Front. Electr. Electron. Eng. China 2011, 6(2): 381–387
DOI 10.1007/s11460-011-0144-0

RESEARCH ARTICLE

Arto KAARNA, Wei LIU, Heikki KÄLVIÄINEN

Development of color density concept with color difference formulas in respect to human vision system

Abstract The aims of this study are to develop the color density concept and to propose the color density based color difference formulas. The color density is defined using the metric coefficients that are based on the discrimination ellipses and the locations of the colors in the color space. The ellipse sets are the MacAdam ellipses in the CIE 1931 xy-chromaticity diagram and the chromaticity-discrimination ellipses in the CIELAB space. The latter set was originally used to develop the CIEDE2000 color difference formula. The color difference can be calculated from the color density for the two colors under consideration. As a result, the color density represents the perceived color difference more accurately, and it could be used to characterize a color by a quantity attribute matching better to the perceived color difference from this color. Resulting from this, the color density concept provides simply a correction term for the estimation of the color differences. In the experiments, the line element formula and the CIEDE2000 color difference formula performed better than the color density based difference measures. The reason behind this is in the current modeling of the color density concept. The discrimination ellipses are typically described with three-dimensional data consisting of two axes, the major and the minor, and the inclination angle. The proposed color density is only a one-dimensional corrector for color differences; thus, it cannot capture all the details of the ellipse information. Still, the color density gives clearly more correct estimations to perceived color differences than Euclidean distances using directly the coordinates of the color space.

Keywords color density, discrimination ellipses, color difference formulas, color vision

Received August 30, 2010; accepted February 24, 2011

Arto KAARNA (✉), Wei LIU, Heikki KÄLVIÄINEN
Machine Vision and Pattern Recognition Laboratory, Department of Information Technology, Faculty of Technology Management, Lappeenranta University of Technology, P.O. Box 20, FI-53851 Lappeenranta, Finland
E-mail: arto.kaarna@lut.fi

1 Introduction

A color difference describes the distance between two colors in the color space. Although color difference formulas provide precise ways for predicting the perceived color differences, the performances of these formulas depend on the perceptual data provided for testing. The color differences calculated in these formulas consist of three components: hue, lightness, and chroma differences; consequently, modeling of the color space with a compatible color difference formula may become complicated. Moreover, human perceived color differences can be represented with chromaticity discrimination ellipses. Therefore, a new concept will be developed. The concept is named as the color density, and it could represent the human perceived color difference quantitatively. Secondly, new color difference formulas derived from the color density concept are proposed. In general, the color density will give an intuitive comprehension of the color differences in the color space.

The MacAdam ellipses show the human vision sensitivity to small color differences in the CIE 1931 xy-chromaticity diagram [1] (see Fig. 1). The chromaticity discrimination ellipse set, which was used for deriving the CIEDE2000 color difference formula, in this article denoted as the CIEDE2000 ellipse set, consists of the BFD-P, Witt, RIT-DuPont, and Leeds data sets [2] (see Fig. 2). These four ellipse sets were fixed for different visual color difference measurements.

Each ellipse represents equal perceived chromaticity differences; in other worlds, the chromaticity differences inside the ellipse cannot be perceived. For each ellipse, the color matching can be expressed as

$$g_{11}(\mathrm{d}x)^2 + 2g_{12}\mathrm{d}x\mathrm{d}y + g_{22}(\mathrm{d}y)^2 = 1, \tag{1}$$

where $\mathrm{d}x$ and $\mathrm{d}y$ are the differences of the coordinates between the ellipse center and any point on the ellipse

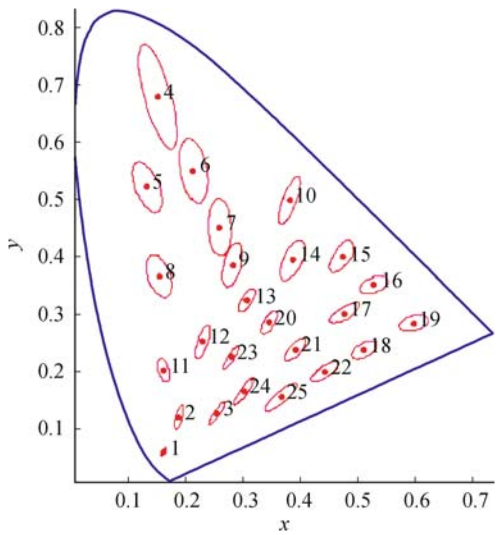

Fig. 1 MacAdam ellipses. Each ellipse is drawn 10 times larger to its real size

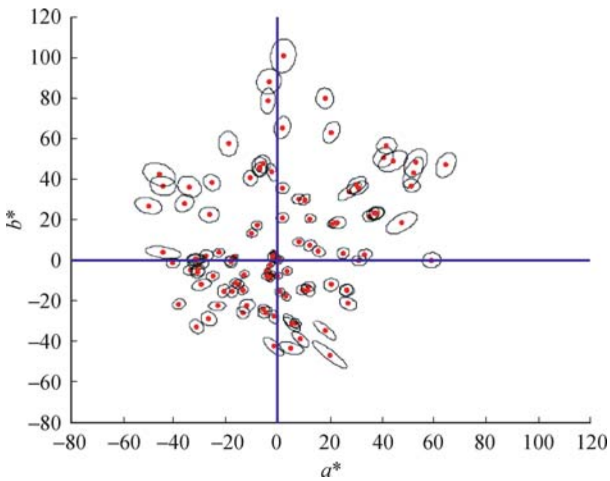

Fig. 2 Ellipse set used for deriving the CIEDE2000 color difference formula

boundary, and the metric coefficients g_{ik} are constant for each ellipse. The coordinates x and y are calculated according to the CIE 1931 definitions [3].

The sizes, shapes, and orientations of the MacAdam ellipses vary systematically over the CIE 1931 xy-chromaticity diagram: the ellipses at the bottom left corner, in the blue area, are the smallest ones, and the angles of inclination are also the smallest ones. Both the sizes and the inclination angles become larger from the bottom to the top and reach the maximum at the top left corner, in the green area [1].

Some clear trends can be seen also for the ellipses in the CIELAB space shown in Fig. 2: the ellipses close to the gray axis (around $-10 < a^* < 10$, $-15 < b^* < 15$) are the smallest; then, ellipse sizes increase as the chromaticities increase. Nearly all ellipses point toward the gray axis, except for those in the blue area, particularly around $a^* = 10$, $b^* = -40$ [4]. a^* and b^* are defined as the chromaticity coordinates in the CIELAB space [3].

The purpose in this study was to develop a concept called the color density, which exists everywhere in the color space. Then, the color density would give a rough estimation of perceived color differences from each color. The color density is derived from the metric coefficients g_{ik} defined using the chromaticity-discrimination ellipses. In this study, two models for calculating the metric coefficients g_{ik} will be applied. The first model is based on the CIE 1931 xy-chromaticity space and the CIELAB color space. The second model is based on the xyY color space. Here, Y corresponds to the luminous efficiency function in the CIE 1931 modeling [3]. Furthermore, two color difference formulas are developed based on the color density concept. The experimental results of the new formulas are compared with the original line element formula [3] and the CIEDE2000 color difference formula [4]. In the last chapter, the conclusions are given.

2 Computing metric coefficients g_{ik}

For the MacAdam ellipses and the chromaticity ellipses in the CIELAB color space, the metric coefficients g_{ik} can be calculated from ellipse parameters as

$$g_{11} = \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}, \tag{2}$$

$$g_{12} = \sin\theta\cos\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right), \tag{3}$$

$$g_{22} = \frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2}, \tag{4}$$

where a and b are the lengths of the major semi-axis and the minor semi-axis of the ellipse, and θ is the inclination angle between the major semi-axis and the horizontal axis, like the x-coordinate in the CIE xy-chromaticity space [1].

The metric coefficients g_{ik} of the chromaticity ellipses in the xyY color space could be calculated also as follows [5]: in the CIELAB color space, the chromaticity difference ΔC ($\Delta Y = 0$) is defined as

$$\Delta C = \sqrt{(\Delta a^*)^2 + (\Delta b^*)^2}, \tag{5}$$

and

$$a^* = 500\left[\left(\frac{X}{X_n}\right)^{1/3} - \left(\frac{Y}{Y_n}\right)^{1/3}\right], \tag{6}$$

$$b^* = 200\left[\left(\frac{Y}{Y_n}\right)^{1/3} - \left(\frac{Z}{Z_n}\right)^{1/3}\right], \tag{7}$$

where $X = (x/y)Y$, $Z = (z/y)Y$, and $z = 1 - x - y$. The reference white in this calculation is $X_n = 94.811$, $Y_n = 100.00$ and $Z_n = 107.304$. The partial derivatives are then

$$a_x = \frac{\partial a^*}{\partial x} = 500\left(\frac{Y}{X_n}\right)^{1/3}\frac{1}{3}\left(\frac{x}{y}\right)^{-2/3}\frac{1}{y}, \tag{8}$$

$$a_y = \frac{\partial a^*}{\partial y} = 500\left(\frac{Y}{X_n}\right)^{1/3}\frac{1}{3}\left(\frac{x}{y}\right)^{-2/3}\left(-\frac{x}{y^2}\right), \quad (9)$$

and

$$b_x = \frac{\partial b^*}{\partial x} = -200\left(\frac{Y}{Z_n}\right)^{1/3}\frac{1}{3}\left(\frac{z}{y}\right)^{-2/3}\left(-\frac{1}{y}\right), \quad (10)$$

$$b_y = \frac{\partial b^*}{\partial y} = -200\left(\frac{Y}{Z_n}\right)^{1/3}\frac{1}{3}\left(\frac{z}{y}\right)^{-2/3}\left(\frac{-y-z}{y^2}\right). \quad (11)$$

The chromaticity difference from Eq. (5) becomes now

$$\Delta C = [(a_x^2 + b_x^2)(\mathrm{d}x)^2 + 2(a_x a_y + b_x b_y)\mathrm{d}x\mathrm{d}y + (a_y^2 + b_y^2)(\mathrm{d}y)^2]^{1/2}, \quad (12)$$

and then the metric coefficients are found as

$$g_{11} = a_x^2 + b_x^2, \quad (13)$$

$$g_{12} = a_x a_y + b_x b_y, \quad (14)$$

$$g_{22} = a_y^2 + b_y^2. \quad (15)$$

3 Defining color density

For developing the color density, two new values — $g_{\max}$ and $g_{\min}$, which are considered as coefficients along the major semi-axes a and minor semi-axes b of the ellipse — should be calculated first. The computations of $g_{\max}$ and $g_{\min}$ are analogous to the von Mises yield criteria modeling [6] in material science where the goal is to find a univariate descriptor for a multidimensional stress field. Correspondingly, using the components g_{11}, g_{12}, and g_{22}, $g_{\max}$ and $g_{\min}$ become now

$$g_{\max} = \frac{g_{11} + g_{22}}{2} + \sqrt{\left(\frac{g_{11} - g_{22}}{2}\right)^2 + g_{12}^2}, \quad (16)$$

and

$$g_{\min} = \frac{g_{11} + g_{22}}{2} - \sqrt{\left(\frac{g_{11} - g_{22}}{2}\right)^2 + g_{12}^2}, \quad (17)$$

where the values of g_{11}, g_{12}, and g_{22} can be calculated from Eqs. (2)–(4) or from Eqs. (13)–(15). Then, the components $g_{\max}$ and $g_{\min}$ in the two main directions are combined to a single value g_{comb} and this value becomes the color density. The color density g_{comb} is now defined as

$$g_{\mathrm{comb}} = \sqrt{g_{\max}^2 - g_{\max}g_{\min} + g_{\min}^2}, \quad (18)$$

in accordance with the von Mises criterion modeling. This derivation gives the one-dimensional color density concept, i.e., the density in a given location in the color space is constant for all directions.

4 Defining color difference formulas based on color density

The color density is now acting as a corrector term for a difference of two colors originating from a chromaticity or a color space. There are many ways to utilize the color density; next, two approaches are given.

Chromaticity differences $\mathrm{d}s_1$ and $\mathrm{d}s_2$ from color C1 to color C2 are defined as

$$\mathrm{d}s_1 = \sqrt{g_{\max,\mathrm{m}}(\mathrm{d}x_\mathrm{e})^2 + g_{\min,\mathrm{m}}(\mathrm{d}y_\mathrm{e})^2}, \quad (19)$$

and

$$\mathrm{d}s_2 = \sqrt{g_{\mathrm{comb,m}}(\mathrm{d}x)^2 + g_{\mathrm{comb,m}}(\mathrm{d}y)^2}, \quad (20)$$

where $g_{\max,\mathrm{m}}$ and $g_{\min,\mathrm{m}}$ are the mean values of $g_{\max}$ and $g_{\min}$ of the two colors, $\mathrm{d}x_\mathrm{e}$ and $\mathrm{d}y_\mathrm{e}$ are the distances along the major semi-axes a and minor semi-axes b of the ellipse with C1 as the ellipse center. In Eq. (19), the color density g is mixed with the ellipse information to the given chromaticities or colors. In Eq. (20), $g_{\mathrm{comb,m}}$ is the mean value of the two g_{comb} values for the two colors; $\mathrm{d}x$ and $\mathrm{d}y$ are the distances of the chromatic coordinates for C1 and C2. In Eq. (20), the color density acts more directly as a corrector term. The required information on the color density everywhere in the space is received through interpolation of the ellipse parameters (a, b, θ) or the metric coefficients over the chromaticity or color space.

5 Experiments

The experiments were performed in two steps using the CIE 1931 xy-chromaticity diagram, the CIELAB color space, and the xyY color space. First, the g_{comb} values were calculated and interpolated over the space. Second, the color difference formulas were compared. Three experimental measurements were performed and the obtained results were compared to the line element formula and the CIEDE2000 color difference formula.

The line element $\mathrm{d}s$ is defined as [3]

$$(\mathrm{d}s)^2 = g_{11}(\mathrm{d}U_1)^2 + 2g_{12}\mathrm{d}U_1\mathrm{d}U_2 + g_{22}(\mathrm{d}U_2)^2 + 2g_{23}\mathrm{d}U_2\mathrm{d}U_3 + g_{33}(\mathrm{d}U_3)^2 + 2g_{31}\mathrm{d}U_3\mathrm{d}U_1, \quad (21)$$

where the coefficients g_{ik} may be continuous functions on the coordinates U_1, U_2, and U_3.

A color difference ΔE in CIEDE2000 is defined as [4]

$$\Delta E = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 + \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T\left(\frac{\Delta C'}{k_C S_C}\right)\left(\frac{\Delta H'}{k_H S_H}\right)}, \quad (22)$$

where the coefficients k and S account for lightness, chroma, and hue components of the colors. They also compensate the perceptual non-uniformity of the CIELAB color space.

5.1 g_{comb} for three color spaces

The definition of the color density is based on the ellipse information, i.e., only for certain locations in the color space. To extend the color density over the whole color space, then interpolation is needed.

To get the color densities of all colors over the CIE 1931 xy-chromaticity diagram, two sets of values were used for the interpolation of g_{comb}. The first set was calculated from the 25 MacAdam ellipses, and the second set was calculated from the estimated g_{ik} values along the color gamut [1]. Figures 3 and 4 show the interpolated color densities over the CIE 1931 xy-chromaticity diagram. The color density g_{comb} reaches the maximum in the blue area (the red colored part of the surface/contours, x~0.2 and y~0.05, in Figs. 3 and 4), and the minimum color density g_{comb} occurred in the green area (the blue colored part of the surface/contours, x~0.1 and y~0.8, in Figs. 3 and 4). The intermediate color densities were in the area between blue and green, and they became larger from the bottom area to the top area, in the yellow-blue direction in the xy-chromaticity space. This showed the same variation trend as the size changes for the MacAdam ellipses. As such, we can conclude that, in the CIE 1931 xy-chromaticity diagram, the larger the color differences perceived from a color, the larger the color density is for that color.

The CIEDE2000 ellipse set was the only available information in the xyY and in the CIELAB color spaces. For this reason, the interpolations were performed only inside the areas that were enclosed by the CIEDE2000 ellipse set. The BFD-P, Rit-DuPont, and Witt data sets that contained 116 ellipses in total were used for the interpolation in this study.

The color densities in the xyY and the CIELAB color spaces are four dimensional; thus, the visualization of the values for the color density becomes more complicated. Now, isosurfaces were used to visualize the color density values. Figures 5 and 6 show the isosurfaces of the color density in the xyY and in the CIELAB color spaces, respectively. In Fig. 5, the isovalues increase from the bottom to the top. This indicates that the color densities increased while the illumination levels increased. Therefore, for reaching the same color densities, the isosurfaces are curved upwards near the green colors. The two isosurfaces with the lowest isovalues are flatter than the others. In the CIELAB color space, the isovalues become larger from the outside to the center (see Fig. 6).

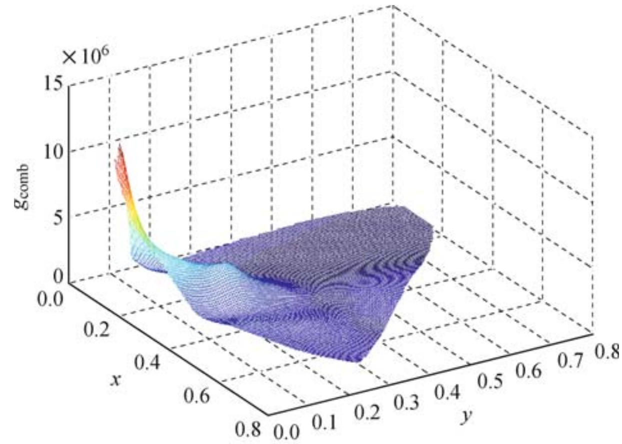

Fig. 3 Surface of color density (g_{comb}) over CIE 1931 xy-diagram

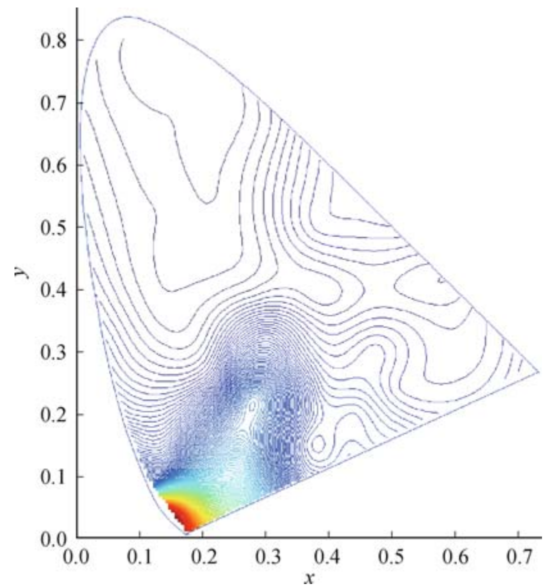

Fig. 4 Contours of color density (g_{comb}) over CIE 1931 xy-diagram

In other words, neutral colors near the center have the largest color density, and the larger the distances from the neutral center, the smaller the color density becomes. This trend agreed well with the ellipse size variations of the CIEDE2000 ellipse set used in this study.

5.2 Comparison of color difference formulas

Three experiments were designed for evaluating the color density based color difference formulas given in Eqs. (19) and (20). The first experiment, called Experiment 1, was performed inside the ellipses. See Fig. 7 for the geometric setting in this experiment. Since we were measuring distances from the ellipse center to the ellipse boundary, the expected chromaticity or color differences should have been now one constant for all ellipses.

The second experiment, called Experiment 2, was performed inside circles. The centers of the ellipses were

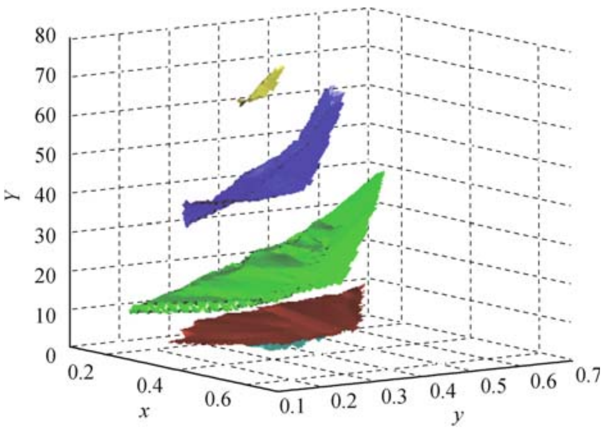

Fig. 5 Isosurfaces of color density in xyY color space. The isovalues are 0.25 (just visible with the lowest values on Y), 0.5, 2.0, 5.0, and 8.0 for the surfaces from bottom to top

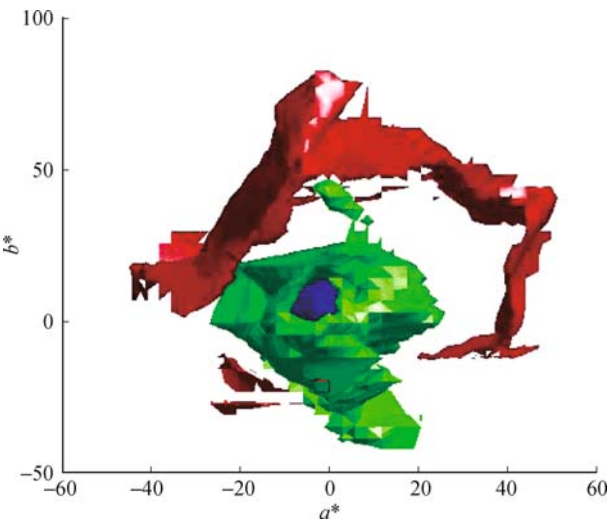

Fig. 6 Isosurfaces of color density plotted in a^*b^* diagram. The isovalues are 2.0, 1.2, and 0.5 for blue, green, and red surfaces

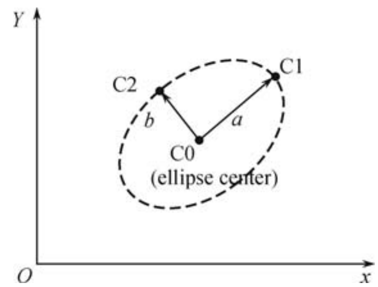

Fig. 7 Visualization of Experiment 1. The color differences are calculated from C0 (the ellipse center) to C1 (point on the ellipse boundary along the major semi-axes a) and to C2 (point on the ellipse boundary along the minor semi-axes b)

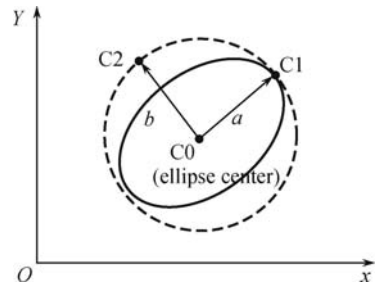

Fig. 8 Visualization of Experiment 2 (the radius of circle is a). The color differences are calculated from C0 (the ellipse center) to C1 (point on the circle boundary along the major semi-axes a) and to C2 (point on the circle boundary along the minor semi-axes b)

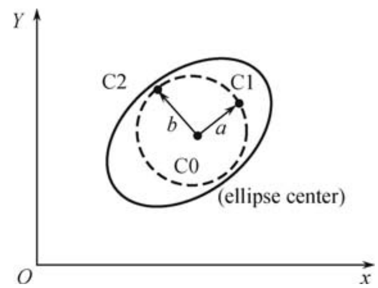

Fig. 9 Visualization of Experiment 3 (constant radius). The color differences were calculated from C0 (the ellipse center) to C1 (point on the circle boundary along the major semi-axes a) and to C2 (point on the circle boundary along the minor semi-axes b). This figure corresponds to a larger ellipse than the circle radius. In other locations of the chromaticity space, the circle could also become larger than the ellipse

taken as the centers of the circles, and the radii of the circles were the lengths of the corresponding major semi-axis a; thus, the sizes of the circles varied in various locations in the chromaticity or color space. See Fig. 8 for the setting of Experiment 2. Since the circle was larger than the ellipse in the direction C0→C2, then the expected differences should have been also larger in the direction C0→C2 than in the direction C0→C1.

The third experiment, called Experiment 3, was also performed with circles. The ellipse centers were taken as the circle centers, and a constant radius for the circles was applied everywhere; see Fig. 9 for the setting. The constant radius was the mean value of all ellipse major semi-axis a. This mean value was found equal to 0.0034 in the CIE 1931 xy-chromaticity diagram and 2.28 in the CIELAB color space. With Experiment 3, we could compare chromaticity differences in various locations of the chromaticity space. The circle remained constant everywhere in the space, and as such, the expected chromaticity differences should have been larger in locations where the ellipse sizes were smaller.

Five MacAdam ellipses (ellipses #1, #12, #16, #7, and #4) with different sizes and locations and five CIE ellipses [7] were selected for performing Experiments 1–3. Table 1 collects the experimental results for the MacAdam ellipse set.

In Experiments 2 and 3, the experimental results for the line element indicate that the color differences on the major semi-axis were always smaller than on the minor semi-axis. However, Eq. (19) (column ds_1) showed opposite trends, and Eq. (20) (column ds_2) gave constant differences in two different directions. This shows the typical behavior of the univariate density: it is not able to adapt to various directions in the space but remains constant for all directions. The same is seen from Experiment 1. In the direction C0→C1, the trend in differences is more correct than for the Euclidean differences, but the differences in the two directions C0→C1 and C0→C2 are very different. The line element is producing most correct results in the xy-space. The proposed difference calculation given in Eq. (20) ($\mathrm{d}s_2$) proved to act better than difference according to Eq. (19) ($\mathrm{d}s_1$). The amplification of the differences in the direction C0→C2 is not strong enough, but in the direction C0→C1 the amplification is stronger. This means that the color density is emphasizing the major semi-axis more than the minor semi-axis.

Table 2 collects the results of Experiments 1–3 in the CIELAB color space. In Experiment 1, CIEDE2000 formula calculated nearly constant color differences in the two directions, which were close to the definition of the discrimination ellipses. In Experiments 2 and 3, the CIEDE2000 formula showed that the color differences on the major semi-axis were smaller than the minor semi-axis, which occurred both inside and outside ellipses. Equations (19) and (20) showed similar results as performed with the MacAdam ellipses.

In comparison with the line element equation and the CIEDE2000 formula, although Eq. (20) relied more on the Euclidean distance between two color centers, it is consistent with the relation between color densities and human perceived color differences. Now, the larger the color density for a color was, the larger the color differences were to be perceived. In general, in CIELAB space, the differences $\mathrm{d}s_1$ and $\mathrm{d}s_2$ performed better than in xy-chromaticity space. The trends were more similar to ΔE_{00}. In Experiment 1, there was still a difference in the two directions, but it was not as large as in xy-space. There was also the correction phenomena present compared to the Euclidean distance ($\mathrm{d}s$). Experiments 2 and 3 still showed that the color density is not amplifying the direction C0→C2 sufficiently.

Table 1 Color density based chromaticity differences from the MacAdam ellipses in xy-space

ellipse	g_{comb} /10^6	direction	Experiment 1				Experiment 2				Experiment 3			
			$\mathrm{d}s_1^{1)}$	$\mathrm{d}s_2^{2)}$	$\mathrm{d}s^{3)}$	LEE$^{4)}$	$\mathrm{d}s_1^{1)}$	$\mathrm{d}s_2^{2)}$	$\mathrm{d}s^{3)}$	LEE$^{4)}$	$\mathrm{d}s_1^{1)}$	$\mathrm{d}s_2^{2)}$	$\mathrm{d}s^{3)}$	LEE$^{4)}$
#1	7.5668	C0→C1	5.87	2.33	0.0008	1.00	2.42	2.33	0.0008	1.00	9.64	7.05	0.0034	4.00
		C0→C2	0.17	0.96	0.0003	1.00	1.01	2.33	0.0008	2.43	4.11	6.99	0.0034	9.71
#12	0.3610	C0→C1	11.8	3.36	0.0031	1.00	3.43	3.36	0.0031	1.00	3.76	2.80	0.0034	1.08
		C0→C2	0.08	0.98	0.0009	1.00	0.99	3.36	0.0031	4.17	1.08	2.80	0.0034	3.75
#16	1.1860	C0→C1	3.99	1.90	0.0026	1.00	2.00	1.90	0.0026	1.00	2.61	1.89	0.0034	1.30
		C0→C2	0.25	0.95	0.0013	1.00	1.00	1.89	0.0026	2.50	1.32	1.88	0.0034	2.61
#7	0.2326	C0→C1	6.28	2.42	0.0050	1.00	2.51	2.41	0.0050	1.00	2.51	1.25	0.0034	0.68
		C0→C2	0.16	0.96	0.0020	1.00	0.99	2.41	0.0050	3.44	1.70	1.25	0.0034	1.70
#4	0.1839	C0→C1	17.4	4.11	0.0096	1.00	4.19	4.11	0.0096	1.00	1.48	1.11	0.0034	0.35
		C0→C2	0.05	0.99	0.0023	1.00	0.98	4.13	0.0096	2.00	0.35	1.11	0.0034	1.48

1) $\mathrm{d}s_1$ was calculated using Eq. (19).
2) $\mathrm{d}s_2$ was calculated using Eq. (20).
3) $\mathrm{d}s$ is calculated as the Euclidean distance $\mathrm{d}s = \sqrt{(\mathrm{d}x)^2 + (\mathrm{d}y)^2}$, where $\mathrm{d}x$ and $\mathrm{d}y$ are the distances in x- and y-directions between the two chromaticities.
4) LEE values were calculated from the equation for a line element, Eq. (21) [3].

Table 2 Color density based chromaticity difference from CIE chromaticity ellipses in CIELAB color space

ellipse	g_{comb}	direction	Experiment 1				Experiment 2				Experiment 3			
			$\mathrm{d}s_1^{1)}$	$\mathrm{d}s_2^{2)}$	$\mathrm{d}s^{3)}$	$\Delta E_{00}^{4)}$	$\mathrm{d}s_1^{1)}$	$\mathrm{d}s_2^{2)}$	$\mathrm{d}s^{3)}$	$\Delta E_{00}^{4)}$	$\mathrm{d}s_1^{1)}$	$\mathrm{d}s_2^{2)}$	$\mathrm{d}s^{3)}$	$\Delta E_{00}^{4)}$
#1	3.8284	C0→C1	1.86	1.75	0.95	0.92	1.86	1.75	0.95	0.92	4.11	3.87	2.28	2.15
		C0→C2	0.53	0.87	0.48	0.72	1.05	1.65	0.95	1.42	2.42	3.87	2.28	3.30
#2	0.6657	C0→C1	1.96	1.86	2.29	0.80	1.96	1.86	2.29	0.80	1.95	1.85	2.28	0.80
		C0→C2	0.70	0.92	1.16	0.75	1.38	1.80	2.29	1.48	1.37	1.79	2.28	1.48
#3	0.4574	C0→C1	2.01	1.91	2.83	0.88	2.01	1.91	2.83	0.88	1.62	1.54	2.28	0.71
		C0→C2	0.50	0.95	1.40	0.94	0.95	1.98	2.83	1.88	0.76	1.59	2.28	1.52
#4	0.6201	C0→C1	1.79	1.68	2.15	0.88	1.79	1.68	2.15	0.88	1.90	1.79	2.28	0.93
		C0→C2	0.57	0.94	1.20	0.80	1.04	1.69	2.15	1.45	1.10	1.79	2.28	1.53
#5	1.0654	C0→C1	4.21	4.08	3.47	0.92	4.21	4.08	3.47	0.92	2.86	2.79	2.28	0.59
		C0→C2	0.29	1.01	0.95	0.87	1.20	3.53	3.47	3.09	0.75	0.75	2.28	2.05

1) $\mathrm{d}s_1$ was calculated using Eq. (19).
2) $\mathrm{d}s_2$ was calculated using Eq. (20).
3) $\mathrm{d}s$ is calculated as the Euclidean distance $\mathrm{d}s = \sqrt{(\mathrm{d}a)^2 + (\mathrm{d}b)^2}$, where $\mathrm{d}a$ and $\mathrm{d}b$ are the distances in a- and b-directions between the two colors.
4) ΔE_{00} were calculated from the CIEDE2000 color difference formula given in Eq. (22) [4].

6 Conclusions

The color density derived in this study revealed variation trends for the MacAdam ellipses and the CIEDE2000 ellipse set. Furthermore, the dependence between the color density and the illumination levels in the xyY color space found in this work was consistent with the results from Melgosa et al. [8] showing the influence of the luminance on the size of discrimination ellipses.

In general, the color with a larger color density yields larger perceived color differences. However, it is not adequate in evaluating the color differences from a color in various directions. Furthermore, none of the color density based color difference formulas performed better than the original line element formula or the CIEDE2000 color difference formula. According to this study, each color could be characterized by the color density, which estimates perceived color differences from this color only at a very rough level. The color densities illustrated in Figs. 3–6 give an intuitive vision how the color differences act in various parts of the color space. Through the interpolation, the color density also becomes a continuous description corresponding to the perceived differences.

The color density concept packs all the ellipse information in one corrector term. The ellipses are defined using three variables, namely the lengths of the two axes and the inclination angle. The density is given with a univariate term only. This means that some information is lost, but at the same time the calculations become simpler. Also, the density surface gives a rough understanding of the correctness of the chromaticity of color difference calculation using the chromaticity or color coordinates directly. The color density is thus correcting those Euclidean estimates from the coordinates.

Our future work includes the description of the color spaces more precisely to further develop the color density concept. Also, the difference equations require new enhancements depending on the density concept. Especially, the main shortcoming, namely the one-dimensional density, will be reconsidered to construct a multidimensional density.

References

1. MacAdam D L. Specification of small chromaticity differences. Journal of the Optical Society of America, 1943, 33(1): 27–30
2. Luo M R, Rigg B. Chromaticity-discrimination ellipses for surface colours. Color Research and Application, 1986, 11(1): 25–42
3. Wyszecki G, Stiles W S. Color Science Concept and Methods, Quantitative Data and Formulae. 2nd ed. New York: John Wiley & Sons Inc, 2000, 654–658
4. Luo M R, Cui G, Rigg B. The development of the CIE2000 color-difference formula: CIEDE2000. Color Research and Application, 2001, 26(5): 340–350
5. MacAdam D L. Metric coefficients for CIE color-difference formulas. Color Research and Application, 1985, 10(1): 45–49
6. von Mises R. Mechanik der Festen Körper im plastisch deformablen Zustand. Göttingen, Mathematisch-Physikalische Klasse, 1913, 4(1): 582–592
7. Robertson A R. CIE guidelines for coordinated research on colour differences evaluations. Color Research and Application, 1978, 3(3): 149–151
8. Melgosa M, Pérez M M, EI Moraghi A, Hita E. Color discrimination results from a CRT device: influence of luminance. Color Research and Application, 1999, 24(1): 38–44

Arto KAARNA received his M.Sc. degree in mechanical engineering in 1980, Licenciate of Science degree in information technology in 1990, and Doctor of Science (Tech.) degree in information technology in 2000, all at Lappeenranta University of Technology (LUT), Finland. He is currently a senior lecturer at the Department of Information Technology, LUT. His main research interests are in digital image processing and in color science. He is a member of Pattern Recognition Society of Finland (member of IAPR) and Institute of Electrical and Electronic Engineers (IEEE).

Wei LIU received her Master of Science (Tech.) degree at Lappeenranta University of Technology, Lappeenranta, Finland, in 2009. Her main research interest is in color science.

Heikki KÄLVIÄINEN received his M.Sc. degree and Ph.D. degree (Doctor of Technology) in Computer Science in 1989 and 1994, respectively, from Department of Information Technology in Lappeenranta University of Technology (LUT), Lappeenranta, Finland. Since 1996, Kälviäinen has been a Professor of Computer Science. Currently, he is a Head of Department of Information Technology and a Head of Machine Vision and Pattern Recognition Laboratory. His primary research interests include machine vision, pattern recognition, and image processing and image analysis. Prof. Kälviäinen belongs to the governing board of IAPR, and he is a member of ACM, IEEE, and SPIE.