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RESEARCH ARTICLE

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Remanufacturing planning based on constrained ordinal optimization

Abstract Resource planning for a remanufacturing system is in general extremely difficult in terms of problem size, uncertainties, complicated constraints, etc. In this paper, we present a new method based on constrained ordinal optimization (COO) for remanufacturing planning. The key idea of our method is to estimate the feasibility of plans by machine learning and to select a subset with the estimated feasibility based on the procedure of horse racing with feasibility model (HRFM). Numerical testing shows that our method is efficient and effective for selecting good plans with high probability. It is thus a scalable optimization method for large scale remanufacturing planning problems with complicated stochastic constraints.

Keywords remanufacturing systems, constrained ordinal optimization (COO), simulation-based optimization, machine learning

1 Introduction

Planning remanufacturing system with high efficiency has drawn considerable attention due to its significant economic impact [1–3]. A typical remanufacturing process includes disassembling a module or unit with reusable parts, cleaning and refurbishing repairable parts and replacing unusable parts by new ones. It is then reassembled to become a unit fully equivalent in its performance to a completely new unit. The remanufacturing system considered in this paper is based on a practical engine repair shop, as shown in Fig. 1. The planning problem for this system is very complicated, including discrete states, probabilistic constraints and uncertainties in cost performance described in details in our previous work [4]. In this paper, we will propose a new optimization method based on this concern with high generality for solving a wide range of similar problems.

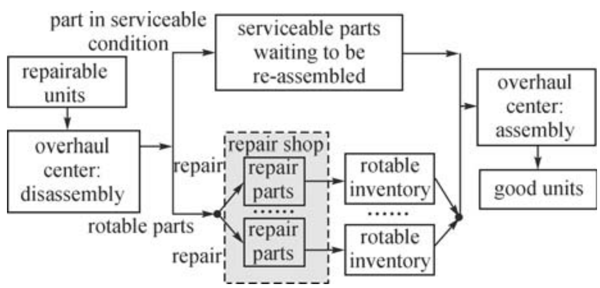

Fig. 1 Structure of remanufacturing system

Many methods including the effective methods of multi-stage stochastic programming have been developed to deal with complex stochastic optimization problems in manufacturing planning and scheduling [5]. However, the issues of the multi-stage planning, fork-join structure, great computational efforts of the huge state space, uncertainties, etc. are very challenging [6]. The remanufacturing problem in this paper is more complicated since the objective function and constraints cannot be evaluated or determined analytically without simulation as will be seen in Sect. 2. As a result, the multi-stage stochastic programming methods can hardly be applied to solve this problem.

In the literature, computational intelligence methods, such as data mining, genetic algorithms, and neural networks [7–10] were applied to analyze and optimize one or more segments of a remanufacturing system such as disassembly [11,12]. A remanufacturing system was seldom

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taken as a whole and interactions among the activities of the segments were generally not considered.

In the current practice, time-consuming simulation is performed to evaluate the remanufacturing plans, which are obtained heuristically. "Since simulation-based optimization became possible to analyze random systems using computers, scientists and engineers have sought the means to optimize systems using simulation models"[13]. In fact, simulation could be the only available approach for planning complicated remanufacturing systems.

In order to overcome the difficulties in complicated planning problems, the ordinal optimization (OO) method offers an efficient simulation-based optimization approach [14,15]. It is based on the concept that the "order" of the planning decisions is much easier to determine than the "value" and is exponentially convergent versus the simulation efforts [16,17], and the "goal softening" by seeking the "good enough with high probability" will exponentially lessen the computational load [18]. OO method is particularly attractive for stochastic discrete optimization since it is immune to large noise with affordable computational complexity [19–21]. However, incorporating constraints efficiently is one of the major challenges in applying the regular unconstrained OO method [22–24]. Based on the feasibility model generated by the machine learning method for planning remanufacturing systems [4], the framework of the constrained ordinal optimization (COO) approach for the simulation-based optimization problems has been constructed [25]. The simplest blind picking (BP) selection rule is designed to solve discrete constrained optimization problems.

Clearly, the feasibility model is more time-saving than the penalty method commonly used in computational intelligent approaches in terms of handling constraints since the former makes use of the structure information of the problem and can directly exclude infeasible plans. However, building a feasibility model needs prior knowledge or that obtained by machine learning. The problem structure information can also be utilized to design the selection rule. Since BP does not use the observed performance, it supplies an upper bound to the size of the selected subset [14,15]. The horse racing (HR) selection rule screens out some observed bad plans from consideration, and thus usually requires a smaller selected subset [26].

In this paper, the COO approach with the HR selection rule is developed to solve the remanufacturing planning problem. The key idea of our method is to estimate the feasibility of plans by machine learning and to select a subset with the estimated feasibility based on the procedure of "horse racing with feasibility model (HRFM)". In the HRFM scheme the candidate plans are compared by "quick simulation" with a small number of replications. A number of plans with the best estimated performance and feasibility are selected, forming the selected subset and the best plan is then obtained within this set by more accurate simulation (with more replications). The combination of feasibility model with the HR rule is not trivial, since there is usually a correlation between the feasibility and the performance of a plan. We use a correlation factor to quantify this correlation, and present a general procedure to determine the selected subset as a function of the correlation factor.

The numerical testing results for a practical remanufacturing system show that the HRFM method presented in this paper is more efficient to meet the same required alignment probability in comparison with brute force simulation and the blind picking with feasibility model (BPFM) selection method.

2 Problem formulation

The problem considered in this paper is motivated by the daily planning tasks encountered in an actual engine repair shop. Enough amount of the processing capacity and the spare part inventories are reserved to satisfy the probabilistic cycle time requirement. However, the reserved capacity and inventories should not be excessive in order to save the associated costs. The cycle time of an asset to be repaired depends on the capacity and inventory level allocated in every quarter of a year and is random due to uncertain arrival and worn-out "precondition".

To avoid the proprietary issue, we formulate the following simplified remanufacturing planning problem with the key characteristics preserved. To facilitate presentation, the notations are given first as

ω : total part surplus;
ψ : non-negative cost coefficients;
$C_{\rm c}$: total allocated capacity;
C : the total cost consisting of capacity cost $\alpha C_{\rm c}$ and inventory cost $V(O,\omega)$;
$V(I,\omega)$: inventory cost related to the inventory I and uncertain difference ω;
$C_{i,j}^{\rm cap}$: allocated capacity for part i in quarter j;
$C_i^{\min}$: lower bound of repairing capacity for part i;
$C_i^{\max}$: upper bound of repairing capacity for part i;
ΔC_i : limit of capacity change in consecutive quarters for part i;
$C_j^{\rm cap}$: total available capacity in quarter j;
$\delta C_{i,j}^{\rm cap}$: capacity adjustment for part i in quarter j;
d : required cycle time (days);
T_i : the planning horizon (days);
k : index for repairable asset;
m : number of quarters;
n : number of part types;
$O_{i,j}$: order for new part i in quarter j;

$\Pr(\cdot)$: probability;
$P_{\rm rc}$: required probability of the cycle time;
$T_{\rm c}(k)$: remanufacturing cycle time of asset k;
$\omega_{i,j}$: surplus of part i in quarter j, i.e., the difference between the number of the repaired part i, the number of order for new part i and the number of the assembly requests for part i in quarter j;
$p_{i,j}$: number of repaired part i in quarter j;
$q_{i,j}$: number of the assembly requests for part i in quarter j;
$I_{i,j}$: inventory for part i in quarter j;
ϑ_k : finish time of asset k.

The remanufacturing problem is formulated as a stochastic optimization problem in Eqs. (1)–(6) as

$$\min_{C_{\rm c},I} C = \psi C_{\rm c} + E_\omega V(I,\omega), \tag{1}$$

where E_ω represents the expectation over ω,

$$C_{\rm c} = \sum_{i=1}^{n}\sum_{j=1}^{m} C_{i,j}^{\rm cap}, \tag{2}$$

$$I = \sum_{i=1}^{n}\sum_{j=1}^{m} I_{i,j}, \tag{3}$$

$$I_{i,j} = I_{i,j-1} + \omega_{i,j-1}, \tag{4}$$

$$\omega = \sum_{i=1}^{n}\sum_{j=1}^{m} \omega_{i,j}, \tag{5}$$

$$\omega_{i,j} = p_{i,j-1} + O_{i,j-1} - q_{i,j-1}, \tag{6}$$

subject to probabilistic cycle time requirement (7) and the repairing capacity constraints (8)–(11):

$$\Pr(T_{\rm c}(k) \leqslant d | \vartheta_k < T_{\rm t}) \geqslant P_{\rm rc}, \tag{7}$$

$$C_{i,j}^{\rm cap} \in [C_i^{\min}, C_i^{\max}], \tag{8}$$

$$|\delta C_{i,j}^{\rm cap}| \leqslant \Delta C_i, \tag{9}$$

$$\sum_{i=1}^{n} C_{i,j}^{\rm cap} \leqslant C_j^{\rm cap}, \tag{10}$$

where $T_{\rm t}$ is a given constant deadline of the total time.

The activities in the remanufacturing system are under the following assumptions:

1) the inter-arrival time of assets follows a Poisson distribution with rate λ_i in quarter i;

2) the part repair time complies triangular distribution;

3) the repairing capacity is adjustable (controllable) for each quarter;

4) the buffer capacities of disassembly, repair, order and assembly are not considered or assumed infinite;

5) disassembling, assembling and shipping times are assumed deterministic but the buffering time are uncertain, and the shipping time is neglected.

Suppose the cycle time $T_{\rm c}$ of asset k is calculated based on its finish time ϑ_k (completion of assembly, which is related with allocated capacity, inventory and uncertainties in every quarter) and arrival time and the random inter-arrival time between asset $l-1$ and asset l is $a(l)$ and initial time is zero. The cycle time is calculated as

$$T_{\rm c}(k) = \vartheta_k(C_{\rm c}, I, \omega) - \sum_{l=1}^{k} a(l), \quad l = 1, 2, \ldots, k. \tag{11}$$

One of the major difficulties in solving this problem is that the expectation in the cost function and the conditional probability in the probabilistic cycle time constraint cannot be evaluated analytically as a function of the decision variables. The main reason is that the model does not satisfy Markovian assumptions or regenerative assumptions required by analytical methods. Furthermore, the following features of the problem add additional dimensions of difficulties:

1) presence of rotable inventories with adjustable repairing capacities;

2) lack of event synchronization caused by the rotable inventory;

3) fork-join disassembly and assembly of the repair shop.

Clearly, the above problem is a very complicated stochastic discrete optimization problem. In fact, due to presence of rotable inventories, efficient simulation of the system is a non-trivial task. After being disassembled, the parts to be repaired will be processed by a set of parallel machines whose processing times have a triangle (non-exponential) distribution. The repaired parts will then go into the rotable inventory or buffer ready for re-assembling without sequence. That is, the part being disassembled first could be re-assembled last. This causes the events in the system to be interdependent. As a result, updating a state in simulation has to be performed globally and every individual part has to be tracked. The simulation on this process turns out to be very time consuming when the inventory level and number of parallel machines are large. As we reported in Ref. [4], simulation by the commercial software ED in actual industrial application is computationally very expensive.

Given the difficulties in applying stochastic programming methods or brute force Monte Carlo simulation, we shall develop an efficient simulation-based optimization method — COO to solve this problem.

3 Method of constrained ordinal optimization (COO)

3.1 Framework

For convenience of presentation, consider the following general optimization problem with stochastic cost and constraints in Eqs. (12) and (13):

$$\min_{\theta\in\Theta} \equiv \mathrm{E}(L(x(t;\theta,\xi))), \tag{12}$$

$$\text{s.t. } h_i(\theta) \equiv \mathrm{E}(L_i(x(t;\theta,\xi))) \leqslant 0, \quad i=1,2,\ldots,m, \tag{13}$$

where θ is the candidate plans in planning space Θ, L, the performance functional of the sample path of a discrete dynamic state $x(t)$, and ξ, i.e., all the uncertainties or "random noises" of the problem [4], E is the expectation operator and $h_i(\cdot)$ is the constraint measure.

In OO, a selection rule or method is needed to select subset S with certain evaluation scheme (crude algorithms, heuristics and even BP) to guarantee the desired degree of "matching" called alignment level k, and the confidence of achieving a certain alignment level with good enough subset G is referred to as the alignment probability P_{A}:

$$P_{\mathrm{A}} = \Pr\{|G\cap S| \geqslant k\}. \tag{14}$$

The difficulty of the COO problem is that direct selection of S in Θ is inefficient since there are many infeasible plans that cannot be excluded beforehand. Consequently, the alignment level cannot be estimated by the regular OO method without going through time-consuming simulation since the quantity of infeasible plans in S is unknown. In other words, the uncertain number of infeasible plans in Θ and S is the main difficulty of COO approach.

In order to overcome the above difficulty, we apply the feasibility model generated by machine learning in Ref. [4] to exclude infeasible plans and obtain an approximate feasible set Θ_{f} in the search space Θ. The feasibility model can be integrated in the selection method in

$$\min_{\theta\in\Theta_{\mathrm{f}}} J(\theta) \tag{15}$$

to obtain a selected subset S_{f} in Θ_{f}. It is easy to see that applying OO to set Θ_{f} to get S_{f} will be better than applying regular OO to Θ, since no time-consuming simulation is needed to exclude infeasible plans. The degree of improvement and the size of the selected subset S_{f} can be quantified with respect to the accuracy of the feasibility model.

Assume the probability that the feasibility of a plan is correctly classified by the feasibility model is P_{f}. Although some infeasible plans in Θ_{f} and S_{f} may be classified as feasible plans incorrectly, the densities of feasible plans in S_{f} (or Θ_{f}) are higher than S (or Θ) if P_{f} is larger than one half. This means that more good plans can be found in S_{f} than in S when S_{f} and S are with the same size.

The COO approach is the evolution of the OO methodology which is suitable for constrained stochastic optimization problems. It is noted that classifying a feasible or infeasible plan is ordinal. All the advantages of OO apply here, i.e., selecting Θ_{f} from Θ can be very effective since only approximate feasibility is required. Moreover, "imperfectness" of the feasibility model is consistent with the "crude model" concept in OO. Although feasibility classification with the feasibility model may give erroneous results, the model could be very robust apply to a group of candidates.

3.2 Algorithm and selection method

3.2.1 Feasibility model

A rough sets theory (RST) [27] based machine learning approach with reinforcement process is developed to generate the rules from training dataset for determining feasibility as reported in Ref. [4]. The knowledge base of these rules constitutes the feasibility model. Feasible plans are discerned and picked out by the feasibility model without simulation.

3.2.2 Subset selection and optimization

The effectiveness of the OO method depends on how the selected subset S is generated. The simplest selection method is BP method [14] without the consideration of performance value of plans. Suppose the size of feasible design space Θ_{f} is F, the alignment probability that at least k designs in the selected subset S_{f} belong to good enough subset G has been given by our previous work on BPFM method:

$$P_{\mathrm{A}}(|G\cap S_{\mathrm{f}}| \geqslant k) = \sum_{j=k}^{\min(g,s_{\mathrm{f}})} \sum_{i=0}^{s_{\mathrm{f}}-j} \frac{\binom{g}{j}\binom{F-g}{s_{\mathrm{f}}-i-j}}{\binom{F}{s_{\mathrm{f}}-i}} \binom{s_{\mathrm{f}}}{i} \times (P_{\mathrm{f}})^{s_{\mathrm{f}}-i}(1-P_{\mathrm{f}})^{i}, \tag{16}$$

where $s_{\mathrm{f}} = |S_{\mathrm{f}}|$ and $g = |G|$.

A more sophisticated selection method — HR integrated with the feasibility model is developed and compared with BPFM method in this paper. HR is based on the performance estimates, possibly inaccurate and crude, in planning space. The plans selected by the HR rule can be considered as having all plans with noisy performance competing to be selected, similar to N horses running in a race with the current leading one being selected but possibly falling behind later. The selection of good enough plans in S is determined by selecting the best s plans based on their estimated performance values with "inaccurate but quick simulation".

3.3 Horse racing (HR) for unconstrained problems

The idea of the HR selection is to assign the best plans by performing a rough simulation as the selected subset. This is similar to selecting the current leading horse as the one that would be more likely to win. The key issue is how large the selected subset should be in order to include good enough plans.

In HR method, ordered performance curve (OPC) is an important reference for subset selection. It is a non-decreasing curve created by plotting the performance or objective values from the smallest to the largest versus the plan indexes [14,26]. Unfortunately, the actual OPC is generally unknown. However, it can be categorized into five general classes and approximated by a function specified by two parameters [26][1)].

If class of the underlying OPC is known by prior knowledge, the size s of the selected subset S depends on:

1) alignment level, k;

2) size of the good enough subset, $|G| = g$;

3) size of planning space, N;

4) value of the required alignment probability P_{A};

5) noise characteristics, $\xi(\cdot)$, reflecting the inaccuracy of temporary leading in "horse racing";

6) type of OPC reflecting the ordered performance values from the smallest to largest versus corresponding plans.

Assuming $N = 1000$, $P_{\mathrm{A}} = 0.95$, and uniform noise density $\xi(\cdot) \sim [-W, W]$ without losing generality; the size s only depends on g and k. We can estimate s by regression based on the data generated by Monte Carlo simulation [26]:

$$s(g, k) = e^{Z_0} k^{\rho} g^{\gamma} + \eta, \tag{17}$$

where Z_0, ρ, γ and η are coefficients depending on N, C, P_{A} and $\xi(\cdot)$.

3.4 Horse racing with feasibility model (HRFM) for constrained problems

To obtain the size of the selected subset quantitatively, we enhance the HR method for constrained problems. Let the subset S_{f} be the one with the infeasible plans excluded by the feasibility model. Its size can be obtained similarly to the unconstrained case as

$$s_{\mathrm{f}}(g, k; N, C, \xi(\cdot), P_{\mathrm{A}}, d_{\mathrm{f}}, P_{\mathrm{f}}, \rho_{FO}) \tag{18}$$

with three additional parameters:

1) P_{f}, the probability reflecting the accuracy of the feasibility model, and is the most important performance measurement of the feasibility model.

2) $d_{\mathrm{f}} = F/N$, the estimated density of feasible plans in planning space, where F is the number of feasible plans discerned by the feasibility model. It can be estimated with the actual sample data or experience. The required size of selected subset would be smaller if the density of feasible plans in planning space is higher.

3) The correlation between the feasibility curve (FC) and the OPC, defined in Eq. (19) as feasibility-performance correlation coefficient (ρ_{FO}) to reflect the distribution of feasibility with the ordered plans.

$$\rho_{FO} = \frac{\sum_{i=1}^{N} \frac{F_i O_i}{N} - \sum_{i=1}^{N} \frac{F_i}{N} \sum_{i=1}^{N} \frac{O_i}{N}}{\sqrt{\frac{\sum_{i=1}^{N} F_i^2}{N} - \left(\sum_{i=1}^{N} \frac{F_i}{N}\right)^2} \sqrt{\frac{\sum_{i=1}^{N} O_i^2}{N} - \left(\sum_{i=1}^{N} \frac{O_i}{N}\right)^2}}, \tag{19}$$

where F_i is a binary variable representing the feasibility of plan i and O_i is the cost of plan i. The correlation ρ_{FO} can also be estimated with the actual sample data.

We define FC with "1" representing a feasible plan and "0" an infeasible plan. FC is a 0-1 rectangle curve, and the relationship between the FC and the OPC, that is where the feasible plans are distributed on the OPC, can clearly seen later in Fig. 3.

The procedure for calculating the size of the selection subset in HRFM is listed in Table 1.

Table 1 Determining S_{f} by regression in HRFM

step	process
1	build feasibility model;
2	generate a corresponding standard OPC for the given model and add noise on the OPC;
3	generate $N \times d_{\mathrm{f}}$ feasible plans that comply with the value of ρ_{FO} between the FC and the OPC;
4	pick feasible plans from the result of Step 3 based on probability P_{f};
5	observe the size of subset selection that meet the required P_{A} for g and k through HR;
6	replicate above Steps 3–6 for different P_{A}, g and k;
7	record all result of subset selection and determine Z_0, ρ, γ and η in Eq. (17) by regression;
8	calculate s_{f} based on Eq. (17) for the required P_{A}, g and k.

Figure 2 demonstrates that the size s_{f} of the selected subset is monotonically decreasing with d_{f}, the density of the feasible plans, P_{f}, the accuracy of the feasibility model and ρ_{FO}, the feasibility-performance correlation between OPC and FC[2)]. This is intuitively true for larger d_{f} or larger P_{f} means more feasible plans are available. Less intuitive is the decreasing of s_{f} with ρ_{FO}. The value

1) The standardized OPC is approximated by $\Lambda(x|\alpha,\beta) = F^{-1}(x|\alpha,\beta) = F(x|1/\alpha,1/\beta)$, where $F(x|\alpha,\beta) = \int_0^y f(z|\alpha,\beta)\mathrm{d}z$, $f(y|\alpha,\beta) = Cy^{\alpha-1}(1-y)^{\beta_1}$, $C = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$, $\Gamma(\cdot)$ is gamma function, where C is one of the five OPC classes. The two-parameter model provides us the flexibility in describing the five OPC categories by varying α and β.

2) For convenience of illustration, the curve in Fig. 2 is s_{f} versus $1/d_{\mathrm{f}}$, $1/P_{\mathrm{f}}$ and $1/\rho_{FO}$ to avoid plotting extended to infinity.

of ρ_{FO} is the measurement of how feasible plans are distributed on OPC. In fact, the value of ρ_{FO} decreases with more feasible plans in the good enough region, and as a result the required size s_{f} of selected subset can be smaller for the same $d_{\mathrm{f}}, P_{\mathrm{f}}$ and P_{A}.

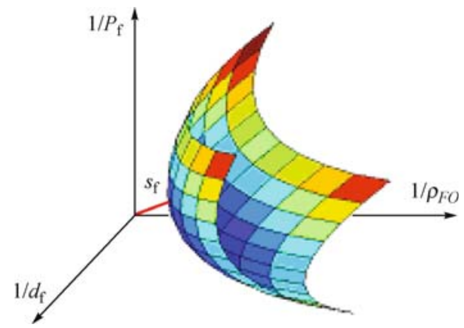

Fig. 2 Relationships between s_{f} and $d_{\mathrm{f}}, P_{\mathrm{f}}, \rho_{FO}$

Intuitively, HRFM method should be better than BPFM method because HRFM is a performance based method.

Proposition 1 *HRFM method is better than BPFM method with a high probability, i.e., for the same requirement of alignment probability,*

$$\Pr(|S_{\mathrm{f}}(\mathrm{HRFM})| \leqslant |S_{\mathrm{f}}(\mathrm{BPFM})|) \geqslant \Pr(|S_{\mathrm{f}}(\mathrm{BPFM})| \leqslant |S_{\mathrm{f}}(\mathrm{HRFM})|).$$

The proof is shown in Appendix A.

It should be noted that for a given feasibility model, what designs will be estimated as feasible designs is a random event, The uncertainty and the randomness cannot simply be combined, since we are always dealing with a specific problem, and we are only interested in whether we will succeed to find good enough designs in this specific problem, not the average successful frequency. We still need to classify the optimization problem according to the relationship between the performance of the infeasible designs and those of the feasible designs.

In general, if the correlation between the FC and the OPC ρ_{FO} is small, many feasible designs can be selected by HRFM method, it is more possible that HRFM method is better than BPFM method. However, because of the imperfectness of feasibility model, it is also possible that BPFM method is better than HRFM method in some extreme situation, e.g., if ρ_{FO} is large, the possibility that HRFM method is better than BPFM method will be low. In the case shown in this paper, because $\rho_{FO} = 0.23$ is very small, it indicates that there are many feasible designs with better performance and many infeasible designs with worse performance. Therefore, it is natural that HRFM method is better than BPFM method as shown in the experimental result in next section.

4 Numerical testing results and analysis

The method is implemented in C++ on a P-IV 2.0 GHz PC. The feasibility model is trained by using Rosetta, the software package for building RST knowledge base.

When building the feasibility model in Step 1 of Table 1, if the reserved capacity $\bar{C}_i$ and controlled inventory level $\bar{I}_i$ in every quarter of a plan make the $P_{\mathrm{b}}(T_{\mathrm{c}}(i) \leqslant d|\eta_k < T_{\mathrm{t}}) \geqslant 95\%$ satisfied, the plan is feasible. The details in building feasibility model can be found in Ref. [4].

Numerical testing following the procedure of Table 1 is shown as follows. The FC curve of 1000 remanufacturing plans is in Fig. 3. Every plan includes 8-dimensional capacity value and 8-dimensional inventory value. There are 654 feasible plans in 1000 plans with $d_{\mathrm{f}} = 0.654$.

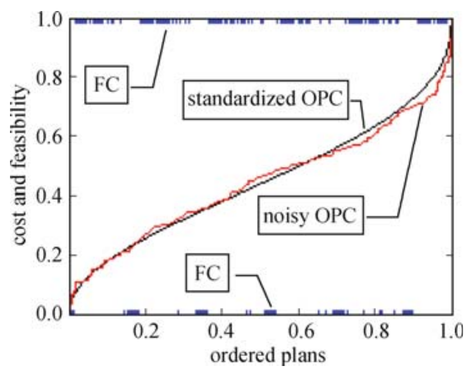

Fig. 3 FC and OPC of remanufacturing plans

The "horse race" is performed by short simulation with only 100 replications and the noisy OPC of the remanufacturing system causes the "rough" curve as seen in Fig. 3. To obtain the coefficients in Eq. (17), the OPC is smoothed by standardized OPC $\Lambda(\cdot)$ with parameters $\alpha = 2$ and $\beta = 2.5$. The feasibility-performance correlation coefficient between FC and OPC is calculated as $\rho_{FO} = 0.23$.

Suppose the required alignment probability $P_{\mathrm{A}} = 95\%$. The size of selected subset is observed for g changing from 20 to 200 at intervals of 10 and k changing from 1 to 10. For the given parameters and requirements, the coefficients are obtained by regression as listed in Table 2.

Table 2 Coefficients Z_0, ρ, γ, η by regression in HRFM

parameters	coefficients	obtained by regression
$N = 1000,\ P_{\mathrm{A}} = 95\%,\ \alpha = 2,$	Z_0	0.2172
$\beta = 2.5,\ \xi(\cdot) \in [-0.01, 0.01],$	ρ	1.1347
$P = 0.654,\ P_{\mathrm{f}} = 80\%,\ \rho_{FO} = 0.23$	γ	0.5027
	η	5.6115

Once the value of Z_0, ρ, γ and η are obtained, the size of subset selection can be calculated by Eq. (17) for the problems with different requirements (e.g., different k, g). The sizes of the selected subset for different alignment level k obtained by HRFM with $N = 1000$, $\alpha = 2$, $\beta = 2.5$, $P_{\mathrm{A}} = 95\%$, $P = 0.654$, $P_{\mathrm{f}} = 80\%$, $\xi(\cdot) \in [-0.01, 0.01]$, $\rho_{FO} = 0.23$, $g = 50$ are listed in Table 3, together with those obtained by Eq. (16) of BPFM. It is seen that for same alignment level, the required size of the selected subset by HRFM is much smaller than that by BPFM. Therefore, the HRFM method is more efficient when evaluating the plans in the selected subset with more replications.

Table 3 Sizes of the selected subsets

k	s_{f}	
	BPFM	HRFM
1	46	15
2	73	26
3	97	37
4	119	49
5	140	61

To make comparison, we try to obtain the "true" good enough plans and their distributions through brute force simulation. Suppose the required size of good enough subset is 50. Through the Monte Carlo simulation with 1000 runs, the index and the performance value of top-50 plans in the planning space is listed in Table 4. The cost range of good enough subset is between [339.16, 355.42].

The alignment between S_{f} and G by HRFM with $k = 1$ is shown in Table 5. It is shown that plan 90, 270, 450 in the selected subset belong to the good enough set G. The best plan in the selected subset with the lowest cost of 339.26.

For comparison, we examine the $s_{\mathrm{f}} = 46$ plans in Table 6 that are selected by the BPFM method based on Eq. (16). It is seen that only Plan 29 with cost of 349.63 is a good enough plan. On the other hand, Plan 90, 270, 450 are in the good enough region for HRFM method with the lowest cost of 339.26. HRFM method selected about three times less plans than the BPFM method, and that in this smaller subset we had three plans that are nearly optimal compared to only one included in BPFM.

The distributions of the selected plans of the HRFM and BPFM method are shown in Figs. 4(a) and 4(b) respectively. It is seen that the selected plans by HRFM are better (lower costs) on average than those by BPFM. Clearly the selected subset of HRFM is smaller with more good enough plans. Therefore HRFM method is more efficient and effective than the BPFM method.

The HRFM method is computationally efficient versus brute force simulation. Fast feasibility determination offsets the time consuming training process. Moreover, training feasibility model and performing regression for Eq. (8) need to be done once too. In our problem of selecting good enough ones among 1000 plans, the computational times of the HRFM method are less than 600 s, much shorter than about 4500 s used by brute and force simulation. Therefore the COO method developed

Table 4 Index and performance value of top-50 plans

item	plans					
plan index	90,	270,	450,	630,	810,	990,
	157,	337,	517,	697,	877,	1,
	194,	374,	554,	734,	914,	136,
	316,	496,	676,	856,	29,	209,
	389,	569,	749,	929,	184,	364,
	544,	724,	904,	43,	223,	403,
	583,	763,	943,	146,	326,	506,
	686,	866,	137,	317,	497,	677,
	857,	143				
corresponding cost	339.16,	339.16,	339.16,	339.16,	339.16,	339.16,
	346.53,	346.53,	346.53,	346.53,	346.53,	348.31,
	348.80,	348.80,	348.80,	348.80,	348.80,	348.84,
	348.84,	348.84,	348.84,	348.84,	349.63,	349.63,
	349.63,	349.63,	349.63,	349.63,	350.10,	350.10,
	350.10,	350.10,	350.10,	351.61,	351.61,	351.61,
	351.61,	351.61,	351.61,	352.02,	352.02,	352.02,
	352.02,	352.02,	354.60,	354.60,	354.60,	354.60,
	354.60,	355.42				

Table 5 Good enough and selected plans by HRFM

item	plans					
plan index	18,	198,	378,	558,	738,	918,
	66,	246,	426,	606,	786,	966,
	90,	***270***,	***450***			
corresponding cost	338.68,	338.69,	338.69,	338.70,	338.73,	338.87,
	338.87,	338.87,	338.88,	338.88,	338.89,	339.15,
	339.26,	339.46,	339.46			

Table 6 Good enough and selected plans by BPFM

item	plans			
plan index	466,	481,	741,	596,
	802,	321,	613,	787,
	992,	271,	58,	401,
	940,	733,	253,	577,
	633,	639,	668,	818,
	470,	406,	238,	93,
	329,	635,	271,	211,
	415,	80,	665,	560,
	966,	628,	241,	412,
	620,	503,	806,	130,
	753,	945,	128,	239,
	29,	262		
corresponding cost	372.6697,	429.0903,	373.5053,	406.9058,
	374.6664,	368.2683,	414.6478,	417.9706,
	391.4767,	383.5325,	361.5789,	379.2839,
	384.5317,	419.2386,	414.6478,	417.4961,
	389.1914,	377.4383,	386.6236,	389.9428,
	413.6675,	368.6778,	361.5789,	389.1914,
	396.0906,	414.9492,	383.5325,	418.6906,
	386.6161,	384.2533,	416.7247,	405.9047,
	338.8750,	411.6919,	392.9731,	394.8953,
	384.2533,	355.4256,	387.4700,	373.1575,
	352.0414,	392.0347,	386.6236,	392.2014,
	349.6347,	374.6664		

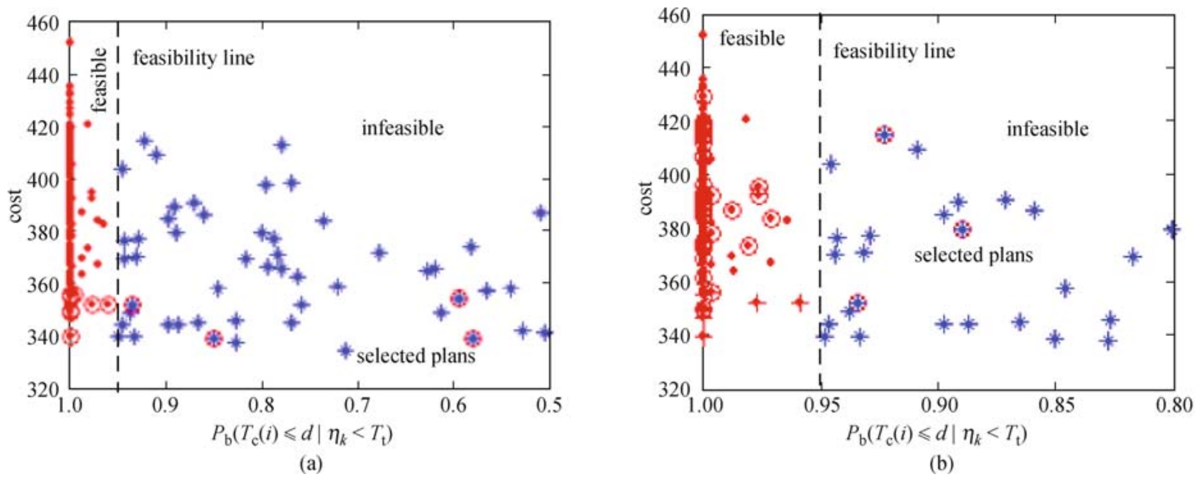

Fig. 4 Distribution of the plans. (a) HRFM; (b) BPFM

in this paper is a promising approach for solving complex remanufacturing planning problems with uncertainties.

5 Conclusions

Planning optimization of a complex remanufacturing system is generally extremely difficult in terms of problem size, uncertainties, stochastic constraints, etc., and in many cases simulation is the only approach available. Although OO is a very efficient simulation-based optimization method, it cannot be applied directly to the remanufacturing planning problems since many infeasible plans cannot be excluded without time-consuming simulation. A new COO approach with feasibility model is efficient and effective to solve the remanufacturing problems. Finding plan feasibility would greatly improve the simulation efficiency and provide powerful "structural information". The method of HRFM is developed to select good enough remanufacturing plans. Numerical testing for a practical remanufacturing system shows that the COO method is very efficient and robust, even when the feasibility model is not very accurate. Furthermore, the COO method presented in the paper is a general approach. Any crude feasibility model even with large noise can be integrated with the COO approach developed in this paper to solve complex discrete and hybrid optimization problems with stochastic constraints.

Appendix A Proof for Proposition 1

If we can prove that the probability of there exist top number designs in $S_{\mathrm{f}}(\mathrm{HRFM})$ is more than the one of $S_{\mathrm{f}}(\mathrm{BPFM})$, the proposition can be proved.

Suppose the selected subset in feasible space of a constrained problem is $S_{\mathrm{f}}(A)$, where A is a selection rule, e.g., BPFM or HRFM, etc.

$$\{\theta_1, \theta_2, \ldots, \theta_{s_{\mathrm{f}}(A)}\}.$$

Because there are still infeasible designs in the selection for the imperfectness of the feasibility model, the actually feasible selected subset is $S_{\mathrm{f}}(A) \cap \Theta_{\mathrm{f}}$, where Θ_{f} is the set of all feasible designs in Θ.

$$\{\theta'_1, \theta'_2, \ldots, \theta'_{s_{\mathrm{f}}(A)\cap\Theta_{\mathrm{f}}}\},$$

where $|S_{\mathrm{f}}(A) \cap \Theta_{\mathrm{f}}| < |S_{\mathrm{f}}|$ and A is the selection rule, e.g., BPFM or HRFM.

For HRFM, suppose the truly feasible selected subset is $S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}$, We regard $S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}$ as the observed top $|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|$ designs of observed feasible space. Strictly, the $|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|$ designs should belong to top $|S_{\mathrm{f}}(\mathrm{HRFM})|$ designs in truly feasible space.

The probability of there exist top $|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|$ designs in $S_{\mathrm{f}}(\mathrm{HRFM})$ is 1.

For BPFM, suppose the selected subset is $S_{\mathrm{f}}(\mathrm{BPFM})$. Let us discuss the size of designs that are top best $|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|$. For BPFM method, the probability that there exist i designs in top $|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|$ is

$$\Pr(|S_{\mathrm{f}}(\mathrm{BPFM}) \cap (S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}})| = i) = \frac{\binom{|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|}{i}\binom{F - |S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|}{|S_{\mathrm{f}}(\mathrm{BPFM})| - i}}{\binom{F}{|S_{\mathrm{f}}(\mathrm{BPFM})|}},$$

where F is the size of feasible space generated by feasibility model. Let $i = |S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|$.

$$\Pr(|S_{\mathrm{f}}(\mathrm{BPFM}) \cap (S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}})| = |S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|) = \frac{\binom{F - |S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|}{|S_{\mathrm{f}}(\mathrm{BPFM})| - |S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|}}{\binom{F}{|S_{\mathrm{f}}(\mathrm{BPFM})|}}.$$

Only if we make sure more than one feasible design is selected, the HRFM method can makes sense. Therefore, we have $|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}| \geqslant 1$.

$$\begin{aligned}&\Pr(|S_{\mathrm{f}}(\mathrm{BPFM}) \cap (S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}})| \\ &\quad = |S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}|) \\ &\leqslant \frac{\binom{F-1}{|S_{\mathrm{f}}(\mathrm{BPFM})|-1}}{\binom{F}{|S_{\mathrm{f}}(\mathrm{BPFM})|}} \leqslant \frac{\binom{F}{|S_{\mathrm{f}}(\mathrm{BPFM})|}}{\binom{F}{|S_{\mathrm{f}}(\mathrm{BPFM})|}} = 1.\end{aligned}$$

Therefore we can prove that the probability of there exist top designs in $S_{\mathrm{f}}(\mathrm{HRFM})$ is more than the one of $S_{\mathrm{f}}(\mathrm{BPFM})$. It means that it only need a smaller size for HRFM method with a higher probability to meet the alignment probability requirement, i.e.,

$$\begin{aligned}&\Pr(|S_{\mathrm{f}}(\mathrm{HRFM})| \leqslant |S_{\mathrm{f}}(\mathrm{BPFM})|) \\ &\quad \geqslant \Pr(|S_{\mathrm{f}}(\mathrm{BPFM})| \leqslant |S_{\mathrm{f}}(\mathrm{HRFM})|).\end{aligned}$$

In the paper, $F = 654$, $P_{\mathrm{f}} = 80\%$, suppose $|S_{\mathrm{f}}(\mathrm{HRFM})| = 20$ and the expected size of $|S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}| = 20 \times 80\% = 16$, let us calculate when $|S_{\mathrm{f}}(\mathrm{BPFM})| = 20$, the probability that there is 16 top designs in BPFM selection is

$$\begin{aligned}&\Pr(|S_{\mathrm{f}}(\mathrm{BPFM}) \cap (S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}})| \\ &\quad = |S_{\mathrm{f}}(\mathrm{HRFM}) \cap \Theta_{\mathrm{f}}| = 20 \times 0.8 = 16) \\ &= 1.09 \times 10^{-28}.\end{aligned}$$

Q.E.D.

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