E_bCTW$ and $2TW=N$, the signal amplitude A in Eq. (22) is determined by

$$A = \sqrt{\frac{CNN_0}{2K}\cdot\frac{E_b}{N_0}},$$

where C is the theoretical capacity corresponding to E_b/N_0. It is seen from Fig. 2 that the simulated capacities fit the theoretical Shannon capacity very well. This fitting means that the capacity is insensitive to the realization of the transmission matrix $\boldsymbol{G}$. Moreover, the randomness of $\boldsymbol{G}$ implies that the capacity is not dependent on the selection of the transmission waveforms $g_n(t)$. This is not surprising, since the capacity-approaching Gaussian distribution of the transmitted signals is independent of the elements of the transmission matrix $\boldsymbol{G}$ by the central limit theorem.

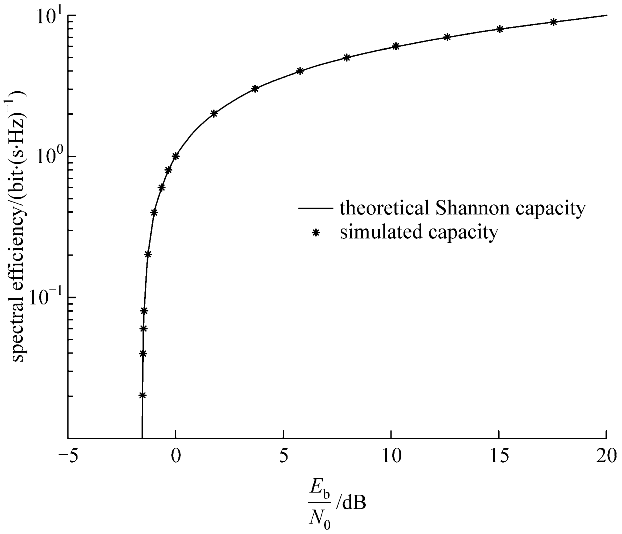

Fig. 2 Simulated and theoretical Shannon capacities in terms of spectral efficiency versus E_b/N_0

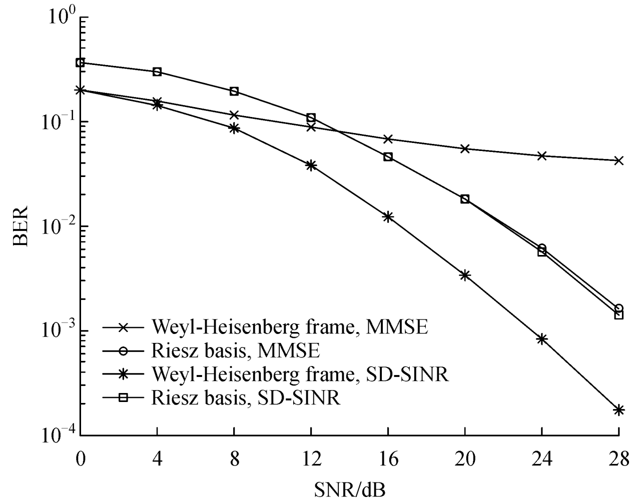

Fig. 3 BER comparison of overcomplete QPSK W-H frame system with $\rho = 1.6$ and incomplete 16QAM Riesz basis system with $\rho = 0.8$ in the T-F dispersive Rayleigh fading channels (Here, the parameters of the exponential delay profile and U-shape Doppler spectrum are set to be $\tau_{\max} = 8$ μs and $f_d = 600$ Hz.)

5.2 Optimized Gabor frame system

It is interesting to compare the performance of the optimized Gabor frame scheme with that of the conventional incomplete Gabor Riesz basis system under the constraint that they have a same spectral efficiency. In the simulation, an exponential power delay profile with the power ratio of the last path to the first one being −18.91 dB and a U-shape Doppler spectrum are assumed. The T-F doubly dispersive channel parameters are set to be $\tau_{\max}$ = 8 μs and f_d = 600 Hz, and the channel state information is assumed to be fully available at the receiver. Moreover, we have a fixed transmission bandwidth $W_B = NF = 1$ MHz. Figure 3 plots the bit error rate (BER) of the Gabor frame system with $\rho = 1.6$ as well as that of the Riesz basis system with $\rho = 0.8$ in the T-F dispersive Rayleigh fading channels. Here, the constellations employed in the Gabor frame system and the Riesz basis system are QPSK and 16QAM, respectively, such that they both have a same spectral efficiency of 3.2 bit/(s·Hz). Both systems are optimally designed according to the matching criteria (21).

It is first seen from Fig. 3 that, for Riesz basis system, the linear minimum mean-square-error (MMSE) detection and the successive detection based on signal-to-inference and noise ratio (SD-SINR) exhibit similar performances. This is because, in this case, the pulses are sufficiently separated in the T-F plane, such that the inter-pulse interference (IPI) are small. Thus, the major impairment comes from the background noise and the SD-SINR gains nearly no advantage over the linear MMSE detection. In the case of Gabor frame system, however, there is a large gap between the performances of the two detection schemes. It is not surprising, since the pulses are put more closely in the T-F plane and the resulting IPI makes the MMSE detection ineffective. Next, let us check the performances between the Gabor frame system and the Riesz basis system. The superiority of the Gabor frame system to the Riesz basis system is apparent in Fig. 3. Specifically, in the low signal-to-noise ratio (SNR) region, no matter what detection scheme is exploited the Gabor frame systems outperform the corresponding Riesz basis systems. This is because the Riesz basis systems with higher-order constellation are more sensitive to the background noise. In the high SNR region, the Gabor frame system with SD-SINR still gains an advantage of about 5 dB in SNR over the Riesz basis system. It can be drawn a conclusion from these results that the increase of spectral efficiency by increasing signaling efficiency is more effective than by employing higher-order constellations as in the conventional communication systems.

6 Conclusions and discussion

By regarding signal transmission as tiling of the T-F plane, we have studied the digital transmission problem from function space theory. By classifying all possible finite-alphabet symbol vectors into appropriate subspaces, we have proven that overcomplete frames can be used to construct perfect transmultiplexing systems, which relaxes the conventional orthogonality and biorthogonality requirements of modulation pulses. The proposed frame transmission system can achieve a signaling efficiency larger than unity, which serves as a bridge between digital modulation and Shannon capacity.

In the framework of digital communication via Gabor frames, we have also presented the pulse shaping and T-F lattice matching criteria jointly for some representative scattering channels from the viewpoint of energy perturbation minimization. Simulation results demonstrate the performance of the new system. A comparison between the proposed system and the conventional Gabor Riesz system shows that the digital transmission system based on Gabor frame is more pertinent to future mobile services. However, due to the overcompleteness and correlation of the pulses in a frame system, symbol interference is not avoidable. While the optimal maximum likelihood sequence detection (MLSD) can completely eliminate such undesirability, it is computationally prohibitive. Therefore, how to design effective and efficient detection schemes for Gabor frame transmission systems is an important direction in our future research.

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