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RESEARCH ARTICLE

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Tighter bounds of multihop relayed communications in Nakagami-*m* fading

Abstract Closed-form bounds for the end-to-end performance of multihop communications with non-regenerative relays over Nakagami-*m* fading channels are investigated. Upper and lower bounds of the end-to-end signal-to-noise ratio (SNR) are first developed by using the monotonicity. Then, the probability density functions (PDFs), the cumulative distribution functions, and the moment-generating functions (MGFs) of the bounds are derived. Using these results, the bounds for the outage and average bit error probability (ABEP) are obtained. Numerical and simulation results are executed to validate the tightness of the proposed bounds.

Keywords average bit error probability (ABEP), multihop relayed communications, Nakagami-*m* fading, outage probability

1 Introduction

Multihop transmission is a well-known technique that has the advantage of extending the coverage with low power at the transmitter. The cooperative diversity concept, where the mobile users cooperate each other for exploiting the benefits of spatial diversity without antenna arrays, has gained widespread concern [1,2].

In the past few years, the performance analysis of multihop wireless communications systems over fading channels has been an important field of research. Reference [3] has presented and analyzed four channel models for multihop wireless communications. References [4–7] have studied the evaluation of the performance of dual-hop wireless systems with non-regenerative (channel state information (CSI)-assisted or fixed gain) relays over several common fading channels. For multihop wireless systems, Ref. [8] has presented an analytical framework for the evaluation of the end-to-end outage probability with CSI-assisted relays over Nakagami-*m* fading channels. References [9,10] have obtained closed-form lower bounds for the outage probability and the average bit error probability (ABEP); however, these lower bounds are not tight enough in medium and high signal-to-noise ratio (SNR) values. To the best of our knowledge, closed-form upper bounds of the performance have never been addressed yet.

In this paper, we present upper and lower bounds of the end-to-end SNR of multihop communications with CSI-assisted relays operating over independent, non-identical (i.n.i.d), Nakagami-*m* fading channels. From the results of analysis, the upper bound of the end-to-end SNR is much tighter than that proposed in Ref. [10], especially for medium and high average SNR values. Furthermore, the lower bound of the end-to-end SNR is the first time to be proposed.

2 Bounds for end-to-end SNRs

2.1 System and channel model

Consider an N-hop communication system which operates over i.n.i.d Nakagami-*m* fading channels. Intermediate terminals relay the signal from one hop to the next. For a non-regenerative (CSI-assisted) system, these intermediate relays amplify and forward the received signal from the

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preceding node without any sort of decoding. It is assumed that the source terminal is transmitting a signal with an average power normalized to unity. Then, the end-to-end SNR can be written as [8]

$$\gamma_{\mathrm{end}}=\frac{\prod_{i=1}^{N}h_i^2G_{i-1}^2}{\sum_{i=1}^{N}N_{0,i}\left(\prod_{j=i+1}^{N}G_{i-1}^2h_j^2\right)},\tag{1}$$

where h_i is the fading amplitude of the *i*th hop, $N_{0,i}$ is the one-sided power spectral density at the input of the *i*th relay, and G_i is the gain of the *i*th relay with $G_0=1$.

The gain of the CSI-assisted relay is proposed in Refs. [4,5] as

$$G_i^2=\frac{1}{h_i^2}.\tag{2}$$

This kind of relay served as a benchmark for all practical multihop systems with non-regenerative relays, and its performance, in the high SNR region, is equal to the performance of the CSI-assisted relays which satisfy the average power constraint, with an amplifying gain given by [1]

$$G_i^2=\frac{1}{h_i^2+N_{0,i}}.\tag{3}$$

By applying Eq. (2) to Eq. (1), the end-to-end SNR becomes

$$\gamma_{\mathrm{end}}=\left(\sum_{i=1}^{N}\frac{1}{\gamma_i}\right)^{-1},\tag{4}$$

where $\gamma_i=h_i^2/N_{0,i}$ is the instantaneous SNR. Since h_i is modeled as a Nakagami-*m* random variable (RV), γ_i is a Gamma-distributed RV with the following probability density function (PDF):

$$f_{\gamma_i}(\gamma)=\frac{m_i^{m_i}}{\overline{\gamma}_i^{m_i}\Gamma(m_i)}\gamma^{m_i-1}\mathrm{e}^{-\frac{m_i\gamma}{\overline{\gamma}_i}},\tag{5}$$

where $\Gamma(\cdot)$ is the Gamma function [11 (Eq. (8.310.1))], $m_i\geqslant 1/2$ is the fading parameter, and $\overline{\gamma}_i=E(h_i^2)/N_{0,i}$ is the average SNR of the *i*th hop.

2.2 Upper and lower bounds of end-to-end SNRs

It is hard to get closed-form expressions for the statistics of Eq. (4), so we will present its upper and lower bounds which are more mathematically tractable forms. Let $\gamma_{(N)}\geqslant\gamma_{(N-1)}\geqslant\cdots\geqslant\gamma_{(1)}\geqslant 0$ be the order statistics obtained by arranging the $\{\gamma_i\}_{i=1}^N$ in decreasing order of magnitude. With the help of $\gamma_{(i)}\to\infty,\ i>1$ and $\gamma_{(i)}\to\gamma_{(1)},\ i>1$, since Eq. (4) is a monotone increasing function of $\gamma_{(i)}$, its upper and lower bounds can be given by

$$\frac{\min(\gamma_1,\gamma_2,\ldots,\gamma_N)}{N}=\frac{\gamma_{\mathrm{u}}}{N}=\gamma_{\mathrm{l}}\leqslant\gamma_{\mathrm{end}}\leqslant\gamma_{\mathrm{u}}=\min(\gamma_1,\gamma_2,\ldots,\gamma_N).\tag{6}$$

The upper bound in Eq. (6) is adopted in Refs. [7,12] which involve only dual-hop.

3 Performance analysis

3.1 Outage probability

In a noise-limited system, the outage probability is defined as the probability that the instantaneous SNR falls below a predetermined threshold value (γ_0). For the non-regenerative multihop systems of interest, it can be shown that the bounds of the end-to-end outage probability can be expressed as

$$P(\gamma_{\mathrm{u}}\leqslant\gamma_0)\leqslant P_{\mathrm{out}}\leqslant P(\gamma_{\mathrm{u}}\leqslant N\gamma_0),\tag{7}$$

where

$$P(\gamma_{\mathrm{u}}\leqslant\gamma_0)=P\Big(\min(\gamma_1,\gamma_2,\ldots,\gamma_N)\leqslant\gamma_0\Big)=1-\prod_{i=1}^{N}\left(\frac{1}{\Gamma(m_i)}\Gamma\left(m_i,\frac{\gamma_0}{\beta_i}\right)\right)\tag{8}$$

with $\beta_i\triangleq\overline{\gamma}_i/m_i$ and $\Gamma(\cdot,\cdot)$ denoting the incomplete gamma function [11 (Eq. (8.350.2))].

3.2 ABEP

The PDF of γ_{u} can be found by taking the derivative of Eq. (8) with respect to γ_0, yielding

$$p_{\gamma_{\mathrm{u}}}(\gamma)=\frac{1}{\prod_{i=1}^{N}\Gamma(m_i)}\sum_{j=1}^{N}\left[\left(\frac{1}{\beta_j}\right)^{m_j}\gamma^{m_j-1}\mathrm{e}^{-\frac{\gamma}{\beta_j}}\prod_{\substack{i=1\\ i\neq j}}^{N}\Gamma\left(m_i,\frac{\gamma}{\beta_i}\right)\right].\tag{9}$$

For $N=2$, Eq. (9) is reduced to the formula presented by Ref. [8].

In order to get closed-form expressions for the moment-generating functions (MGFs), $M_X(s)=\mathrm{E}(\mathrm{e}^{-sX})$, where $\mathrm{E}(\cdot)$ denotes the statistical average operator; we will consider two different situations, respectively.

For $N=2$, the MGFs of the bounds with the help of Ref. [11 (Eq. (6.455.1))] can be written in closed forms as

$$M_{\gamma_u}(s) = \frac{\Gamma(m_1+m_2)\left(\frac{1}{\beta_1}\right)^{m_1}\left(\frac{1}{\beta_2}\right)^{m_2}}{\Gamma(m_1)\Gamma(m_2)\left(\frac{1}{\beta_1}+\frac{1}{\beta_2}+s\right)^{m_1+m_2}}\cdot\left[\frac{1}{m_1}\,{}_2F_1\left(1,\, m_1+m_2;\, m_1+1;\, \frac{\frac{1}{\beta_1}+s}{\frac{1}{\beta_1}+\frac{1}{\beta_2}+s}\right) + \frac{1}{m_2}\,{}_2F_1\left(1,\, m_1+m_2;\, m_2+1;\, \frac{\frac{1}{\beta_2}+s}{\frac{1}{\beta_1}+\frac{1}{\beta_2}+s}\right)\right] \tag{10}$$

and

$$M_{\gamma_l}(s) = \frac{\Gamma(m_1+m_2)\left(\frac{2}{\beta_1}\right)^{m_1}\left(\frac{2}{\beta_2}\right)^{m_2}}{\Gamma(m_1)\Gamma(m_2)\left(\frac{2}{\beta_1}+\frac{2}{\beta_2}+s\right)^{m_1+m_2}}\cdot\left[\frac{1}{m_1}\,{}_2F_1\left(1,\, m_1+m_2;\, m_1+1;\, \frac{\frac{2}{\beta_1}+s}{\frac{2}{\beta_1}+\frac{2}{\beta_2}+s}\right) + \frac{1}{m_2}\,{}_2F_1\left(1,\, m_1+m_2;\, m_2+1;\, \frac{\frac{2}{\beta_2}+s}{\frac{2}{\beta_1}+\frac{2}{\beta_2}+s}\right)\right], \tag{11}$$

where $m_i > 1/2$ and ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is Gauss hypergeometric function defined in Ref. [11 (Eq. (9.100))].

For $N > 2$, we will assume that m_j will take only integer values in order to get closed-form expressions.

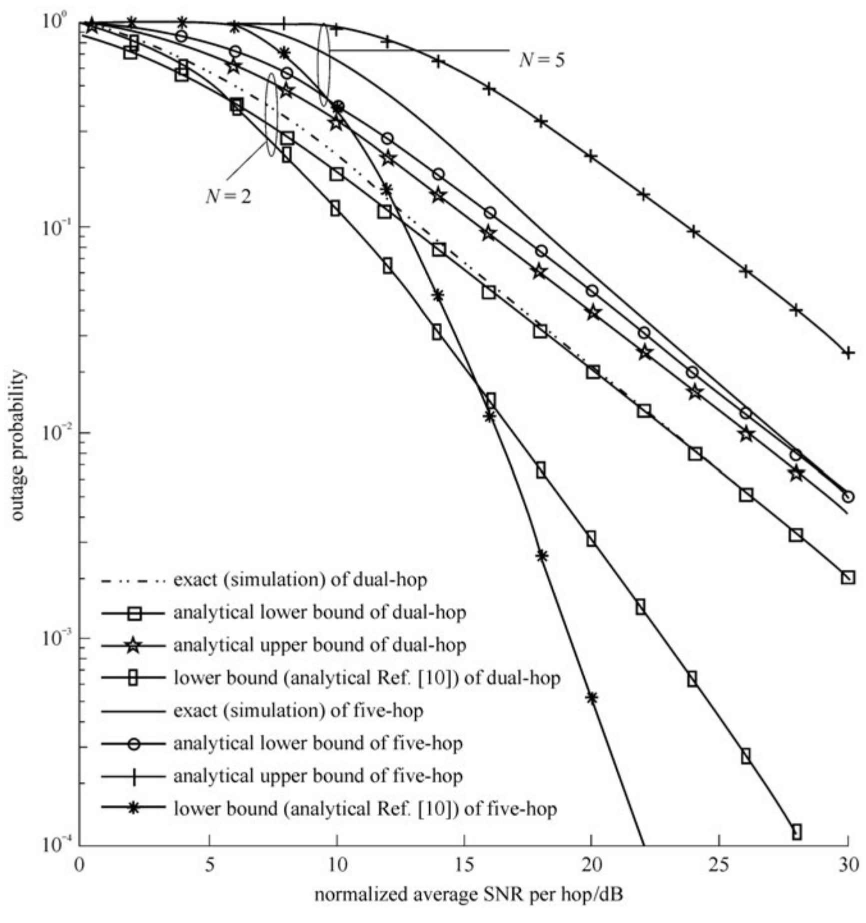

Fig. 1 Outage probability bounds for relaying systems in i.n.i.d Nakagami-m fading channels ($\bar{\gamma}_i = \bar{\gamma}$, $m_i = 1$)

Reference [13] indicates that the Nakagami-m PDF with an integer-order fading parameter is an alternative form of the classical Nakagami-m PDF. Then, the PDF of γ_u with the aid of Ref. [11 (Eq. (8.352.2))] can be given by

$$p_{\gamma_u}(\gamma) = \sum_{j=1}^{N}\sum_{p_1=0}^{m_1-1}\cdots\sum_{p_{j-1}=0}^{m_{j-1}-1}\sum_{p_{j+1}=1}^{m_{j+1}-1}\cdots\sum_{p_N=1}^{m_N-1}\frac{D_j}{C_j}\gamma^{A_j-1}\mathrm{e}^{-B\gamma}, \tag{12}$$

where

$$A_j = p_1 + p_2 + \cdots + p_{j-1} + m_j + p_{j+1} + p_N,$$

$$B = \sum_{i=1}^{N}\frac{1}{\beta_i},$$

$$C_j = p_1!p_2!\cdots p_{j-1}!(m_j-1)!p_{j+1}!\cdots p_N!,$$

and

$$D_j = \left(\frac{1}{\beta_1}\right)^{p_1}\left(\frac{1}{\beta_2}\right)^{p_2}\cdots\left(\frac{1}{\beta_{j-1}}\right)^{p_{j-1}}\left(\frac{1}{\beta_j}\right)^{m_j}\left(\frac{1}{\beta_{j+1}}\right)^{p_{j+1}}\cdots\left(\frac{1}{\beta_N}\right)^{p_N}.$$

Using Eq. (12), the closed-form expressions of the bounds can be obtained as

$$M_{\gamma_u}(s) = \sum_{j=1}^{N}\sum_{p_1=0}^{m_1-1}\cdots\sum_{p_{j-1}=0}^{m_{j-1}-1}\sum_{p_{j+1}=1}^{m_{j+1}-1}\cdots\sum_{p_N=1}^{m_N-1}\frac{D_j\Gamma(A_j)}{C_j(S+B)^{A_j}} \tag{13}$$

and

$$M_{\gamma_l}(s) = \sum_{j=1}^{N}\sum_{p_1=0}^{m_1-1}\cdots\sum_{p_{j-1}=0}^{m_{j-1}-1}\sum_{p_{j+1}=1}^{m_{j+1}-1}\cdots\sum_{p_N=1}^{m_N-1}\frac{N^{A_j}D_j\Gamma(A_j)}{C_j(S+NB)^{A_j}}. \tag{14}$$

Interestingly enough, both expressions can be easily evaluated due to the fact that only simple functions are included.

Using the MGFs of the bounds in closed forms as in Eqs. (10), (11), (13) and (14), the average symbol error probability can be evaluated for a wide variety of M-ary modulation (such as M-ary phase-shift keying (M-PSK), M-ary differential phase-shift keying (M-DPSK), and M-ary quadrature amplitude modulation (M-QAM)). For instance, the ABEP of binary DPSK is given by

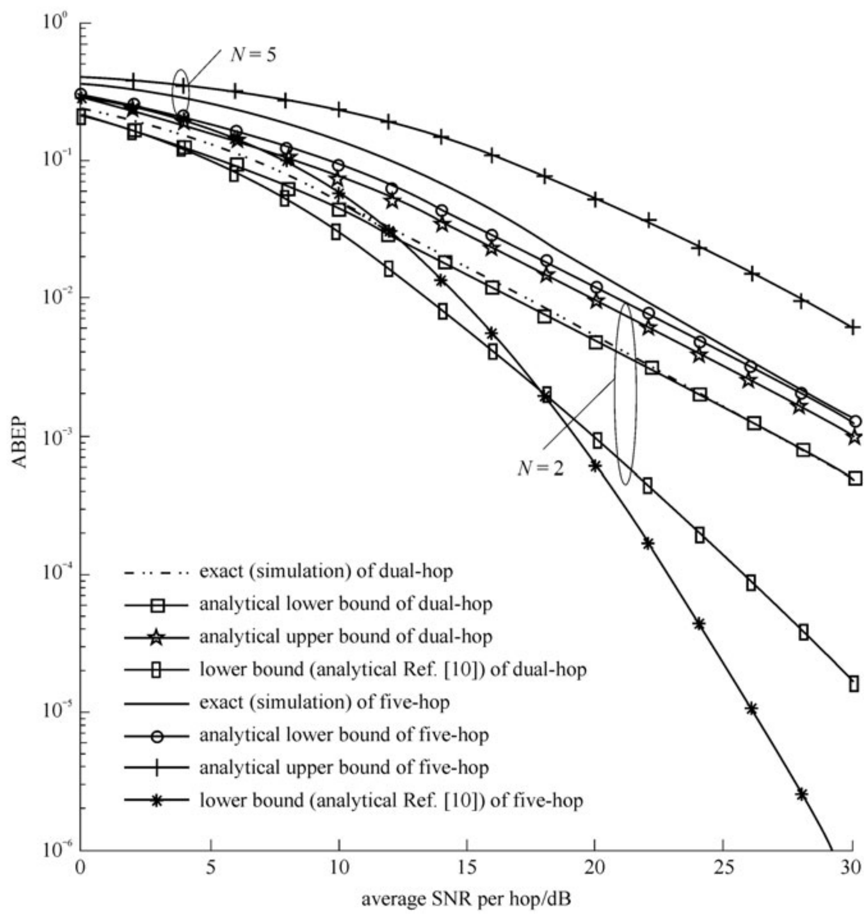

Fig. 2 BPSK error bounds for relaying systems in i.n.i.d Nakagami-m fading channels ($\bar{\gamma}_i = \bar{\gamma}$, $m_i = 1$)

$p_b(E) = \frac{1}{2}M_\gamma(1)$, so the bounds of $p_b(E)$ can be given by

$$\frac{1}{2}M_{\gamma_u}(1) \leqslant p_b(E) \leqslant \frac{1}{2}M_{\gamma_l}(1).$$

4 Introduction numerical results

In this section, we show numerical and simulation results of the analytical outage probability and ABEP. Figures 1 and 2 depict the outage probability and ABEP of dual-hop and five-hop relaying systems, respectively, including the lower bounds presented in Ref. [10] for comparison. It is clear from the two figures that the analytical lower bounds obtained in this paper are much tighter than those proposed in Ref. [10], especially for medium and high average SNR values. Furthermore, the tightness of the lower bounds improves as the average SNR increases. It is also shown that the analytical upper bounds are tight, especially in the low average SNR. In addition, the distance of upper and lower bounds maintains stability as the average SNR increases.

5 Conclusion

Closed-form bounds for the end-to-end performance of multihop communications with CSI-assisted relays operating over Nakagami-*m* fading channels are investigated. Upper and lower bounds of the end-to-end SNR are developed by using the monotonicity. The PDFs, the cumulative distribution functions, and the MGFs of the bounds are derived. Using these results, the bounds for the outage and average bit error probability are obtained. Finally, numerical and simulation results are executed to validate the tightness of the proposed bounds.

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