

Weihao LIU, Shenggang LIU

PIC simulation study of electron gun with rotational surface cathode

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Abstract The electron gun with rotational surface cathode is presented to improve the laminarity of the converging electron beams in this paper, and the function expression of the rotational surfaces is given. The results of particle in cell (PIC) simulation indicate that anode-hole spherical aberration is the major cause for the nonlaminarity of the electron beams. By properly choosing the size of the shape, rotational surface cathode can effectively counteract the effect of the anode-hole spherical aberration and enhance the laminarity of the electron beams. The theoretical analysis was carried out for the explanation of the phenomenon that appeared in the PIC simulation.

Keywords rotational surface cathode, laminarity, anode-hole spherical aberration

1 Introduction

Converging Pierce guns are widely used in electronic devices of microwave, millimeter-wave, terahertz-wave, etc. According to the principle of rectilinear flow, which was presented by Pierce in 1940, converging axial symmetry electron flow is formed by cutting sphere diode [1]. So, the shape of cathode of converging electron gun is traditional spherical, which is the residue part of the sphere diode. Yet, both results of the theoretical analysis and afterwards experiments indicated that electron beams from the spherical cathode are far from the laminar flows of Pierce. The electron trajectories cross with each other seriously, especially at the edge of the beam, which lead to the uneven distribution of the electrons and current density in the beam [2–5]. This nonlaminarity has a bad influence

to the efficiency, stability, and noises factors of electronic devices.

To improve the laminarity of the electron beams, Oscar Heil presented a cathode whose outer edge is conical to substitute the spherical cathode in 1960s [6]. Yet, this kind of cathode was difficulty to manufacture at that time. With the numerically controlled machines using the cathode fabrication, it is now feasible for cathode to take different shapes. So, the un-spherical cathode is again run in the sight of the researchers. In 2004, Brian et al. presented a kind of algorithm of choosing cathode shape to optimize the quality of electron beams [6]. Yet, they did not give out the functional expression of the cathode surface and the cathode shapes are still too arbitrary for manufacture.

Still to improve the laminarity of the electron beam, we present the rotational surface cathode and give out its function expression also. Both theoretical analysis and particle in cell (PIC) simulation indicate that this kind of cathode can effectively improve the laminarity of the converging electron beams.

2 Simulation of electron gun with spherical cathode

According to Pierce's principle of rectilinear flow, the electron trajectories in the electron beam are straight lines or curves traveling side by side without any intersection. This kind of electron beam is called laminar flow, which is the ideal state of electron beam transporting. Nonlaminarity means intersection of electron trajectories during the beam transporting, which will lead to the uneven distribution of electrons and current densities in the cross-section of the beam.

Several factors could be the candidates of resulting nonlaminarity, such as initial transverse velocity, over compression of electron beam and the spherical aberration of anode hole [7,8]. In this paper, we are determined to find out the most significant factor of nonlaminarity and to handle it. The method we use is PIC simulation. PIC

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Weihao LIU (✉), Shenggang LIU

Center of Terahertz Science and Technology, School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu 610054, China
E-mail: liuw hao@foxmail.com

simulation for electron gun allows people not only to watch the evolution of the phase space of the electrons but also to measure the distribution of electrons and current density at different places. It is proven to be a very useful tool for researching and electron gun designing [9–12].

The structural and working parameters of the electron gun in our simulation are listed in Table 1. Here, R_c is radius of curvature of cathode, θ is the half-angle of cathode, D is the distance between cathode and anode, r_a is the radius of anode hole, and r_w is the radius of electron path. The way of electron emission we chose is thermionic emission, in which the initial transverse velocities are Maxwell distribution. The focusing magnetic is a step function at the waist of the electron beam as shown in Fig. 1(a). The physical model and simulation results are given in Fig. 1, in which, Fig. 1(a) is the distribution of electron phase space and focusing magnetic; Fig. 1(b) is the distribution of electrons along the radial direction in the focusing region; Fig. 1(c) is the distribution of current density along the radial direction at different places of axial direction (from point ‘a’ to ‘o’ in Fig. 1(a)). It can be seen

from Fig. 1(c) that the current density is almost uniform in the region between cathode and anode (from point ‘a’ to ‘e’ in Fig. 1(a)), which is also called gun-region, and the current density at the cathode surface is about 6 A/cm^2 . But after passing through the anode hole, the distribution of current density displays a shape of dent in the center, which means that the current density in the outer part of beam is higher than the inner part. The distribution of electrons in the beam displayed in Fig. 1(b) also shows this phenomenon. This phenomenon appears due to the intersection of the electron trajectories at the edge of the beam. In another word, this is the nonlaminar flow that should be avoided. It can also be found from the results above that the anode-hole spherical aberration is the primary contributor of nonlaminarity.

The spherical aberration is caused by the first-order approximation of the axial symmetry field when studying the electron beams. The tolerance of approximation aggrandizes with the increase of the distance of the electron to the axis. So, in the electron beam, the tolerance of the outer electrons is larger than the inner ones, which

Table 1 Structural and working parameters of spherical surface

working parameters			structural parameters				
voltage/kV	current/A	beam waist/mm	R_c /mm	θ (°)	D /mm	r_a /mm	r_w /mm
17	0.3	0.3	9.6	7.8	4.8	0.7	0.5

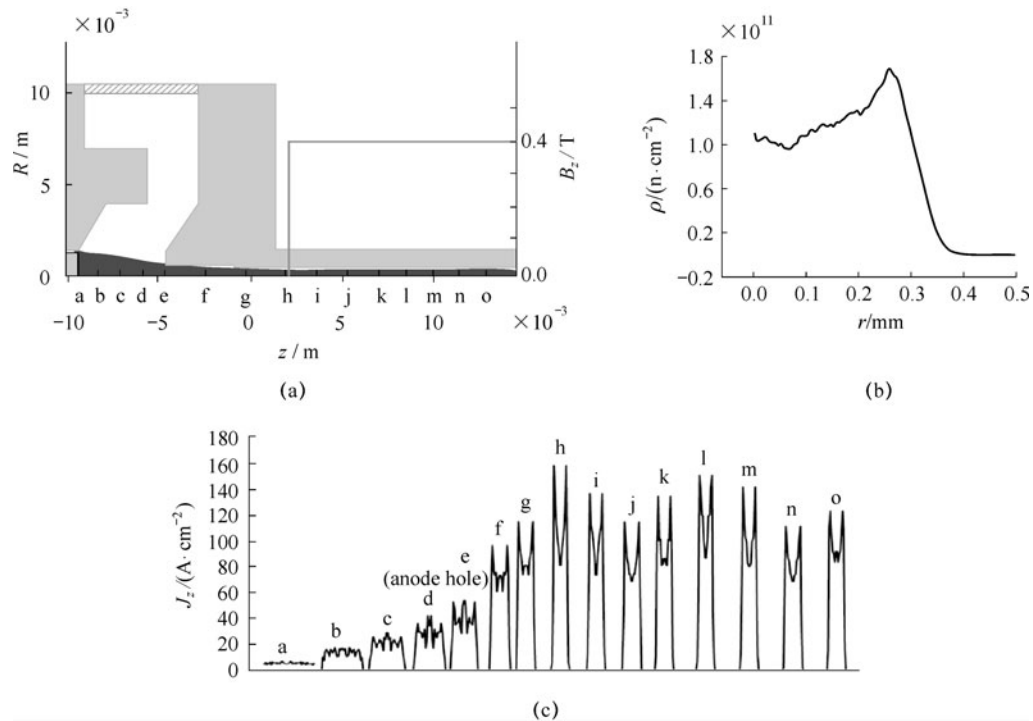


Fig. 1 Physical model and results of simulation. (a) Focusing magnetic and electron phase space; (b) distribution of electrons in radial direction; (c) radial distribution of current density

leads to the focusing force of the outer electrons is less than the inner ones at the anode hole. This difference of focusing force leads to the divergence of the inner electrons is larger than the outer ones [7]. Their trajectories cross with each other at the edge of the beam, which leads to the current density of the outer beam is larger than the inner beam, like the simulation results in Fig. 1.

3 Simulation of electron gun with rotational surface cathode

As mentioned above, spherical aberration of the anode hole is caused by the more serious divergence of the inner electrons than the outer ones. Rotational surface cathode can control the initial convergence of the electrons at any places of the cathode by changing the curvature of the cathode. In other words, the rotational surface can make the outer electrons diverge more than the inner ones, which can be against the spherical aberration of anode hole. The curvature at the edge of the cathode is kept the same with that of the spherical cathode for avoidance of over-compression of the electrons at the beam edge. The schematic diagram of the rotational surface cathode is shown in Fig. 2.

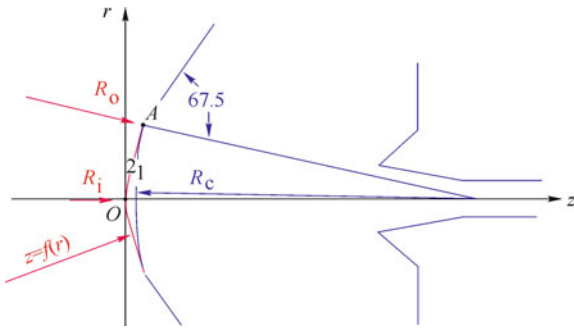


Fig. 2 Structural of electron gun with rotational surface cathode (blue line '1' is spherical cathode and red line '2' is rotational surface)

Quartic polynomial curve is used as the baseline of the rotational surface cathode in this paper. So, the function expression can be defined as follows in the coordinate (z, r) of Fig. 2:

$$z = f(r) = ar^4 + br^3 + cr^2 + dr + e, \quad (1)$$

where a , b , c , d , and e are undetermined coefficients. According to axial symmetry of the rotational surface, $z(r) = z(-r)$, we know $b = 0$ and $d = 0$. Because the curve passes the origin of the coordinate, we know $e = 0$. Then, Eq. (1) turns to be

$$z = ar^4 + cr^2. \quad (2)$$

Two equations are still required to determine the unknown coefficients a and c . In fact, the radius of curvature at two points of the curve can be enough. We choose the point of edge and the center of the curve (point 'A' and 'O' in Fig. 2); their radius of curvature can be expressed as follows:

$$\left. \frac{(1 + z'(r)^2)^{3/2}}{z''(r)} \right|_{r=r_A} = R_o, \quad (3)$$

$$\left. \frac{(1 + z'(r)^2)^{3/2}}{z''(r)} \right|_{r=0} = R_i, \quad (4)$$

where R_i and R_o are the radius of the curvature at the inner and outer points of the rotational surface as indicated in Fig. 2, which means radius of curvature of the edge point and the center point, respectively. As we have assumed the radius of the edge is kept the same as the spherical cathode, then R_o can be determined by $R_o = R_c$. R_i can be determined by the center point's curvature. To counteract the spherical aberration, the radius of curvature of the inner parts should smaller than that of the outer parts, that is to say, $R_i < R_o$. In optimization, the value of R_i should be adjusted to get the best quality of electron beams.

The other parameters of electron gun are kept unchanged. The spherical cathode is replaced by rotational surface cathode. We do PIC simulation again for four different values of R_i , which come in four different shapes of electron guns. The values of R_i and corresponding values of a and c in Eq. (2) are listed in Table 2.

Table 2 Parameters of rotational surface cathode

R_i/mm	a	c
8.0	-0.00084	0.0625
7.0	-0.00170	0.0714
6.5	-0.00220	0.0770
6.0	-0.00280	0.0833

Figure 3 gives the simulation results of these four electron guns. It shows that, when R_i is chosen to be 8.0 mm, the distribution of the electrons and current densities are almost the same as the spherical cathode: the dents of the inner current density are distinct, which means that the spherical aberration still not be restrained. Yet, the density of inner electron beam increases correspondingly with the reduction of the value of R_i . When R_i is set at 6.5 mm, the distribution of electrons and current densities are almost uniform in the radial direction, which means that the spherical aberration is counteracted effectively and the laminarity is much improved. When R_i reduces to 6.0 mm, the inner current density larger than the outer in reverse and the distribution of the current density take on the

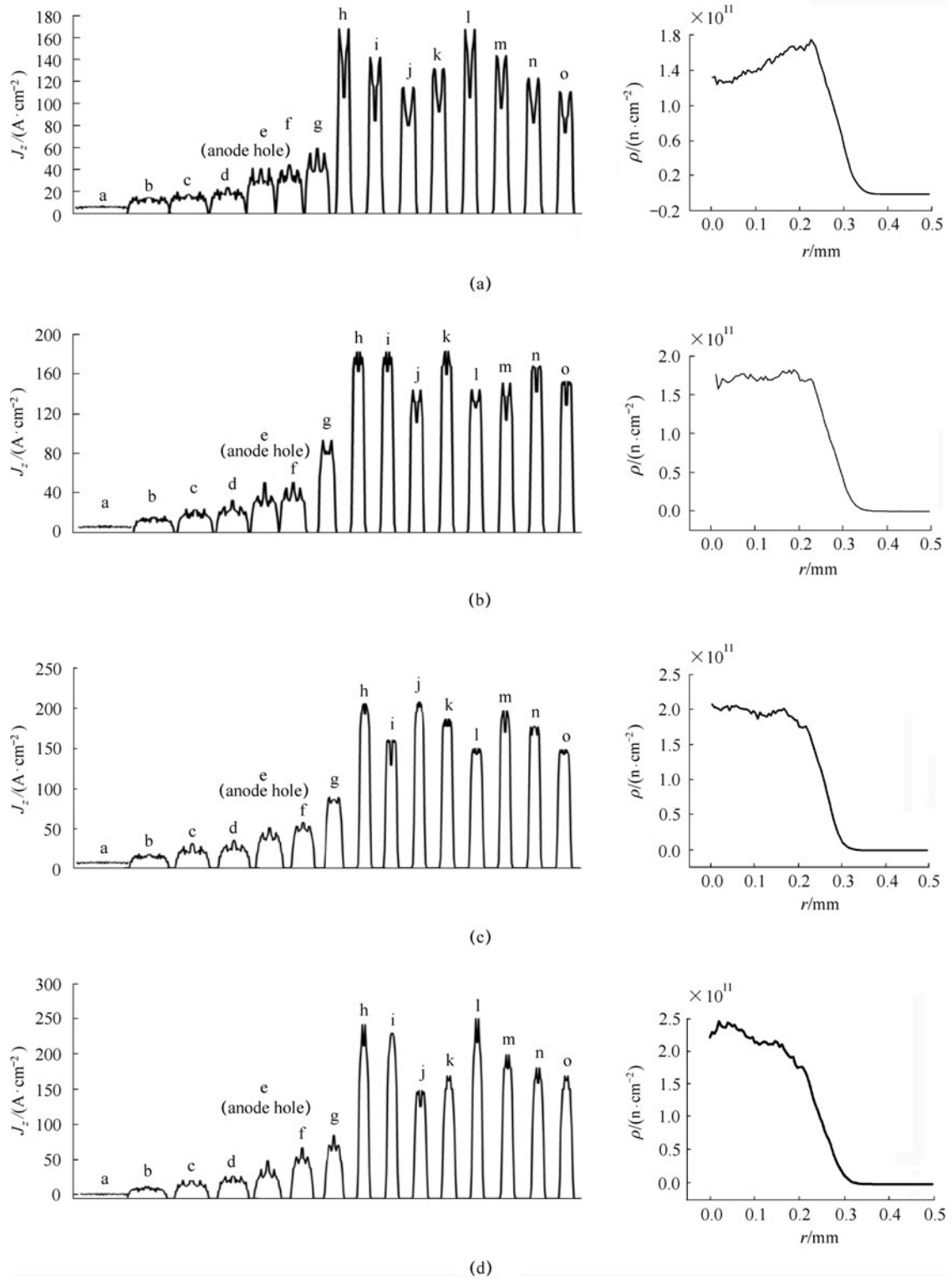


Fig. 3 Results of simulation of electron guns with rotational surface cathode. (a) $R_i = 8.0$ mm; (b) $R_i = 7.0$ mm; (c) $R_i = 6.5$ mm; (d) $R_i = 6.0$ mm (The left parts of (a)–(d) are radial distributions of current density and the right parts are radial distributions of charge density for these four cases, respectively)

shape of Gauss distribution, which means that new nonuniform occurs in the beam. The following analysis will argue that this resulted from over-compression of the inner electrons.

Now let us analyze the relations between the laminarity of the beam and the shape of cathode in detail. In the traditional electron gun with spherical cathode, the difference of divergence between outer and inner electrons result in the trajectories intersection occurring at the edge of the beam, as shown in Fig. 4(a): outer trajectory *A* intersects with inner trajectories *B* and *C* in the equipotential region after passing through the anode hole.

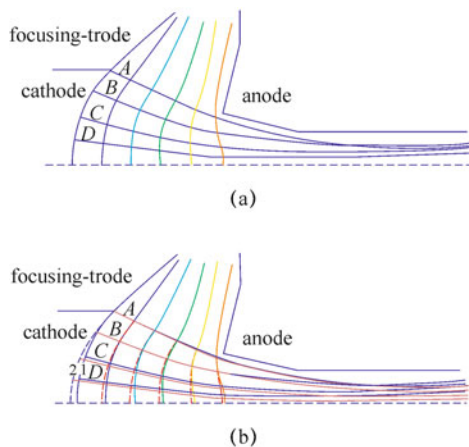


Fig. 4 Distribution of electric field and electron trajectories. (a) Spherical cathode; (b) rotational surface cathode

When the shape of cathode changes to rotational surface (from real line 1 to dashed line 2 in Fig. 4(b)), the distribution of static electric field changed (equipotential lines change from real lines to dashed lines). As the curvature of the edge is kept unchanged, the field distribution at the edge of beam is almost unchanged, which leads to the electron trajectory *A* is kept almost the same with it in the spherical cathode. Yet, in the inner parts of the beam, the equipotential line bent toward the cathode, as displayed in Fig. 4(b), which leads to the compression of the inner electrons more serious than the outer ones, reducing the effects of spherical aberration of the anode hole. The inner trajectories *B*, *C*, and *D* change from the blue lines to red lines as in Fig. 4(b). As can be seen in the figure, the trajectories do not cross with each other anymore and the laminarity of the electron beam is improved a lot.

Figure 5 indicates the change of the electron beam transportation with the change of radius of curvature at the center of the cathode. Figure 5(a) shows the case when the change of center curvature is not enough to counteract the spherical aberration, in which the trajectories cross with each other and the nonlaminarity is obvious, just like the

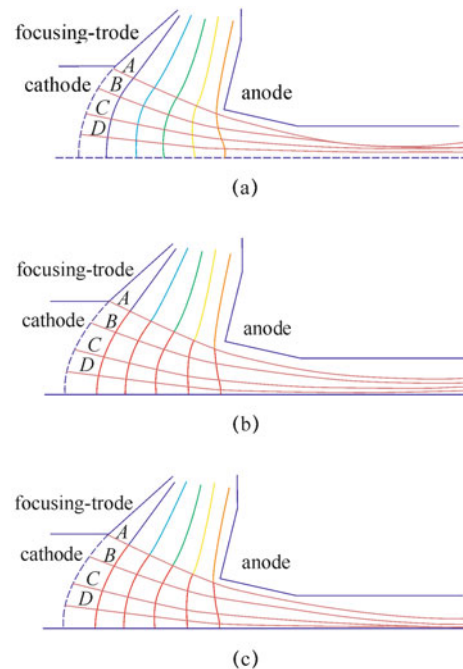


Fig. 5 Distribution of electric field and electron trajectories in different electron guns. (a) Larger R_i ; (b) right R_i ; (c) smaller R_i

case of $R_i = 8.0$ mm in the simulation. Figure 5(b) shows the case when R_i reduces to the point that can counteract the spherical aberration exactly, the laminarity of the electron beam is improved a lot as in the case of $R_i = 6.5$ mm in simulation. Figure 5(c) shows the case when R_i reduces too much, the inner electrons are compressed to the axis, which leads to the current density of the inner part is higher than the outer part, just like the case of $R_i = 6.0$ mm in simulation.

The simulation and analysis above indicate that the laminarity of the converge electron beams can be much improved by rightly choosing the radius of curvature at the center point of the rotational surface cathode.

4 Discussion

1) Simulations were also carried out by electronics optics software EGUN [13] for the former four electron guns, as shown in Fig. 6, which agree well with the results of PIC simulation.

2) As mentioned above, the change of the cathode shape will affect the distribution of the static electric field of the gun region, which then affects the emitting current of the cathode according to the theory of space charge limit flow. Table 3 shows the current of electron beam change with the value of R_i . The current reduces a little as R_i is reduced and the current of the rotational surface is less than that of spherical surface, because the reduction of R_i enlarges the gap between the cathode and the anode in

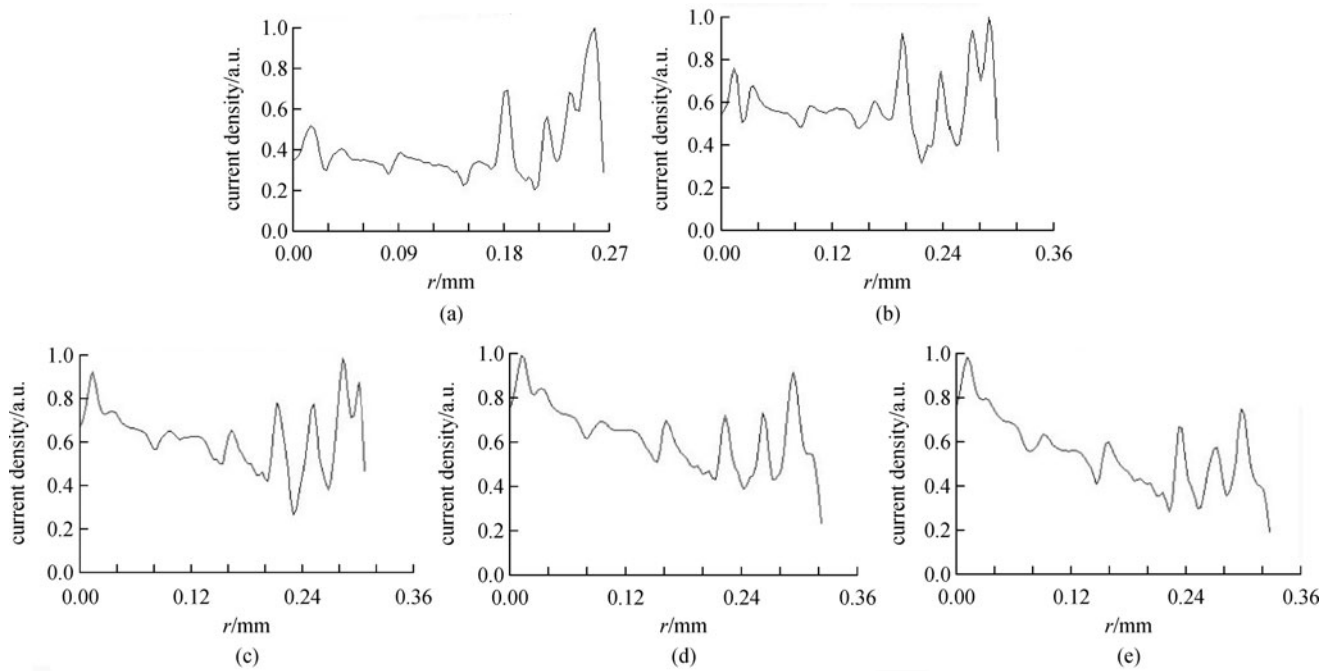


Fig. 6 Distribution of current density at beam waist of different guns. (a) Spherical cathode; (b) $R_i = 8.0$ mm; (c) $R_i = 7.0$ mm; (d) $R_i = 6.5$ mm; (e) $R_i = 6.0$ mm

Table 3 Emitting current of different shapes of guns

	spherical cathode		rotational surface cathode		
	$R_i = 9.6$ mm	$R_i = 8.0$ mm	$R_i = 7.0$ mm	$R_i = 6.5$ mm	$R_i = 6.0$ mm
beam current/A	0.374	0.365	0.354	0.351	0.348
error	0%	2.4%	5.3%	6.1%	6.9%

some extent, which leads to the reduction of electron current.

3) The rotational surface can also be determined by polynomial function of higher order, whose method is the same but needs for radius of curvature of more points (not only the edge point and the center point).

5 Conclusion

The anode-hole spherical aberration is the major factor for nonlaminarity of electron beams in spherical cathode, which is validated by PIC simulation in this paper. The rotational surface cathode was initiated to replace the traditional spherical cathode to reduce the effect of anode-hole spherical aberration and to improve the laminarity of the converging electron beams. The functional expression is also determined by choosing the radius of curvature of the center point of the cathode. Both the results of theory analysis and PIC simulation indicate that the laminarity of electron beams can be improved a lot by choosing the curvature of the cathode properly.

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